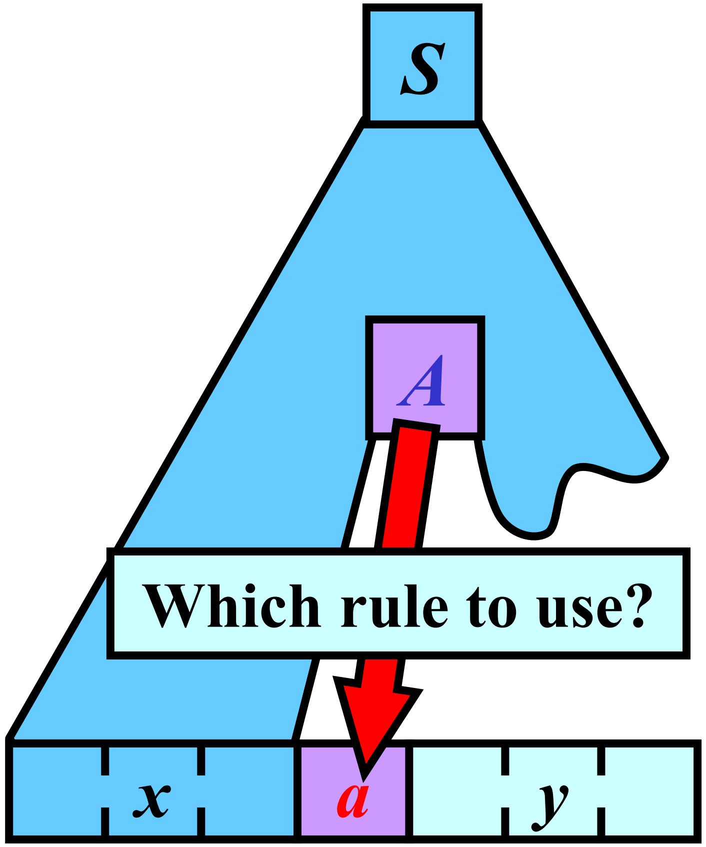
Deterministic Top-Down Parsing

Chapter 4

Top-Down Parsing: Introduction

Problem:



Basic idea:

Table:

α	...	*a*	...
...			
A		α(*A*, *a*)	
...			

Use rule r: $A \to x$

Question: Could you construct this table for **any** CFG?

Answer: NO

A Table-Based Selection of a Rule

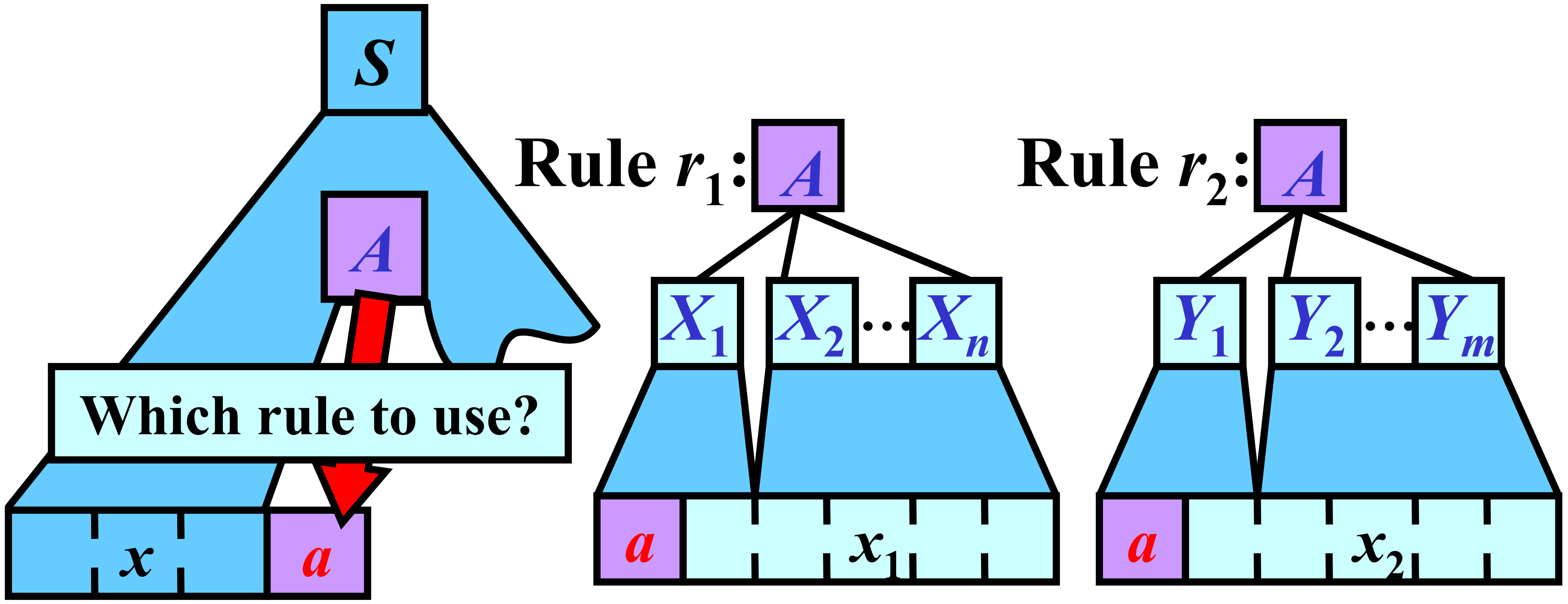


Table:

α	...	*a*	...
...			
A		α(*A*, *a*)	
...			

?

Use rule r_1: $A \to X_1X_2\ldots X_n$

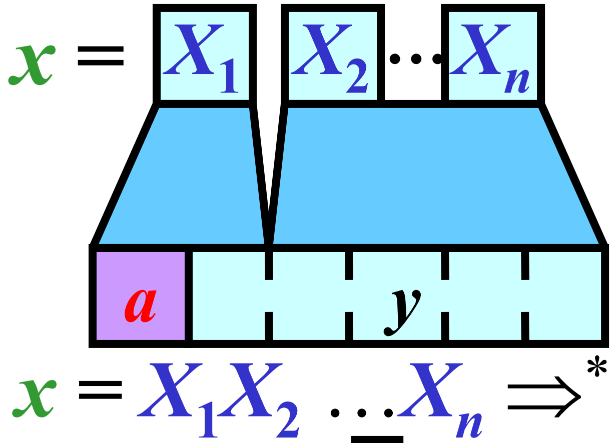
Use rule r_2: $A \to Y_1Y_2\ldots Y_m$

Set *First*

Gist: *First*(x) is the set of all terminals that can begin a sentential form derivable from x.

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $x \in (N \cup T)^*$,we define the set *First*(x) as *First*(x) $= \{a: a \in T, x \Rightarrow^* ay; y \in (N \cup T)^*\}$.

Illustration:



$$x = X_1 X_2 \ldots X_n \Rightarrow^* ay$$

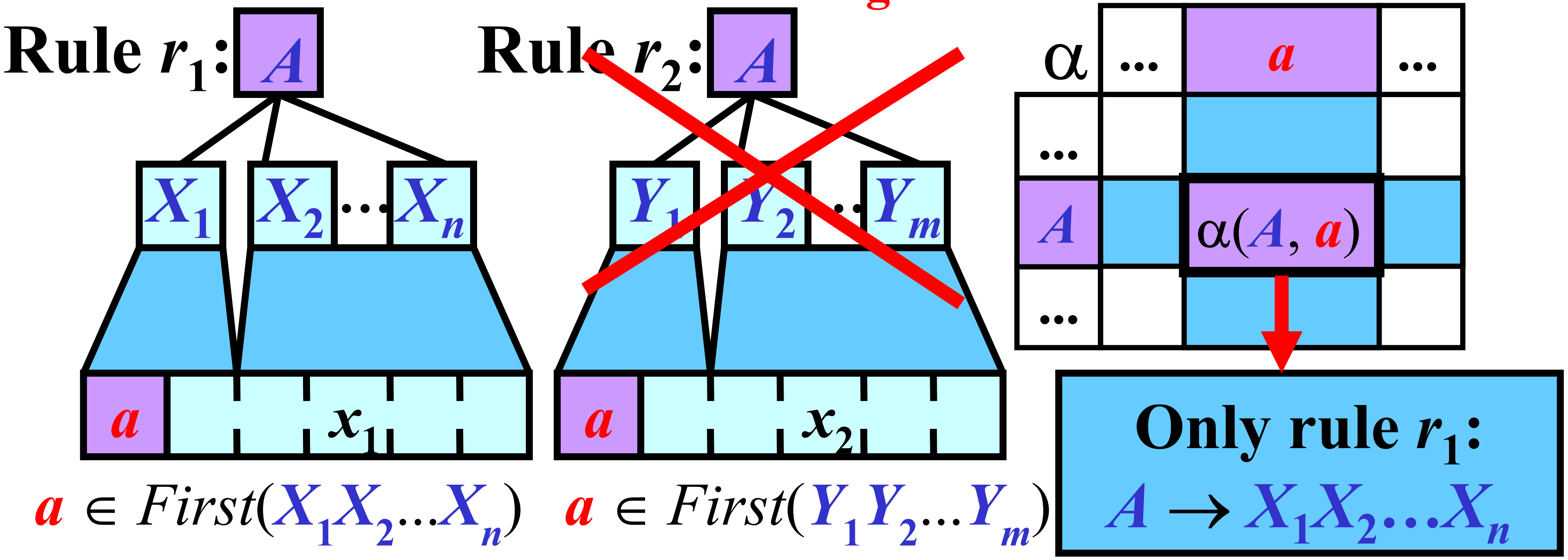
$$\Downarrow$$

$$a \in First(x)$$

LL Grammars without ε-rules

Definition: Let $G = (N, T, P, S)$ be a CFG <u>without ε-rules</u>. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** rule $A \to X_1X_2\ldots X_n \in P$ such that $a \in First(X_1X_2\ldots X_n)$

Illustration: **Ruled out in an LL grammar** **Table:**



*S*imple *P*rogramming *L*anguage (SPL)

1:	<prog>	→	begin <st-list>
2:	<st-list>	→	<stat> ; <st-list>
3:	<st-list>	→	end
4:	<stat>	→	read id
5:	<stat>	→	write <item>
6:	<stat>	→	id := add (<item> <it-list>
7:	<it-list>	→	, <item> <it-list>
8:	<it-list>	→	)
9:	<item>	→	int
10:	<item>	→	id

Note: *G*$_{SPL}$ is LL grammar

Example:

```
begin
   read i;
   j := add(i, 1);
   write j;
end
```

∈ SPL

Algorithm: *First*(X)

- **Input:** $G = (N, T, P, S)$ **without ε-rules**
- **Output:** *First*(X) for every $X \in N \cup T$

- **Method:**
- for each $a \in T$: *First*(a) := $\{a\}$
- **Apply the following rule until no *First* set can be changed:**
- if $A \to X_1X_2\ldots X_n \in P$, then add *First*($X_1$) to *First*($A$)

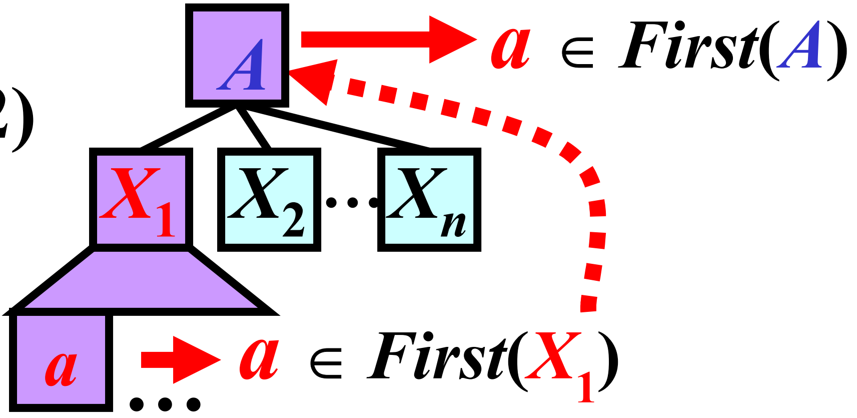
Illustration:

1) for each $a \in T$:

$First(a) := \{a\}$

because $a \Rightarrow^0 a$

2)



First(*X*) for SPL: Example

First(**begin**) := {**begin**}	*First*(**id**) := {**id**}	*First*(**,**) := {**,**}
First(**end**) := {**end**}	*First*(**int**) := {**int**}	*First*(**(**) := {**(**}
First(**read**) := {**read**}	*First*(**:=**) := {**:=**}	*First*(**)**) := {**)**}
First(**write**) := {**write**}	*First*(**add**) := {**add**}	*First*(**;**) := {**;**}

<item> → id ∈ *P*: add *First*(**id**) to *First*(<item>)
<item> → int ∈ *P*: add *First*(**int**) to *First*(<item>)
Summary: *First*(<item>) = {**id**, **int**}

<it-list> →) ∈ *P*: add *First*(**)**) to *First*(<it-list>)
<it-list> → , … ∈ *P*: add *First*(**,**) to *First*(<it-list>)
Summary: *First*(<it-list>) = {**)**, **,**}

<stat> → id … ∈ *P*: add *First*(**id**) to *First*(<stat>)
<stat> → write … ∈ *P*: add *First*(**write**) to *First*(<stat>)
<stat> → read … ∈ *P*: add *First*(**read**) to *First*(<stat>)
Summary: *First*(<stat>) = {**id**, **write**, **read**}

<st-list> → end ∈ *P*: add *First*(**end**) to *First*(<st-list>)
<st-list> → <stat> … ∈ *P*: add *First*(<stat>) to *First*(<st-list>)
Summary: *First*(<st-list>) = {**id**, **write**, **read**, **end**}

<prog> → begin … ∈ *P*: add *First*(**begin**) to *First*(<prog>)
Summary: *First*(<prog>) = {**begin**}

Construction of LL Table

α	…	a	…
…			
A		$\alpha(A, a)$	
…			

$\alpha(A, a) = A \rightarrow X_1X_2\ldots X_n \in P$
if $a \in First(X_1)$; otherwise,
$\alpha(A, a)$ is blank $\Rightarrow$ **ERROR**

Task: LL table for SPL

	id	int	:=	…
<prog>				
<st-list>	2 $\leftarrow$ id $\in$ *First*(<stat>)			
<stat>	6 $\leftarrow$ id $\in$ *First*(id)			
<it-list>				
<item>	10 $\leftarrow$ id $\in$ *First*(id)			

Construct the rest analogically.

Rule r: $A \rightarrow X_1X_2\ldots X_n$	$First(X_1)$
1: <prog> $\rightarrow$ begin …	{begin}
2: <st-list> $\rightarrow$ <stat>…	{id, write, read}
3: <st-list> $\rightarrow$ end	{end}
4: <stat> $\rightarrow$ read …	{read}
5: <stat> $\rightarrow$ write …	{write}
6: <stat> $\rightarrow$ id …	{id}
7: <it-list> $\rightarrow$, …	{,}
8: <it-list> $\rightarrow$)	{)}
9: <item> $\rightarrow$ int	{int}
10: <item> $\rightarrow$ id	{id}

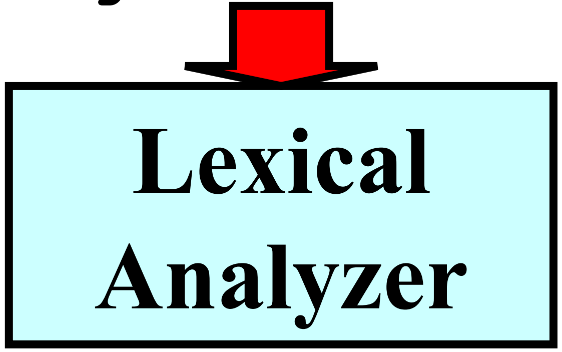
Parsing Based on LL Table: Example

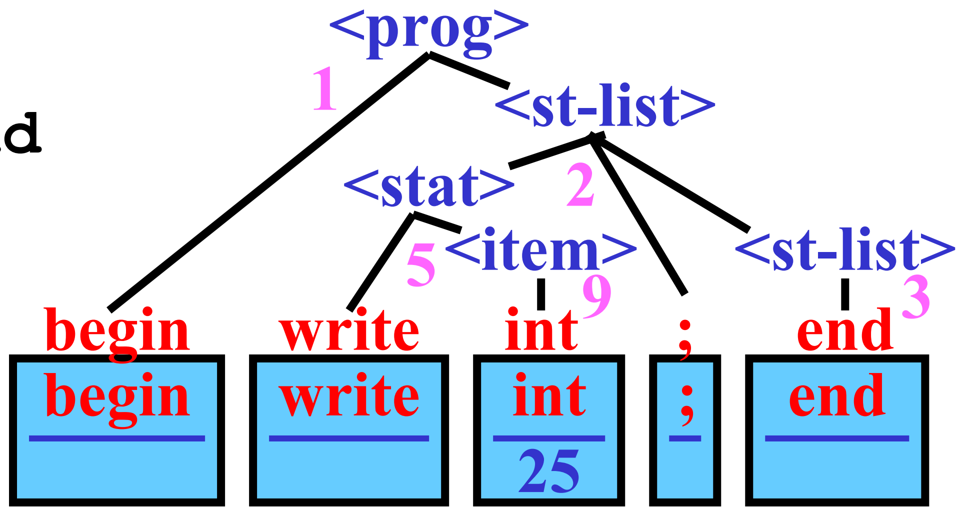
1:	<prog> → begin <st-list>		6:	<stat> → id := add (...	
2:	<st-list> → <stat> ; <st-list>		7:	<it-list> → , <item> <it-list>	
3:	<st-list> → end		8:	<it-list> →)	
4:	<stat> → read id		9:	<item> → int	
5:	<stat> → write <item>		10:	<item> → id	

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

```
begin write 25; end
```

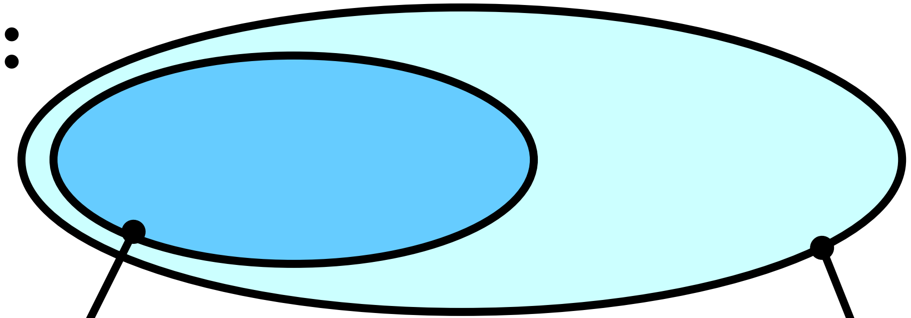




LL Grammars: Useful Transformations

Generaly: CFG are stronger that LL grammars

Illustration:



The family of languages generated by LL grammars $\subset$ **The family of languages generated by CFGs**

- **Some** CFGs can be converted to equivalent LL grammars

Basic conversions:

1) Factorization

2) Left recursion replacement

Note: A rule of the form $A \rightarrow Ax$, where $A \in N$, $x \in (N \cup T)^*$ is called a *left recursive rule.*

Factorization

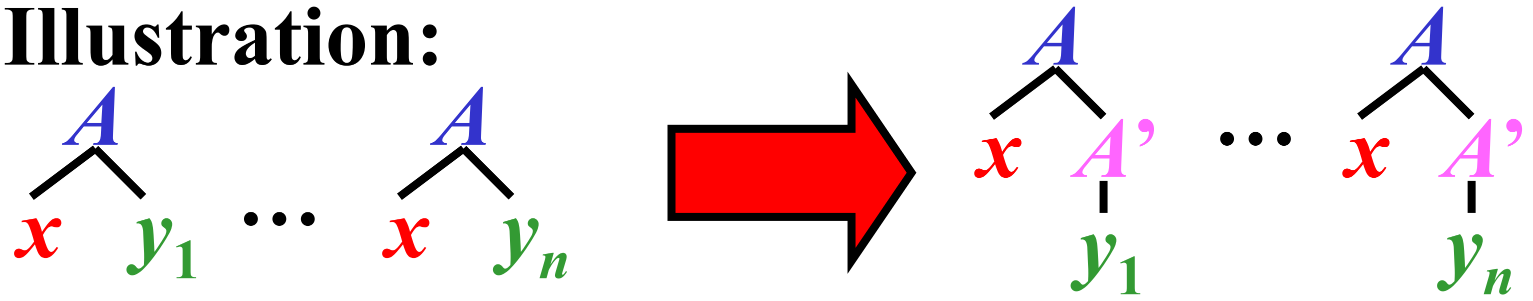
Idea: Replace rules of the form

$A \to xy_1$, $A \to xy_2$, … , $A \to xy_n$ with

$A \to xA'$, $A' \to y_1$, $A' \to y_2$, … , $A' \to y_n$,

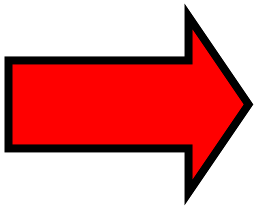
where A' is a new nonterminal

Illustration:



Example:

<stat> → write id
<stat> → write int

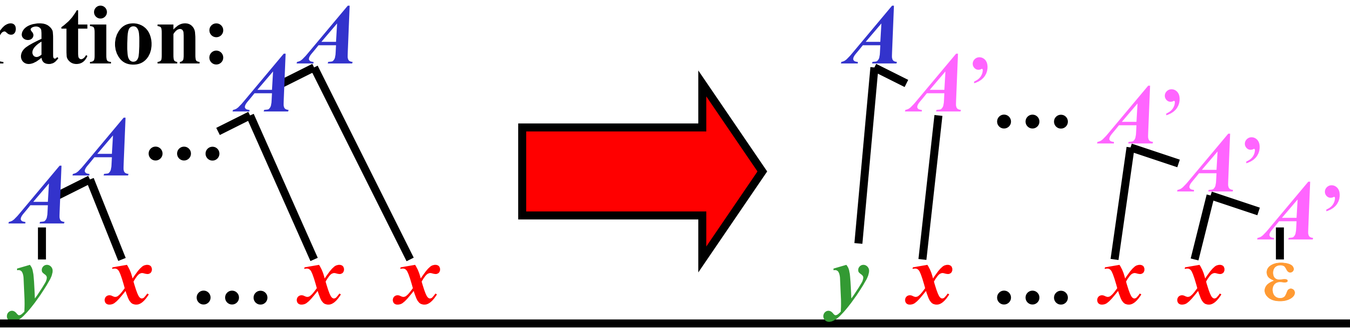


<stat> → write <item>
<item> → id
<item> → int

Left Recursion Replacement

Idea: Replace rules of the form $A \to Ax$, $A \to y$ with $A \to yA'$, $A' \to xA'$, $A' \to \varepsilon$, where A' is a new nonterminal.

Illustration:



Example:

$$
\left.\begin{array}{l} E \to E+T \\ E \to T \end{array}\right\} \Rightarrow \quad E \to TE', E' \to +TE', E' \to \varepsilon
$$

$$
\left.\begin{array}{l} T \to T*F \\ T \to F \end{array}\right\} \Rightarrow \quad T \to FT', T' \to *FT', T' \to \varepsilon
$$

$F \to (E)$ $F \to (E)$

$F \to i$ $F \to i$

LL Grammars with ε-rules: Introduction

Why ε-rules?

- elimination of the left recursion introduces ε-rule
- ε-rules often make the language specification clearer

Simplification of this part:

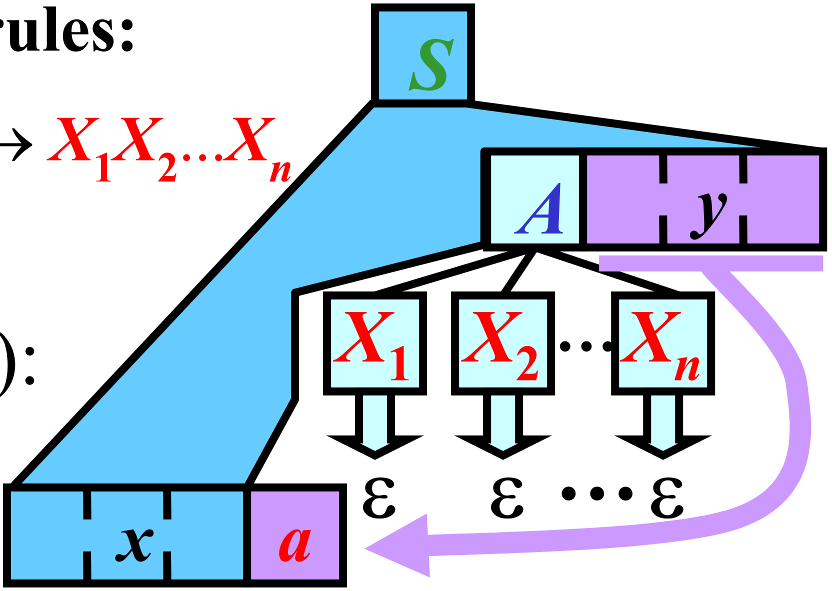
Assume that every input string of tokens ends with $.

Note: $ acts as an *end marker*.

Main problem with ε-rules:

Rule ***r***: $A \rightarrow X_1X_2\ldots X_n$

Maybe: $a \notin First(A)$:



Note: We must define other sets: *Empty*, *Follow* and *Predict.*

Grammar for Arithmetical Expressions

- $G_{expr3} = (N, T, P, \boldsymbol{E})$, where
- $N = \{\boldsymbol{E}, \boldsymbol{E'}, \boldsymbol{T}, \boldsymbol{T'}, \boldsymbol{F}\}$,
- $T = \{\boldsymbol{i}, \mathbf{+}, \mathbf{*}, \mathbf{(}, \mathbf{)}\}$,
- $P = \{$ 1: $\boldsymbol{E} \to \boldsymbol{TE'}$, 2: $\boldsymbol{E'} \to \mathbf{+}\boldsymbol{TE'}$,
  3: $\boldsymbol{E'} \to \varepsilon$, 4: $\boldsymbol{T} \to \boldsymbol{FT'}$,
  5: $\boldsymbol{T'} \to \mathbf{*}\boldsymbol{FT'}$, 6: $\boldsymbol{T'} \to \varepsilon$,
  7: $\boldsymbol{F} \to \mathbf{(}\boldsymbol{E}\mathbf{)}$, 8: $\boldsymbol{F} \to \boldsymbol{i}$ $\}$

Example:

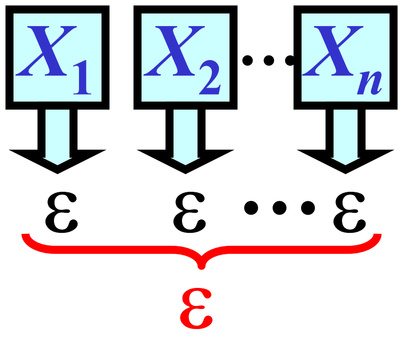
$$(\boldsymbol{i} + \boldsymbol{i})*(\boldsymbol{i} + \boldsymbol{i}) \in \boldsymbol{L}(\boldsymbol{G}_{expr3})$$

Set *Empty*

Gist: *Empty*(*x*) is the set that include ε if *x* derives the empty string; otherwise, *Empty*(*x*) is empty

Definition: Let $G = (N, T, P, S)$ be a CFG.
$Empty(x) = \{\varepsilon\}$ if $x \Rightarrow^* \varepsilon$; otherwise,
$Empty(x) = \emptyset$, where $x \in (N \cup T)^*$.

Illustration: $x =$



$$x = X_1 X_2 \ldots X_n \Rightarrow^* \varepsilon$$

$$\Downarrow$$

$$Empty(x) = \{\varepsilon\}$$

Algorithm: *Empty*(X)

- **Input:** $G = (N, T, P, S)$
- **Output:** *Empty*(X) for every $X \in N \cup T$

- **Method:**
- **for each** $a \in T$: *Empty*(a) := $\emptyset$
- **for each** $A \in N$:

```
        if $A \to \varepsilon \in P$ then $Empty(A) := \{\varepsilon\}$
                                 else $Empty(A) := \emptyset$
```

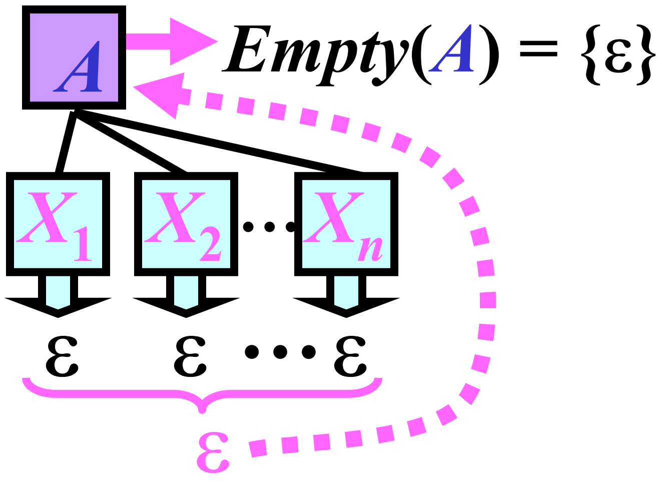
- **Apply the following rule until no *Empty* set can be changed:**
  - **if** $A \to X_1X_2\ldots X_n \in P$ **and** *Empty*(X_i) $= \{\varepsilon\}$ for all $i = 1,\ldots, n$ **then** *Empty*(A) $= \{\varepsilon\}$

Previous Algorithm: Illustration

1) for each $a \in T$: $Empty(a) := \emptyset$ because $a \not\Rightarrow^* \varepsilon$
2) for each r: $A \to \varepsilon \in P$: $Empty(A) := \{\varepsilon\}$ because $A \Rightarrow^1 \varepsilon$ $[r]$

3) Apply the following rules until no *Empty* set can be changed:

- **if** $A \to X_1X_2 \dots X_n \in P$ **and** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\dots,n$ **then** $Empty(A) = \{\varepsilon\}$



$Empty(X)$ for G_{expr3}: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 1: $E \rightarrow TE'$, 2: $E' \rightarrow +TE'$, 3: $E' \rightarrow \varepsilon$, 4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$, 6: $T' \rightarrow \varepsilon$, 7: $F \rightarrow (E)$, 8: $F \rightarrow i$ $\}$

Initialization:

$Empty(i)$	$:= \emptyset$	$Empty(E)$	$:= \emptyset$
$Empty(+)$	$:= \emptyset$	$Empty(E')$	$:= \{\varepsilon\}$
$Empty(*)$	$:= \emptyset$	$Empty(T)$	$:= \emptyset$
$Empty(()$	$:= \emptyset$	$Empty(T')$	$:= \{\varepsilon\}$
$Empty())$	$:= \emptyset$	$Empty(F)$	$:= \emptyset$

- **No *Empty* set can be changed.**

Algorithm: *First*(*X*)

- **Input:** $G = (N, T, P, S)$
- **Output:** $First(X)$ for every $X \in N \cup T$

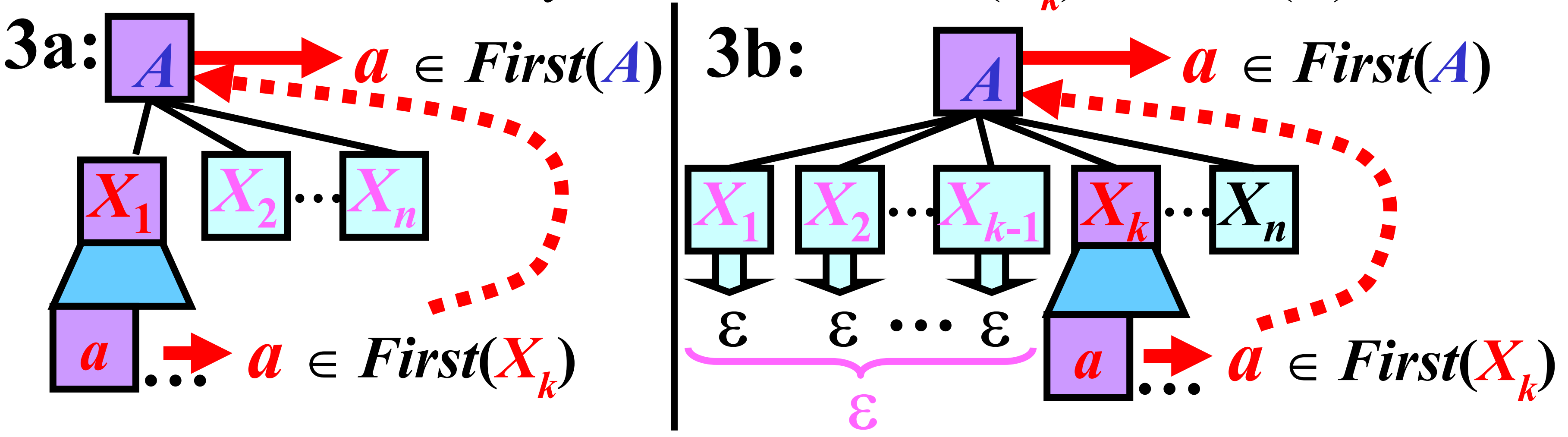
- **Method:**
- **for each** $a \in T$: $First(a) := \{a\}$
- **for each** $A \in N$: $First(A) := \emptyset$
- **Apply the following rule until no *First* set can be changed:**
- **if** $A \rightarrow X_1X_2\dots X_{k-1}X_k\dots X_n \in P$ **then**
  - add all symbols from $First(X_1)$ to $First(A)$
  - **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\dots, k\text{-}1$, where $k \leq n$ **then** add all symbols from $First(X_k)$ to $First(A)$

Previous Algorithm: Illustration

1) for each $a \in T$: *First*(a) := {a} because $a \Rightarrow^0 a$
2) for each $A \in N$: *First*(A) := ∅ (inicialization)

3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $A \to X_1X_2 \dots X_{k-1}X_k \dots X_n \in P$ **then**
  - **3a)** add all symbols from *First*(X_1) to *First*(A)
  - **3b) if** *Empty*(X_i) = {ε} for all $i = 1,\dots, k$-1, where $k < n$ **then** add all symbols from *First*(X_k) to *First*(A):



$First(X)$ for G_{expr3}: Example

Initialization:

$First(\,i\,)$	$:= \{i\}$	$First(E)$	$:= \emptyset$
$First(\,+\,)$	$:= \{+\}$	$First(E')$	$:= \emptyset$
$First(\,*\,)$	$:= \{*\}$	$First(T)$	$:= \emptyset$
$First(\,(\,)$	$:= \{\,(\,\}$	$First(T')$	$:= \emptyset$
$First(\,)\,)$	$:= \{\,)\,\}$	$First(F)$	$:= \emptyset$

$F \to i \in P$: **add** $First(i) = \{i\}$ **to** $First(F)$

$F \to (E) \in P$: **add** $First(() = \{(\}$ **to** $First(F)$

Summary: $\boldsymbol{First}(F) = \{i, (\}$

$T' \to *FT' \in P$: **add** $First\,(*) = \{*\}$ **to** $First(T')$

Summary: $\boldsymbol{First}(T') = \{*\}$

$T \to FT' \in P$: **add** $First\,(F) = \{i, (\}$ **to** $First(T)$

Summary: $\boldsymbol{First}(T) = \{i, (\}$

$E' \to +TE' \in P$: **add** $First\,(+) = \{+\}$ **to** $First(E')$

Summary: $\boldsymbol{First}(E') = \{+\}$

$E \to TE' \in P$: **add** $First\,(T) = \{i, (\}$ **to** $First(E)$

Summary: $\boldsymbol{First}(E) = \{i, (\}$

- **No *First* set can be changed.**

First(*X*) & *Empty*(*X*) for G_{expr3}: Summary

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \rightarrow TE'$, 2: $E' \rightarrow +TE'$, 3: $E' \rightarrow \varepsilon$, 4: $T \rightarrow FT'$

5: $T' \rightarrow *FT'$, 6: $T' \rightarrow \varepsilon$, 7: $F \rightarrow (E)$, 8: $F \rightarrow i$ $\}$

Set *Empty* for all $X \in N \cup T$:

$Empty(i)$	$:= \emptyset$	$Empty(E)$	$:= \emptyset$
$Empty(+)$	$:= \emptyset$	$Empty(E')$	$:= \{\varepsilon\}$
$Empty(*)$	$:= \emptyset$	$Empty(T)$	$:= \emptyset$
$Empty(()$	$:= \emptyset$	$Empty(T')$	$:= \{\varepsilon\}$
$Empty())$	$:= \emptyset$	$Empty(F)$	$:= \emptyset$

Set *First* for all $X \in N \cup T$:

$First(i)$	$:= \{i\}$	$First(E)$	$:= \{i, (\}$
$First(+)$	$:= \{+\}$	$First(E')$	$:= \{+\}$
$First(*)$	$:= \{*\}$	$First(T)$	$:= \{i, (\}$
$First(()$	$:= \{(\}$	$First(T')$	$:= \{*\}$
$First())$	$:= \{)\}$	$First(F)$	$:= \{i, (\}$

Note: for each $a \in T$: $Empty(a) = \emptyset$, $First(a) = \{a\}$

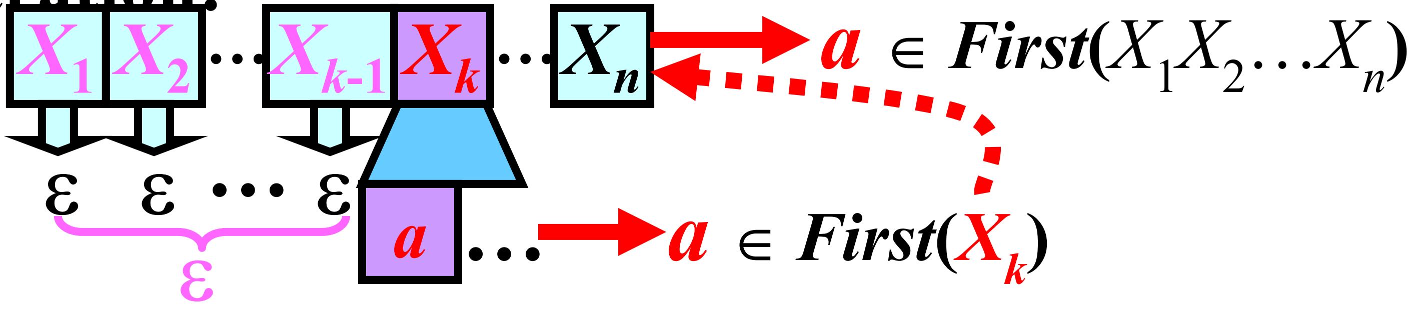
Algorithm: *First*$(X_1X_2\dots X_n)$

- **Input:** $G = (N, T, P, S)$; $First(X)$ & $Empty(X)$ for every $X \in N \cup T$; $x = X_1X_2\dots X_n$, where $x \in (N \cup T)^+$
- **Output:** $First(X_1X_2\dots X_n)$

- **Method:**
- $First(X_1X_2\dots X_n) := First(X_1)$
- **Apply the following rule until nothing can be added to *First*$(X_1X_2 \dots X_{k-1} X_k \dots X_n)$:**
  - **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\dots,k\text{-}1$, where $k \leq n$
    then add all symbols from $First(X_k)$ to $First(X_1X_2\dots X_n)$

! Note: $First(\varepsilon) = \varnothing$

Illustration:


X_1
X_2
X_{k-1}
X_k
X_n
ε ε … ε
ε
a
$a \in First(X_1X_2\dots X_n)$
$a \in First(X_k)$

$First(X_1X_2 \ldots X_n)$: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \to TE'$, 2: $E' \to +TE'$, 3: $E' \to \varepsilon$, 4: $T \to FT'$
5: $T' \to *FT'$, 6: $T' \to \varepsilon$, 7: $F \to (E)$, 8: $F \to i$ $\}$

Set *Empty* & *First* for all $X \in N$:				
	$Empty(E)$	$:= \varnothing$	$First(E)$	$:= \{i, (\}$
	$Empty(E')$	$:= \{\varepsilon\}$	$First(E')$	$:= \{+\}$
	$Empty(T)$	$:= \varnothing$	$First(T)$	$:= \{i, (\}$
	$Empty(T')$	$:= \{\varepsilon\}$	$First(T')$	$:= \{*\}$
	$Empty(F)$	$:= \varnothing$	$First(F)$	$:= \{i, (\}$

Task: $First(E'T'FET)$

1) $First(\underline{E'}T'FET) := First(E') = \{+\}$

2) $First(E'\underline{T'}FET)$: add $First(T') = \{*\}$ to $First(E'T'FET)$

$Empty(E') = \{\varepsilon\}$

3) $First(E'T'\underline{F}ET)$: add $First(F) = \{i, (\}$ to $First(E'T'FET)$

$Empty(E') = Empty(T') = \{\varepsilon\}$

Summary: $First(E'T'FET) = \{+, *, i, (\}$

Algorithm: $Empty(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $Empty(X)$ for every $X \in N \cup T$;
  $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $Empty(X_1X_2\ldots X_n)$

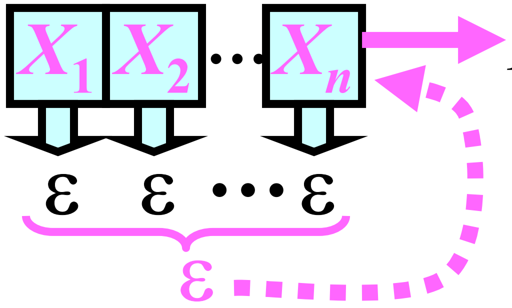
- **Method:**
- **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots,n$ **then**
  $Empty(X_1X_2\ldots X_n) := \{\varepsilon\}$

  else

  $Empty(X_1X_2\ldots X_n) := \emptyset$

! Note: $Empty(\varepsilon) = \{\varepsilon\}$

Illustration:



$Empty(X_1X_2\ldots X_n) = \{\varepsilon\}$

$Empty(X_1X_2\ldots X_n)$: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 1: $E \to TE'$, 2: $E' \to +TE'$, 3: $E' \to \varepsilon$, 4: $T \to FT'$
5: $T' \to *FT'$, 6: $T' \to \varepsilon$, 7: $F \to (E)$, 8: $F \to i$ $\}$

Set *Empty* for all $X \in N$:	$Empty(E)$	$:= \emptyset$
	$Empty(E')$	$:= \{\varepsilon\}$
	$Empty(T)$	$:= \emptyset$
	$Empty(T')$	$:= \{\varepsilon\}$
	$Empty(F)$	$:= \emptyset$

Task: $Empty(E'T')$

$Empty(E') = Empty(T') = \{\varepsilon\}$, **so** $Empty(E'T') = \{\varepsilon\}$

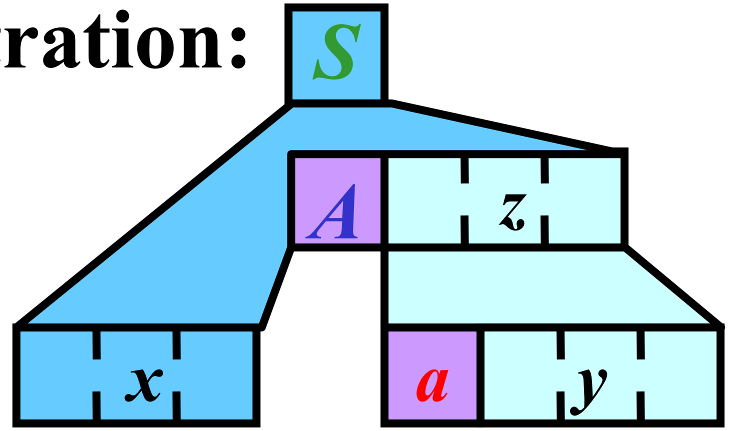
Set *Follow*

Gist: *Follow*(*A*) is the set of all terminals that can come right after *A* in a sentential form of *G*

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \in N$, we define the set *Follow*(A) as

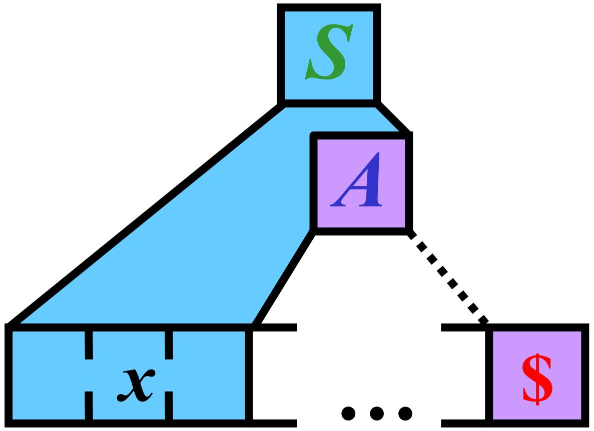
$$Follow(A) = \{a: a \in T, S \Rightarrow^* xAay, x, y \in (N \cup T)^*\}$$
$$\cup \{\$: S \Rightarrow^* xA, x \in (N \cup T)^*\}$$

Illustration:



$S \Rightarrow^* xAz \Rightarrow^* xAay$

$a \in Follow(A)$



$S \Rightarrow^* xA$

$\$ \in Follow(A)$

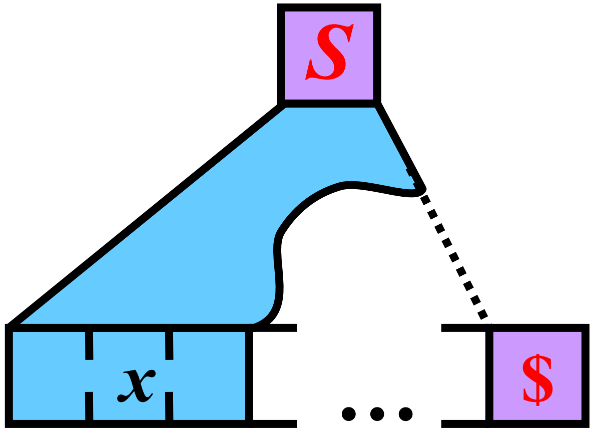
Algorithm: *Follow*(*A*)

- **Input:** $G = (N, T, P, S)$;
- **Output:** $Follow(A)$ for every $A \in N$

- **Method:**
- $Follow(S) := \{\$\}$;
- **Apply the following rules until no *Follow* set can be changed:**
- **if** $A \rightarrow xBy \in P$ **then**
    - **if** $y \neq \varepsilon$ **then**
      add all symbols from $First(y)$ to $Follow(B)$;
    - **if** $Empty(y) = \{\varepsilon\}$ **then**
      add all symbols from $Follow(A)$ to $Follow(B)$;

Previous Algorithm: Illustration

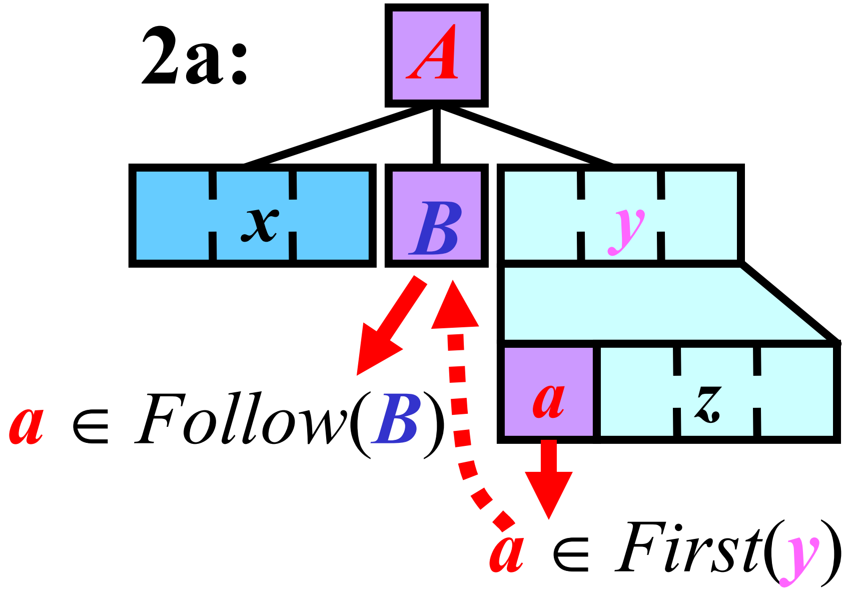
1) $Follow(S) := \{\$\}$



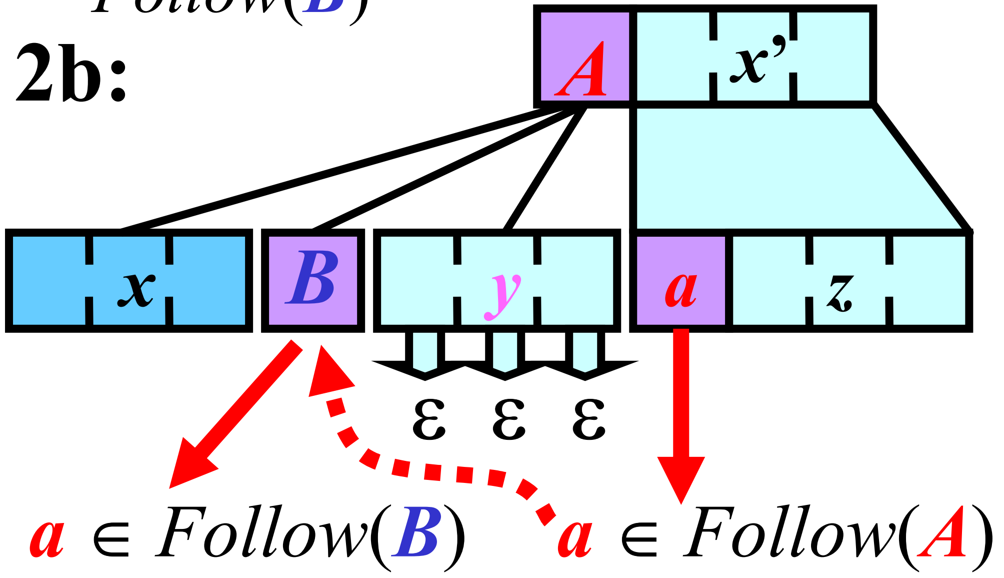
2) Apply the following rules until no *Follow* set can be changed:

- **if** $A \rightarrow xBy \in P$ **then**
  - **2a) if** $y \neq \varepsilon$ **then** add all symbols from $First(y)$ to $Follow(B)$
  - **2b) if** $Empty(y) = \{\varepsilon\}$ **then** add all symbols from $Follow(A)$ to $Follow(B)$

2a:



2b:



Follow(*X*) for G_{expr3}: Example 1/3

First(*E*)	:= {*i*, (}	*Empty*(*E*)	:= ∅	*Follow*(*E*)	:= ∅
First(*E'*)	:= {+}	*Empty*(*E'*)	:= {ε}	*Follow*(*E'*)	:= ∅
First(*T*)	:= {*i*, (}	*Empty*(*T*)	:= ∅	*Follow*(*T*)	:= ∅
First(*T'*)	:= {*}	*Empty*(*T'*)	:= {ε}	*Follow*(*T'*)	:= ∅
First(*F*)	:= {*i*, (}	*Empty*(*F*)	:= ∅	*Follow*(*F*)	:= ∅

0) *Follow*(*E*) := {\$}

1) $F \rightarrow (E \underbrace{)}_{\neq \varepsilon} \in P$: **add** *First*()) = {)} **to** *Follow*(*E*)

Summary: *Follow*(*E*) = {\$,)}

2) $E \rightarrow TE' \underbrace{\;}_{\varepsilon:\ Empty(\varepsilon) = \{\varepsilon\}} \in P$: **add** *Follow*(*E*) = {\$,)} **to** *Follow*(*E'*)

$E \rightarrow T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(*E'*) = {+} **to** *Follow*(*T*)

$E \rightarrow T\underbrace{E'}_{} \in P$: **add** *Follow*(*E*) = {\$,)} **to** *Follow*(*T*)

Empty(*E'*) = {ε}

Summary: *Follow*(*E'*) = {\$,)}, *Follow*(*T*) = {+, \$,)}

$Follow(X)$ for G_{expr3}: Example 2/3

$First(E) := \{i, (\}$	$Empty(E) := \emptyset$	$Follow(E) := \{\$,)\}$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \{\$,)\}$
$First(T) := \{i, (\}$	$Empty(T) := \emptyset$	$Follow(T) := \{+, \$,)\}$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \emptyset$
$First(F) := \{i, (\}$	$Empty(F) := \emptyset$	$Follow(F) := \emptyset$

3) $E' \to +TE'\underbrace{\;}_{} \in P$: **add** $Follow(E') = \{\$,)\}$ **to** $Follow(E')$

ε: $Empty(\varepsilon) = \{\varepsilon\}$

$E' \to +T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** $First(E') = \{+\}$ **to** $Follow(T)$

$E' \to +T\underbrace{E'}_{} \in P$: **add** $Follow(E') = \{\$,)\}$ **to** $Follow(T)$

$Empty(E') = \{\varepsilon\}$

Summary: Nothing is changed

4) $T \to FT'\underbrace{\;}_{} \in P$: **add** $Follow(T) = \{+, \$,)\}$ **to** $Follow(T')$

ε: $Empty(\varepsilon) = \{\varepsilon\}$

$T \to F\underbrace{T'}_{\neq \varepsilon} \in P$: **add** $First(T') = \{*\}$ **to** $Follow(F)$

$T \to F\underbrace{T'}_{} \in P$: **add** $Follow(T) = \{+, \$,)\}$ **to** $Follow(F)$

$Empty(T') = \{\varepsilon\}$

Summary: $Follow(T') = \{+, \$,)\}$, $Follow(F) = \{*, +, \$,)\}$

$Follow(X)$ for G_{expr3}: Example 3/3

$First(E)$	$:= \{i, (\}$	$Empty(E)$	$:= \varnothing$	$Follow(E)$	$:= \{\$,)\}$
$First(E')$	$:= \{+\}$	$Empty(E')$	$:= \{\varepsilon\}$	$Follow(E')$	$:= \{\$,)\}$
$First(T)$	$:= \{i, (\}$	$Empty(T)$	$:= \varnothing$	$Follow(T)$	$:= \{+, \$,)\}$
$First(T')$	$:= \{*\}$	$Empty(T')$	$:= \{\varepsilon\}$	$Follow(T')$	$:= \{+, \$,)\}$
$First(F)$	$:= \{i, (\}$	$Empty(F)$	$:= \varnothing$	$Follow(F)$	$:= \{*, +, \$,)\}$

5) $T' \to *FT'\underbrace{}_{\varepsilon: Empty(\varepsilon) = \{\varepsilon\}} \in P$: **add** $Follow(T') = \{+, \$,)\}$ **to** $Follow(T')$

$T' \to *F\underbrace{T'}_{\neq \varepsilon} \in P$: **add** $First(T') = \{*\}$ **to** $Follow(F)$

$T' \to *F\underbrace{T'}_{Empty(T') = \{\varepsilon\}} \in P$: **add** $Follow(T') = \{+, \$,)\}$ **to** $Follow(F)$

End: Nothing is changed.

Summary:
$Follow(E) := \{\$,)\}$
$Follow(E') := \{\$,)\}$
$Follow(T) := \{+, \$,)\}$
$Follow(T') := \{+, \$,)\}$
$Follow(F) := \{*, +, \$,)\}$

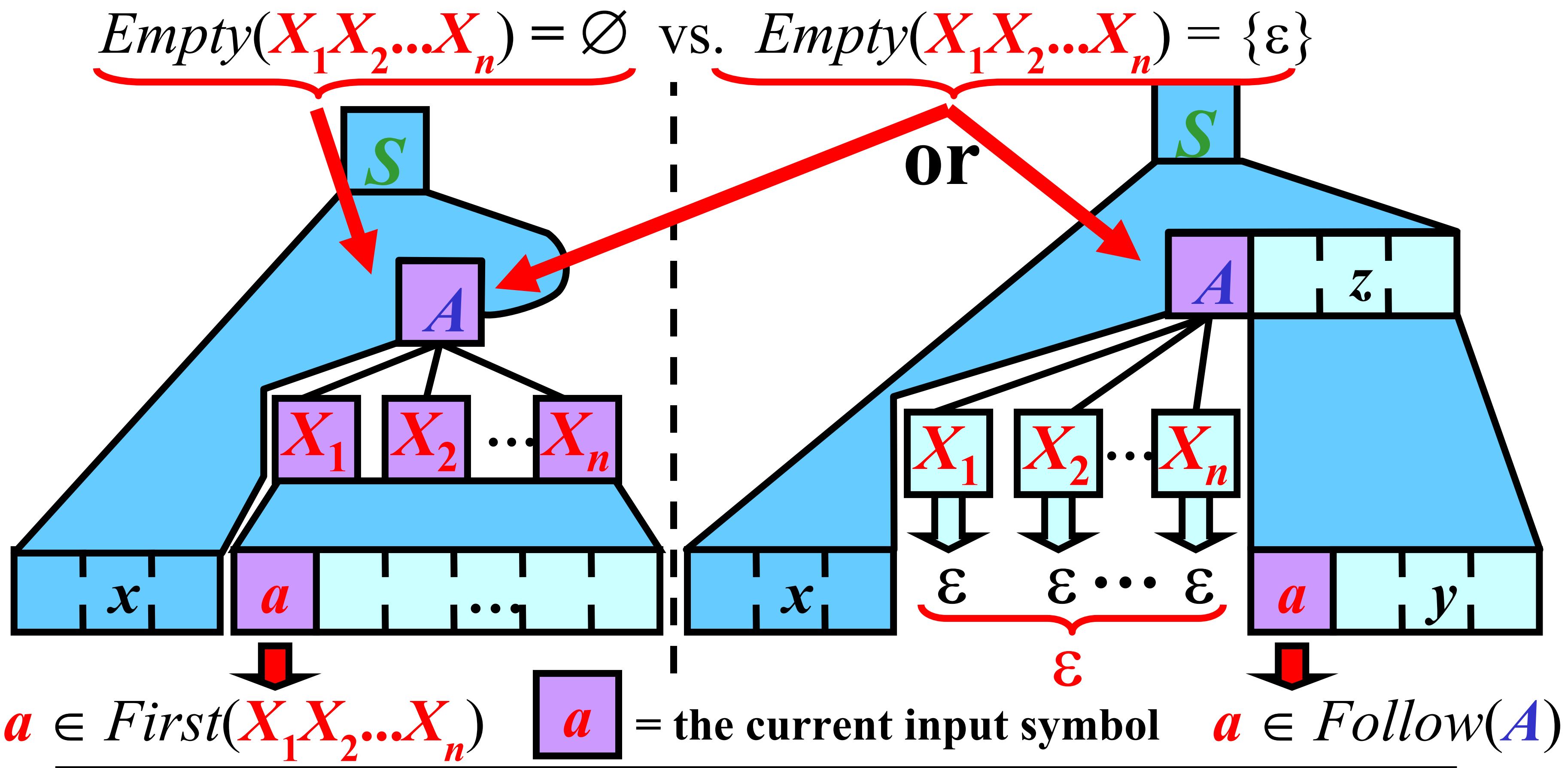
Set *Predict*

Gist: ***Predict*($A \rightarrow x$) is the set of all terminals that can begin a string obtained by a derivation started by using $A \rightarrow x$.**

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \rightarrow x \in P$, we define *Predict*($A \rightarrow x$) so that

- if *Empty*(x) = $\{\varepsilon\}$ then
  Predict($A \rightarrow x$) = *First*(x) $\cup$ *Follow*(A)
- if *Empty*(x) = $\emptyset$ then
  Predict($A \rightarrow x$) = *First*(x)

Set *Predict*($A \rightarrow X_1X_2\ldots X_n$): Illustration



$Empty(X_1X_2\ldots X_n) = \emptyset$
vs.
$Empty(X_1X_2\ldots X_n) = \{\varepsilon\}$
or
S
A
X_1
X_2
…
X_n
x
a
…
S
A
z
X_1
X_2
…
X_n
x
ε
ε
…
ε
ε
a
y
$a \in First(X_1X_2\ldots X_n)$
a
= the current input symbol
$a \in Follow(A)$

Summary: if $Empty(X_1X_2\ldots X_n) = \{\varepsilon\}$ then $Predict(A \rightarrow X_1X_2\ldots X_n) = First(X_1X_2\ldots X_n) \cup Follow(A)$;
otherwise, $Predict(A \rightarrow X_1X_2\ldots X_n) = First(X_1X_2\ldots X_n)$

Predict($A \to x$) for G_{expr3}: Example 1/2

First(E) := {i, (}	*Empty*(E) := ∅	*Follow*(E) := {\$,)}
First(E') := {+}	*Empty*(E') := {ε}	*Follow*(E') := {\$,)}
First(T) := {i, (}	*Empty*(T) := ∅	*Follow*(T) := {+, \$,)}
First(T') := {*}	*Empty*(T') := {ε}	*Follow*(T') := {+, \$,)}
First(F) := {i, (}	*Empty*(F) := ∅	*Follow*(F) := {*, +, \$,)}

1: $E \to TE'$

Empty(TE') = ∅ because *Empty*(T) = ∅

Predict(1) := *First*(TE') = *First*(T) = {i, (}

2: $E' \to +TE'$

Empty($+TE'$) = ∅ because *Empty*(T) = ∅

Predict(2) := *First*($+TE'$) = *First*(+) = {+}

3: $E' \to \varepsilon$

Empty(ε) = {ε}

Predict(3) := *First*(ε) ∪ *Follow*(E') = ∅ ∪ {\$,)} = {\$,)}

4: $T \to FT'$

Empty(FT') = ∅ because *Empty*(F) = ∅

Predict(4) := *First*(FT') = *First*(F) = {i, (}

$Predict(A \rightarrow x)$ for G_{expr3}: Example 2/2

$First(E)$	$:= \{i, (\}$	$Empty(E)$	$:= \emptyset$	$Follow(E)$	$:= \{\$,)\}$
$First(E')$	$:= \{+\}$	$Empty(E')$	$:= \{\varepsilon\}$	$Follow(E')$	$:= \{\$,)\}$
$First(T)$	$:= \{i, (\}$	$Empty(T)$	$:= \emptyset$	$Follow(T)$	$:= \{+, \$,)\}$
$First(T')$	$:= \{*\}$	$Empty(T')$	$:= \{\varepsilon\}$	$Follow(T')$	$:= \{+, \$,)\}$
$First(F)$	$:= \{i, (\}$	$Empty(F)$	$:= \emptyset$	$Follow(F)$	$:= \{*, +, \$,)\}$

5: $T' \rightarrow *FT'$

$Empty(*FT') = \emptyset$ because $Empty(F) = \emptyset$

$Predict(5) := First(*FT') = First(*) = \{*\}$

6: $T' \rightarrow \varepsilon$

$Empty(\varepsilon) = \{\varepsilon\}$

$Predict(6) := First(\varepsilon) \cup Follow(T') = \emptyset \cup \{+, \$,)\} = \{+, \$,)\}$

7: $F \rightarrow (E)$

$Empty((E)) = \emptyset$ because $Empty(E) = \emptyset$

$Predict(7) := First((E)) = First(() = \{(\}$

8: $F \rightarrow i$

$Empty(i) = \emptyset$

$Predict(8) := First(i) = \{i\}$

Construction of LL Table

α	…	a	…
…			
A		$\alpha(A, a)$	
…			

$\alpha(A, a) = A \to X_1X_2\ldots X_n \in P$ if $a \in Predict(A \to X_1X_2\ldots X_n)$; otherwise, $\alpha(A, a)$ is blank.

Task: LL table for G_{expr1}

	i	+	*	(	)	$
E	1					
E'						
T	4					
T'						
F	8					

$i \in Predict(1)$

$i \in Predict(4)$

$i \in Predict(8)$

Construct the rest analogically.

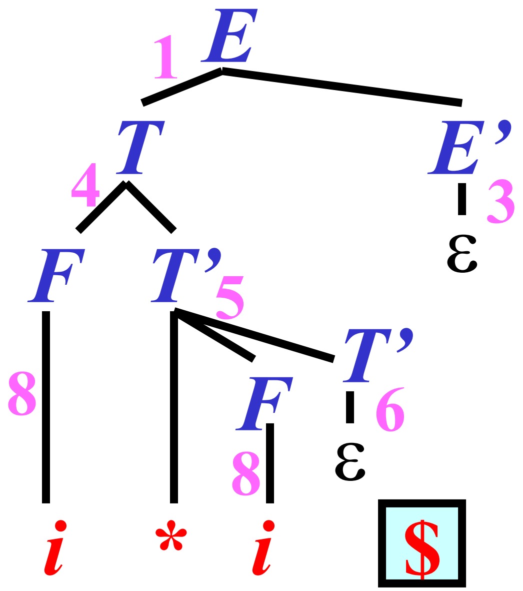
Rule r	***Predict***(r)
1: $E \to TE'$	{i, (}
2: $E' \to +TE'$	{+}
3: $E' \to \varepsilon$	{$,)}
4: $T \to FT'$	{i, (}
5: $T' \to *FT'$	{*}
6: $T' \to \varepsilon$	{+, $,)}
7: $F \to (E)$	{(}
8: $F \to i$	{i}

Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
6: $T' \rightarrow \varepsilon$
7: $F \rightarrow (E)$
8: $F \rightarrow i$

Question: $i * i \in \mathrm{L}(G_{expr3})$?



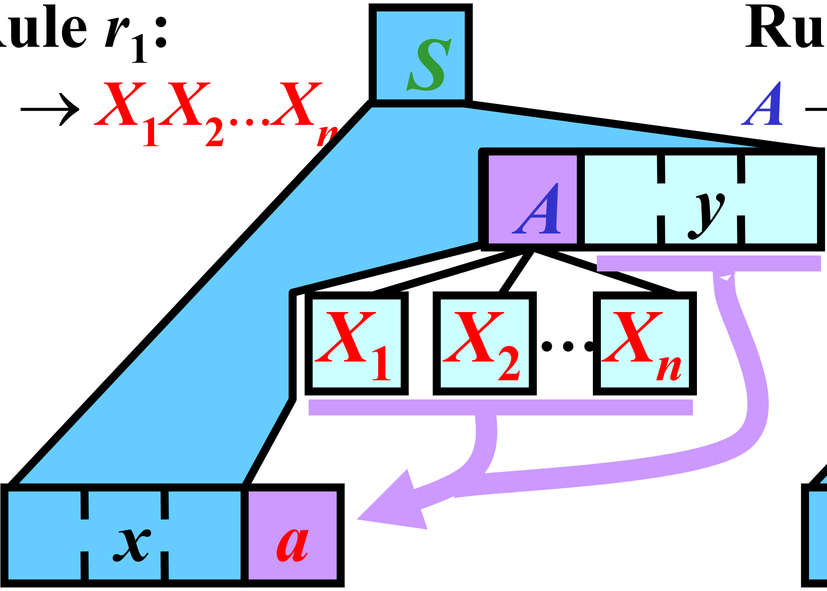
LL Grammars with ε-rules: Definition

Definition: Let $G = (N, T, P, S)$ be a CFG. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** A-rule $A \rightarrow X_1X_2\ldots X_n \in P$ such that $a \in Predict(A \rightarrow X_1X_2\ldots X_n)$

Illustration:

Rule r_1:

$A \rightarrow X_1X_2\ldots X_n$

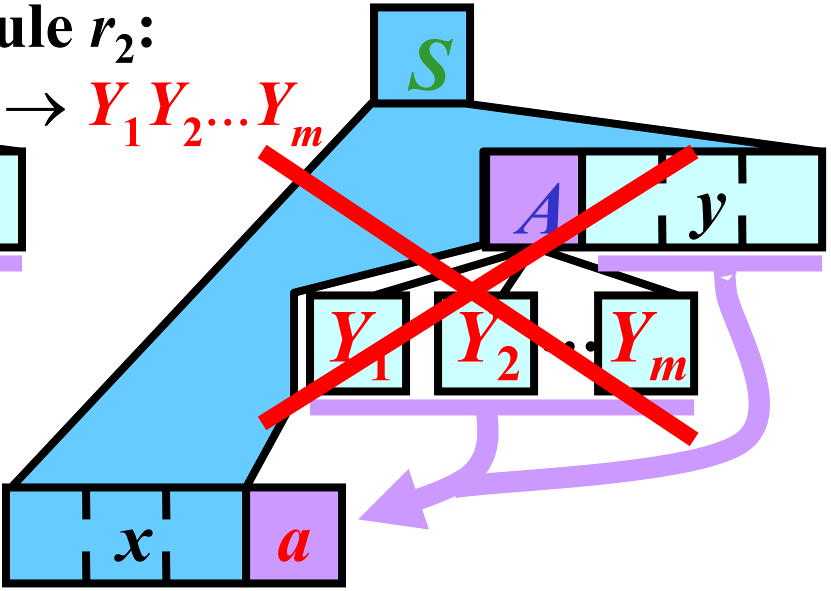


$a \in Predict(A \rightarrow X_1X_2\ldots X_n)$

Ruled out in an LL grammar

Rule r_2:

$A \rightarrow Y_1Y_2\ldots Y_m$



$a \in Predict(A \rightarrow Y_1Y_2\ldots Y_m)$

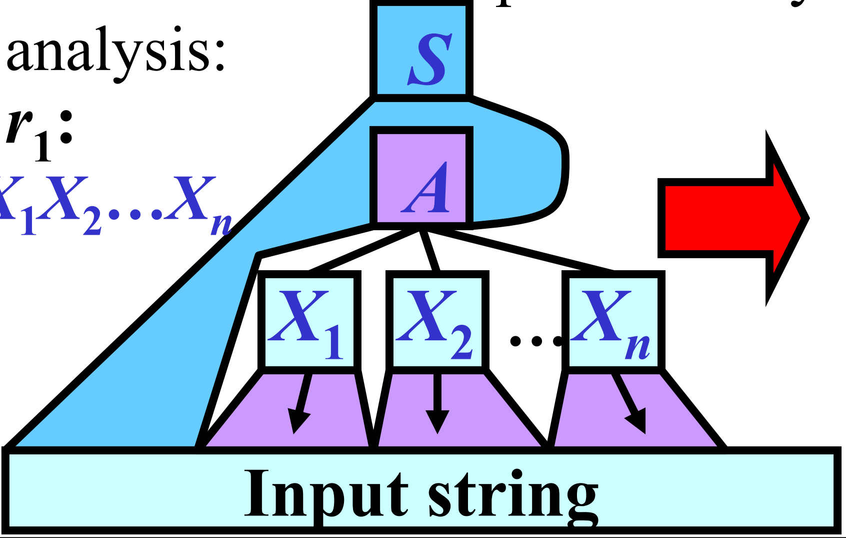
LL Analyzer Implementation

1) Recursive-Descent Parsing

• Each nonterminal is represented by a procedure, which perform its analysis:

Rule r_1:

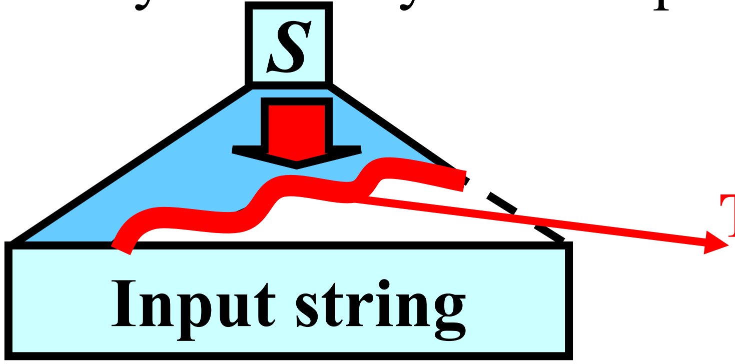
$A \rightarrow X_1X_2\ldots X_n$



```
function A: boolean;
begin
   {X1 analysis}
   {X2 analysis}
          …
   {Xn analysis}
end
```

2) Predictive Parsing

• Table-driven syntax analyzer with pushdown



These symbols are in the pushdown.

Recursive Descent: Example 1/4

```
Procedure GetNextToken;
begin
{ this procedure get the next token to global variable “token”}
end
```

• For $E \in N$: Rule 1: $E \rightarrow TE'$

$\boxed{1}$

```
function E: boolean;
begin
   E := false;
   if token in ['i', '('] then
         { simulation of rule 1: E → TE' }
         E := T and E1;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

• For $T \in N$: Rule 4: $T \rightarrow FT'$

$\boxed{4}$

```
function T: boolean;
begin
   T := false;
   if token in ['i', '('] then
         { simulation of rule 4: T → FT' }
         T := F and T1;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Recursive Descent: Example 2/4

- For E' ∈ N: Rules 2: $E' \rightarrow +TE'$, 3: $E' \rightarrow \varepsilon$

```
function E1: boolean;
begin
   E1 := false;
   if token = '+' then begin
      { simulation of rule 2: E' → +TE' }
      GetNextToken;
      E1 := T and E1;
   end
   else
   if token in [')', '$'] then
      { simulation of rule 3: E' → ε}
      E1 := true;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Recursive Descent: Example 3/4

- For *T'* ∈ *N*: Rules 5: *T'* → **FT'*, 6: *T'* → ε

```
function T1: boolean;
begin
   T1 := false;
   if token = '*' then begin
      { simulation of rule 5: T' → *FT' }
      GetNextToken;
      T1 := F and T1;
   end
   else
   if token in ['+', ')', '$'] then
      { simulation of rule 6: T' → ε}
      T1 := true;
end;
```

5

6

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Recursive Descent: Example 4/4

- For $F \in N$: Rules 7: $F \to (E)$, 8: $F \to i$

```
function F: boolean;
begin
   F := false;
   if token = '(' then begin
      { simulation of rule 7: F → (E) }
      GetNextToken;
      if E then begin
         F := (token = ')');
         GetNextToken;
      end;
   end
   else
   if token = 'i' then begin
      { simulation of rule 8: F → i }
      F := true;
      GetNextToken;
   end;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Main body:

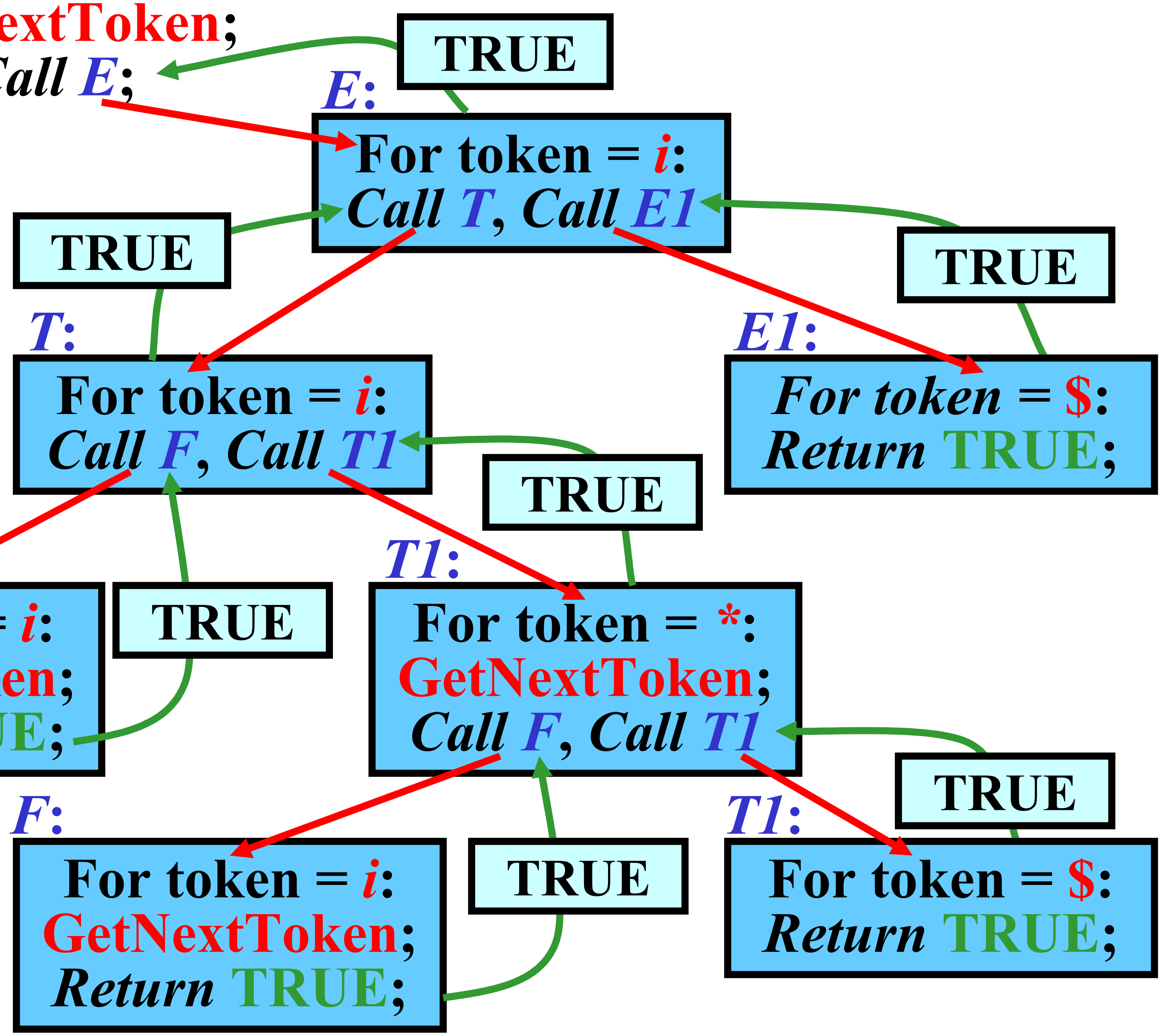
```
begin
   GetNextToken;
   if E then
      write('OK')
   else
      write('ERROR')
end.
```

Recursive Descent: Illustration for *i***i*$

Start: GetNextToken;
Call E;

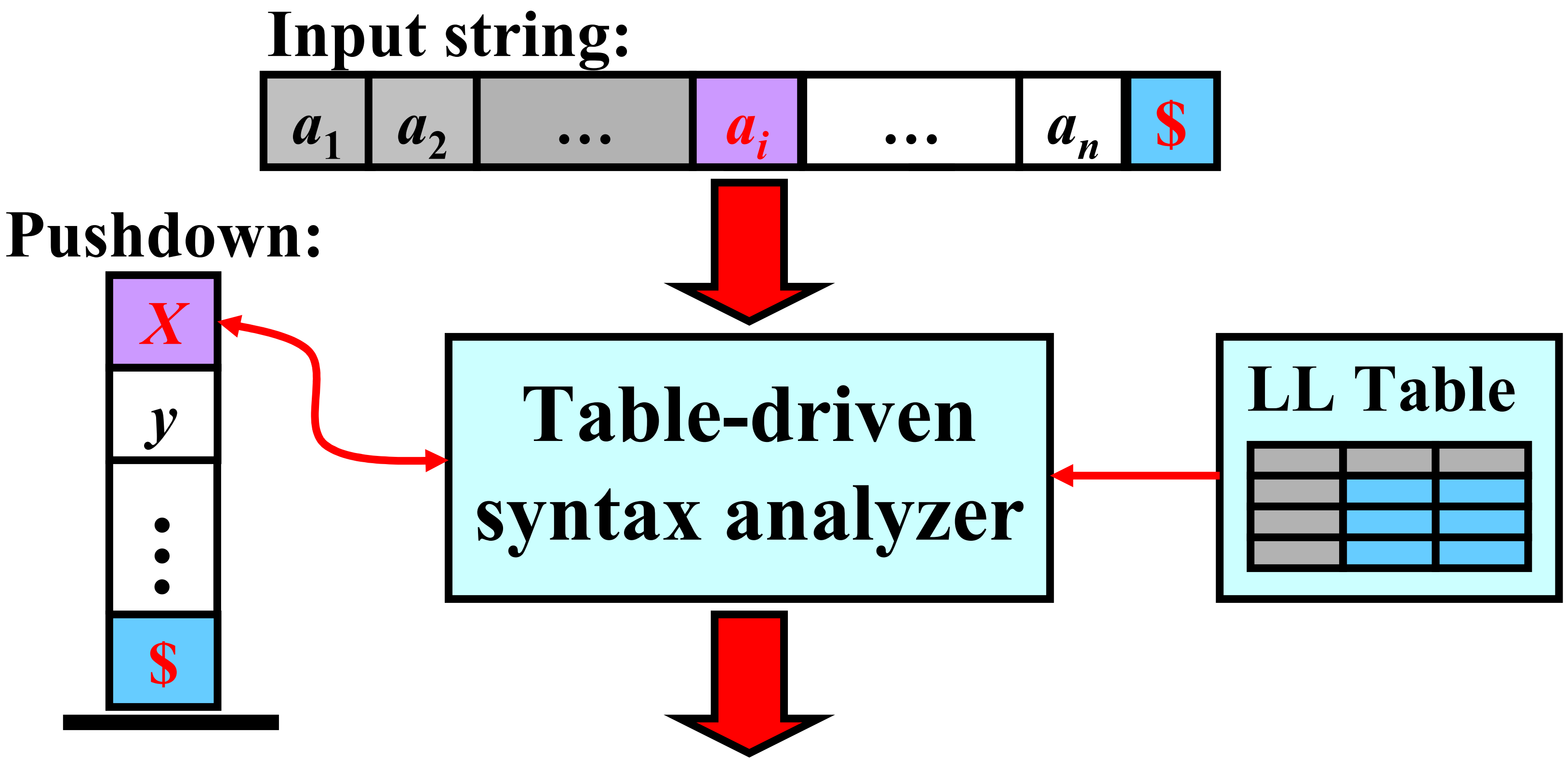
Input string:

i	*	*i*	$



Predictive Parsing

- **Model of table-driven syntax analyzer:**



Left parse = sequence of rules used in the leftmost derivation of the input string.

Table-Driven Parsing: Algorithm

- **Input:** LL-table for $G=(N, T, P, S)$; $x \in T^*$
- **Output:** Left parse of x if $x \in L(G)$; otherwise, error

---

- **Method:**

```
• push($) & push(S) onto the pushdown;
• while the pushdown is not empty do
    • let X = the pushdown top and a = the current token
    • case X of:
        • X = $:    if a = $ then success
                              else error;
        • X ∈ T:    if X = a then pop(X) & read next a from
                                   input string
                              else error;
        • X ∈ N:    if r: X → x ∈ LL-table[X, a] then
                    replace X with reversal(x) on the
                    pushdown & write r to output
                    else error;
  end
```

Table-Driven Parsing: Example

	i	+	*	(	)	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Rules:

1: $E \rightarrow TE'$
2: $E' \rightarrow +TE'$
3: $E' \rightarrow \varepsilon$
4: $T \rightarrow FT'$
5: $T' \rightarrow *FT'$
6: $T' \rightarrow \varepsilon$
7: $F \rightarrow (E)$
8: $F \rightarrow i$

Input string: $i * i$ \$

Pushdown	Input	Rule	Derivation
\$$E$	$i*i$\$	1: $E \rightarrow TE'$	$\underline{E} \Rightarrow \underline{T}E'$
\$$E'T$	$i*i$\$	4: $T \rightarrow FT'$	$\Rightarrow \underline{F}T'E'$
\$$E'T'F$	$i*i$\$	8: $F \rightarrow i$	$\Rightarrow i\underline{T'}E'$
\$$E'T'i$	$i*i$\$		
\$$E'T'$	$*i$\$	5: $T' \rightarrow *FT'$	$\Rightarrow i*\underline{F}T'E'$
\$$E'T'F*$	$*i$\$		
\$$E'T'F$	i\$	8: $F \rightarrow i$	$\Rightarrow i*i\underline{T'}E'$
\$$E'T'i$	i\$		
\$$E'T'$	\$	6: $T' \rightarrow \varepsilon$	$\Rightarrow i*i\underline{E'}$
\$$E'$	\$	3: $E' \rightarrow \varepsilon$	$\Rightarrow i*i$
\$	\$		

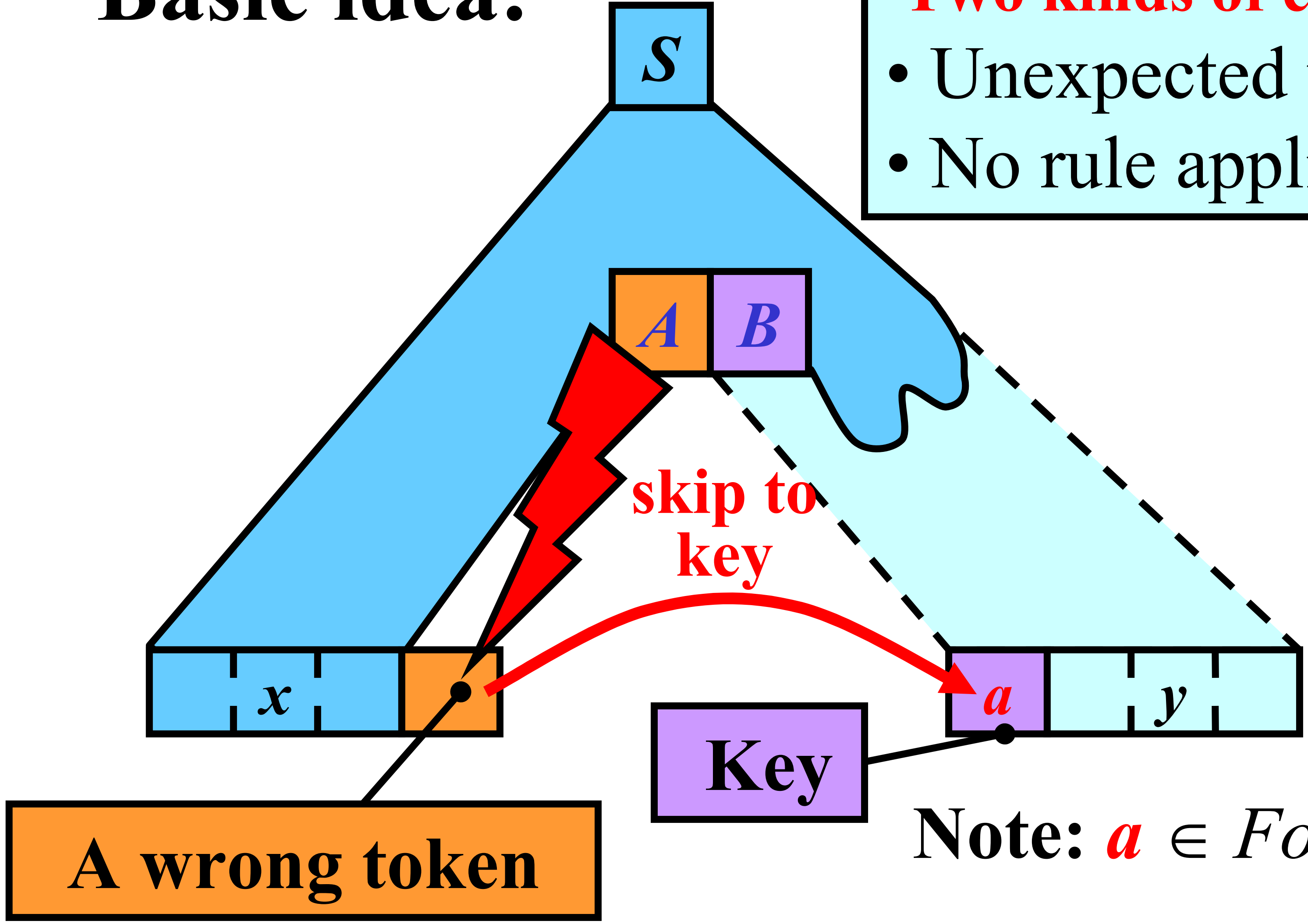
Success

Left parse: 1485863

Handling Errors: Introduction

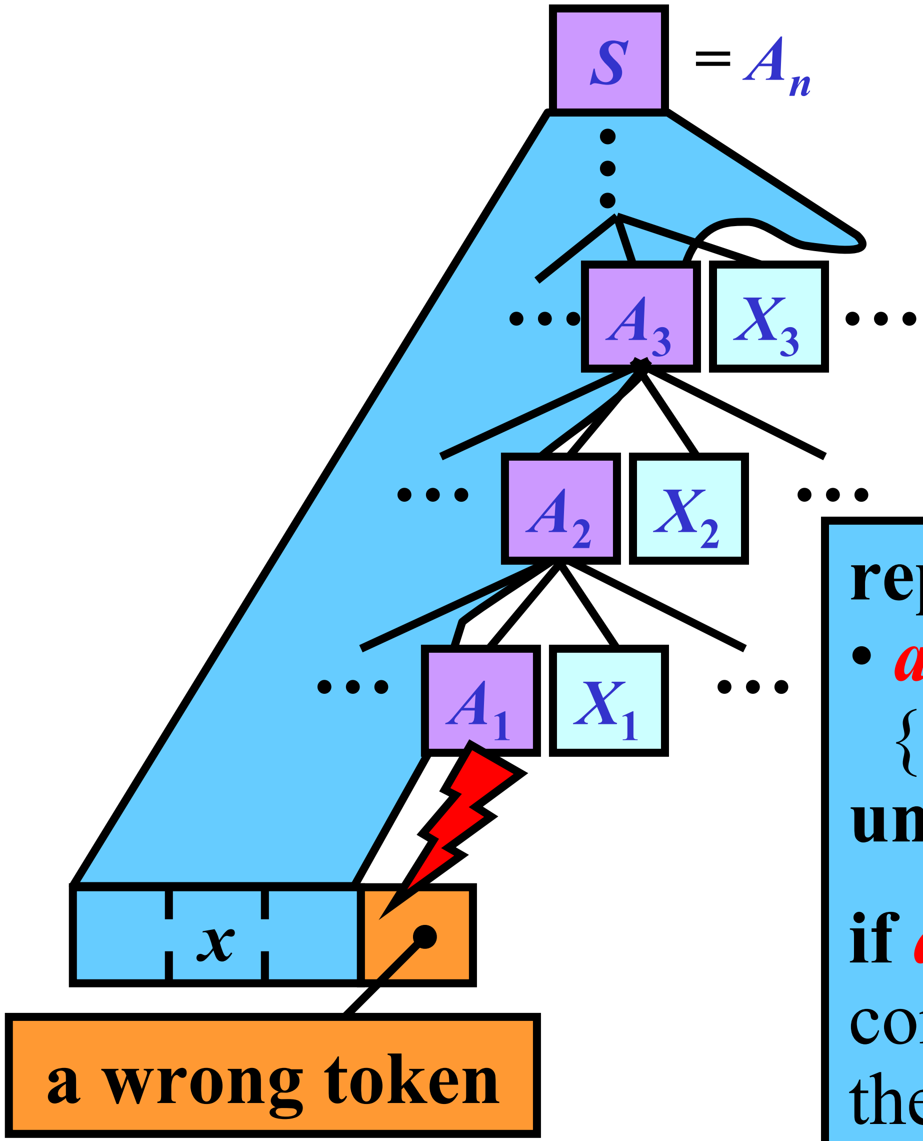
Basic idea:

Two kinds of errors:
- Unexpected token
- No rule applicable



Note: $a \in Follow(A)$

Panic-Mode (Hartmann) Error Recovery



- Let *Context*(A_1) = *Follow*(A_1) ∪ *Follow*(A_2) ∪ … *Follow*(A_n)

repeat

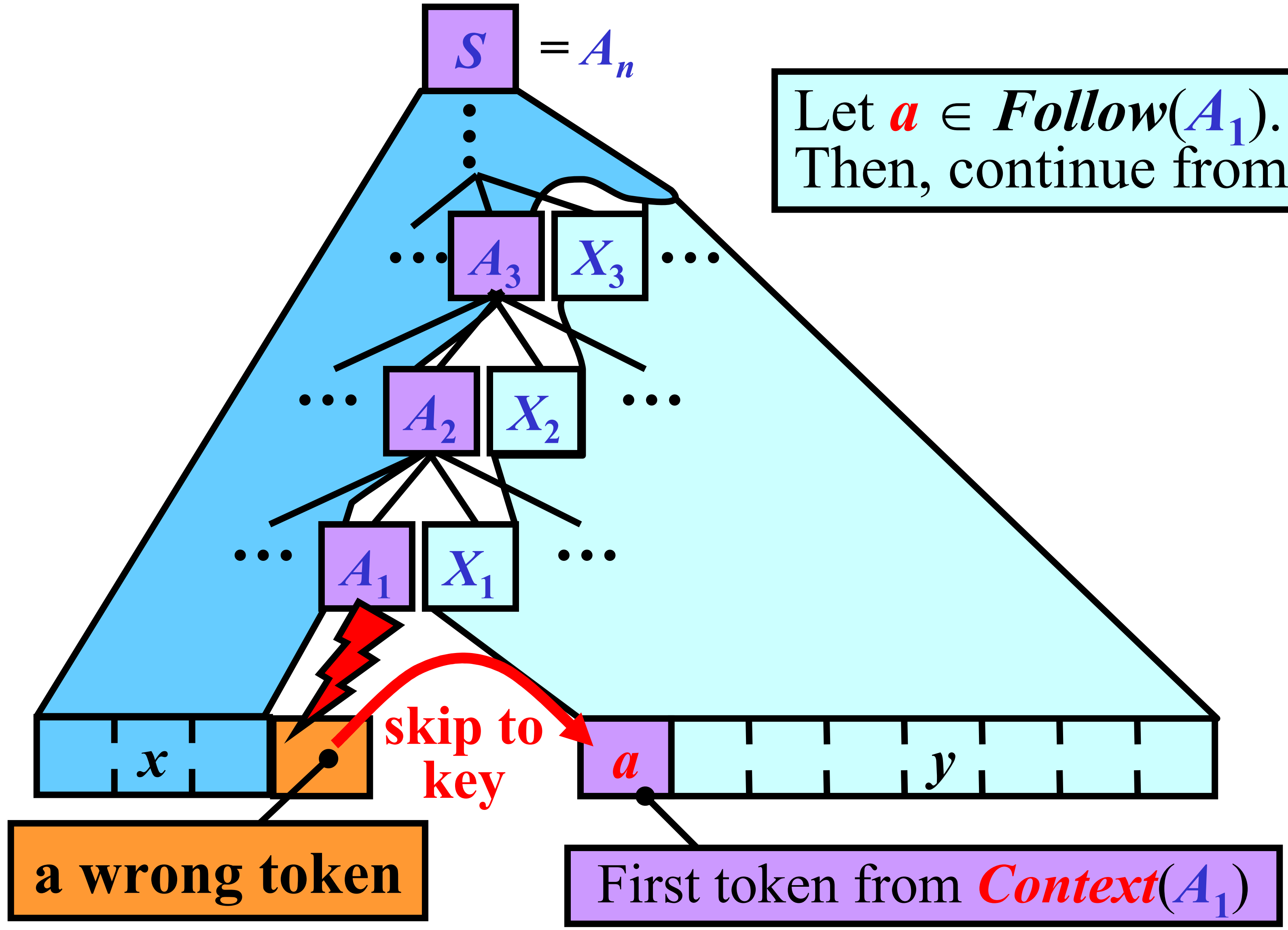
- a := GetNextToken;
  {These tokens are skipped}

until a in *Context*(A_1)

if a in *Follow*(A_i) **then** continue with parsing from the symbol X_i.

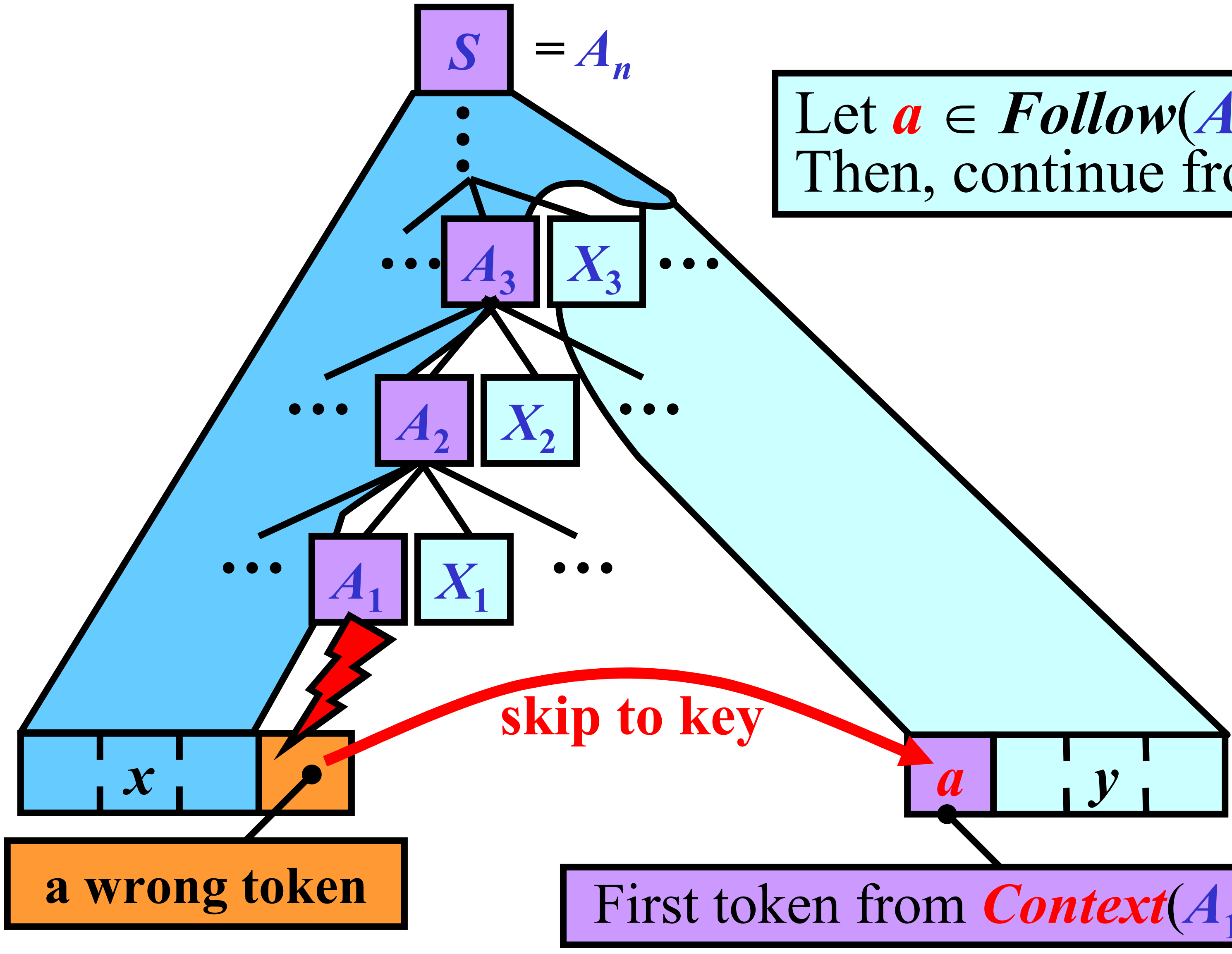
Panic-Mode Recovery: Illustration 1/2

Let $a \in Follow(A_1)$.
Then, continue from X_1



Panic-Mode Recovery: Illustration 2/2

Let $a \in \textbf{\textit{Follow}}(A_3)$.
Then, continue from X_3



Context(*X*) for Predictive Parser: Variant I

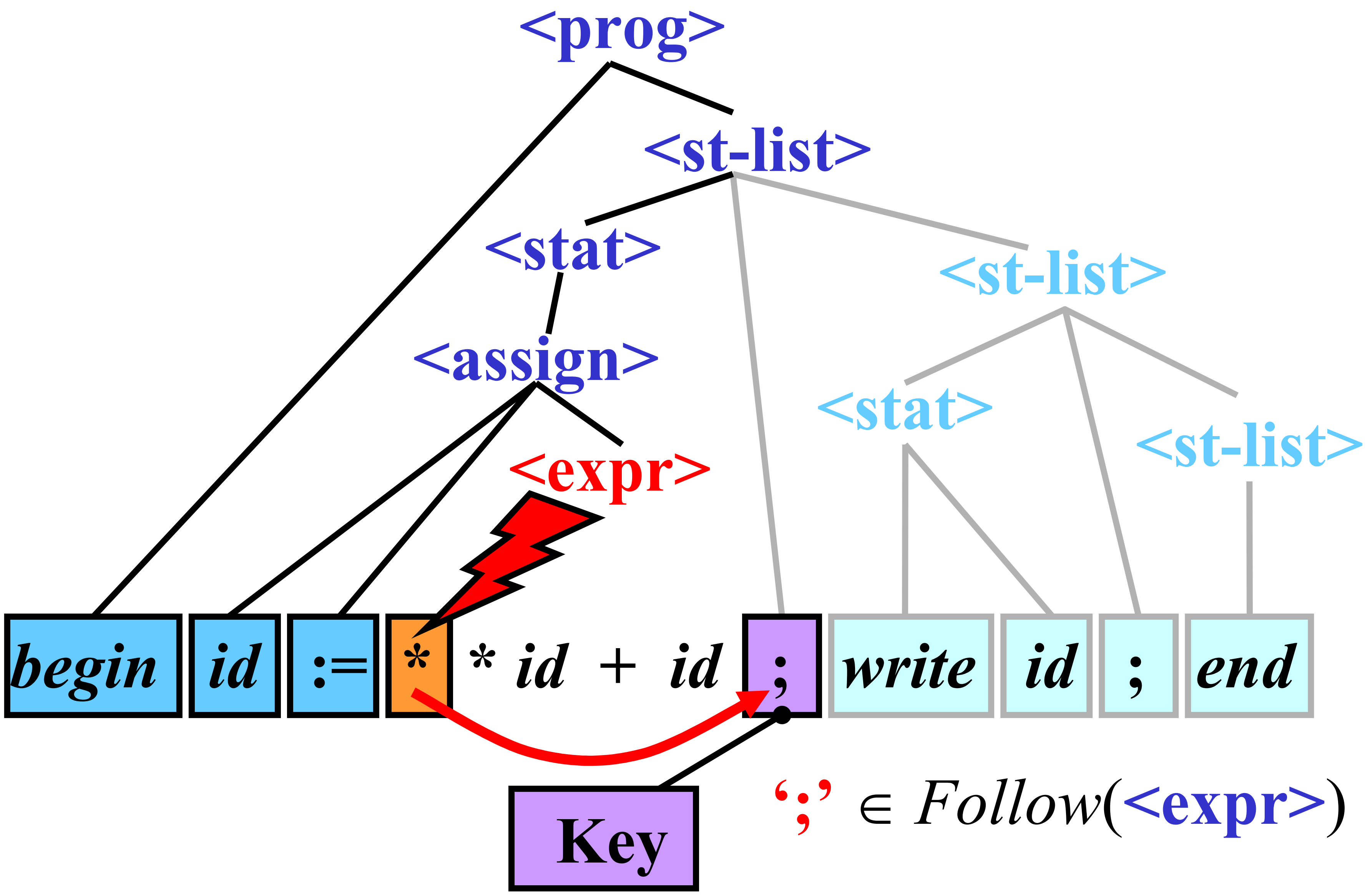
For $G = (N, T, P, S)$,
***Context*(A) = *Follow*(A) for every $A \in N$**

- **Method:**
- Let A be pushdown top & no rule is applicable:
- **repeat**

```
    a := GetNextToken;
    {These tokens are skipped}
until a in Context(A)
```

- pop A from the pushdown;

Variant I: Example



Context(X) for Predictive Parser: Variant II

For $G = (N, T, P, S)$,

Context(A) = ***First***(A) $\cup$ ***Follow***(A) **for every** $A \in N$

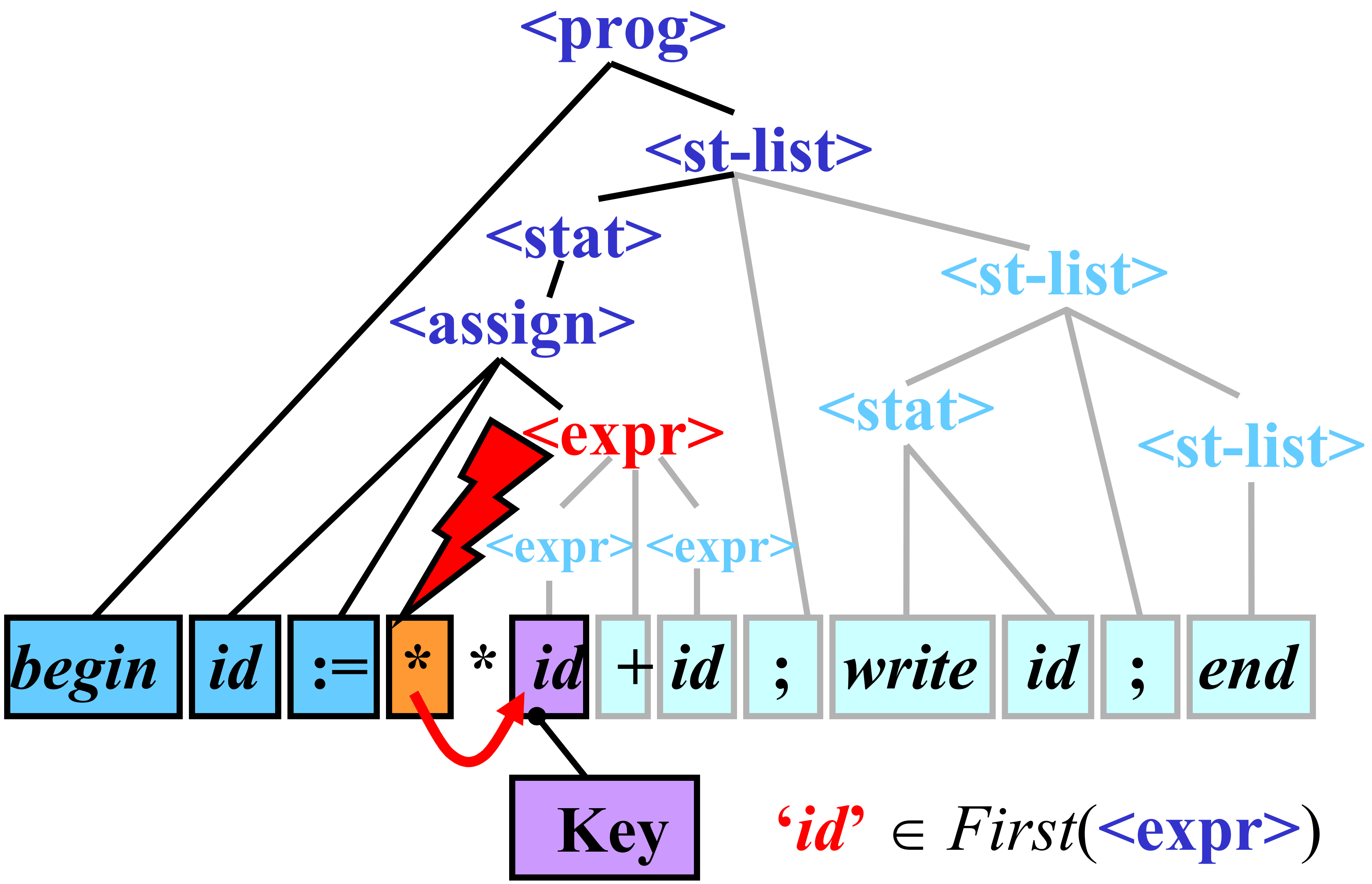
- **Method:**
- Let A be pushdown top & no rule is applicable:
- **repeat**

```
    a := GetNextToken;
    {These tokens are skipped}
  until a in Context(A)
```

- **if** $a \in First(A)$ **then** resume according to A

  else pop A from the pushdown // $a \in Follow(A)$

Variant II: Example



‘*id*’ ∈ *First*(<expr>)