

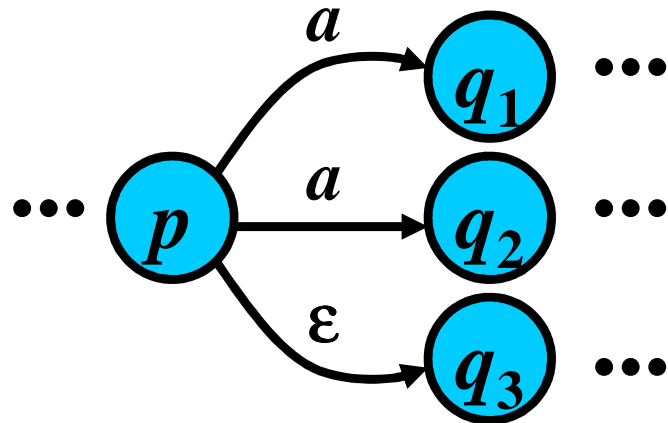
# **Part IV.**

# **Variants of Finite**

# **Automata**

# Theory vs. Practice

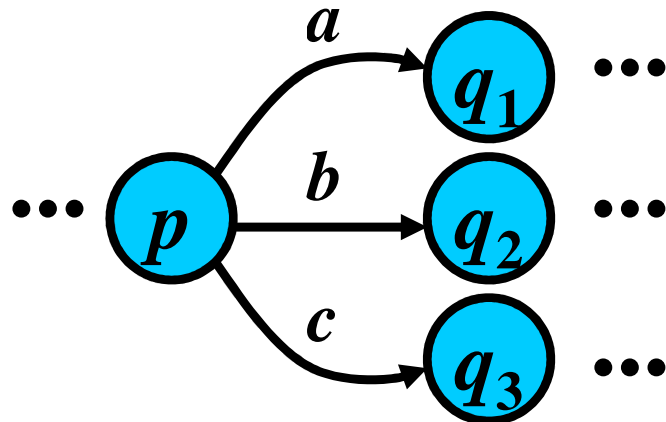
a) Configuration:  $pax$



Next Configuration:  
 $q_1x$  or  $q_2x$  or  $q_3ax$  ?

Theory: 😊 × Practice: ☹️

b) Configuration:  $pax$



Next Configuration:  
 only  $q_1x$

Theory: ☹️ × Practice: 😊

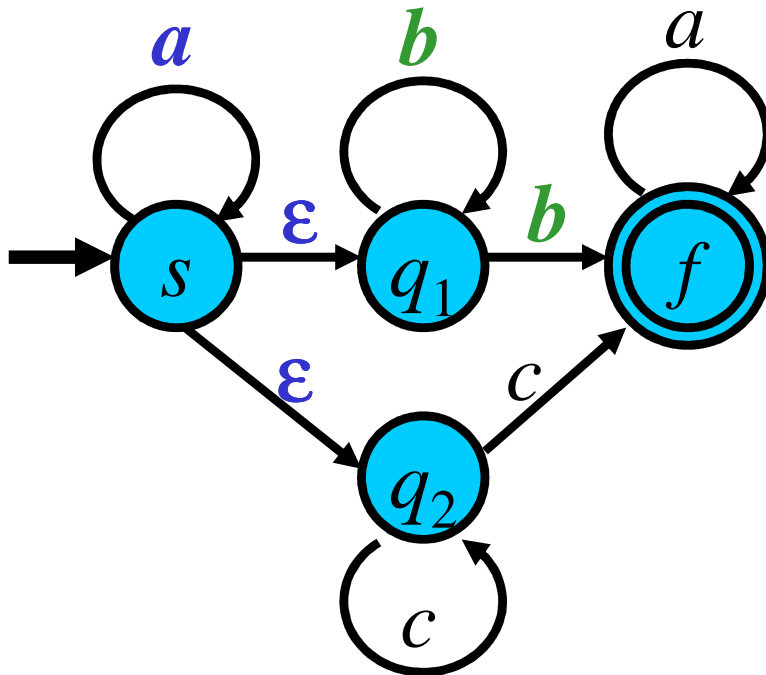
# Use of FA in General

Simulation of all possible moves from every configuration.

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## Example:

FA  $M$  is defined as:



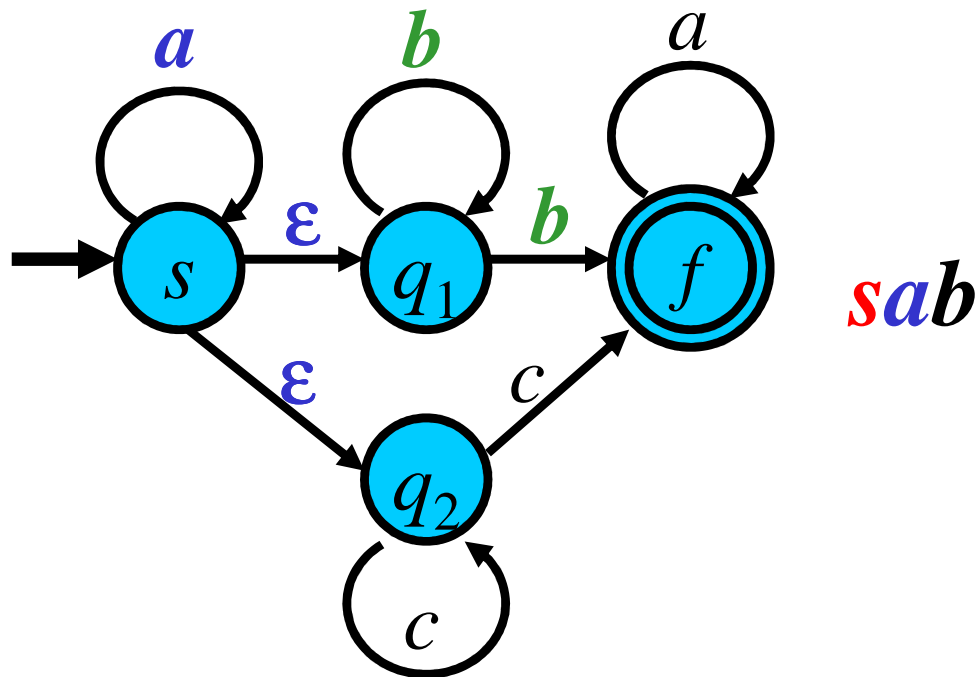
**Question:**  $ab \in L(M)$  ?

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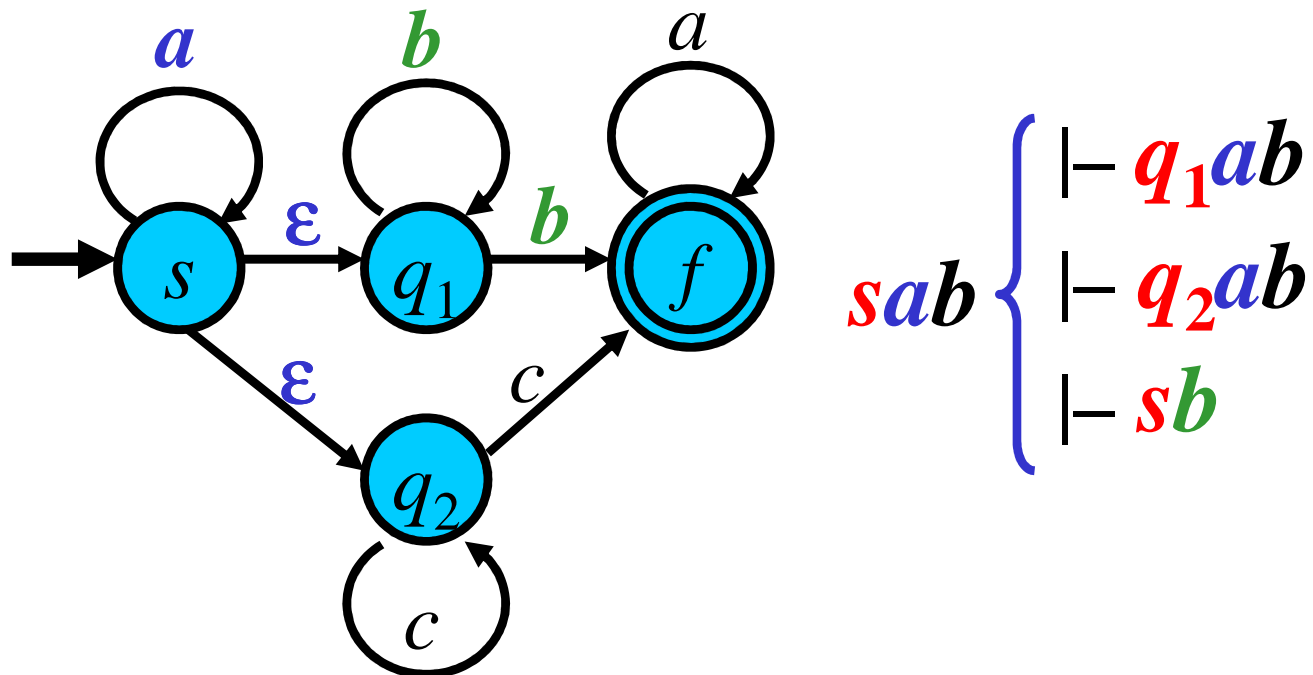
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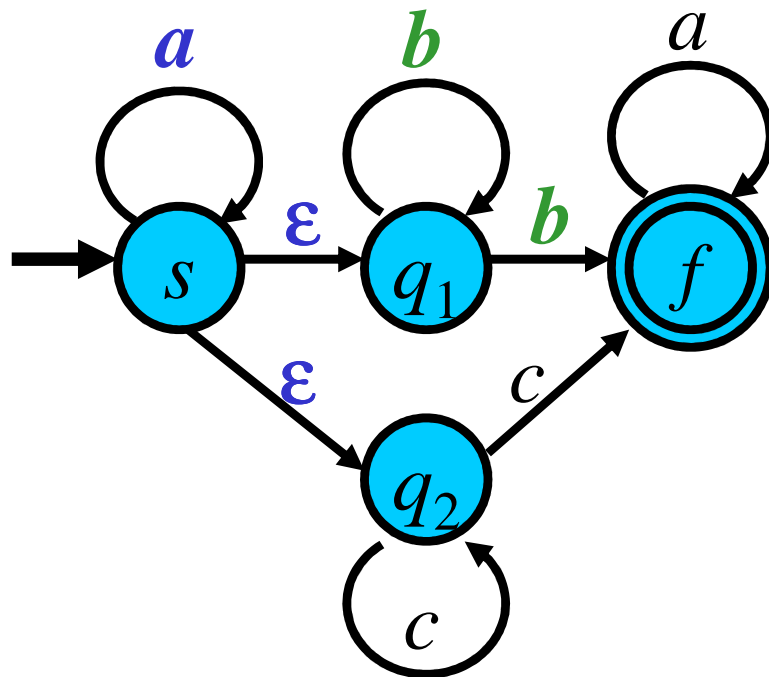
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Simulation of all possible moves from every configuration.

## Example:

FA  $M$  is defined as:



$sab$  {
 

- $\vdash q_1ab$
- $\vdash q_2ab$
- $\vdash sb$

No next configuration

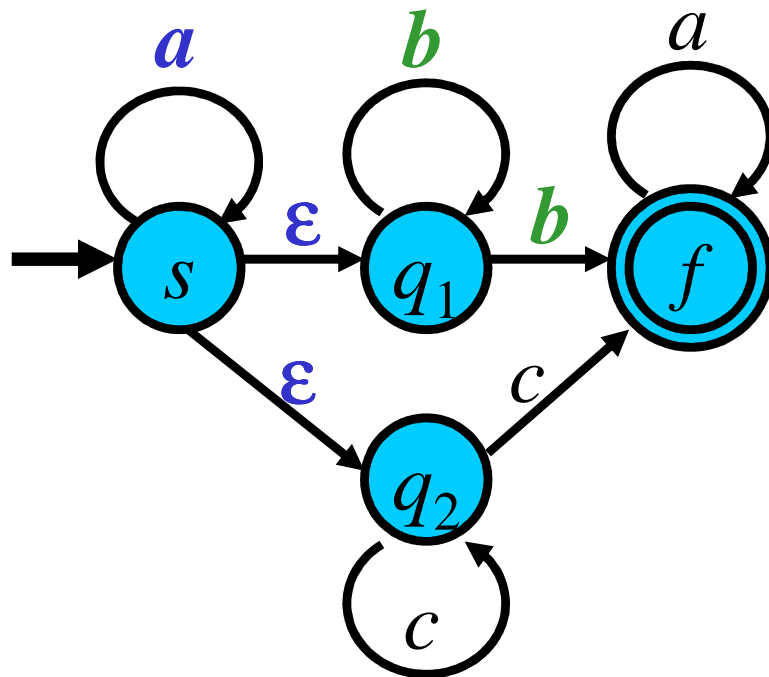
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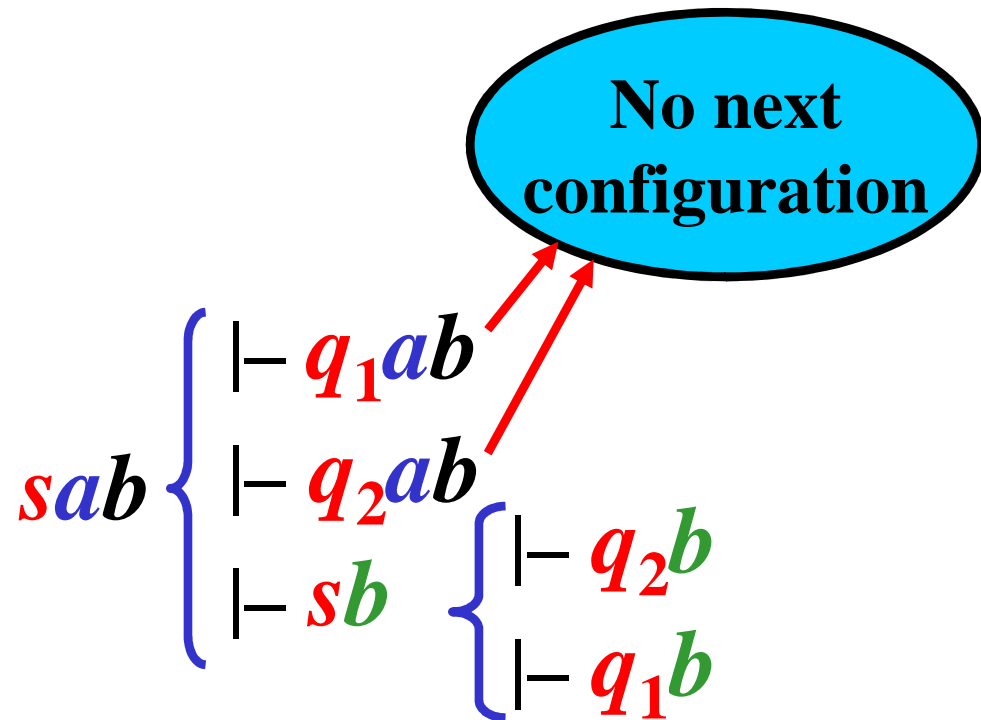
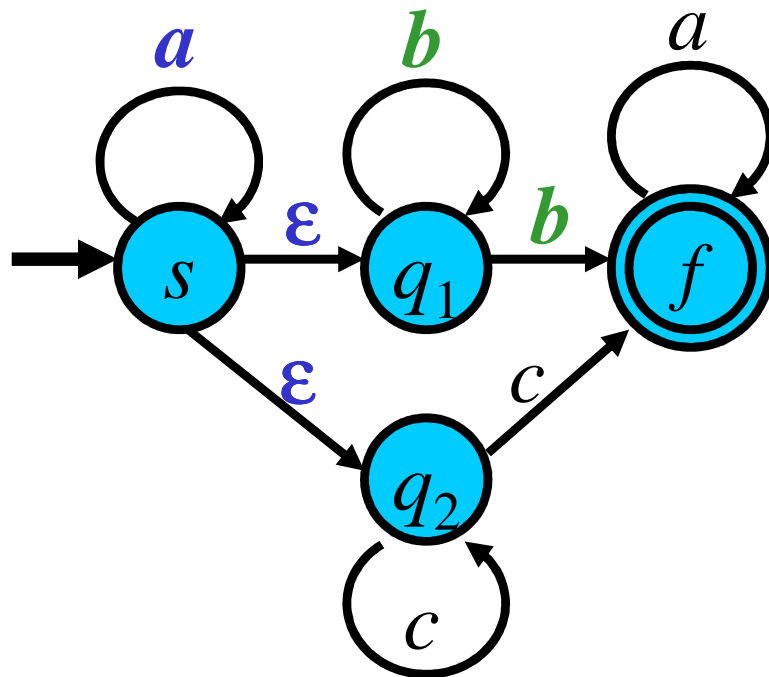
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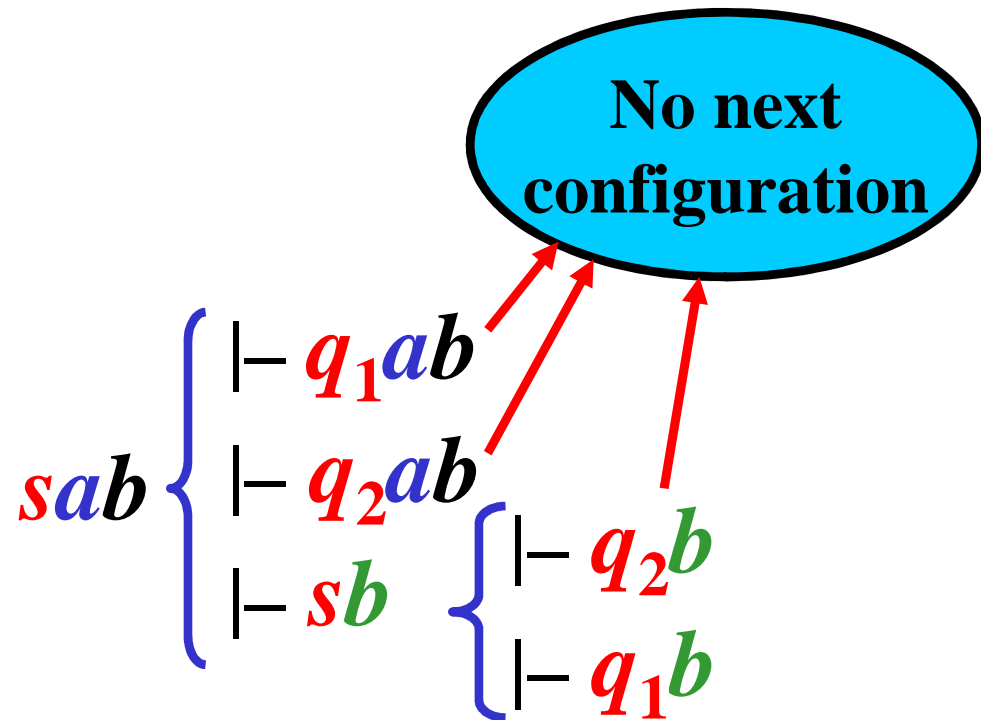
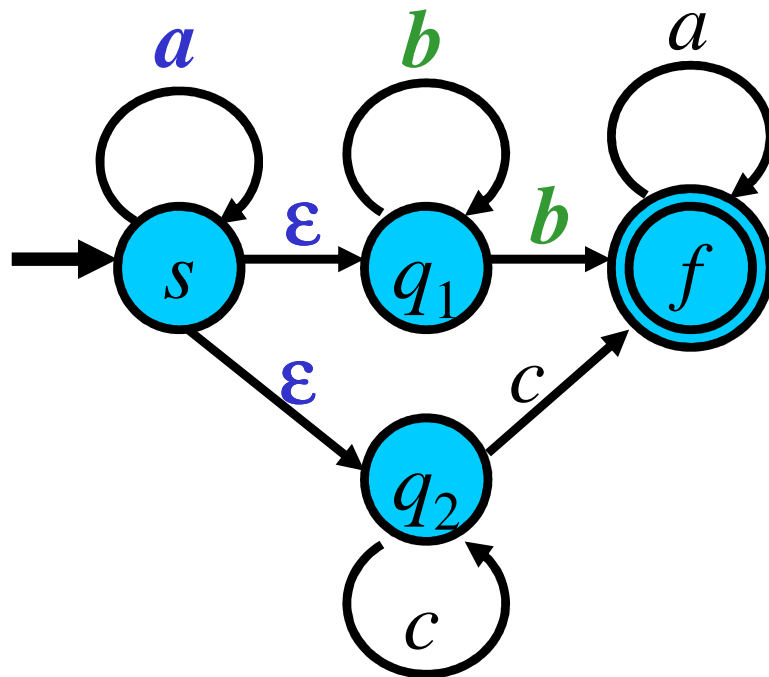


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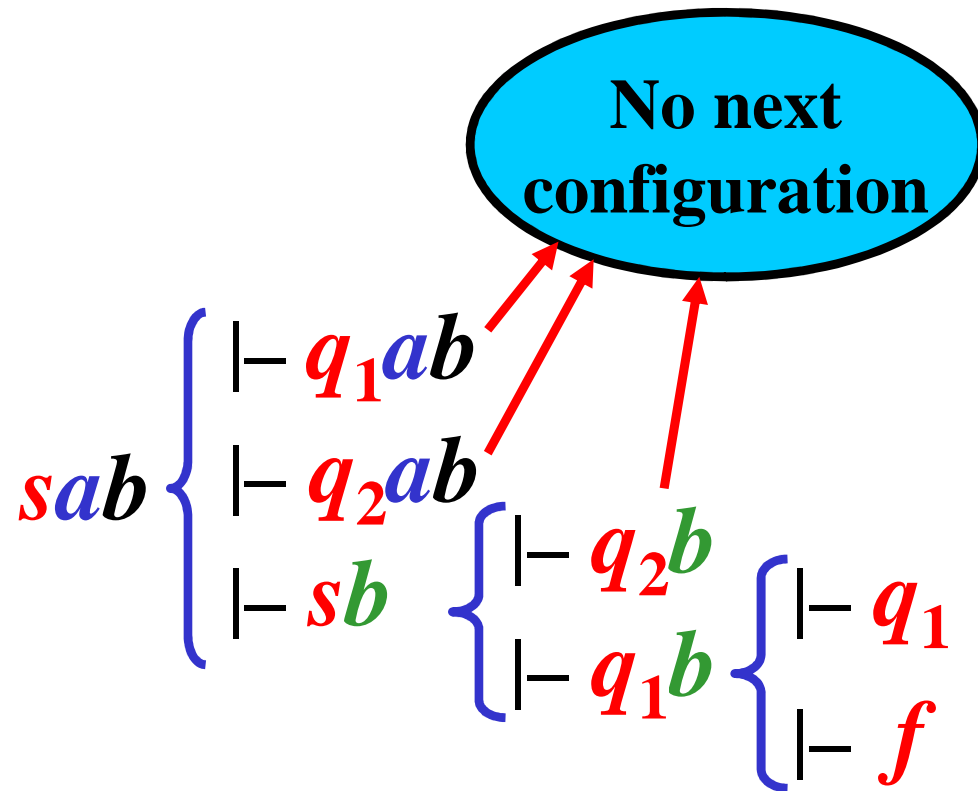
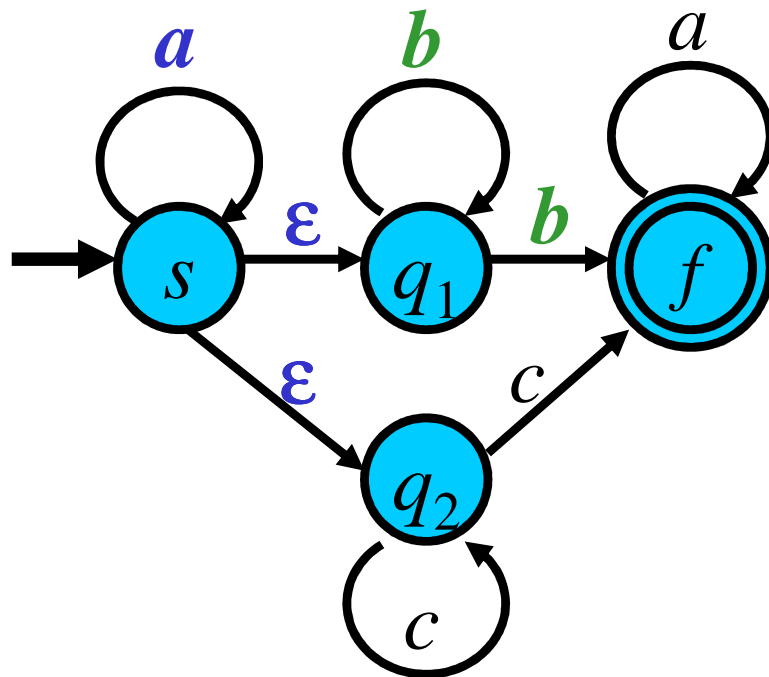
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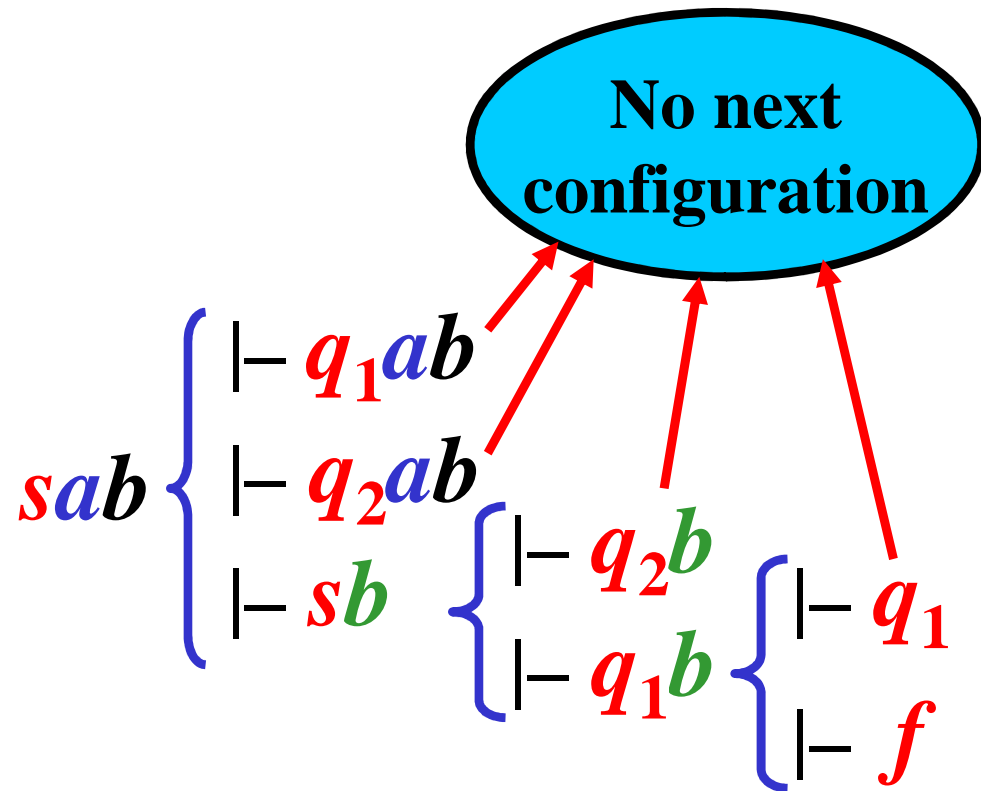
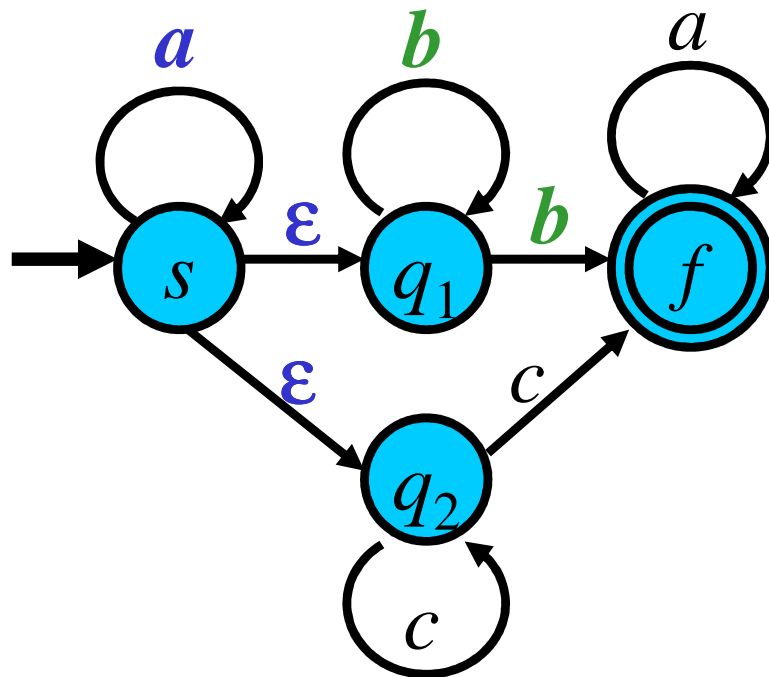
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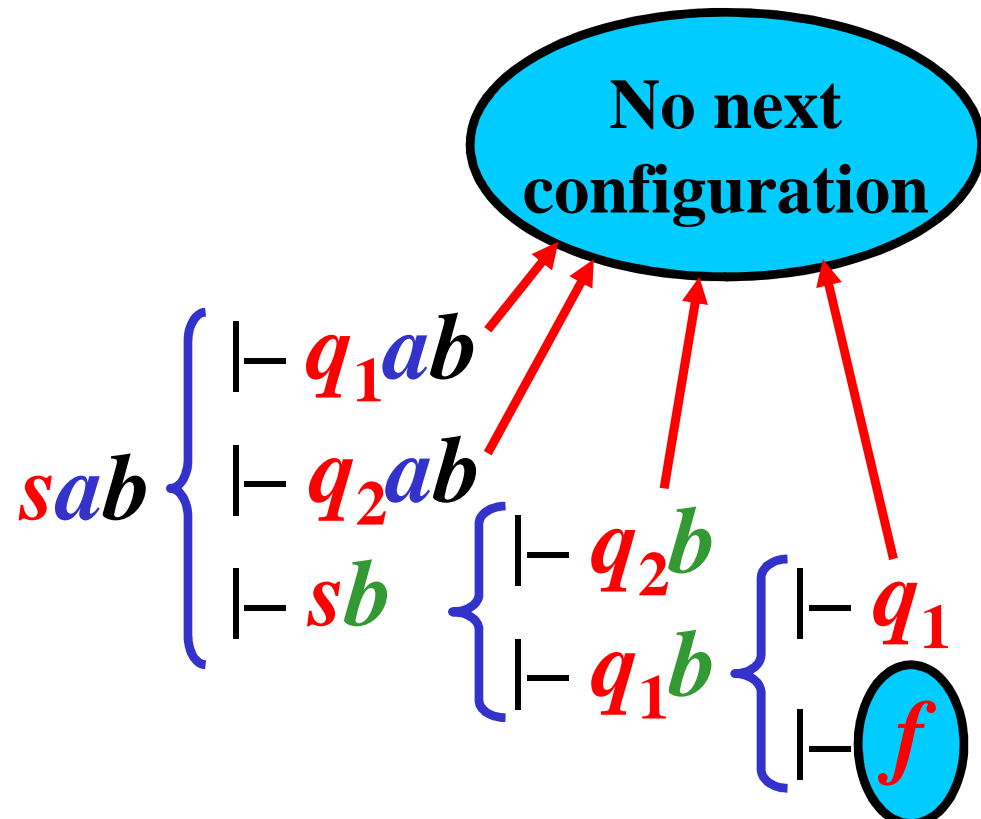
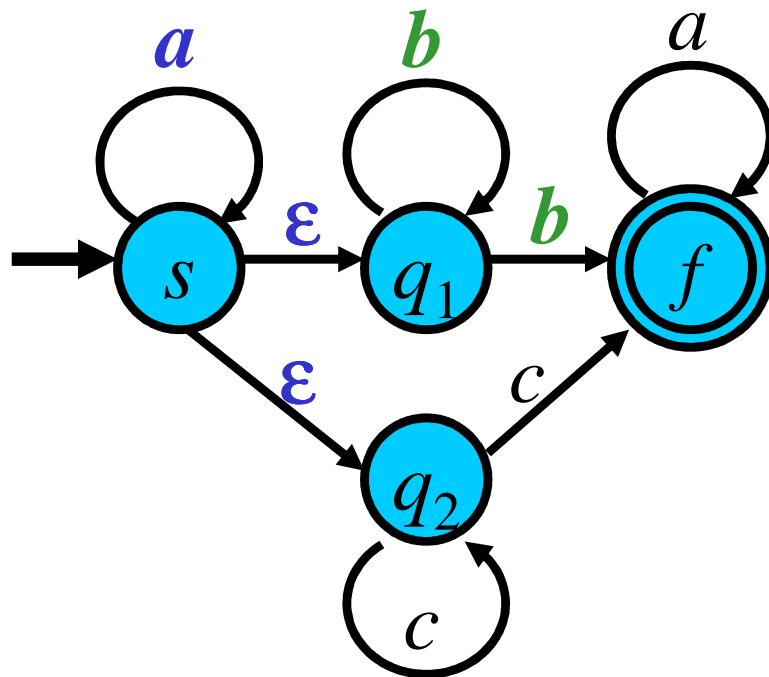
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Simulation of all possible moves from every configuration.

## Example:

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Question:  $ab \in L(M)$  ?

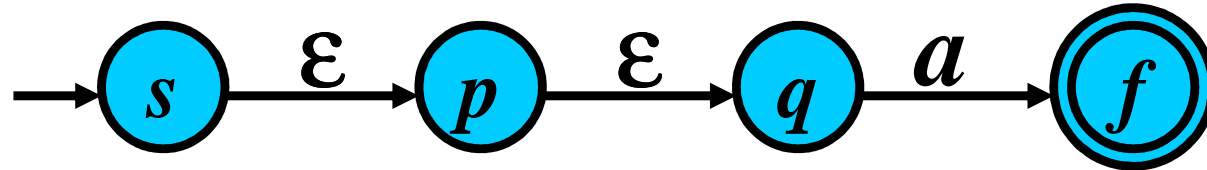
Answer: **YES**,  $ab \in L(M)$   
because  $f \in F$ .

# From FA to DFA in Essence 1/2

**Preference in practice:** *Deterministic FA* (DFA) that makes no more than one move from every configuration.

---

## 1) Gist: Removal of $\epsilon$ -moves

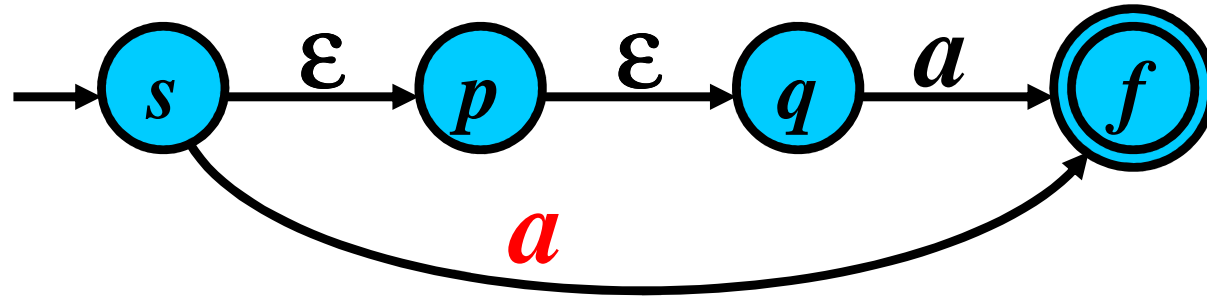


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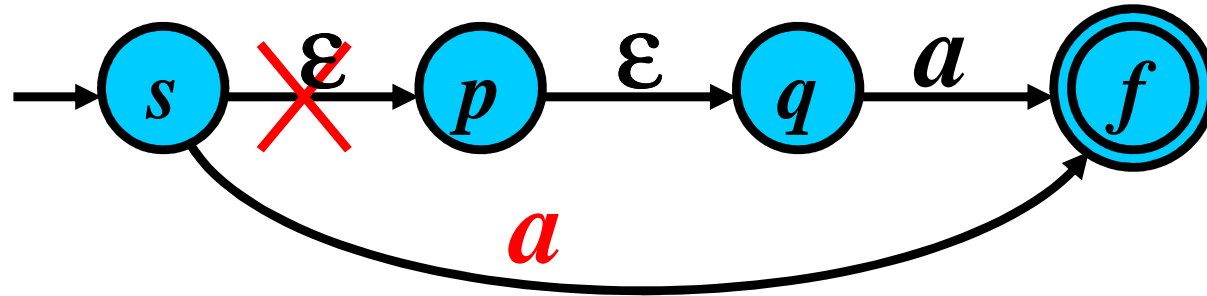


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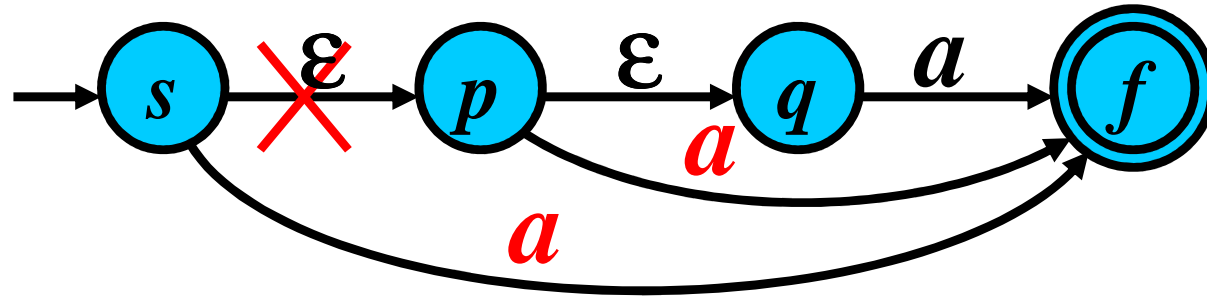


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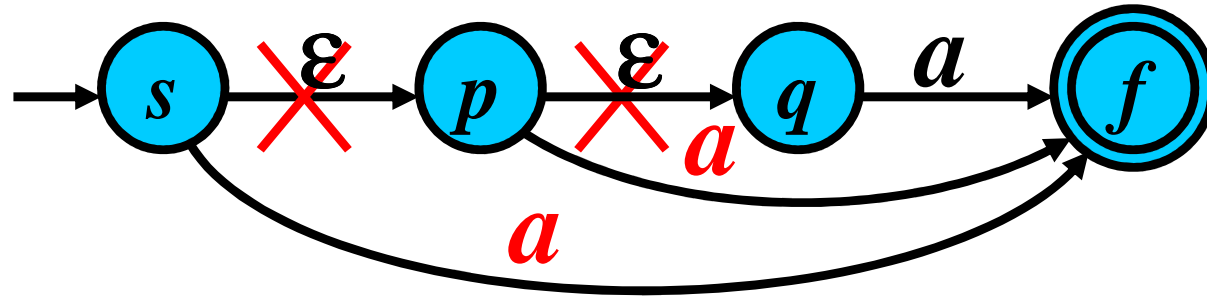


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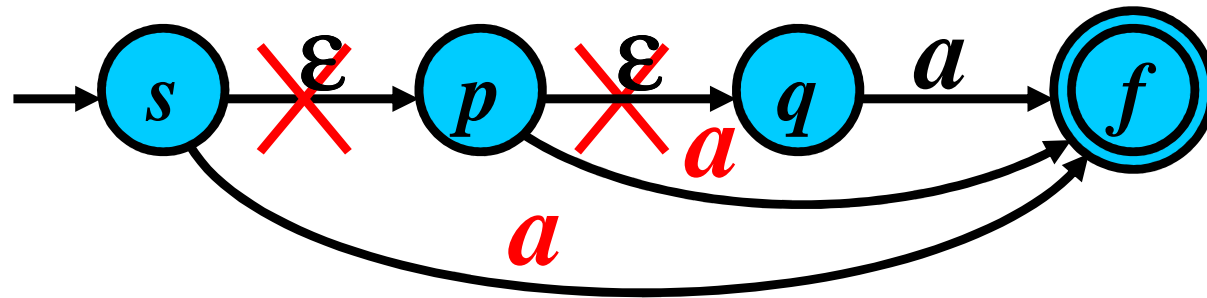
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# From FA to DFA in Essence 1/2

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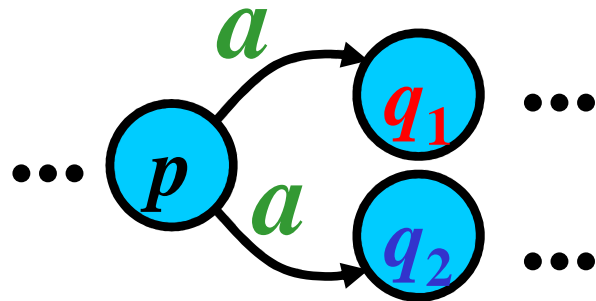


**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be a FA.  $M$  is an  *$\varepsilon$ -free finite automaton* if for all rules  $pa \rightarrow q \in R$ , where  $p, q \in Q$ , holds

$$a \in \Sigma (a \neq \varepsilon)$$

## From FA to DFA in Essence 2/2

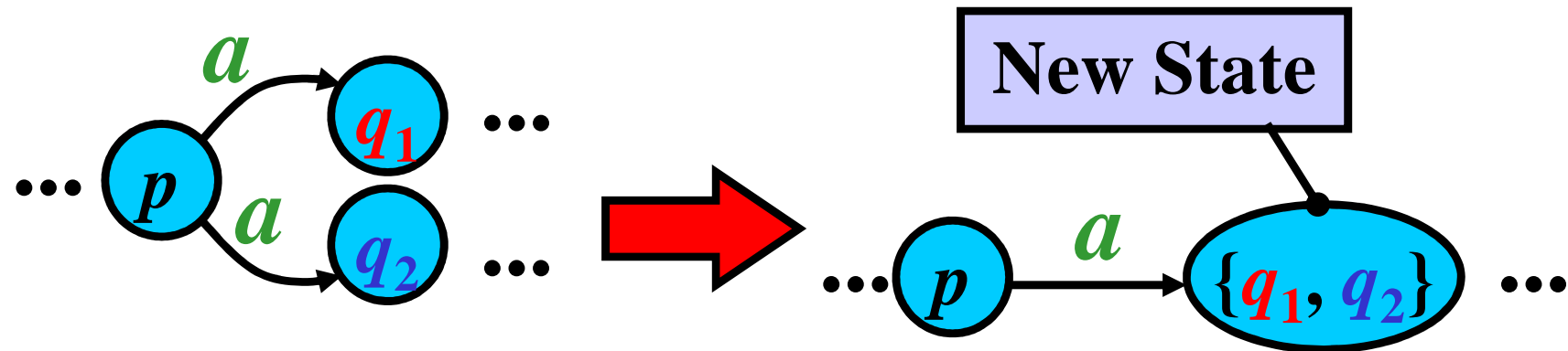
### 2) Gist: Removal of nondeterminism



**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be an  **$\epsilon$ -free FA**.  $M$  is a *deterministic finite automaton* (DFA) if for each rule  $pa \rightarrow q \in R$  it holds that  $R - \{pa \rightarrow q\}$  contains no rule with the left-hand side equal to  $pa$ .

# From FA to DFA in Essence 2/2

## 2) Gist: Removal of nondeterminism

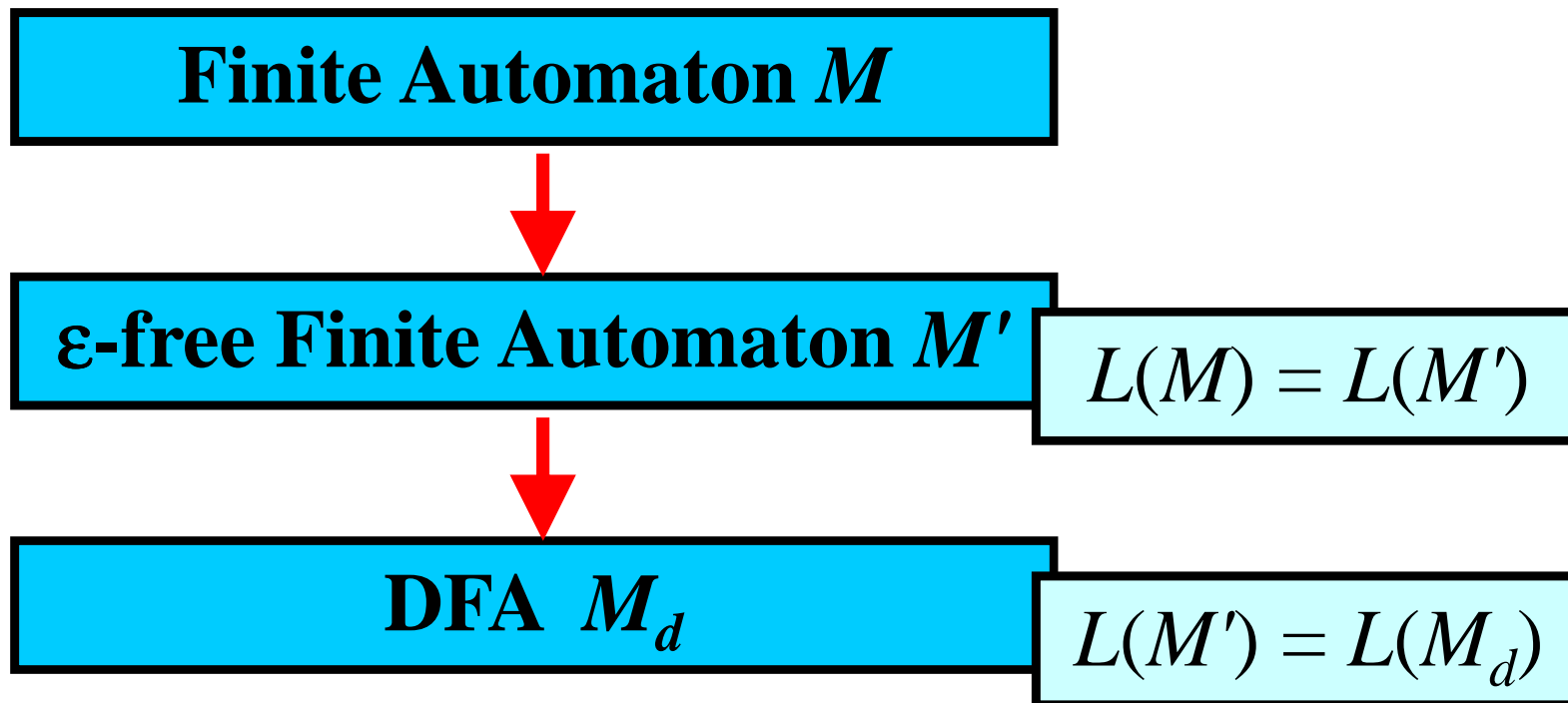


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# Theorem

- For every FA  $M$ , there is an equivalent DFA  $M_d$ .

**Proof** is based on these conversions:

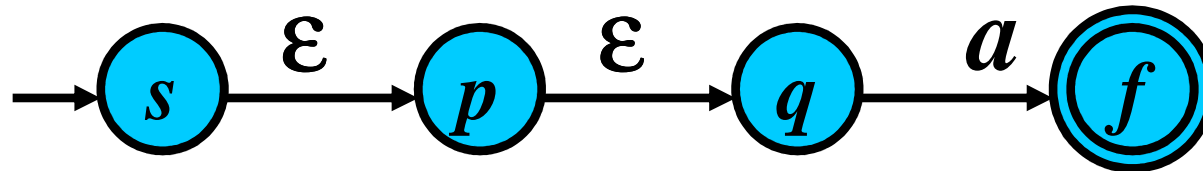


## $\epsilon$ -closure

**Gist:**  $q$  is in  $\epsilon$ -closure( $p$ ) if FA can reach  $q$  from  $p$  without reading.

**Definition:** For every states  $p \in Q$ , we define a set  $\epsilon$ -closure( $p$ ) as  $\epsilon$ -closure( $p$ ) =  $\{q: q \in Q, p \vdash^* q\}$

**Example:**

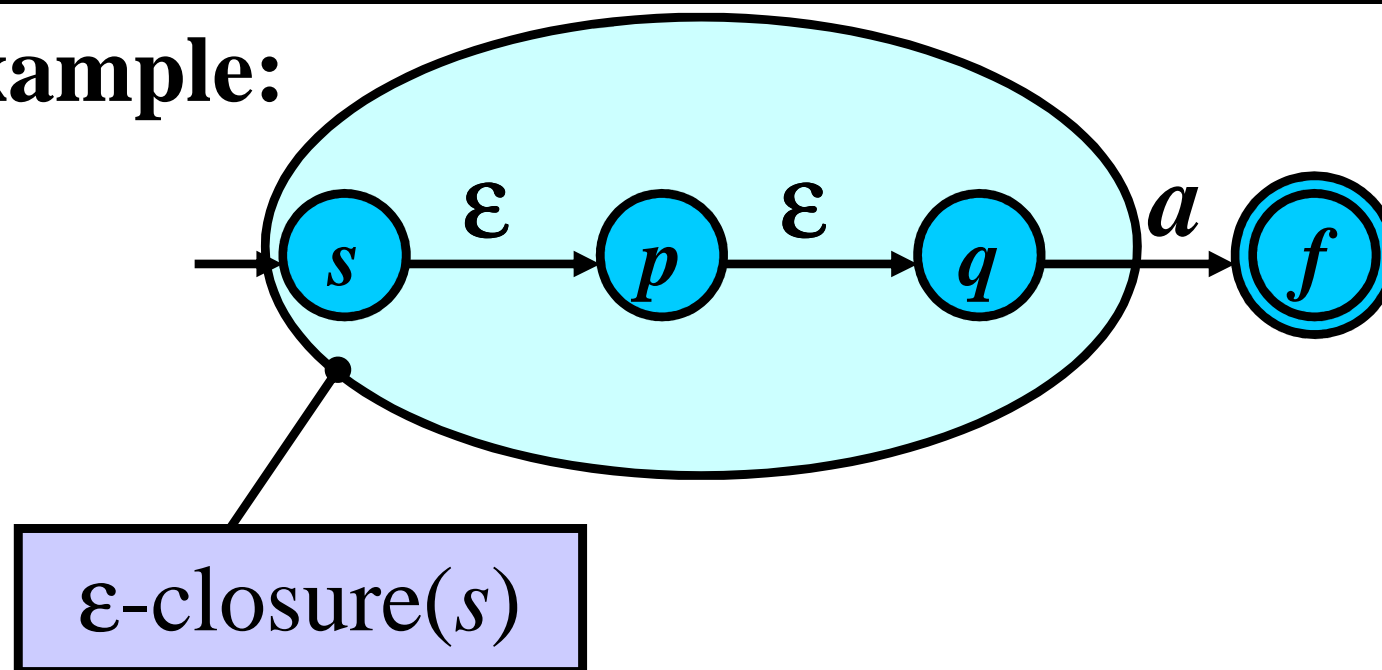


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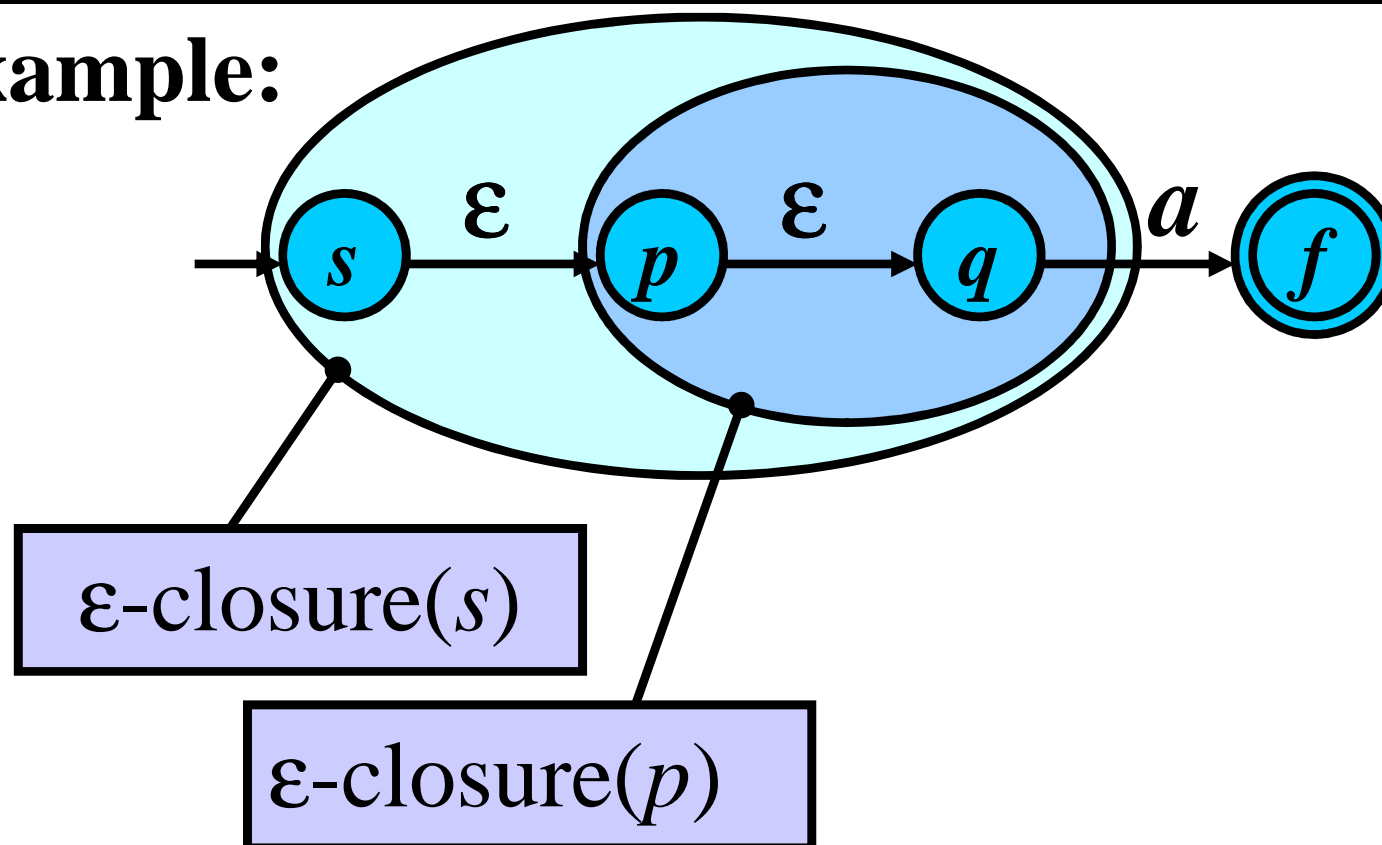


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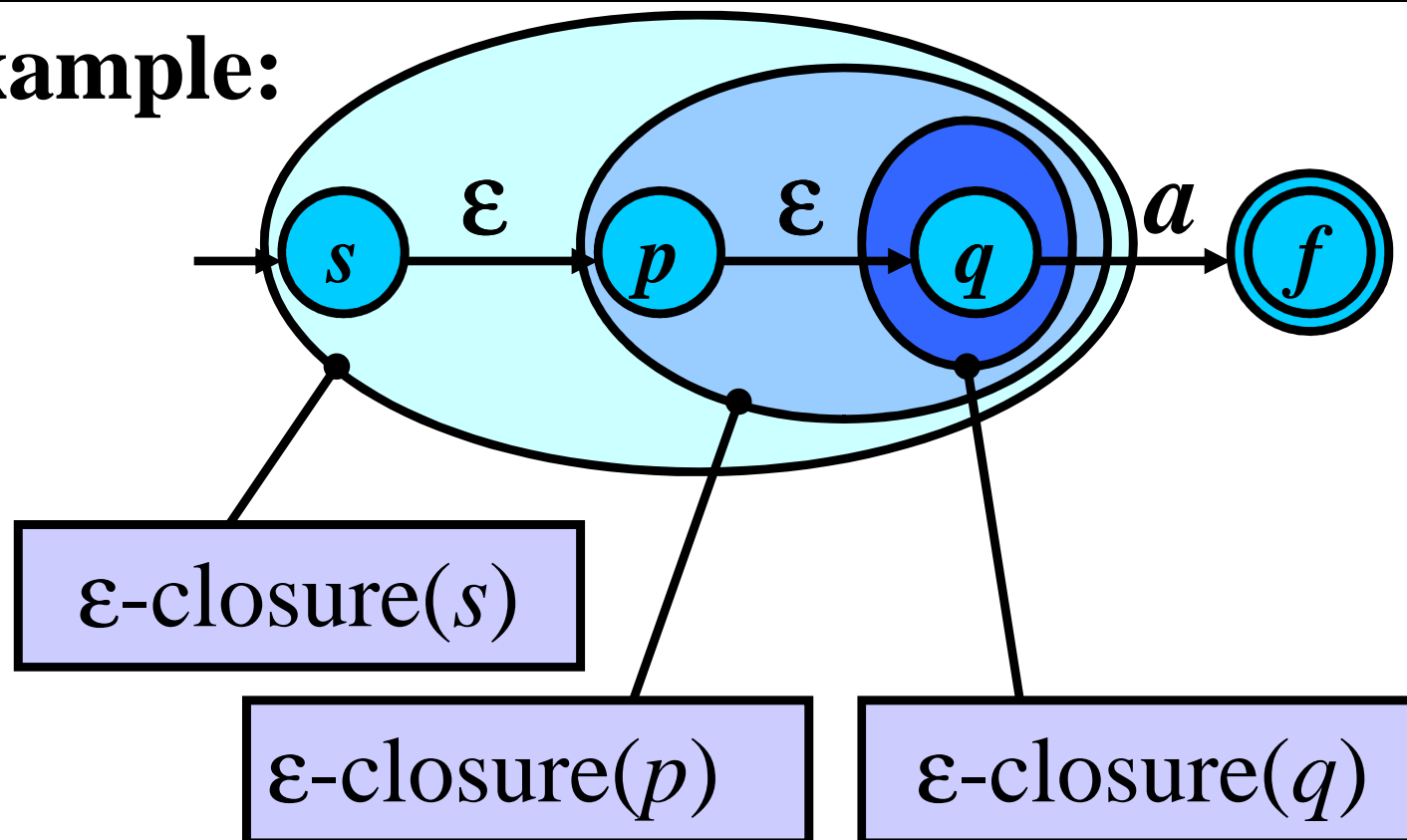


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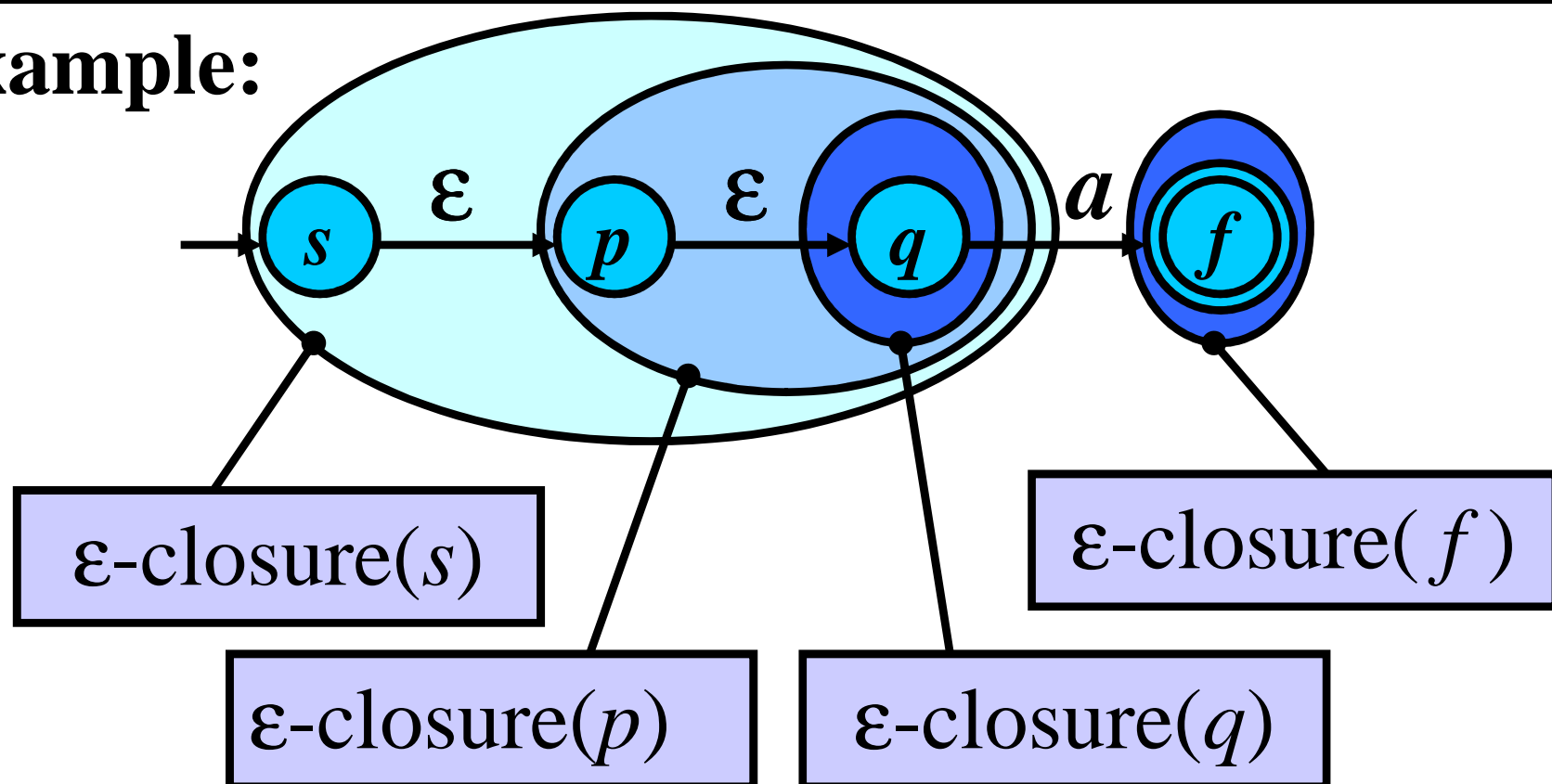


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**Example:**



## Algorithm: $\varepsilon$ -closure

- **Input:**  $M = (Q, \Sigma, R, s, F)$ ;  $p \in Q$
  - **Output:**  $\varepsilon\text{-closure}(p)$
- 

- **Method:**

- $i := 0$ ;  $Q_0 := \{p\}$ ;

- **repeat**

$i := i + 1$ ;

$Q_i := Q_{i-1} \cup \{ p' : p' \in Q, q \rightarrow p' \in R, \\ q \in Q_{i-1} \}$ ;

**until**  $Q_i = Q_{i-1}$ ;

- $\varepsilon\text{-closure}(p) := Q_i$ .

## $\epsilon$ -closure: Example

$M = (Q, \Sigma, R, s, F)$ , where:  $Q = \{s, p, q, f\}$ ,  $\Sigma = \{a\}$ ,  
 $R = \{s \rightarrow p, p \rightarrow q, qa \rightarrow f\}$ ,  $F = \{f\}$

**Task:**  $\epsilon$ -closure( $s$ )

---

## $\epsilon$ -closure: Example

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---

$Q_0 = \{s\}$

---

## ε-closure: Example

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**Task:**  $\varepsilon\text{-closure}(s)$

---


$$Q_0 = \{\textcolor{red}{s}\}$$


---

$$1) \quad \textcolor{red}{s} \rightarrow p'; p' \in Q: \quad \textcolor{red}{s} \rightarrow \textcolor{blue}{p}$$

$$Q_1 = \{\textcolor{red}{s}\} \cup \{\textcolor{blue}{p}\} = \{\textcolor{red}{s}, \textcolor{red}{p}\}$$


---

# $\varepsilon$ -closure: Example

$M = (Q, \Sigma, R, s, F)$ , where:  $Q = \{s, p, q, f\}$ ,  $\Sigma = \{a\}$ ,  
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**Task:**  $\varepsilon$ -closure( $s$ )

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$$Q_0 = \{s\}$$


---

$$1) \quad s \rightarrow p'; p' \in Q: \quad s \rightarrow p$$

$$Q_1 = \{s\} \cup \{p\} = \{s, p\}$$


---

$$2) \quad \begin{array}{l} s \rightarrow p'; p' \in Q: \quad s \rightarrow p \\ p \rightarrow p'; p' \in Q: \quad p \rightarrow q \end{array}$$

$$Q_2 = \{s, p\} \cup \{p, q\} = \{s, p, q\}$$


---

# $\epsilon$ -closure: Example

$M = (Q, \Sigma, R, s, F)$ , where:  $Q = \{s, p, q, f\}$ ,  $\Sigma = \{a\}$ ,  
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$$2) \quad \begin{array}{ll} s \rightarrow p'; p' \in Q: & s \rightarrow p \\ p \rightarrow p'; p' \in Q: & p \rightarrow q \end{array}$$

$$Q_2 = \{s, p\} \cup \{p, q\} = \{s, p, q\}$$


---

$$3) \quad \begin{array}{ll} s \rightarrow p'; p' \in Q: & s \rightarrow p \\ p \rightarrow p'; p' \in Q: & p \rightarrow q \\ q \rightarrow p'; p' \in Q: & \text{none} \end{array}$$

$$Q_3 = \{s, p, q\} \cup \{p, q\} = \{s, p, q\} = Q_2 = \epsilon\text{-closure}(s)$$


---



# Algorithm: FA to $\varepsilon$ -free FA

## Gist: Skip all $\varepsilon$ -moves

- **Input:** FA  $M = (Q, \Sigma, R, s, F)$
- **Output:**  $\varepsilon$ -free FA  $M' = (Q, \Sigma, R', s, F')$

### • Method:

- $R' := \emptyset$ ;
- **for all**  $p \in Q$  **do**
  - $R' := R' \cup \{ pa \rightarrow q : p'a \rightarrow q \in R, a \in \Sigma, p' \in \varepsilon\text{-closure}(p), q \in Q \}$ ;
- $F' := \{ p : p \in Q, \varepsilon\text{-closure}(p) \cap F \neq \emptyset \}$ .

# Algorithm: FA to $\varepsilon$ -free FA

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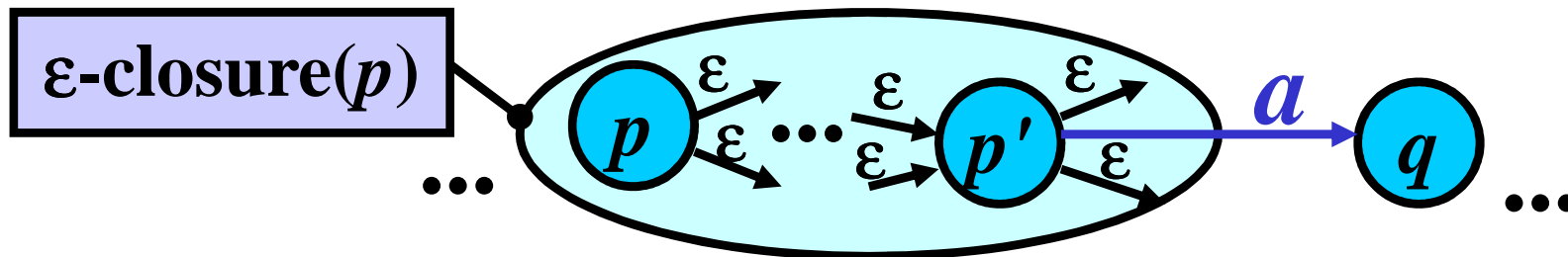
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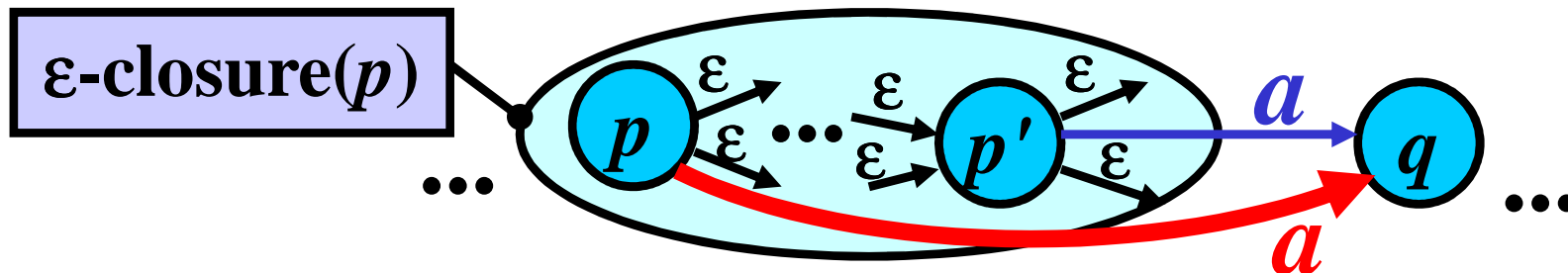
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## FA to $\varepsilon$ -free FA: Example 1/3

$M = (Q, \Sigma, R, s, F)$ , where:

$Q = \{s, q_1, q_2, f\}; \Sigma = \{a, b, c\};$

$R = \{sa \rightarrow s, s \rightarrow q_1, q_1b \rightarrow q_1, q_1b \rightarrow f, s \rightarrow q_2,$   
 $q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\}; F = \{f\}$

---

# FA to $\varepsilon$ -free FA: Example 1/3

$M = (Q, \Sigma, R, s, F)$ , where:

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 $q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\}; F = \{f\}$

---

1) for  $p = s$ :  $\varepsilon\text{-closure}(s) = \{s, q_1, q_2\}$

A.  $sd \rightarrow q', d \in \Sigma, q' \in Q: sa \rightarrow s$

B.  $q_1d \rightarrow q', d \in \Sigma, q' \in Q: q_1b \rightarrow q_1, q_1b \rightarrow f$

C.  $q_2d \rightarrow q', d \in \Sigma, q' \in Q: q_2c \rightarrow q_2, q_2c \rightarrow f$

$R' = \emptyset \cup \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f\}$

# FA to $\varepsilon$ -free FA: Example 2/3

## FA to $\varepsilon$ -free FA: Example 2/3

2) for  $p = q_1$ :  $\varepsilon$ -closure( $q_1$ ) =  $\{q_1\}$

A.  $q_1 d \rightarrow q'$ ;  $d \in \Sigma$ ;  $q' \in Q$ :  $q_1 b \rightarrow q_1$ ,  $q_1 b \rightarrow f$

$R' = R' \cup \{q_1 b \rightarrow q_1, q_1 b \rightarrow f\}$

---

## FA to $\varepsilon$ -free FA: Example 2/3

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$R' = R' \cup \{q_1 b \rightarrow q_1, q_1 b \rightarrow f\}$

---

3) for  $p = q_2$ :  $\varepsilon$ -closure( $q_2$ ) =  $\{q_2\}$

A.  $q_2 d \rightarrow q'$ ;  $d \in \Sigma$ ;  $q' \in Q$ :  $q_2 c \rightarrow q_2$ ,  $q_2 c \rightarrow f$

$R' = R' \cup \{q_2 c \rightarrow q_2, q_2 c \rightarrow f\}$

---



## FA to $\varepsilon$ -free FA: Example 2/3

2) for  $p = q_1$ :  $\varepsilon\text{-closure}(q_1) = \{q_1\}$

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$R' = R' \cup \{q_2 c \rightarrow q_2, q_2 c \rightarrow f\}$

---

4) for  $p = f$ :  $\varepsilon\text{-closure}(f) = \{f\}$

A.  $f d \rightarrow q'$ ;  $d \in \Sigma$ ;  $q' \in Q$ :  $f a \rightarrow f$

$R' = R' \cup \{f a \rightarrow f\}$

---

## FA to $\varepsilon$ -free FA: Example 2/3

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3) for  $p = q_2$ :  $\varepsilon\text{-closure}(q_2) = \{q_2\}$

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4) for  $p = f$ :  $\varepsilon\text{-closure}(f) = \{f\}$

A.  $f d \rightarrow q'$ ;  $d \in \Sigma$ ;  $q' \in Q$ :  $f a \rightarrow f$

$R' = R' \cup \{f a \rightarrow f\}$

---

$R' = \{s a \rightarrow s, s b \rightarrow q_1, s b \rightarrow f, s c \rightarrow q_2, s c \rightarrow f,$   
 $q_1 b \rightarrow q_1, q_1 b \rightarrow f, q_2 c \rightarrow q_2, q_2 c \rightarrow f, f a \rightarrow f\}$

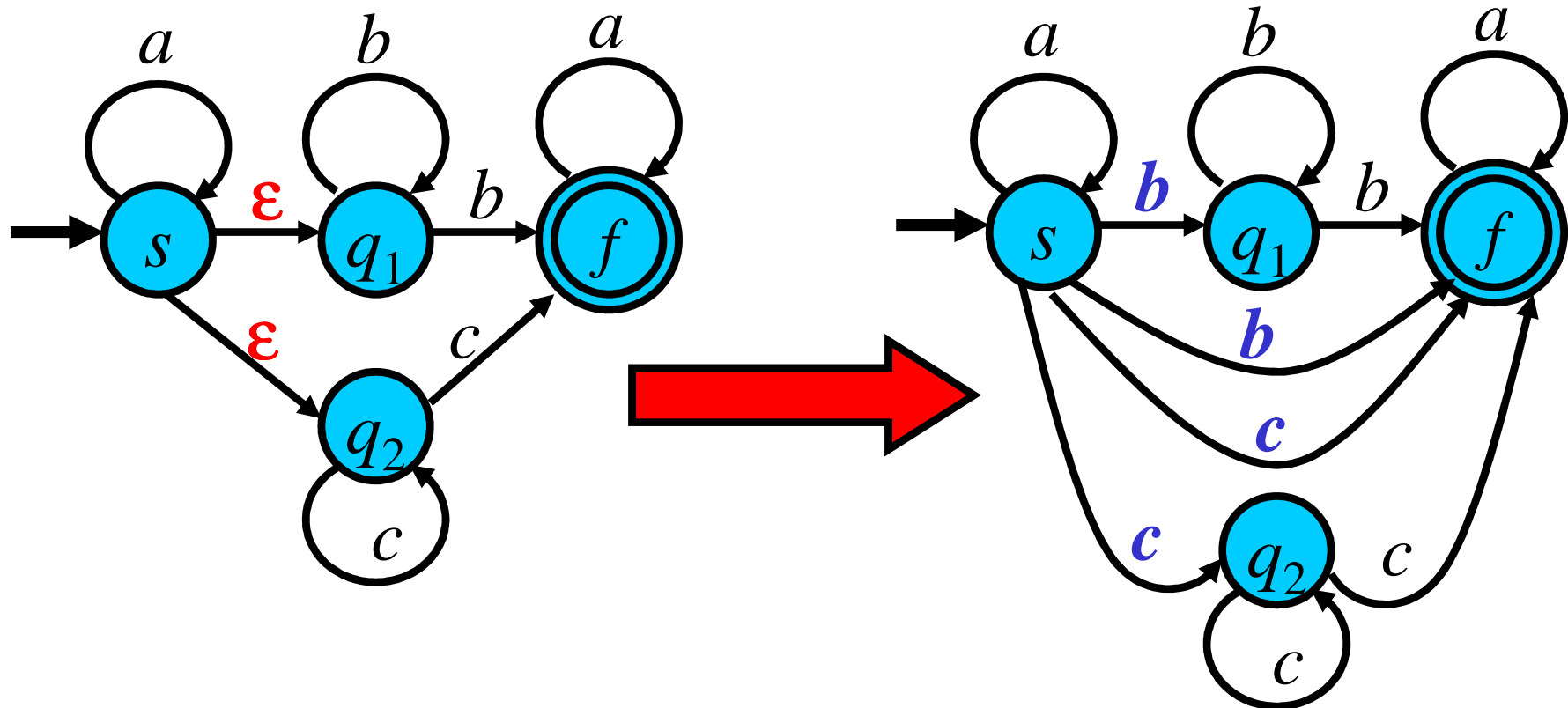
# FA to $\epsilon$ -free FA: Example 3/3

$$\begin{array}{lcl}
 \epsilon\text{-closure}(s) \cap F = \{s, q_1, q_2\} \cap \{f\} = \emptyset \\
 \epsilon\text{-closure}(q_1) \cap F = \{q_1\} \cap \{f\} = \emptyset \\
 \epsilon\text{-closure}(q_2) \cap F = \{q_2\} \cap \{f\} = \emptyset \\
 \epsilon\text{-closure}(f) \cap F = \{f\} \cap \{f\} = \{f\} \neq \emptyset
 \end{array}
 \left. \vphantom{\begin{array}{l} \epsilon\text{-closure}(s) \\ \epsilon\text{-closure}(q_1) \\ \epsilon\text{-closure}(q_2) \end{array}} \right\} F' = \{f\}$$


---

# FA to $\epsilon$ -free FA: Example 3/3

$$\begin{array}{lcl}
 \epsilon\text{-closure}(s) \cap F = \{s, q_1, q_2\} \cap \{f\} = \emptyset \\
 \epsilon\text{-closure}(q_1) \cap F = \{q_1\} \cap \{f\} = \emptyset \\
 \epsilon\text{-closure}(q_2) \cap F = \{q_2\} \cap \{f\} = \emptyset \\
 \epsilon\text{-closure}(f) \cap F = \{f\} \cap \{f\} = \{f\} \neq \emptyset
 \end{array}
 \left. \vphantom{\begin{array}{l} \epsilon\text{-closure}(s) \\ \epsilon\text{-closure}(q_1) \\ \epsilon\text{-closure}(q_2) \\ \epsilon\text{-closure}(f) \end{array}} \right\} F' = \{f\}$$



## Algorithm: $\epsilon$ -free FA to DFA 1/2

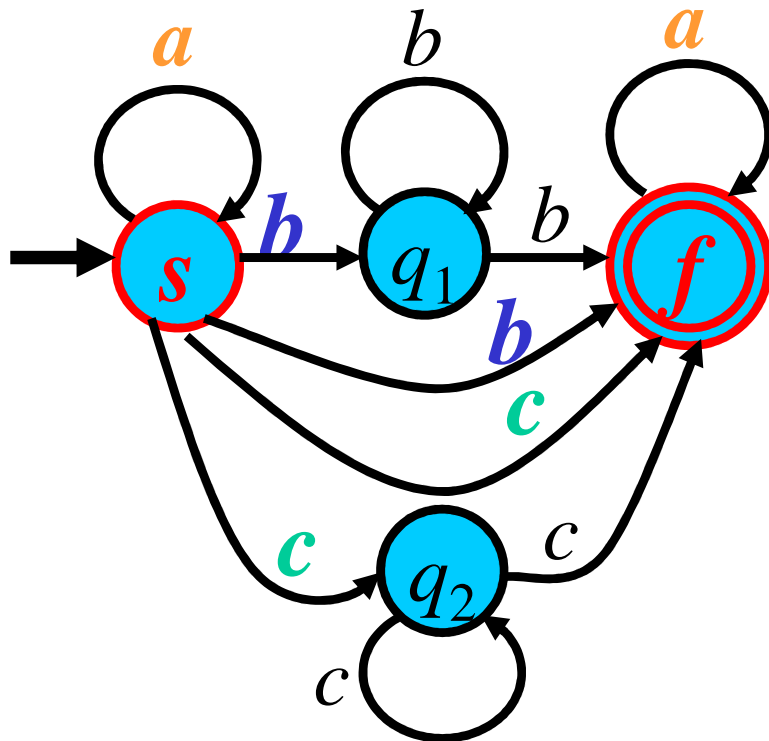
**Gist:** In DFA, make states from all subsets of states in  $\epsilon$ -free FA and move between them so that all possible states of  $\epsilon$ -free FA are simultaneously simulated.

---

# Algorithm: $\epsilon$ -free FA to DFA 1/2

**Gist:** In DFA, make states from all subsets of states in  $\epsilon$ -free FA and move between them so that all possible states of  $\epsilon$ -free FA are simultaneously simulated.

**Illustration:**

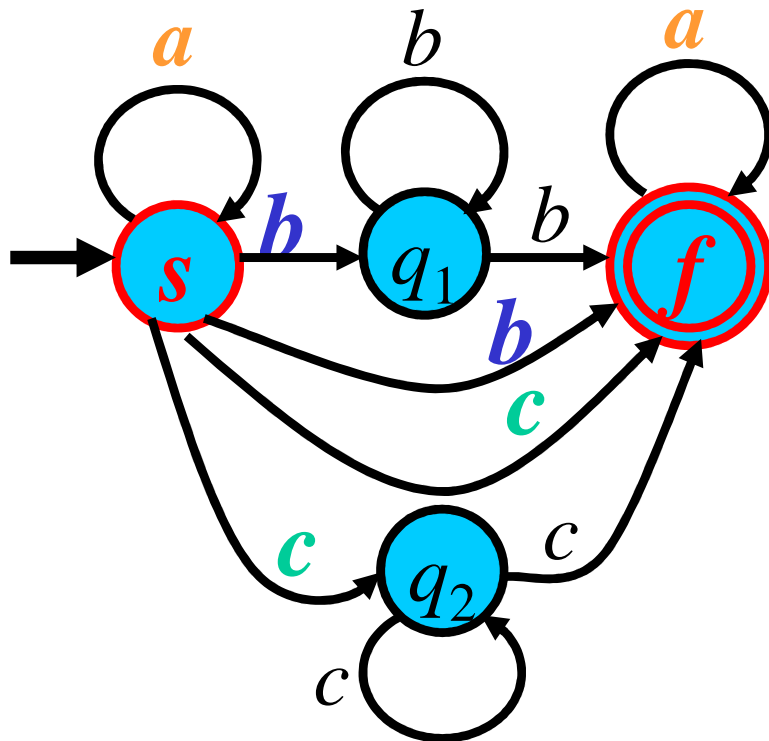


$$Q_{DFA} = \{\{s\}, \{q_1\}, \{q_2\}, \{f\}, \{s, q_1\}, \{s, q_2\}, \{s, f\}, \{q_1, q_2\}, \{q_1, f\}, \{q_2, f\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_2, f\}, \{q_1, q_2, f\}, \{s, q_1, q_2, f\}\}$$

# Algorithm: $\epsilon$ -free FA to DFA 1/2

**Gist:** In DFA, make states from all subsets of states in  $\epsilon$ -free FA and move between them so that all possible states of  $\epsilon$ -free FA are simultaneously simulated.

**Illustration:**



$Q_{DFA} = \{\{s\}, \{q_1\}, \{q_2\}, \{f\}, \{s, q_1\}, \{s, q_2\}, \{s, f\}, \{q_1, q_2\}, \{q_1, f\}, \{q_2, f\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_2, f\}, \{q_1, q_2, f\}, \{s, q_1, q_2, f\}\}$

For state  $\{s\}$ : ...

⋮

For state  $\{s, f\}$ :  $\{s, f\}$

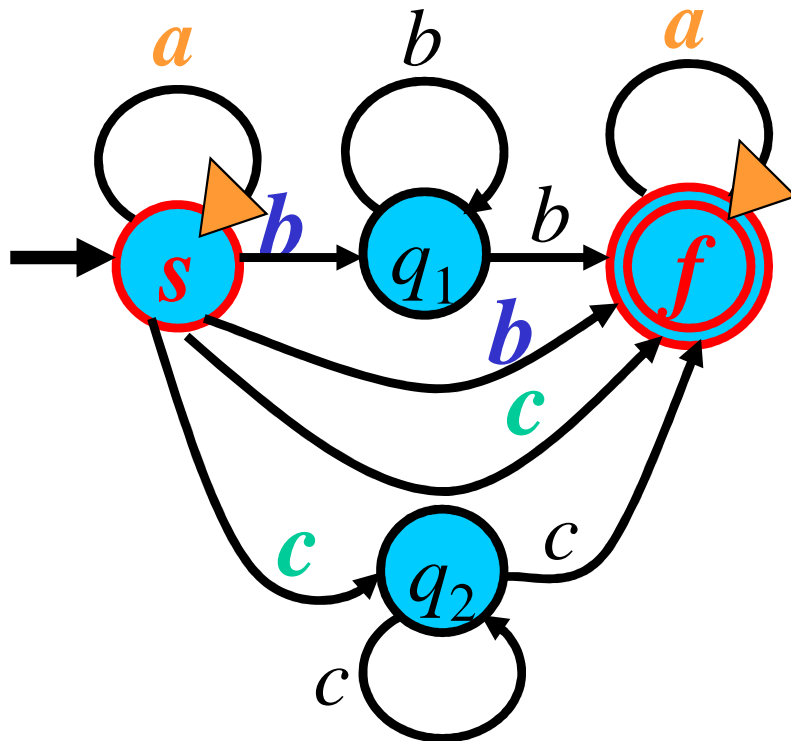
⋮

For state  $\{s, q_1, q_2, f\}$ : ...

# Algorithm: $\epsilon$ -free FA to DFA 1/2

**Gist:** In DFA, make states from all subsets of states in  $\epsilon$ -free FA and move between them so that all possible states of  $\epsilon$ -free FA are simultaneously simulated.

**Illustration:**



$$Q_{DFA} = \{\{s\}, \{q_1\}, \{q_2\}, \{f\}, \{s, q_1\}, \{s, q_2\}, \{s, f\}, \{q_1, q_2\}, \{q_1, f\}, \{q_2, f\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_2, f\}, \{q_1, q_2, f\}, \{s, q_1, q_2, f\}\}$$

For state  $\{s\}$ : ...

⋮

For state  $\{s, f\}$ :

⋮

For state  $\{s, q_1, q_2, f\}$ : ...

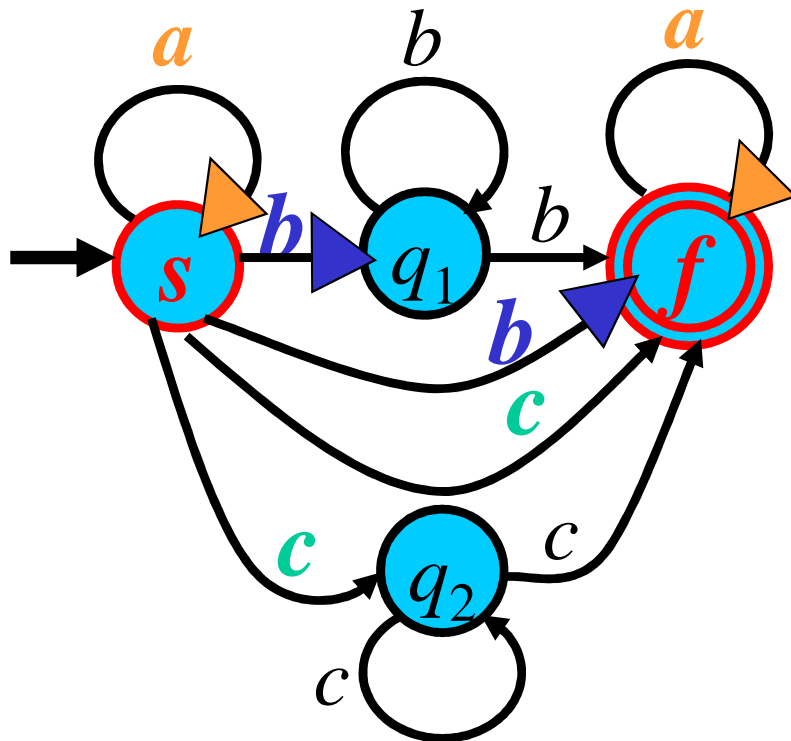




# Algorithm: $\epsilon$ -free FA to DFA 1/2

**Gist:** In DFA, make states from all subsets of states in  $\epsilon$ -free FA and move between them so that all possible states of  $\epsilon$ -free FA are simultaneously simulated.

**Illustration:**



$Q_{DFA} = \{\{s\}, \{q_1\}, \{q_2\}, \{f\}, \{s, q_1\}, \{s, q_2\}, \{s, f\}, \{q_1, q_2\}, \{q_1, f\}, \{q_2, f\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_2, f\}, \{q_1, q_2, f\}, \{s, q_1, q_2, f\}\}$

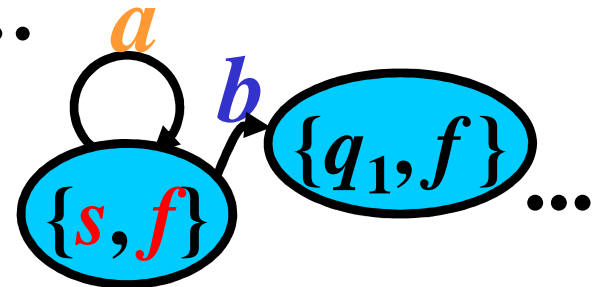
For state  $\{s\}$ : ...

⋮

For state  $\{s, f\}$ :

⋮

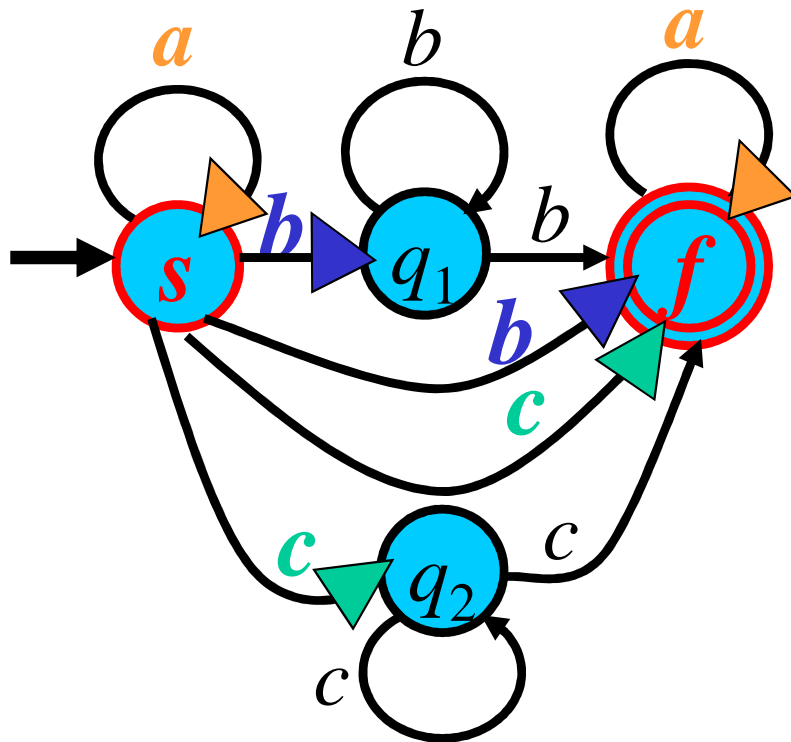
For state  $\{s, q_1, q_2, f\}$ : ...



# Algorithm: $\epsilon$ -free FA to DFA 1/2

**Gist:** In DFA, make states from all subsets of states in  $\epsilon$ -free FA and move between them so that all possible states of  $\epsilon$ -free FA are simultaneously simulated.

**Illustration:**



$Q_{DFA} = \{\{s\}, \{q_1\}, \{q_2\}, \{f\}, \{s, q_1\}, \{s, q_2\}, \{s, f\}, \{q_1, q_2\}, \{q_1, f\}, \{q_2, f\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_2, f\}, \{q_1, q_2, f\}, \{s, q_1, q_2, f\}\}$

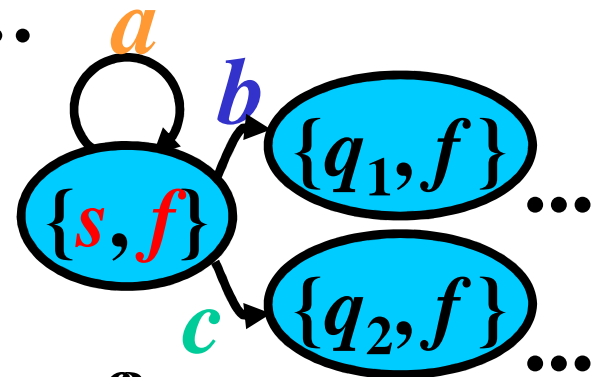
For state  $\{s\}$ : ...

⋮

For state  $\{s, f\}$ :

⋮

For state  $\{s, q_1, q_2, f\}$ : ...



# Algorithm: $\varepsilon$ -free FA to DFA 2/2

- **Input:**  $\varepsilon$ -free FA:  $M = (Q, \Sigma, R, s, F)$
  - **Output:** DFA:  $M_d = (Q_d, \Sigma, R_d, s_d, F_d)$
- 
- **Method:**
  - $Q_d := \{Q' : Q' \subseteq Q, Q' \neq \emptyset\}; R_d := \emptyset;$
  - **for each**  $Q' \in Q_d$ , **and**  $a \in \Sigma$  **do begin**
    - $Q'' := \{q : p \in Q', pa \rightarrow q \in R\};$
    - if**  $Q'' \neq \emptyset$  **then**  $R_d := R_d \cup \{Q'a \rightarrow Q''\};$
  - end**
  - $s_d := \{s\};$
  - $F_d := \{F' : F' \in Q_d, F' \cap F \neq \emptyset\}.$

# $\epsilon$ -free FA to DFA: Example 1/5

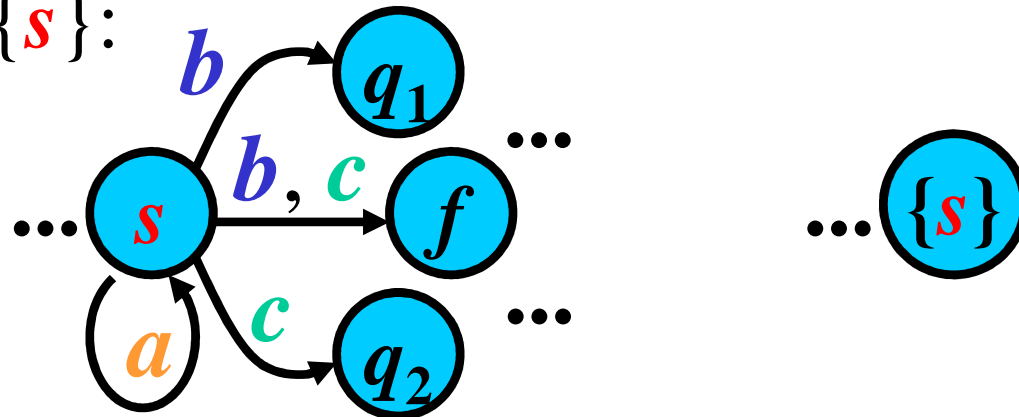
$M = (Q, \Sigma, R, s, F)$ , where:

$Q = \{s, q_1, q_2, f\}; \Sigma = \{a, b, c\}; F = \{f\}$

$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$   
 $q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

$Q_d = \{\{s\}, \{s, q_1\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_1, q_2, f\}, \{s, q_2\}, \{s, q_2, f\},$   
 $\{s, f\}, \{q_1\}, \{q_1, q_2\}, \{q_1, f\}, \{q_1, q_2, f\}, \{q_2\}, \{q_2, f\}, \{f\}\}$

for  $Q' = \{s\}$ :



# $\epsilon$ -free FA to DFA: Example 1/5

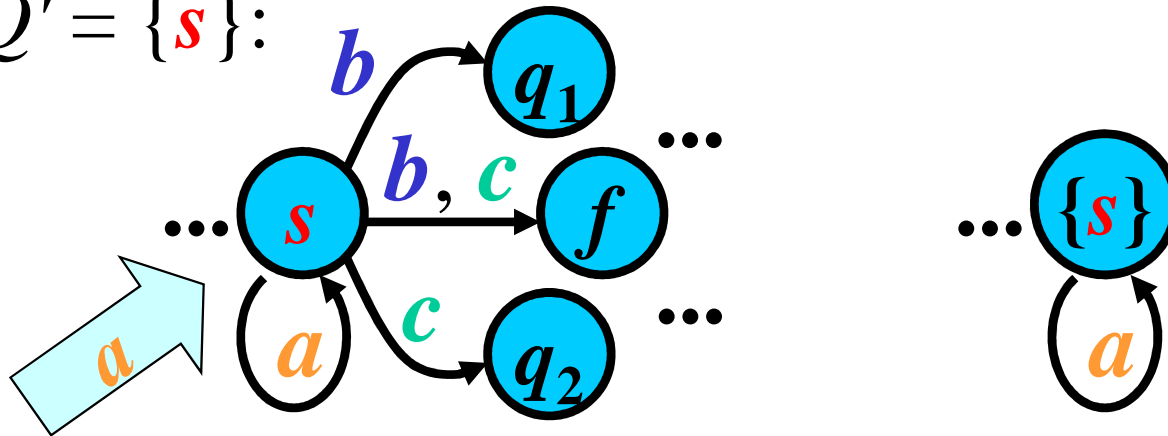
$M = (Q, \Sigma, R, s, F)$ , where:

$Q = \{s, q_1, q_2, f\}; \Sigma = \{a, b, c\}; F = \{f\}$

$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$   
 $q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

$Q_d = \{\{s\}, \{s, q_1\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_1, q_2, f\}, \{s, q_2\}, \{s, q_2, f\},$   
 $\{s, f\}, \{q_1\}, \{q_1, q_2\}, \{q_1, f\}, \{q_1, q_2, f\}, \{q_2\}, \{q_2, f\}, \{f\}\}$

for  $Q' = \{s\}$ :



# $\epsilon$ -free FA to DFA: Example 1/5

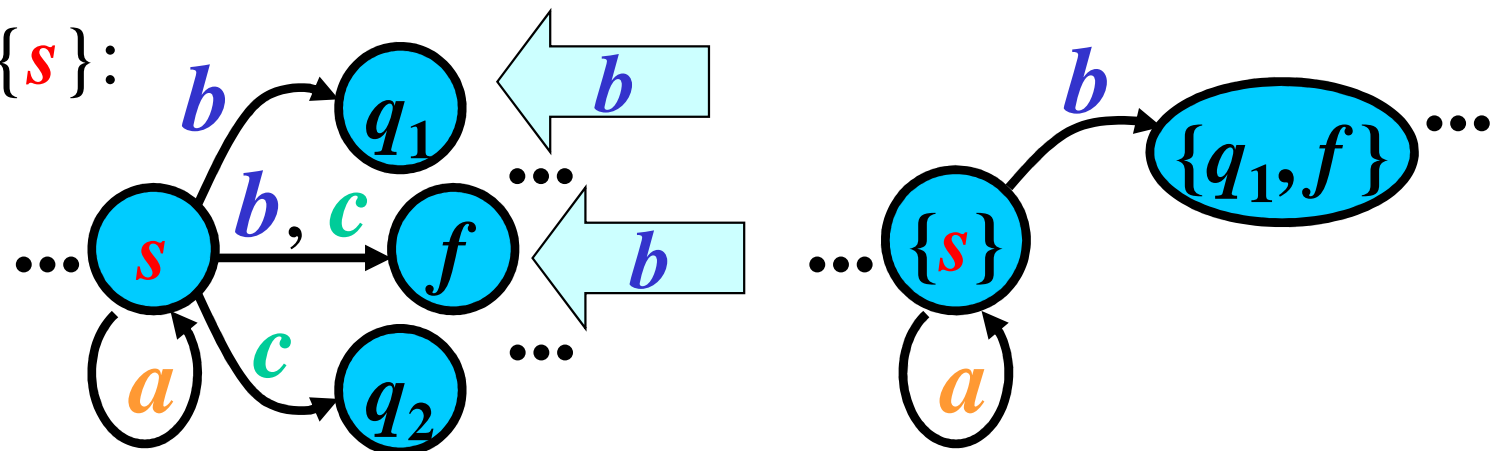
$M = (Q, \Sigma, R, s, F)$ , where:

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$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f, \\ q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

$Q_d = \{\{s\}, \{s, q_1\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_1, q_2, f\}, \{s, q_2\}, \{s, q_2, f\}, \\ \{s, f\}, \{q_1\}, \{q_1, q_2\}, \{q_1, f\}, \{q_1, q_2, f\}, \{q_2\}, \{q_2, f\}, \{f\}\}$

for  $Q' = \{s\}$ :



# $\epsilon$ -free FA to DFA: Example 1/5

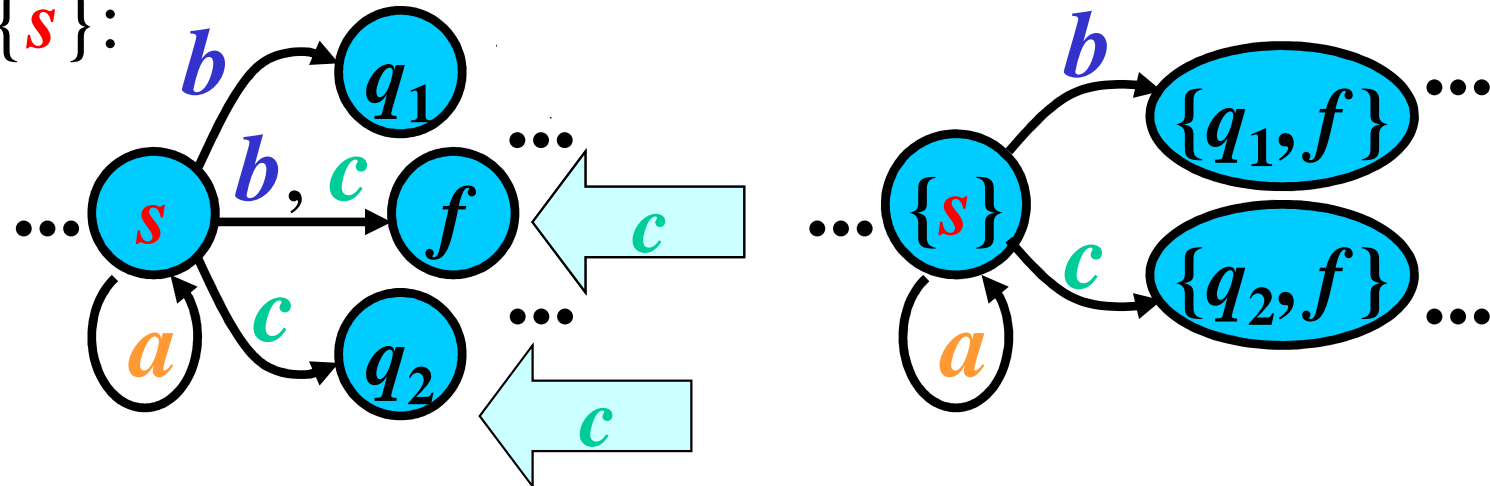
$M = (Q, \Sigma, R, s, F)$ , where:

$Q = \{s, q_1, q_2, f\}; \Sigma = \{a, b, c\}; F = \{f\}$

$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$   
 $q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

$Q_d = \{\{s\}, \{s, q_1\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_1, q_2, f\}, \{s, q_2\}, \{s, q_2, f\},$   
 $\{s, f\}, \{q_1\}, \{q_1, q_2\}, \{q_1, f\}, \{q_1, q_2, f\}, \{q_2\}, \{q_2, f\}, \{f\}\}$

for  $Q' = \{s\}$ :



# $\epsilon$ -free FA to DFA: Example 1/5

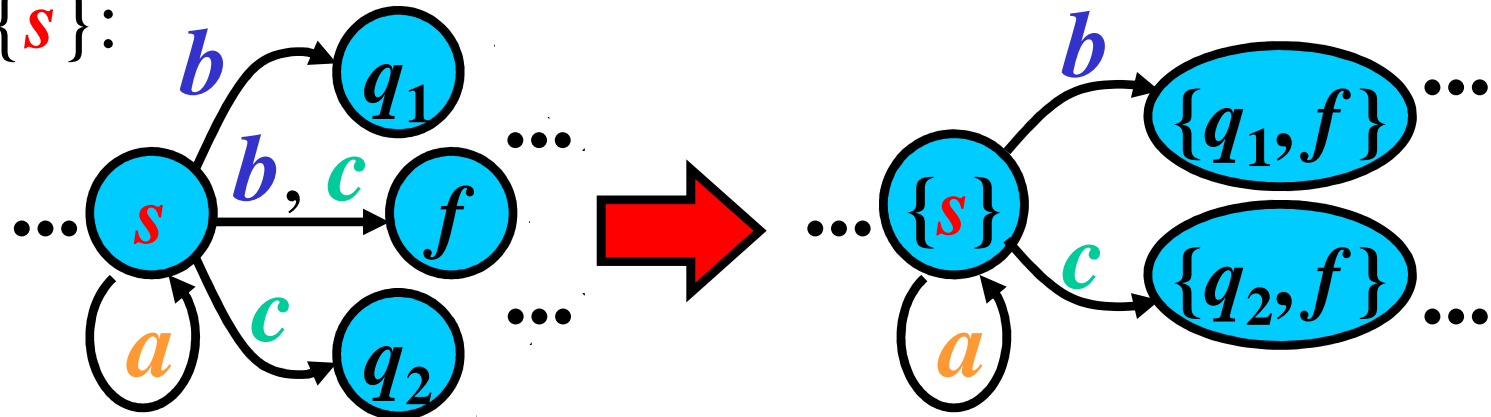
$M = (Q, \Sigma, R, s, F)$ , where:

$$Q = \{s, q_1, q_2, f\}; \Sigma = \{a, b, c\}; F = \{f\}$$

$$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f, \\ q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$$

$$Q_d = \{\{s\}, \{s, q_1\}, \{s, q_1, q_2\}, \{s, q_1, f\}, \{s, q_1, q_2, f\}, \{s, q_2\}, \{s, q_2, f\}, \\ \{s, f\}, \{q_1\}, \{q_1, q_2\}, \{q_1, f\}, \{q_1, q_2, f\}, \{q_2\}, \{q_2, f\}, \{f\}\}$$

for  $Q' = \{s\}$ :

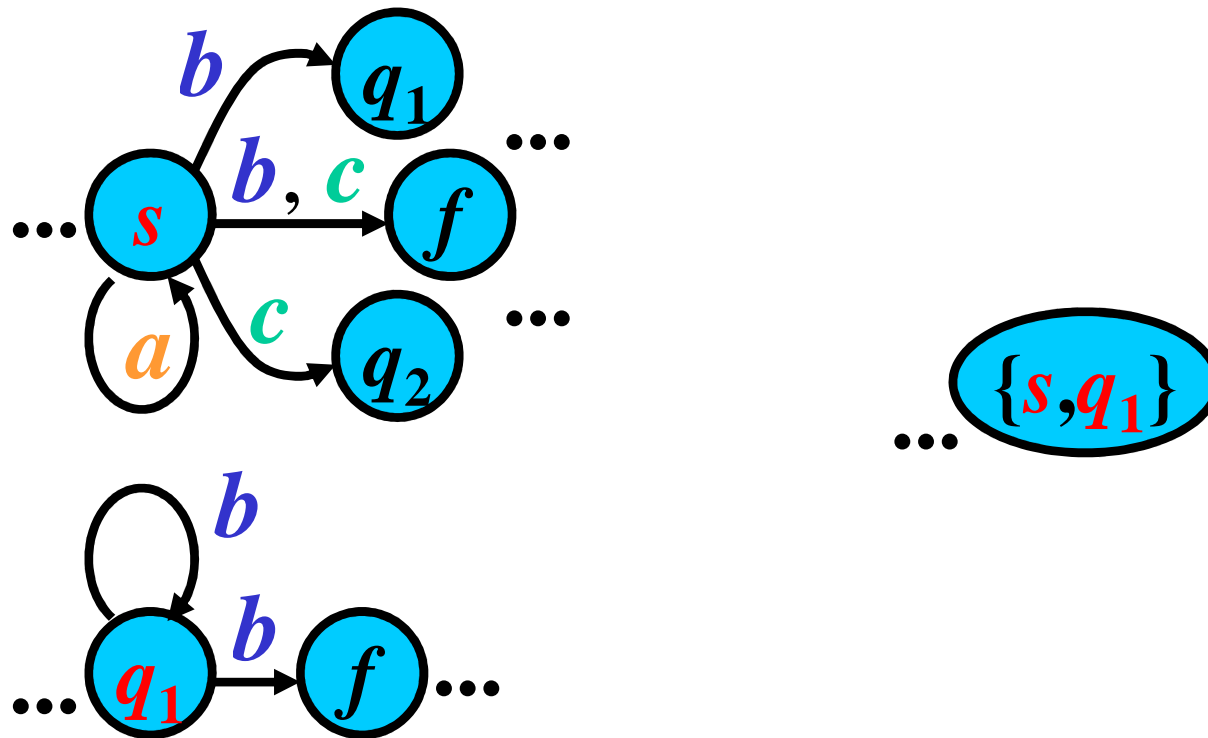


$$R_d = \emptyset \cup \{\{s\}a \rightarrow \{s\}, \{s\}b \rightarrow \{q_1, f\}, \{s\}c \rightarrow \{q_2, f\}\}$$



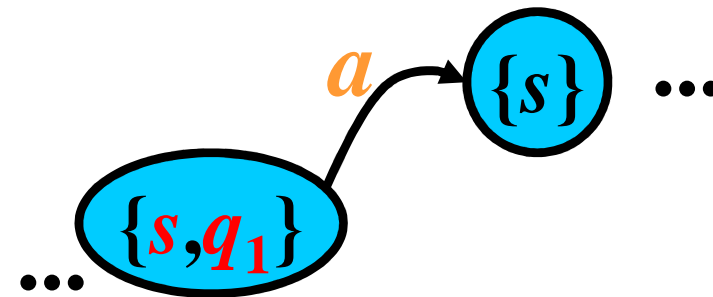
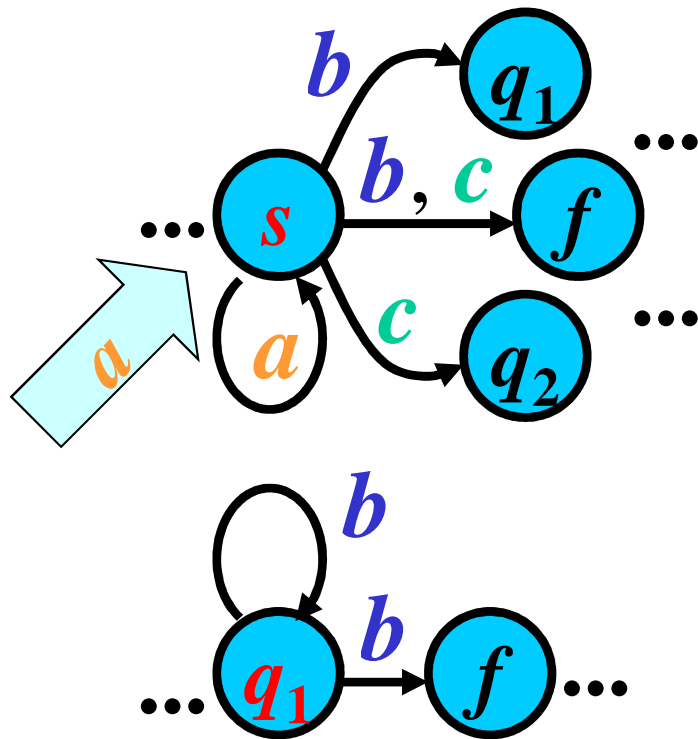
# $\epsilon$ -free FA to DFA: Example 2/5

for  $Q' = \{s, q_1\}$ :



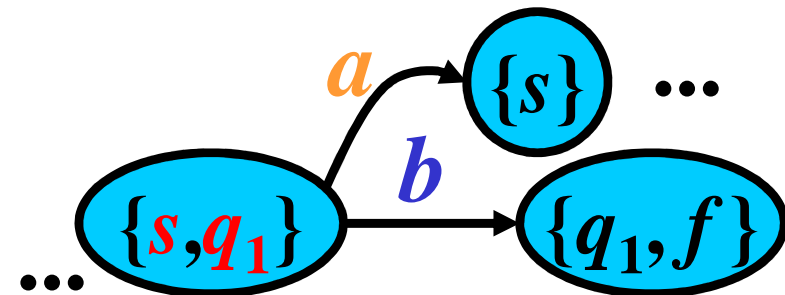
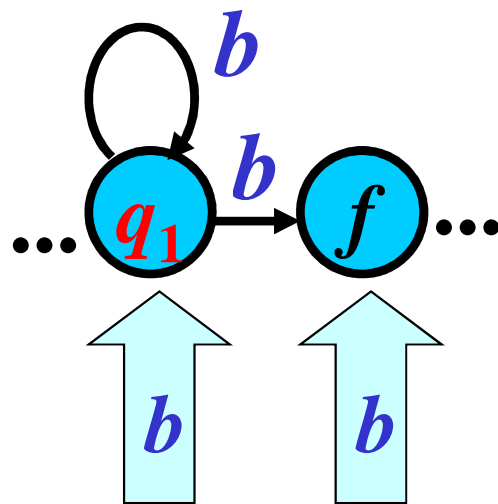
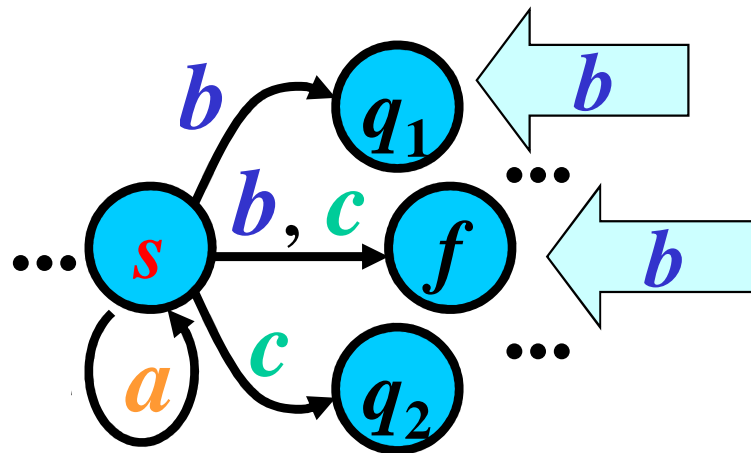
# $\epsilon$ -free FA to DFA: Example 2/5

for  $Q' = \{s, q_1\}$ :



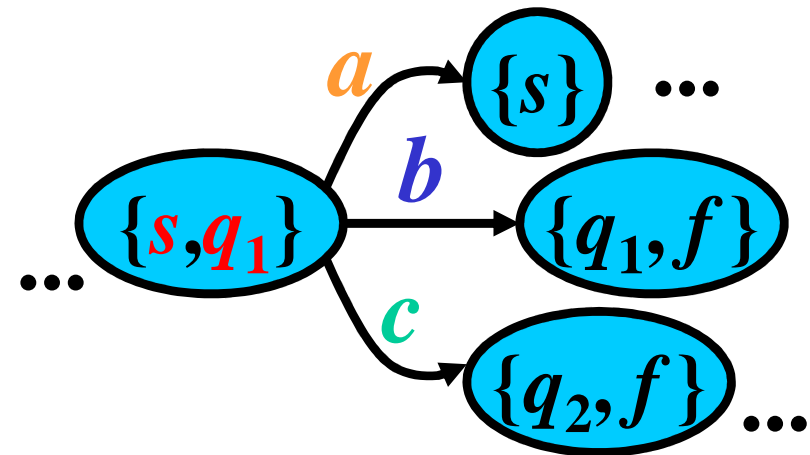
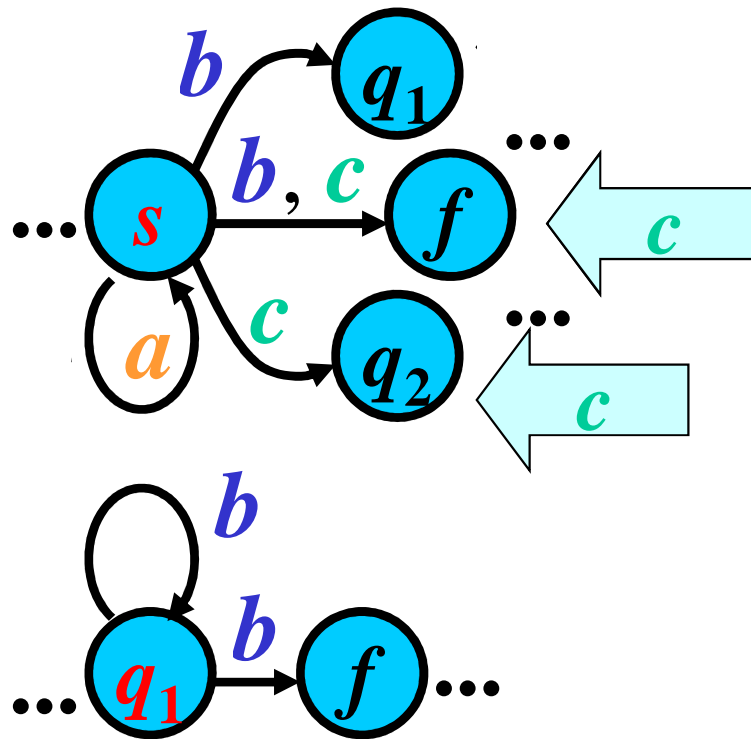
# $\epsilon$ -free FA to DFA: Example 2/5

for  $Q' = \{s, q_1\}$ :



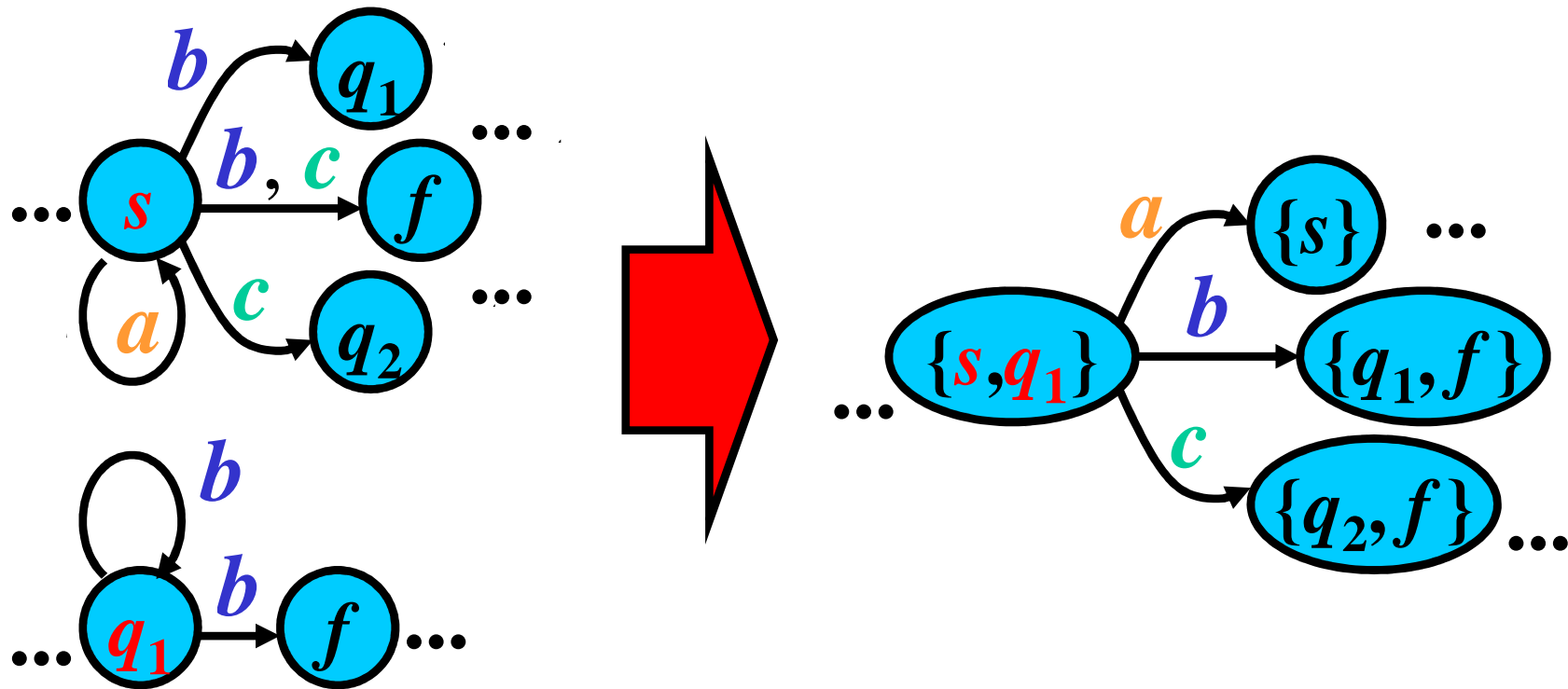
# $\epsilon$ -free FA to DFA: Example 2/5

for  $Q' = \{s, q_1\}$ :



# $\epsilon$ -free FA to DFA: Example 2/5

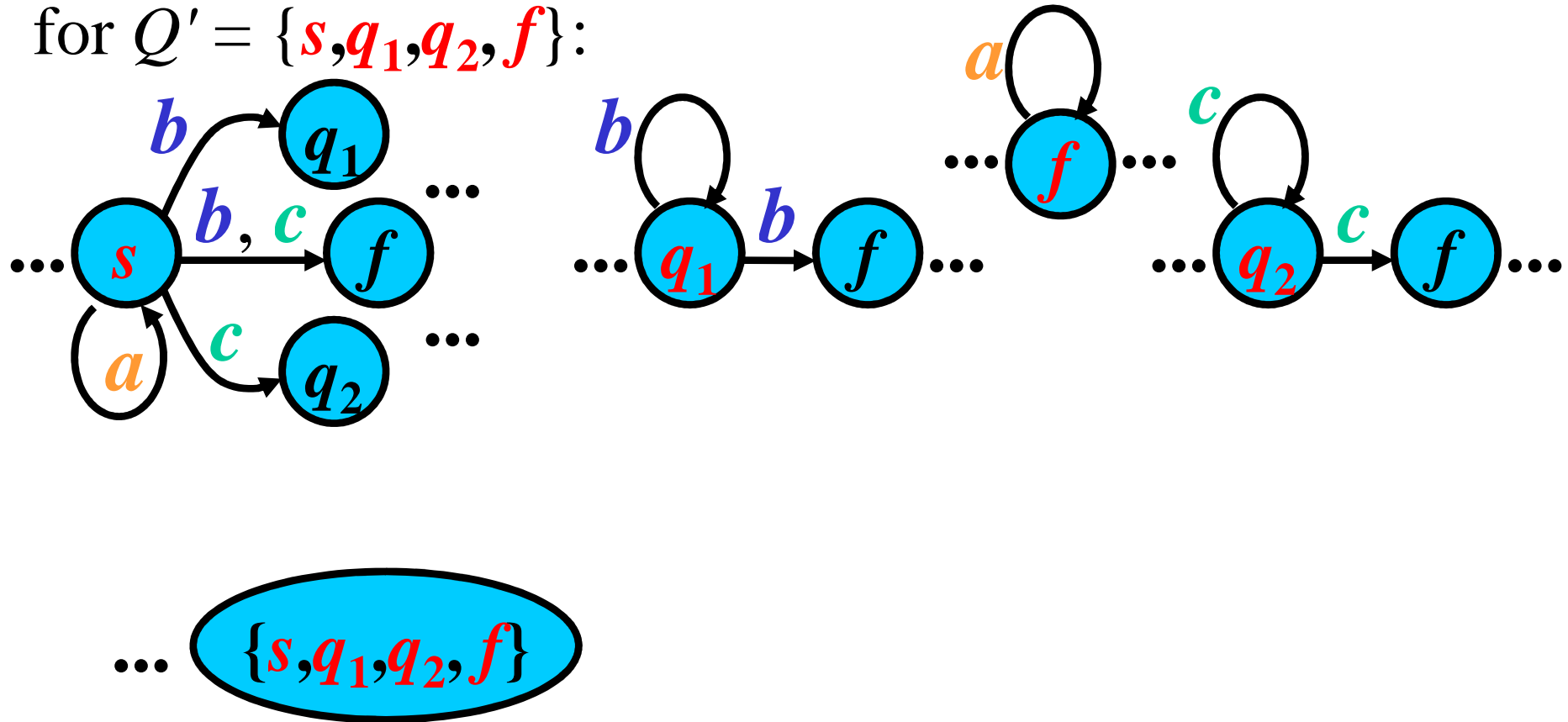
for  $Q' = \{s, q_1\}$ :



$$R_d = R_d \cup \{ \{s, q_1\} a \rightarrow \{s\}, \{s, q_1\} b \rightarrow \{q_1, f\}, \{s, q_1\} c \rightarrow \{q_2, f\} \}$$

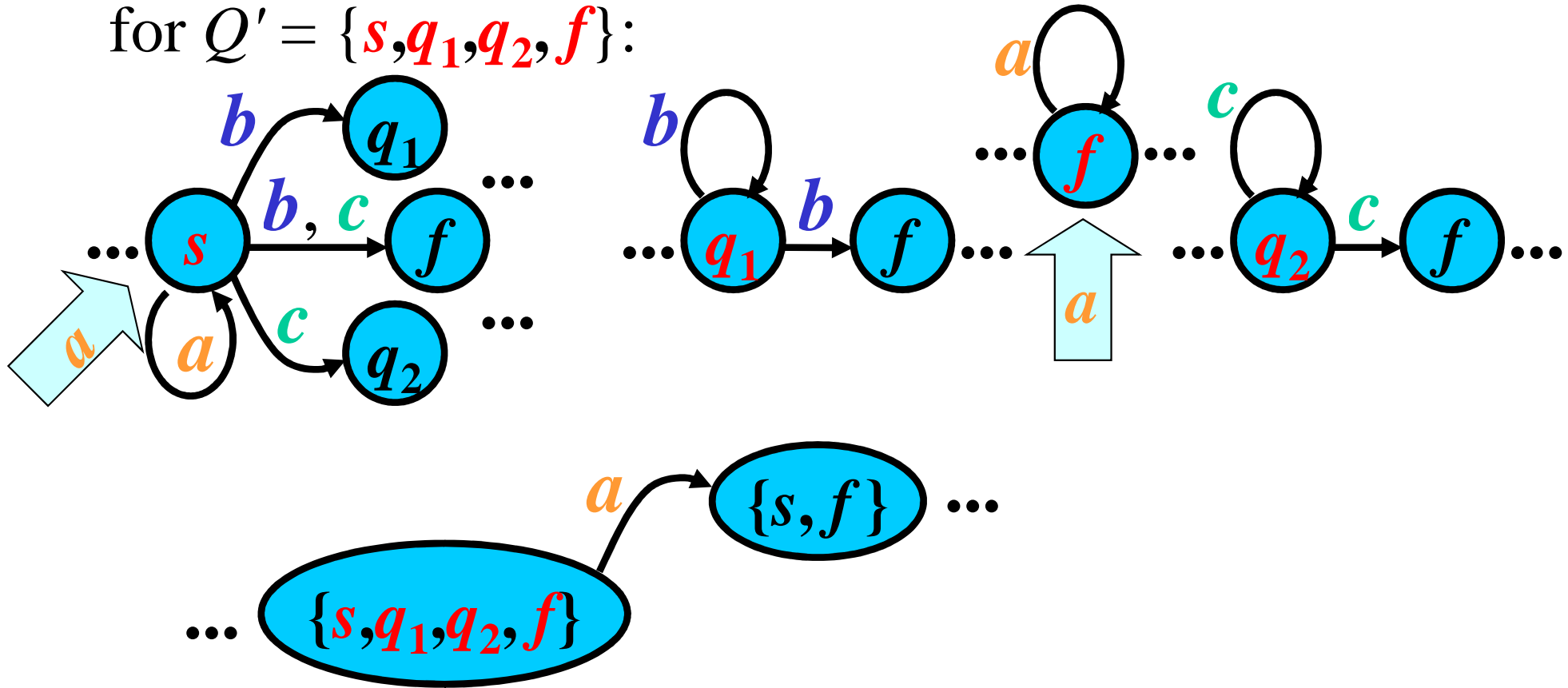
# $\epsilon$ -free FA to DFA: Example 3/5

for  $Q' = \{s, q_1, q_2, f\}$ :



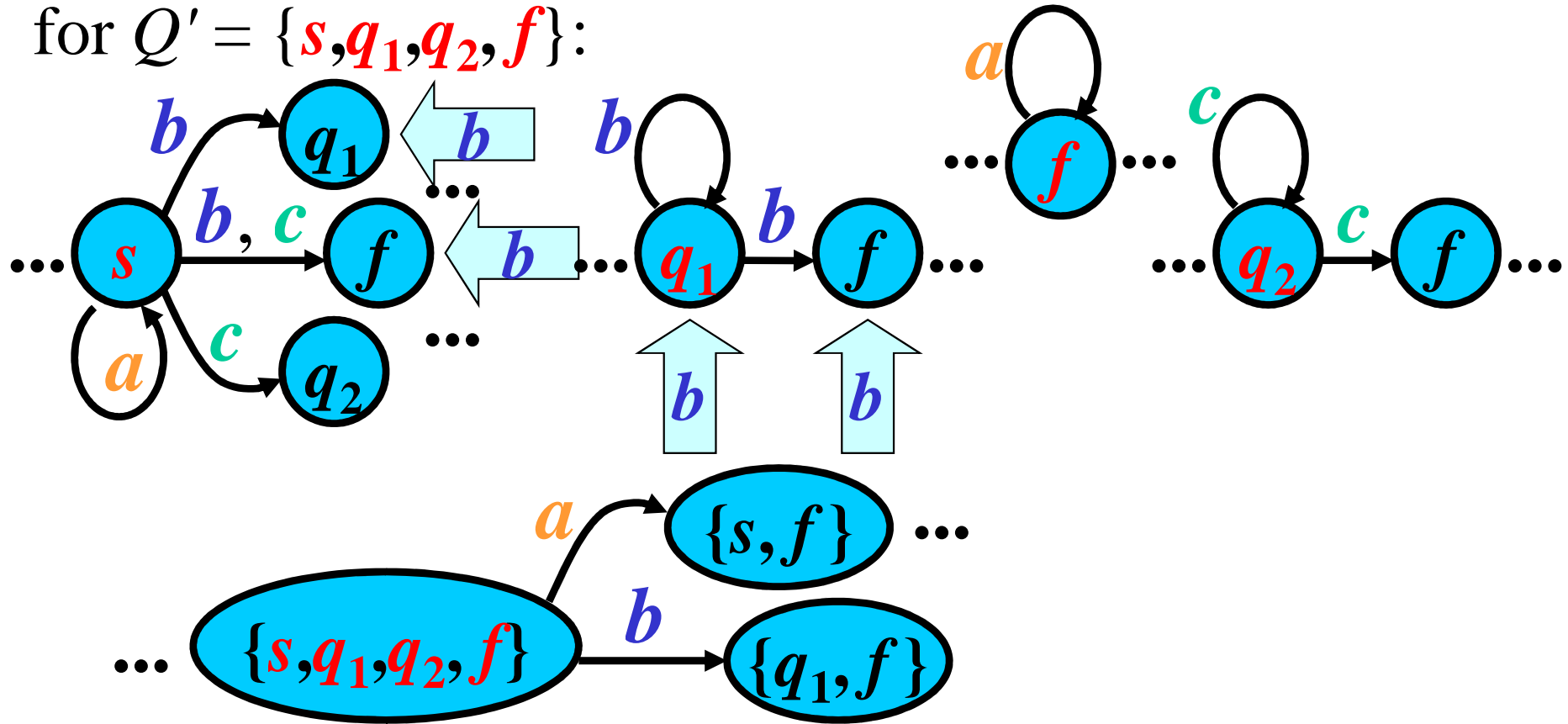
# $\epsilon$ -free FA to DFA: Example 3/5

for  $Q' = \{s, q_1, q_2, f\}$ :



# $\epsilon$ -free FA to DFA: Example 3/5

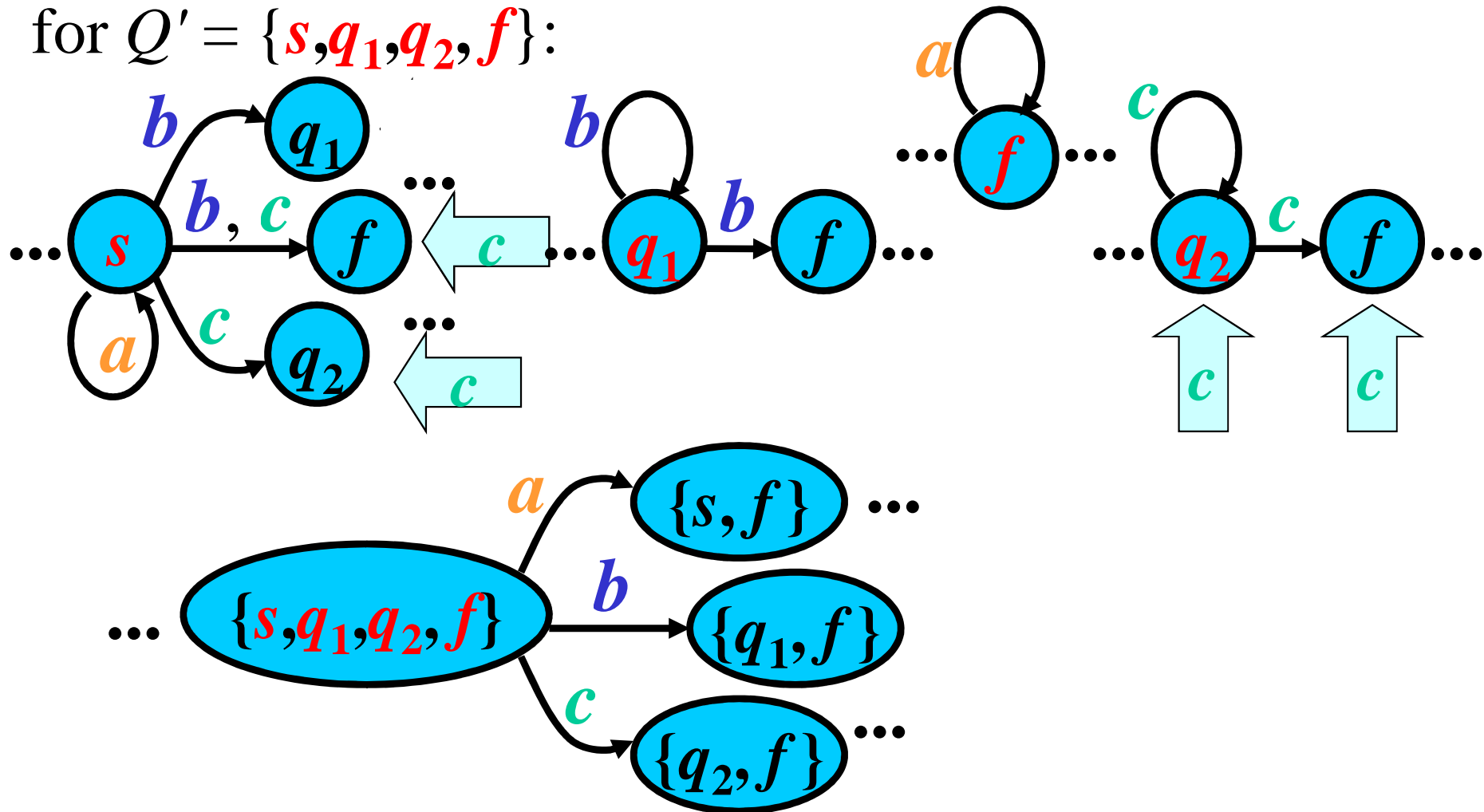
for  $Q' = \{s, q_1, q_2, f\}$ :





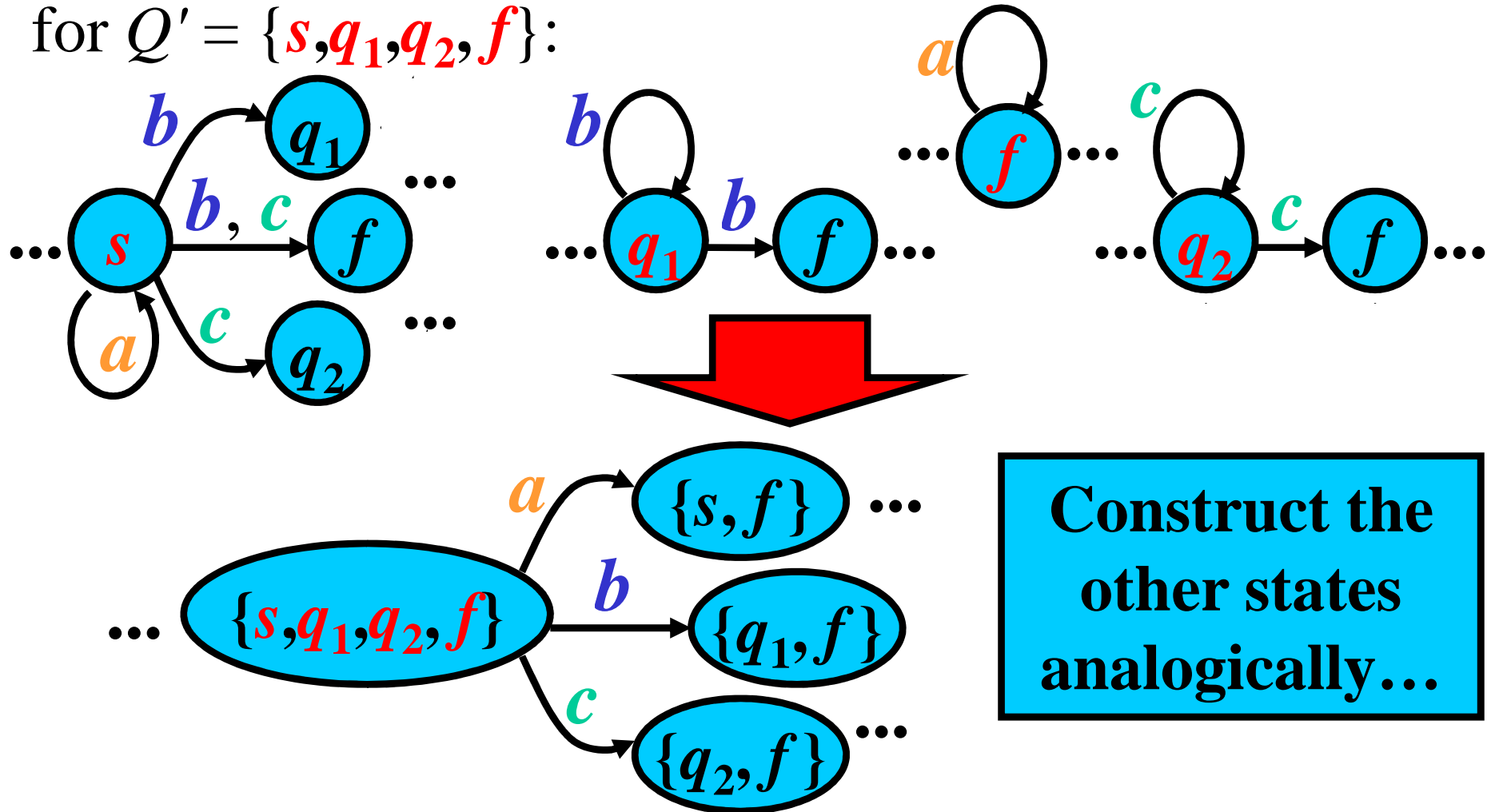
# $\epsilon$ -free FA to DFA: Example 3/5

for  $Q' = \{s, q_1, q_2, f\}$ :



# $\epsilon$ -free FA to DFA: Example 3/5

for  $Q' = \{s, q_1, q_2, f\}$ :



$$R_d = R_d \cup \{ \{s, q_1, q_2, f\} a \rightarrow \{s, f\}, \{s, q_1, q_2, f\} b \rightarrow \{q_1, f\}, \\ \{s, q_1, q_2, f\} c \rightarrow \{q_2, f\} \}$$

# $\varepsilon$ -free FA to DFA: Example 4/5

**Final states:**  $F_d := \{F': F' \in Q_d, F' \cap F \neq \emptyset\}$   
 for  $F = \{f\}$ :

$$\{s\} \cap \{f\} = \emptyset \quad \Rightarrow \quad \{s\} \notin F_d$$

$$\{s, q_1\} \cap \{f\} = \emptyset \quad \Rightarrow \quad \{s, q_1\} \notin F_d$$

$$\{s, q_1, q_2\} \cap \{f\} = \emptyset \quad \Rightarrow \quad \{s, q_1, q_2\} \notin F_d$$

$$\{s, q_1, f\} \cap \{f\} = \{f\} \neq \emptyset \quad \Rightarrow \quad \{s, q_1, f\} \in F_d$$

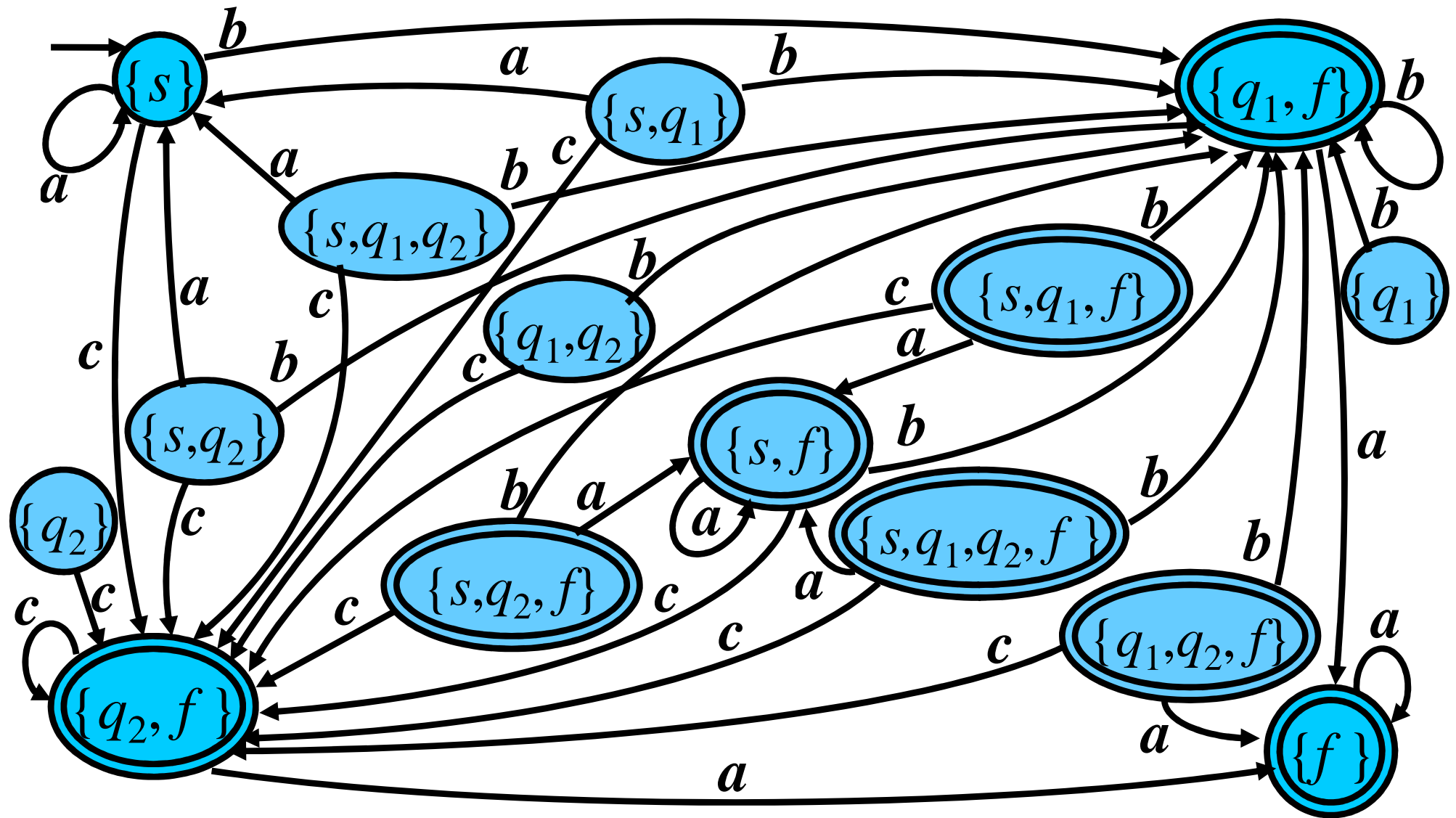
$$\{s, q_1, q_2, f\} \cap \{f\} = \{f\} \neq \emptyset \quad \Rightarrow \quad \{s, q_1, q_2, f\} \in F_d$$

⋮

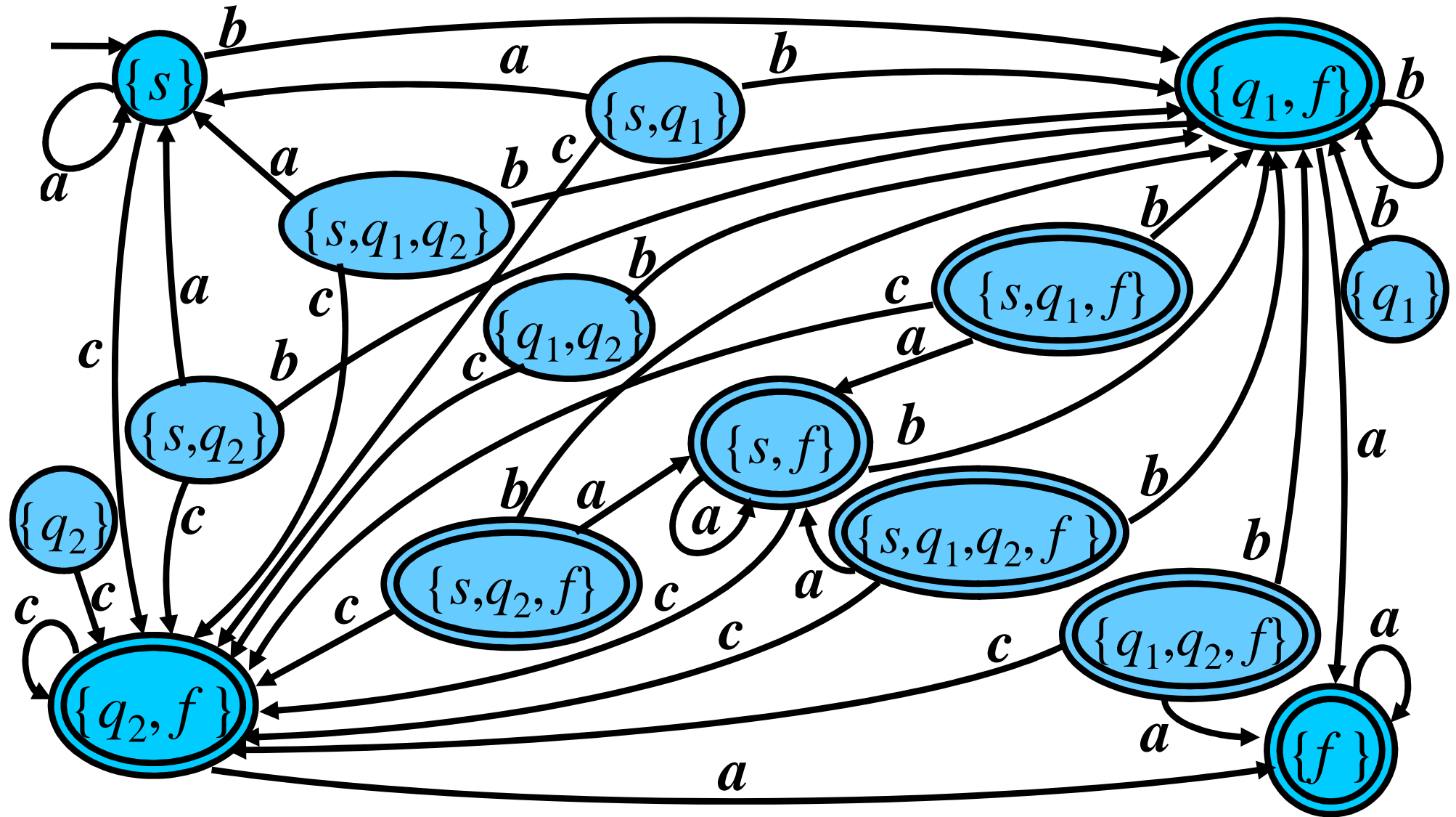
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$$F_d = \{\{s, q_1, f\}, \{s, q_1, q_2, f\}, \{s, q_2, f\}, \{s, f\}, \\ \{q_1, f\}, \{q_1, q_2, f\}, \{q_2, f\}, \{f\}\}$$

# $\epsilon$ -free FA to DFA: Example 5/5

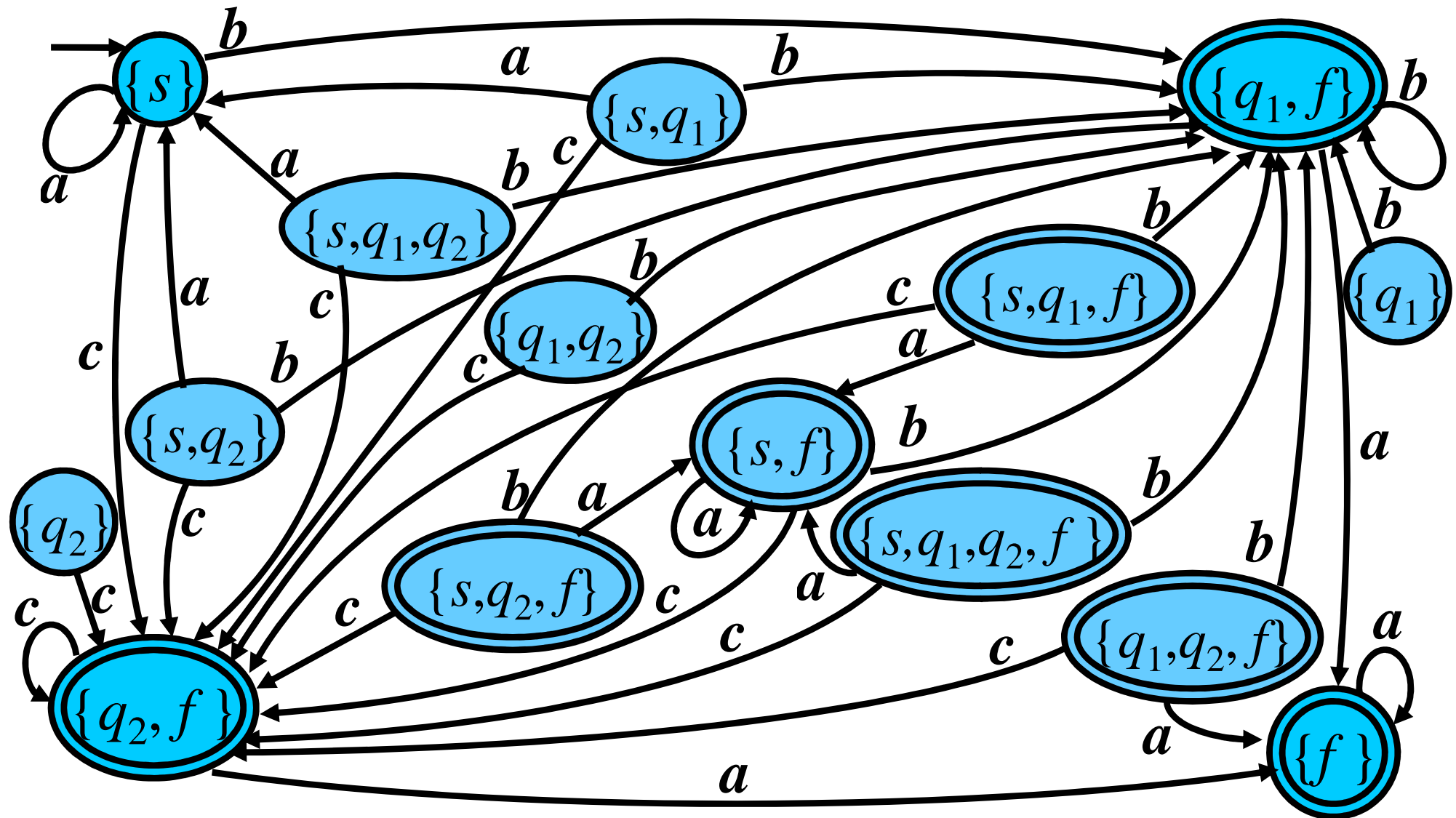


# $\epsilon$ -free FA to DFA: Example 5/5



**Question:** Can we make DFA smaller?

# $\epsilon$ -free FA to DFA: Example 5/5



**Question:** Can we make DFA smaller?

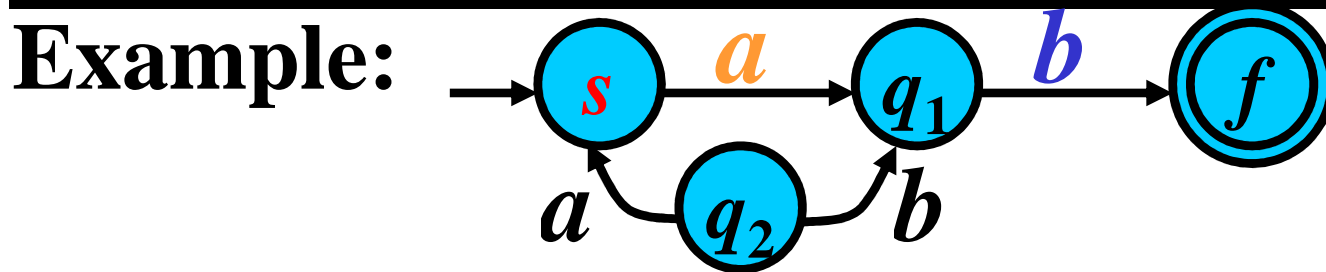
**Answer:** **YES**

# Accessible States

**Gist:** State  $q$  is *accessible* if a string takes DFA from  $s$  (the start state) to  $q$ .

**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be an FA. A state  $q \in Q$  is *accessible* if there exists  $w \in \Sigma^*$  such that  $sw \vdash^* q$ ; otherwise,  $q$  is *inaccessible*.

**Note:** Each inaccessible state can be removed from FA

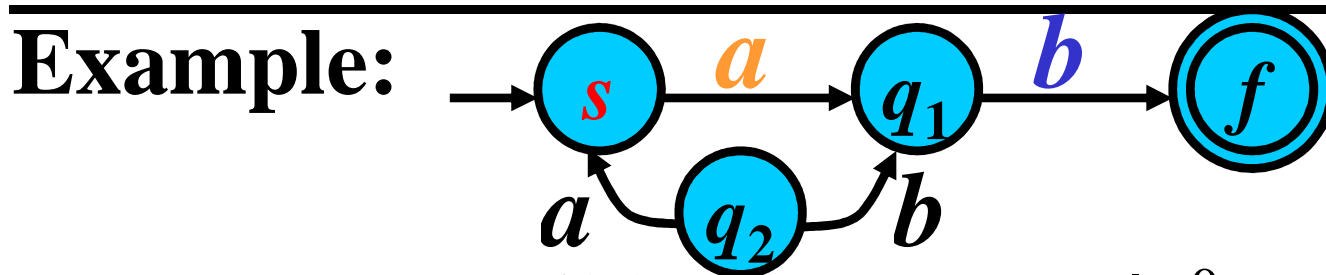


# Accessible States

**Gist:** State  $q$  is *accessible* if a string takes DFA from  $s$  (the start state) to  $q$ .

**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be an FA. A state  $q \in Q$  is *accessible* if there exists  $w \in \Sigma^*$  such that  $sw \vdash^* q$ ; otherwise,  $q$  is *inaccessible*.

**Note:** Each inaccessible state can be removed from FA



State  $s$  - accessible:  $w = \varepsilon$  :  $s \vdash^0 s$

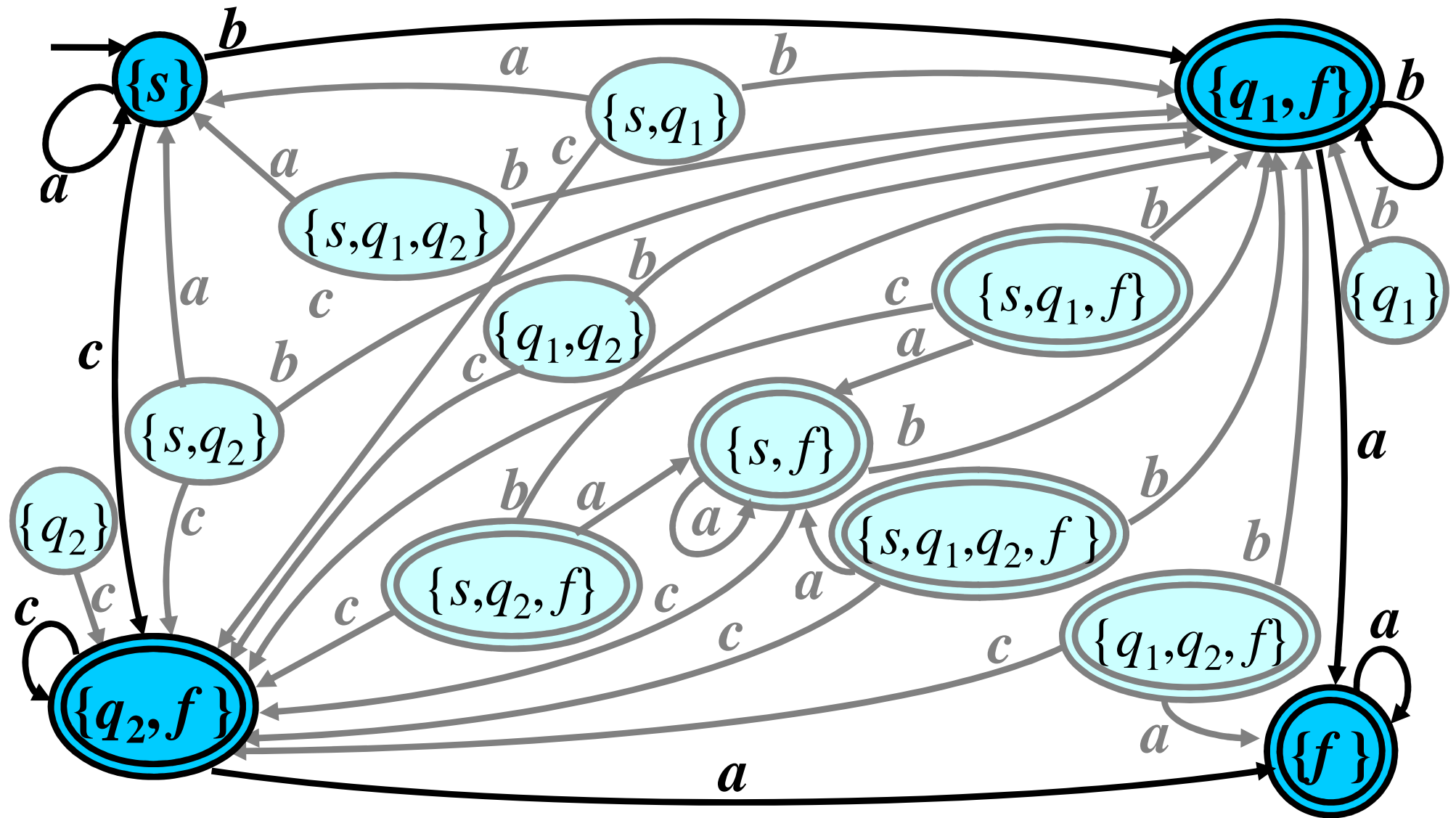
State  $q_1$  - accessible:  $w = a$  :  $sa \vdash q_1$

State  $f$  - accessible:  $w = ab$  :  $sab \vdash q_1 \vdash f$

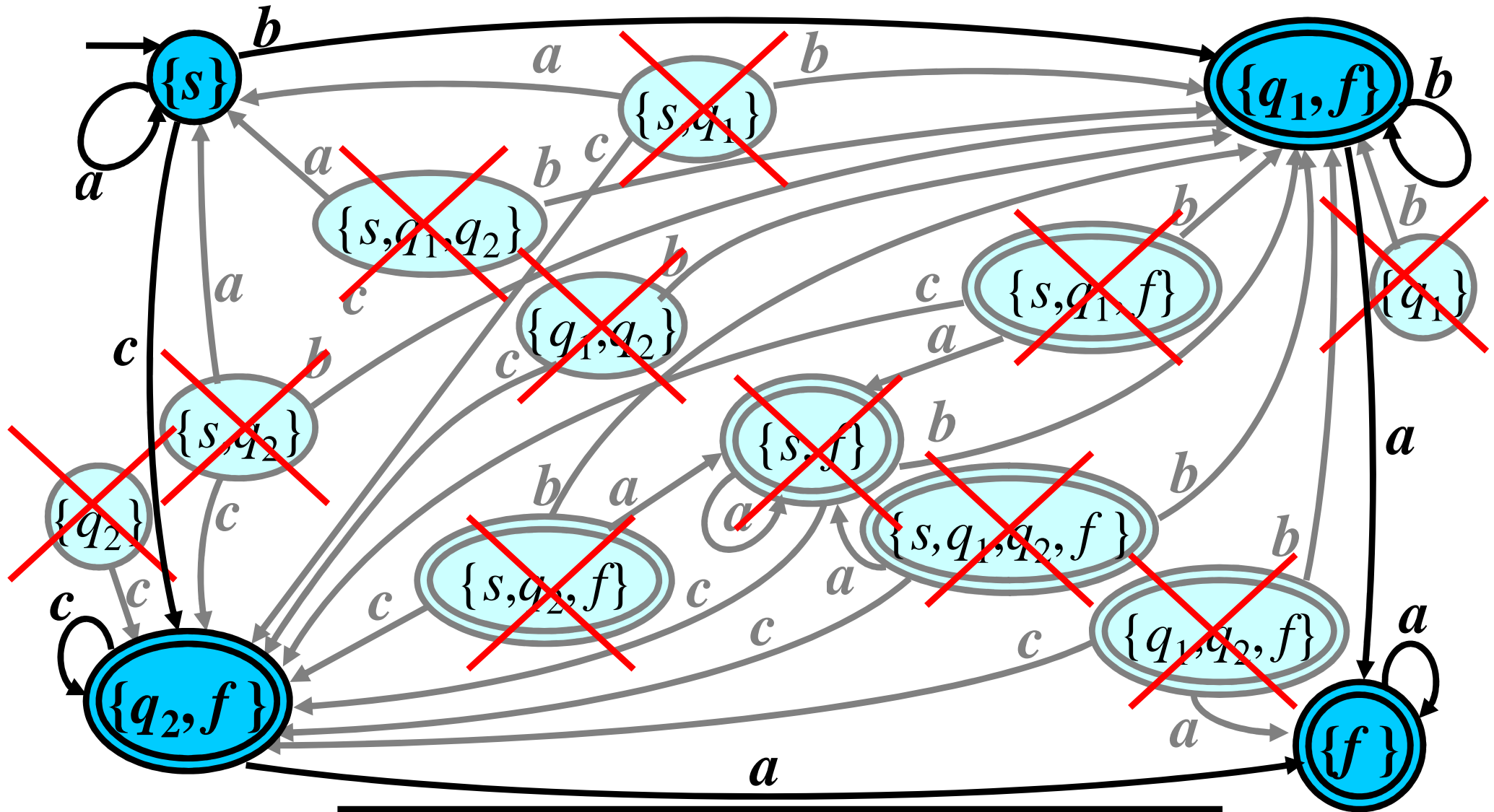
State  $q_2$  - **inaccessible** (there is no  $w \in \Sigma^*$  such that  $sw \vdash^* q_2$ )



# Previous Example



# Previous Example



**Many inaccessible states**

## Algorithm II: $\epsilon$ -free FA to DFA 1/2

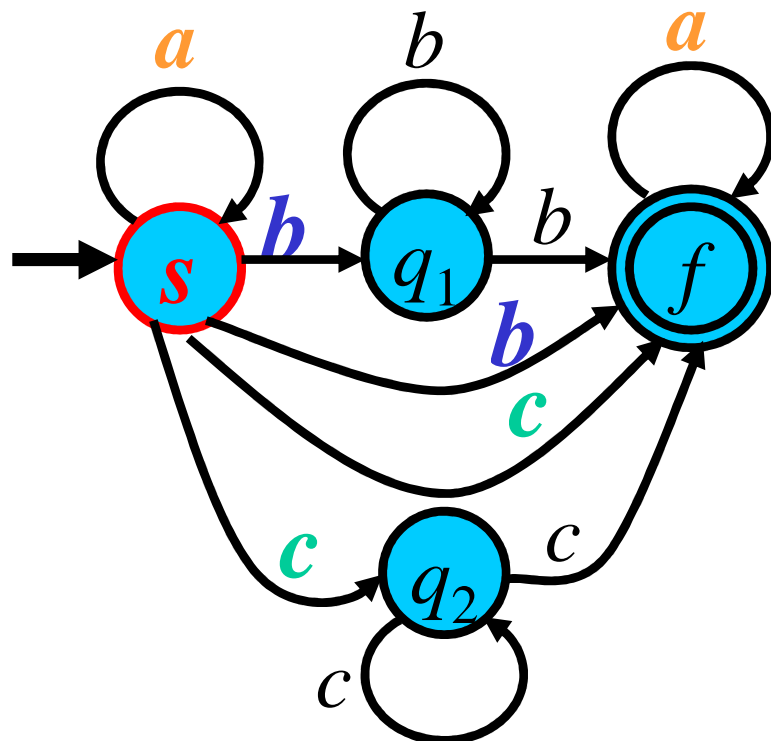
**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

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# Algorithm II: $\epsilon$ -free FA to DFA 1/2

**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

**Illustration:**

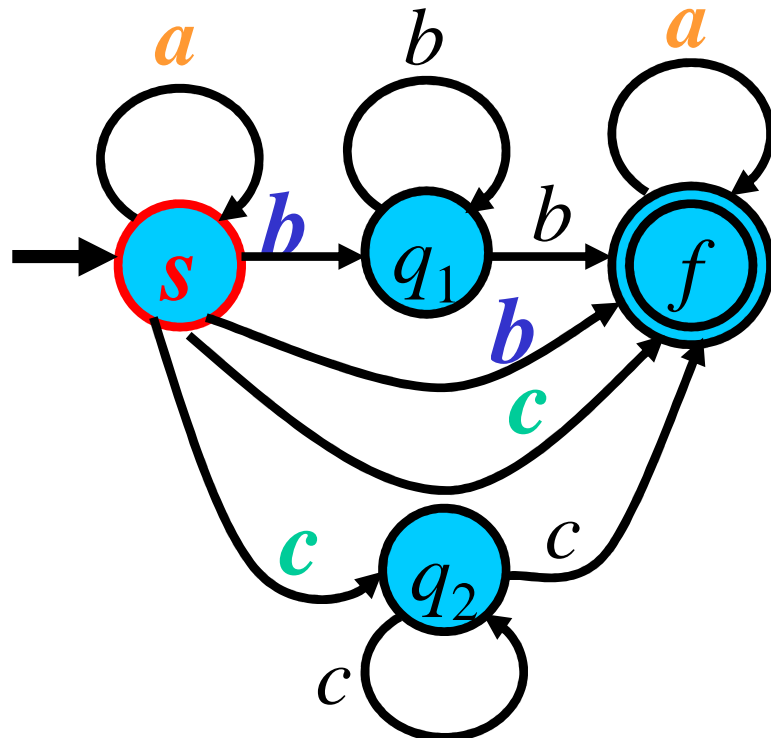


$$Q_{DFA} = \{\{s\}\}$$

# Algorithm II: $\varepsilon$ -free FA to DFA 1/2

**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

**Illustration:**



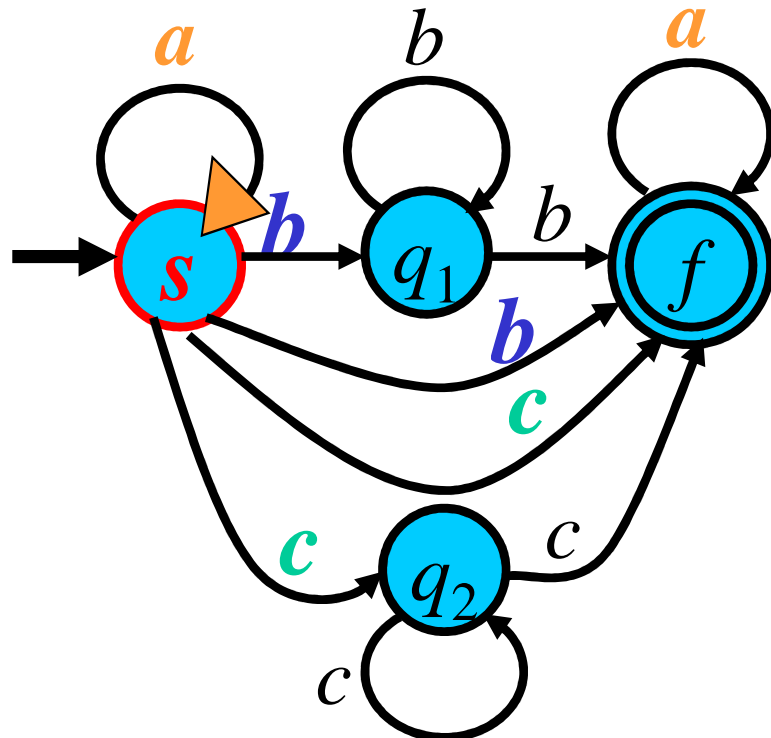
$$Q_{DFA} = \{\{s\}\}$$

For state  $\{s\}$ :  $\{s\}$

# Algorithm II: $\epsilon$ -free FA to DFA 1/2

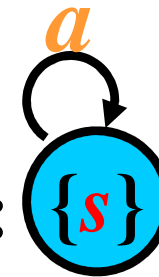
**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

**Illustration:**



$$Q_{DFA} = \{\{s\}\}$$

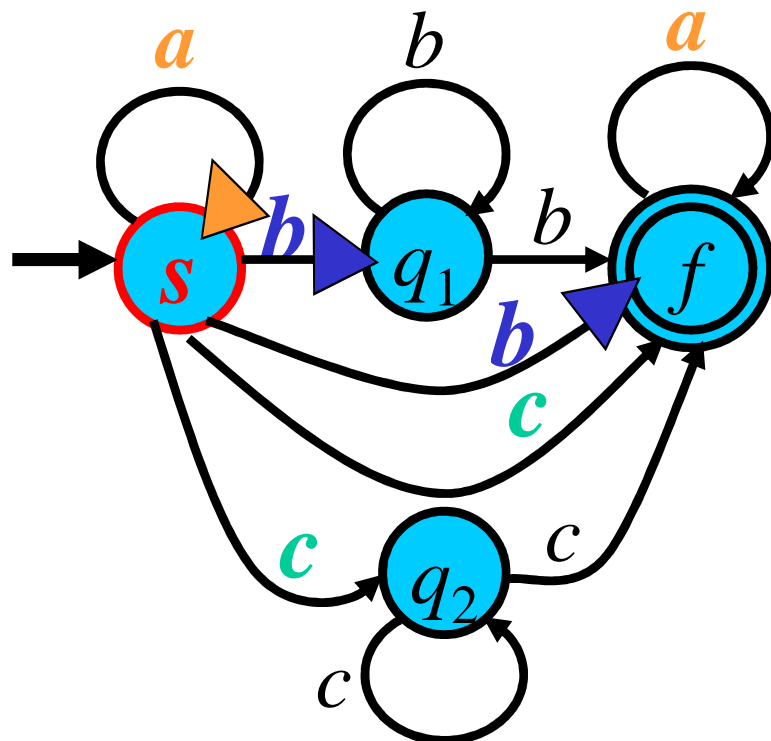
For state  $\{s\}$ :



# Algorithm II: $\epsilon$ -free FA to DFA 1/2

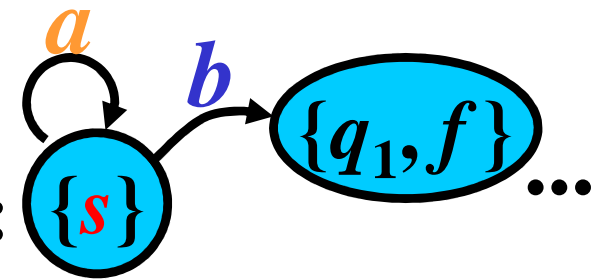
**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

**Illustration:**



$$Q_{DFA} = \{\{s\}\}$$

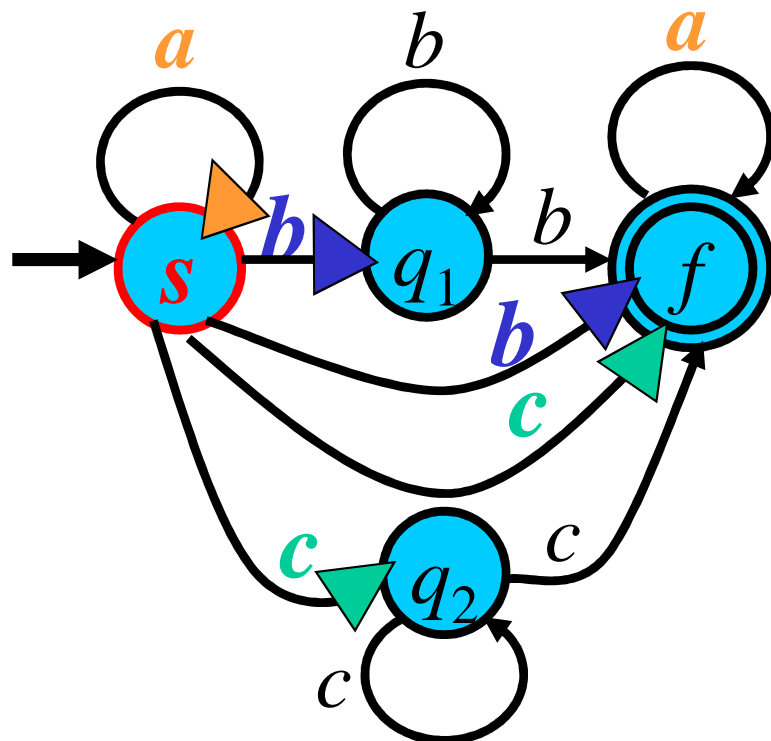
For state  $\{s\}$ :



# Algorithm II: $\epsilon$ -free FA to DFA 1/2

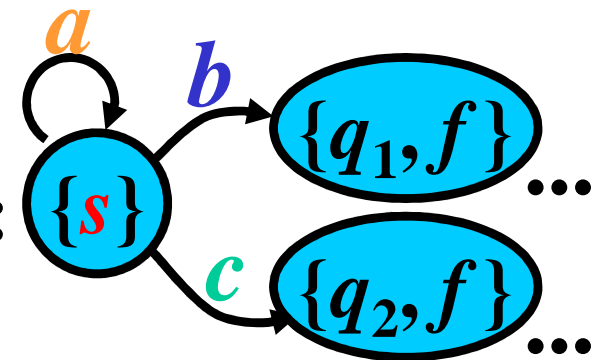
**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

**Illustration:**



$$Q_{DFA} = \{\{s\}\}$$

For state  $\{s\}$ :

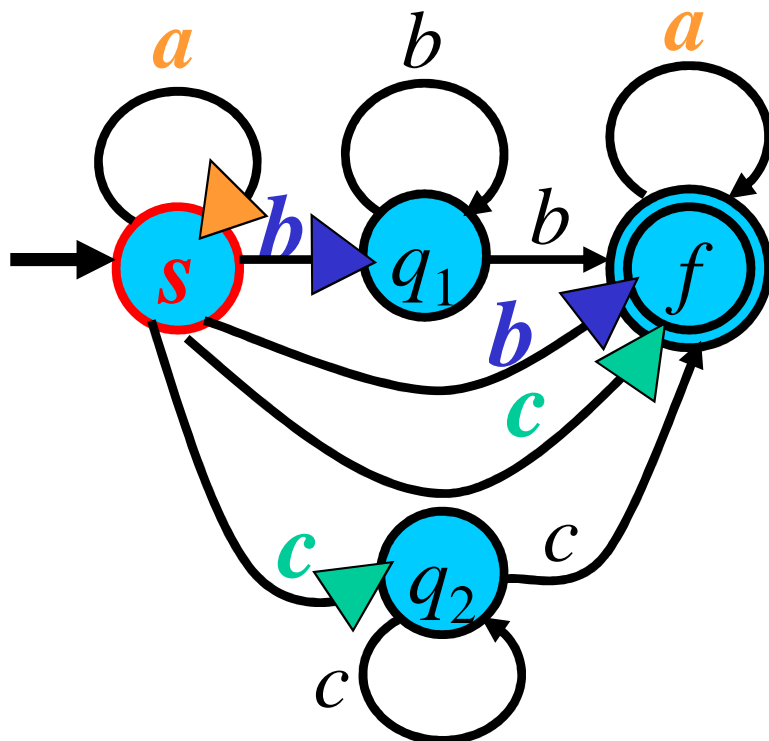




# Algorithm II: $\epsilon$ -free FA to DFA 1/2

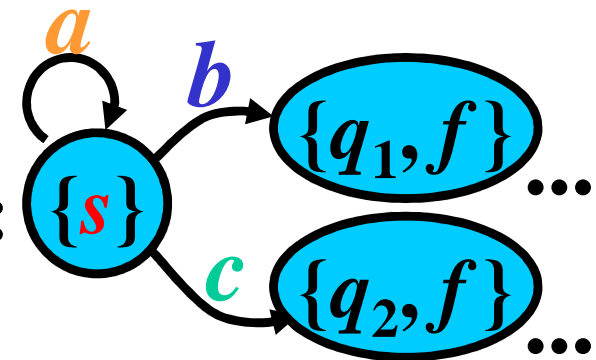
**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

**Illustration:**



$$Q_{DFA} = \{\{s\}\}$$

For state  $\{s\}$ :

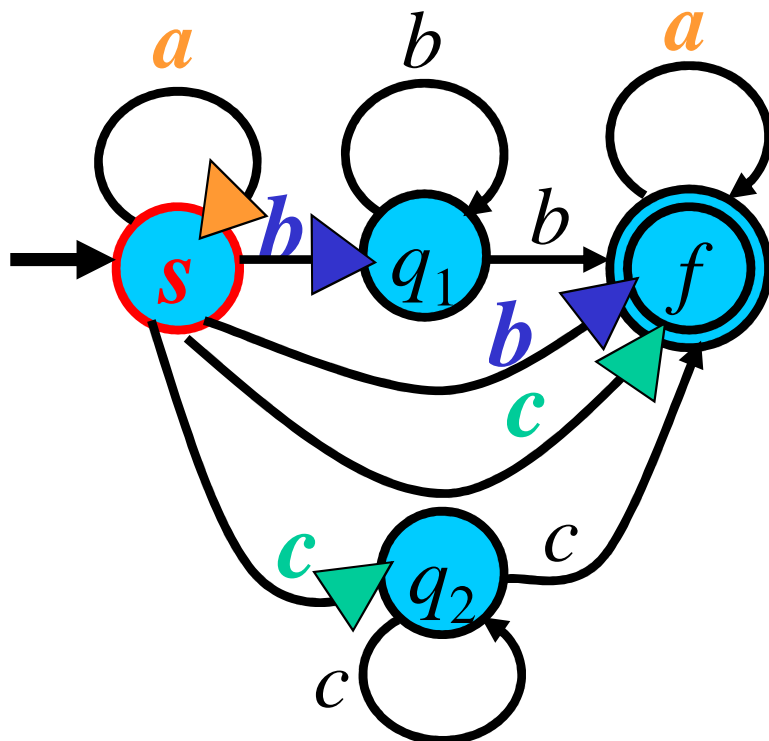


Add new states  $\{q_1, f\}$ ,  $\{q_2, f\}$  to  $Q_{DFA}$

# Algorithm II: $\epsilon$ -free FA to DFA 1/2

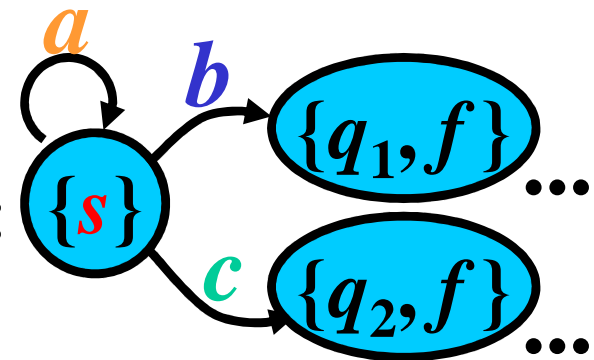
**Gist:** Analogy to the previous algorithm except that only sets of accessible states are introduced.

**Illustration:**



$$Q_{DFA} = \{\{s\}\}$$

For state  $\{s\}$ :



Add new states  $\{q_1, f\}$ ,  $\{q_2, f\}$  to  $Q_{DFA}$

For state  $\{q_1, f\}$ : ...

For state  $\{q_2, f\}$ : ...

Add new states ...

⋮

# Algorithm II: $\varepsilon$ -free FA to DFA 2/2

- **Input:**  $\varepsilon$ -free FA:  $M = (Q, \Sigma, R, s, F)$
- **Output:** DFA:  $M_d = (Q_d, \Sigma, R_d, s_d, F_d)$   
without any inaccessible states

## • Method:

- $s_d := \{s\}; Q_{new} := \{s_d\}; R_d := \emptyset; Q_d := \emptyset; F_d := \emptyset;$
- **repeat**
  - let  $Q' \in Q_{new}; Q_{new} := Q_{new} - \{Q'\}; Q_d := Q_d \cup \{Q'\};$
  - for each**  $a \in \Sigma$  **do begin**
    - $Q'' := \{q: p \in Q', pa \rightarrow q \in R\};$
    - if**  $Q'' \neq \emptyset$  **then**  $R_d := R_d \cup \{Q'a \rightarrow Q''\};$
    - if**  $Q'' \notin Q_d \cup \{\emptyset\}$  **then**  $Q_{new} := Q_{new} \cup \{Q''\}$
  - end;**
  - if**  $Q' \cap F \neq \emptyset$  **then**  $F_d := F_d \cup \{Q'\}$
- until**  $Q_{new} = \emptyset.$

# $\epsilon$ -free FA to DFA: Example 1/3

$M = (Q, \Sigma, R, s, F)$ , where:

$Q = \{s, q_1, q_2, f\}; \Sigma = \{a, b, c\}; F = \{f\}$

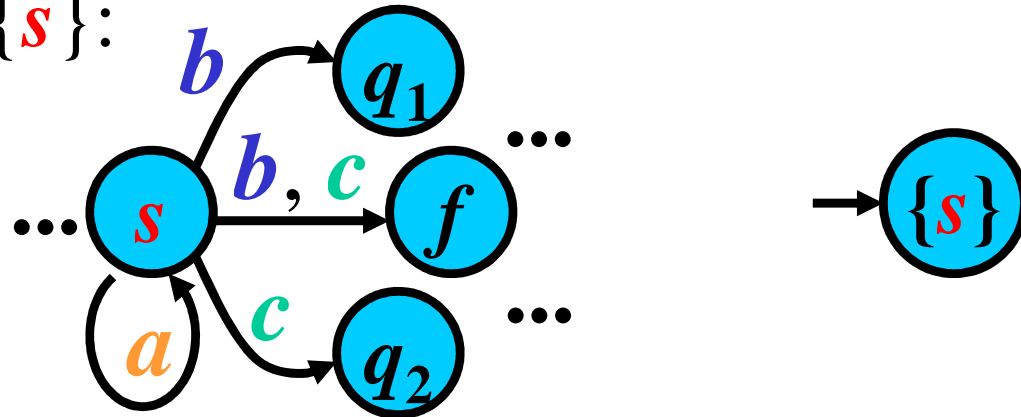
$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$   
 $q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

---

$Q_{new} = \{\{s\}\}; R_d = \emptyset; Q_d = \emptyset; F_d = \emptyset$

---

for  $Q' = \{s\}$ :



# $\epsilon$ -free FA to DFA: Example 1/3

$M = (Q, \Sigma, R, s, F)$ , where:

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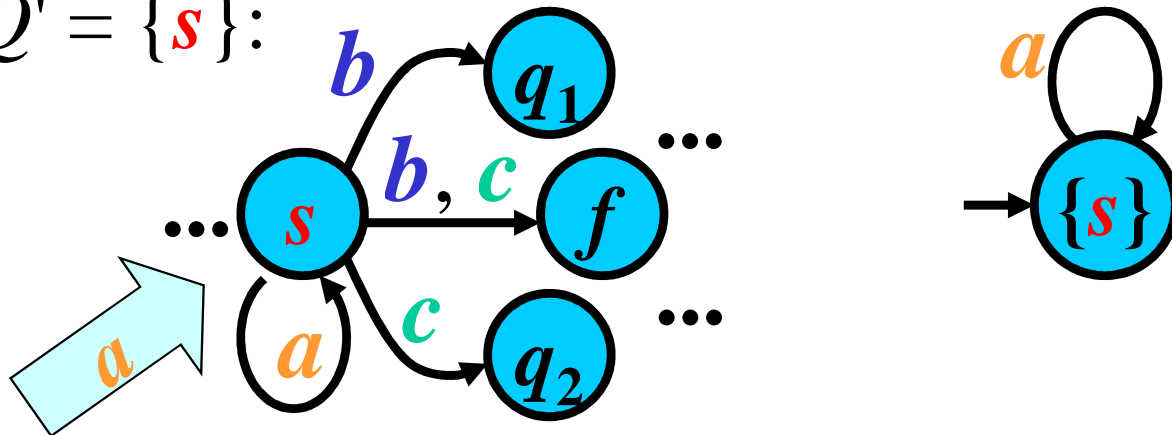
$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$   
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---

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# $\epsilon$ -free FA to DFA: Example 1/3

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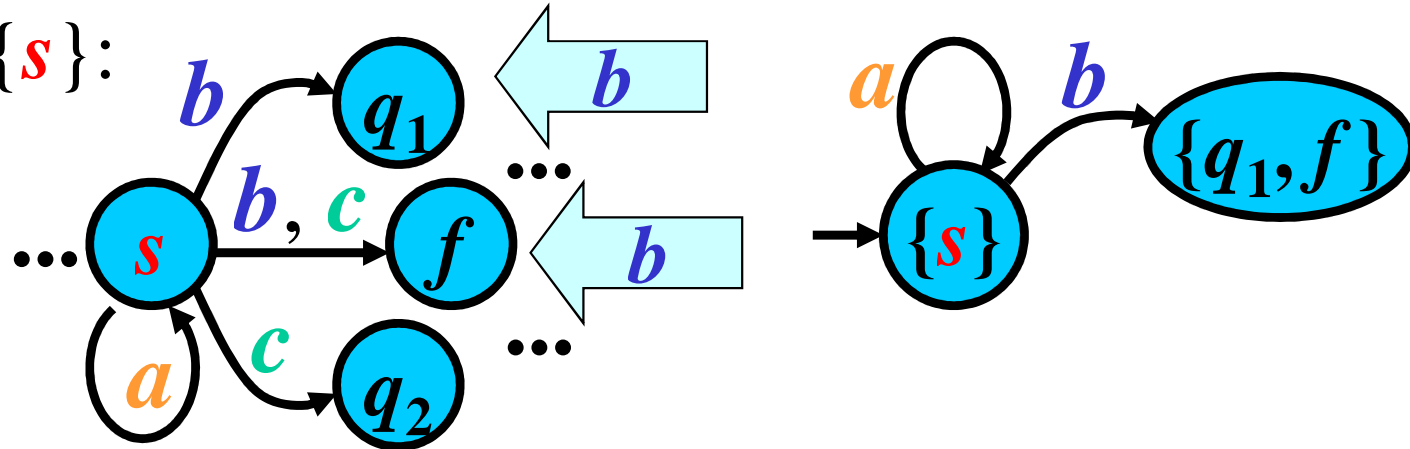
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 $q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

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$Q_{new} = \{\{s\}\}; R_d = \emptyset; Q_d = \emptyset; F_d = \emptyset$

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# $\epsilon$ -free FA to DFA: Example 1/3

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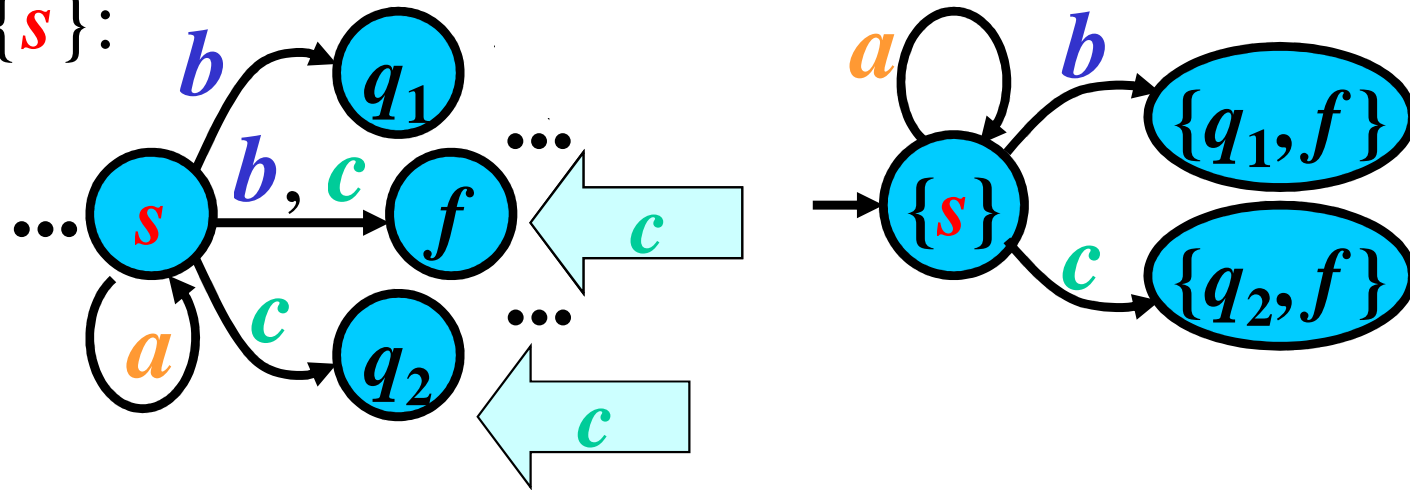
$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$   
 $q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

---

$Q_{new} = \{\{s\}\}; R_d = \emptyset; Q_d = \emptyset; F_d = \emptyset$

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# $\epsilon$ -free FA to DFA: Example 1/3

$M = (Q, \Sigma, R, s, F)$ , where:

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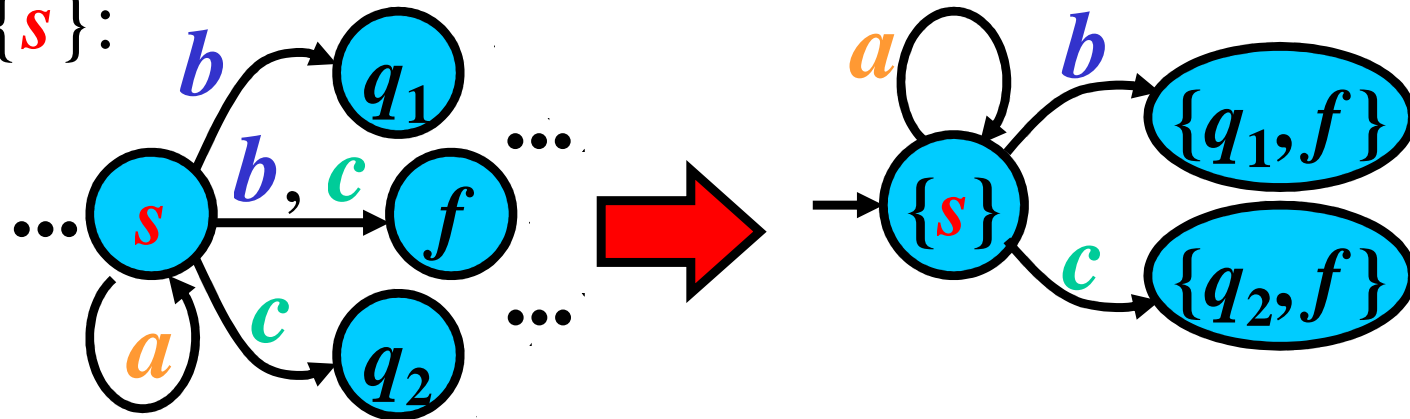
$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f, \\ q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\};$

---

$Q_{new} = \{\{s\}\}; R_d = \emptyset; Q_d = \emptyset; F_d = \emptyset$

---

for  $Q' = \{s\}$ :



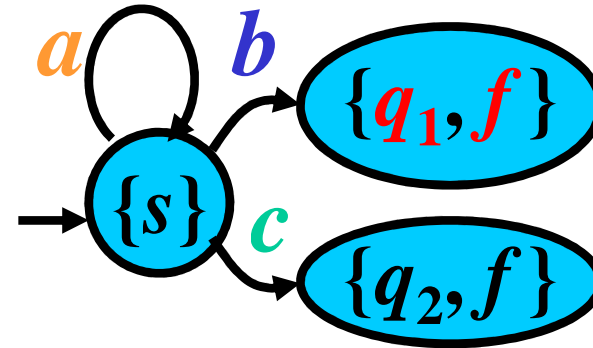
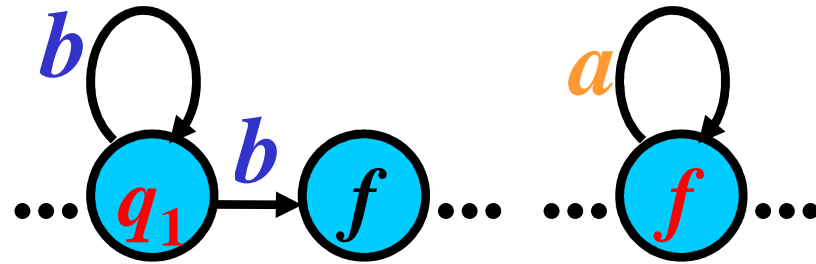
$R_d := \emptyset \cup \{\{s\}a \rightarrow \{s\}, \{s\}b \rightarrow \{q_1, f\}, \{s\}c \rightarrow \{q_2, f\}\}$

$Q_{new} = \{\{q_1, f\}, \{q_2, f\}\}, Q_d = \emptyset \cup \{\{s\}\}, F_d = \emptyset$



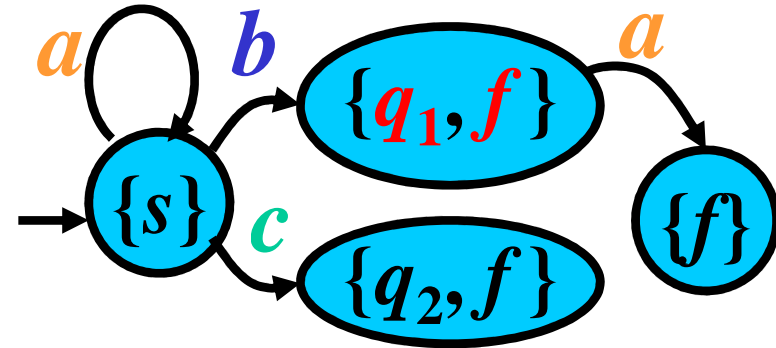
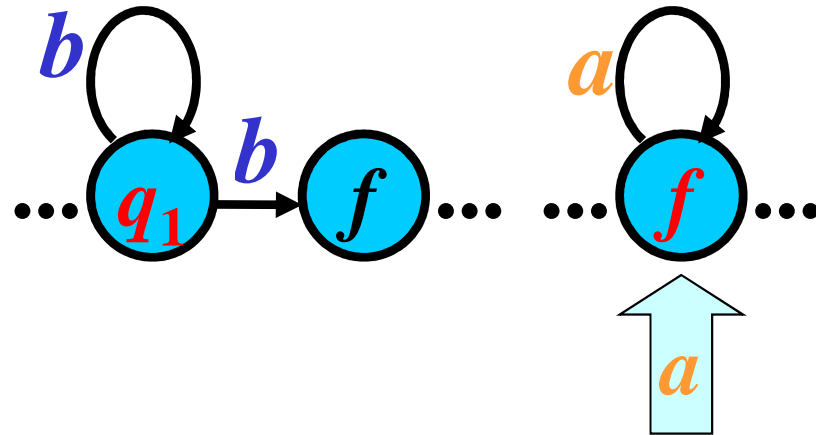
# $\epsilon$ -free FA to DFA: Example 2/3

for  $Q' = \{q_1, f\}$ :



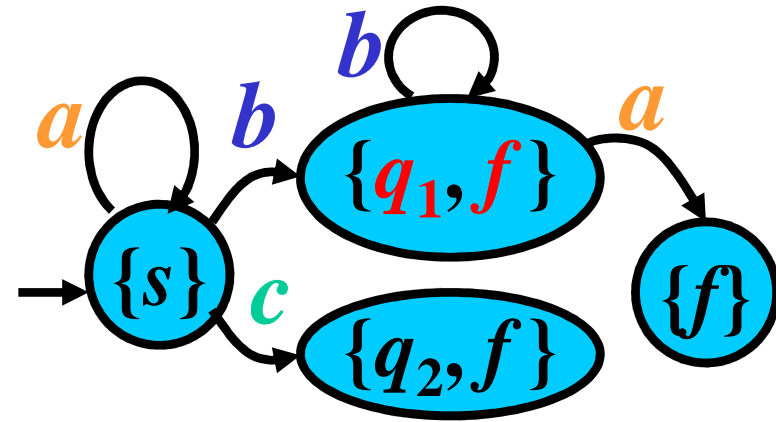
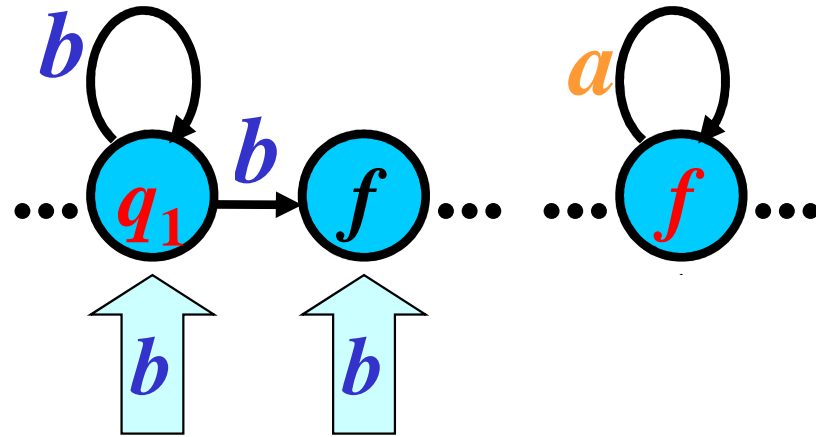
# $\epsilon$ -free FA to DFA: Example 2/3

for  $Q' = \{q_1, f\}$ :



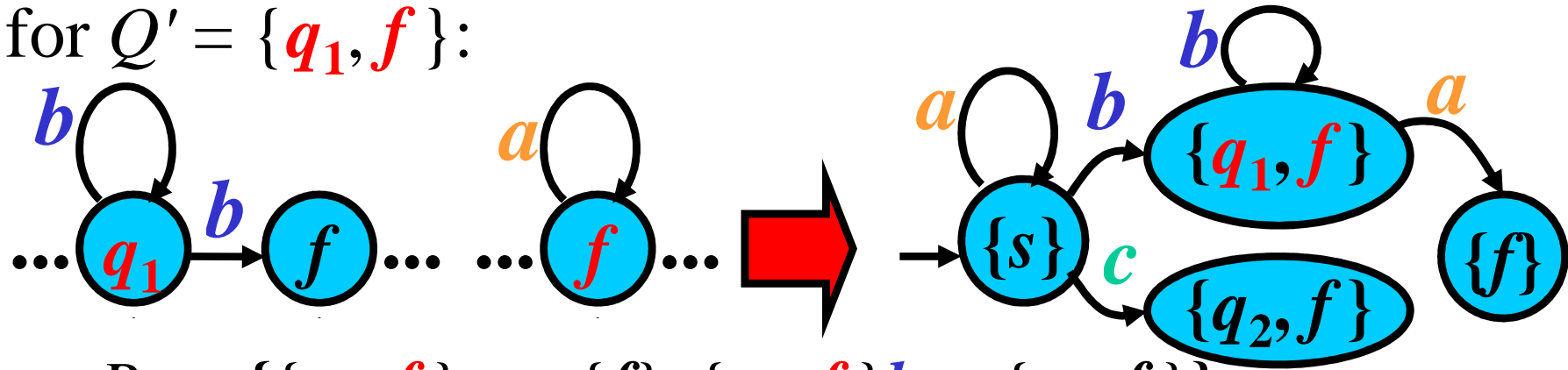
# $\epsilon$ -free FA to DFA: Example 2/3

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# $\epsilon$ -free FA to DFA: Example 2/3

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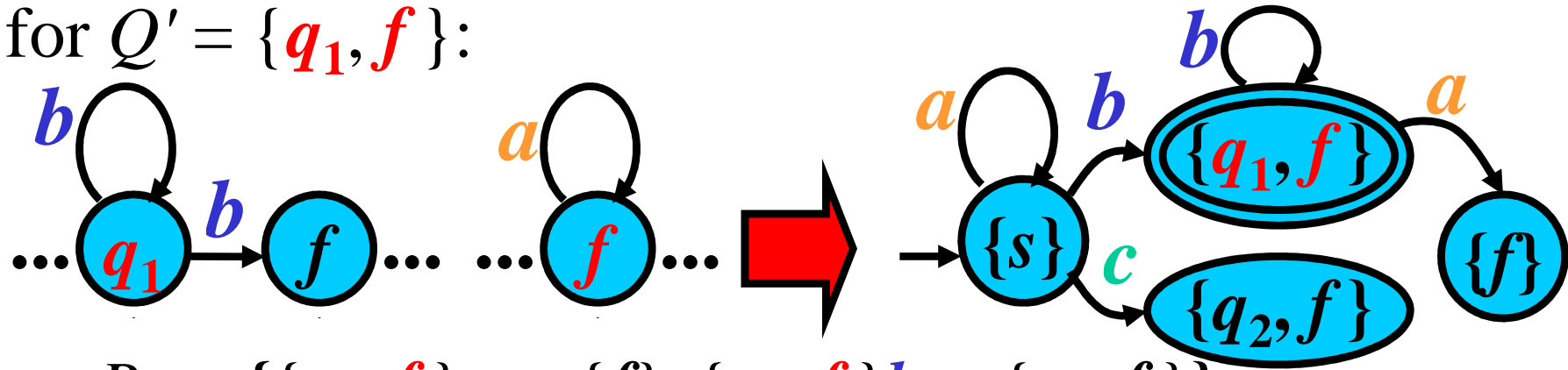


$$R_d := R_d \cup \{\{q_1, f\}a \rightarrow \{f\}, \{q_1, f\}b \rightarrow \{q_1, f\}\}$$

$$Q_{new} = \{\{q_2, f\}, \{f\}\}, Q_d = Q_d \cup \{\{q_1, f\}\},$$

# $\epsilon$ -free FA to DFA: Example 2/3

for  $Q' = \{q_1, f\}$ :

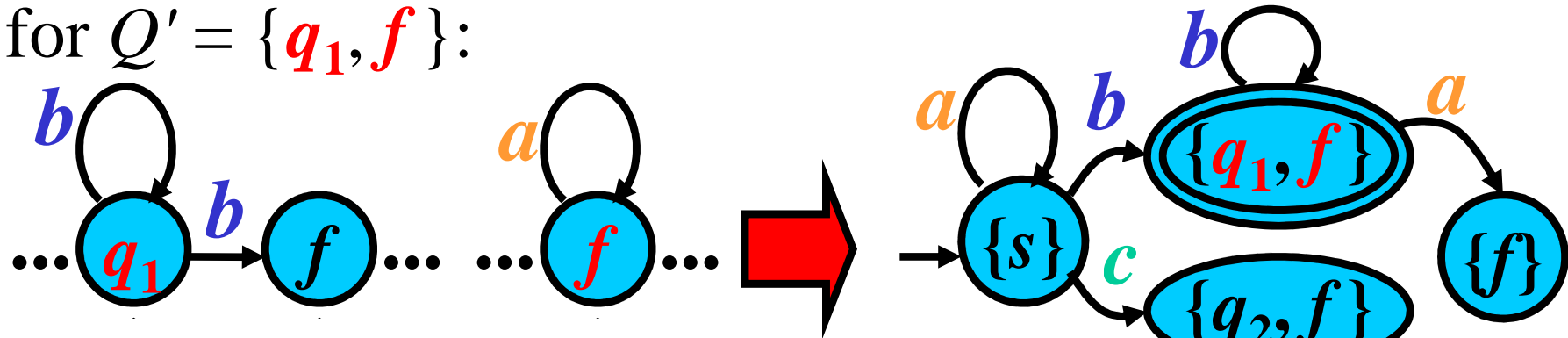


$$R_d := R_d \cup \{ \{q_1, f\} a \rightarrow \{f\}, \{q_1, f\} b \rightarrow \{q_1, f\} \}$$

$$Q_{new} = \{ \{q_2, f\}, \{f\} \}, Q_d = Q_d \cup \{ \{q_1, f\} \}, F_d := \emptyset \cup \{ \{q_1, f\} \}$$

# $\epsilon$ -free FA to DFA: Example 2/3

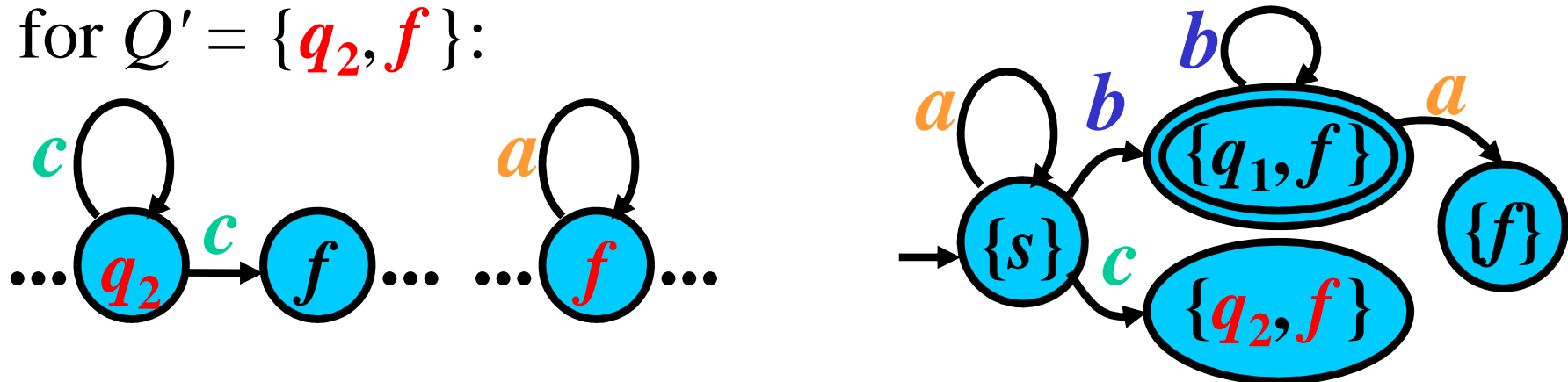
for  $Q' = \{q_1, f\}$ :



$$R_d := R_d \cup \{ \{q_1, f\} a \rightarrow \{f\}, \{q_1, f\} b \rightarrow \{q_1, f\} \}$$

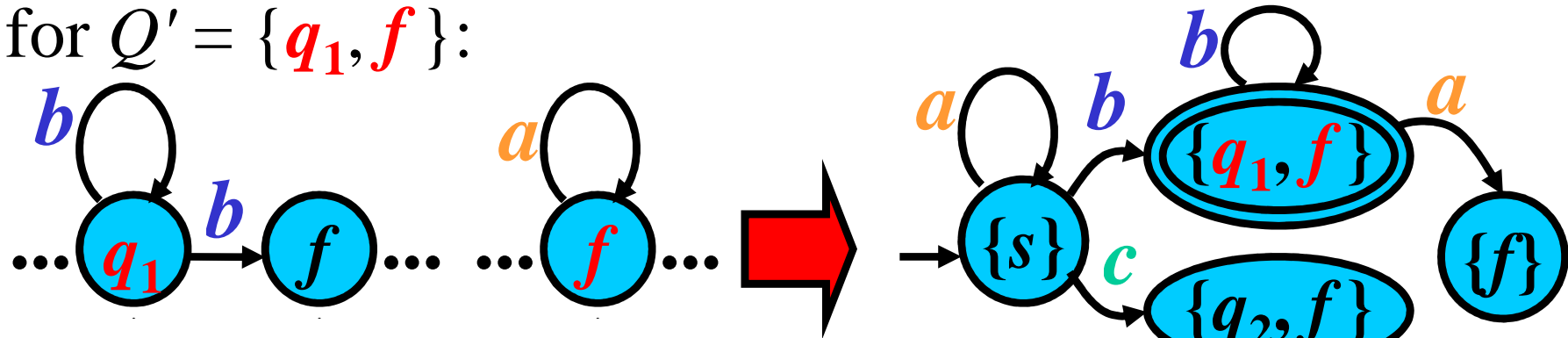
$$Q_{new} = \{ \{q_2, f\}, \{f\} \}, Q_d = Q_d \cup \{ \{q_1, f\} \}, F_d := \emptyset \cup \{ \{q_1, f\} \}$$

for  $Q' = \{q_2, f\}$ :



# $\epsilon$ -free FA to DFA: Example 2/3

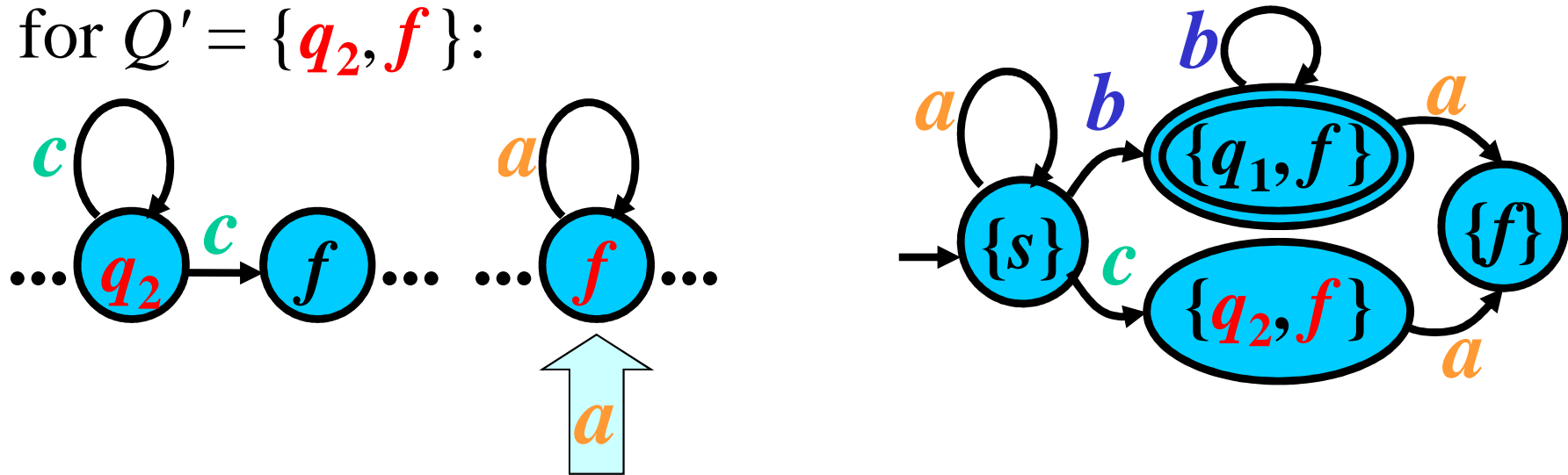
for  $Q' = \{q_1, f\}$ :



$$R_d := R_d \cup \{ \{q_1, f\} a \rightarrow \{f\}, \{q_1, f\} b \rightarrow \{q_1, f\} \}$$

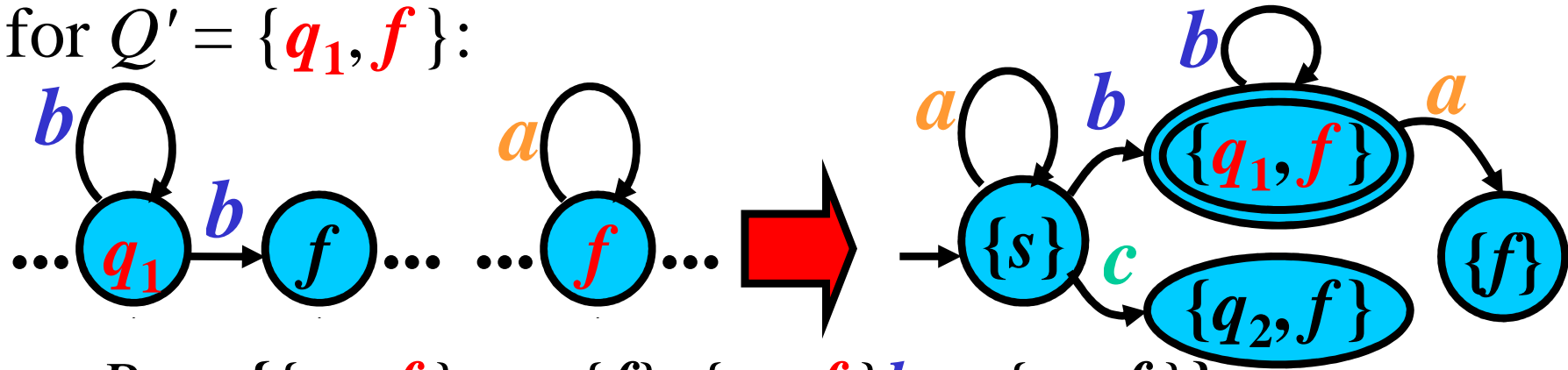
$$Q_{new} = \{ \{q_2, f\}, \{f\} \}, Q_d = Q_d \cup \{ \{q_1, f\} \}, F_d := \emptyset \cup \{ \{q_1, f\} \}$$

for  $Q' = \{q_2, f\}$ :



# $\epsilon$ -free FA to DFA: Example 2/3

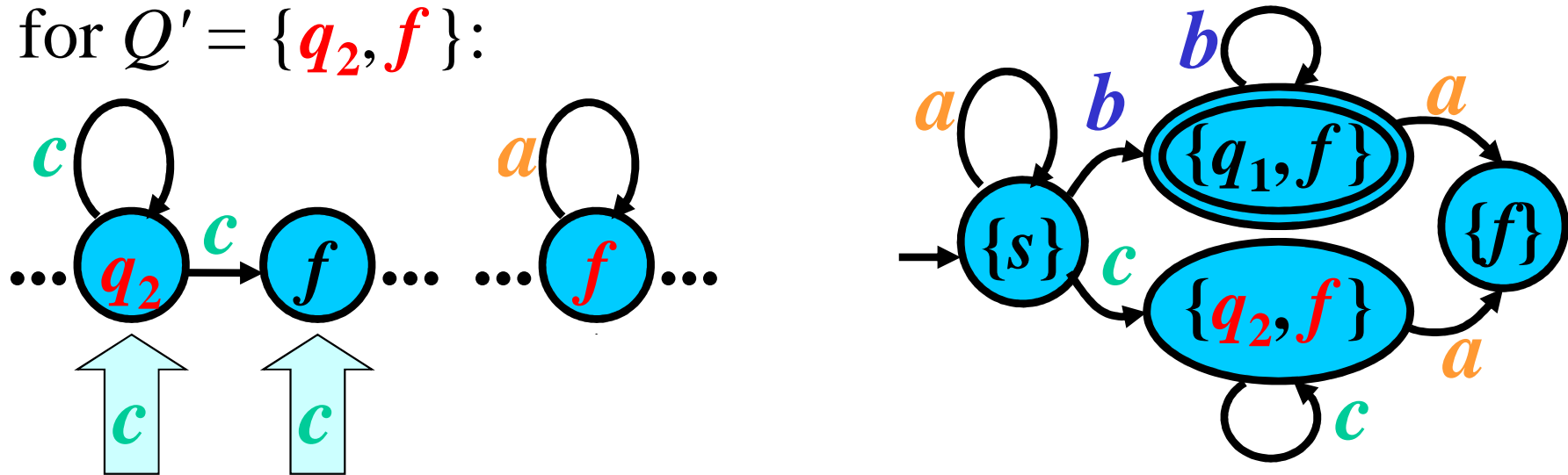
for  $Q' = \{q_1, f\}$ :



$$R_d := R_d \cup \{ \{q_1, f\} a \rightarrow \{f\}, \{q_1, f\} b \rightarrow \{q_1, f\} \}$$

$$Q_{new} = \{ \{q_2, f\}, \{f\} \}, Q_d = Q_d \cup \{ \{q_1, f\} \}, F_d := \emptyset \cup \{ \{q_1, f\} \}$$

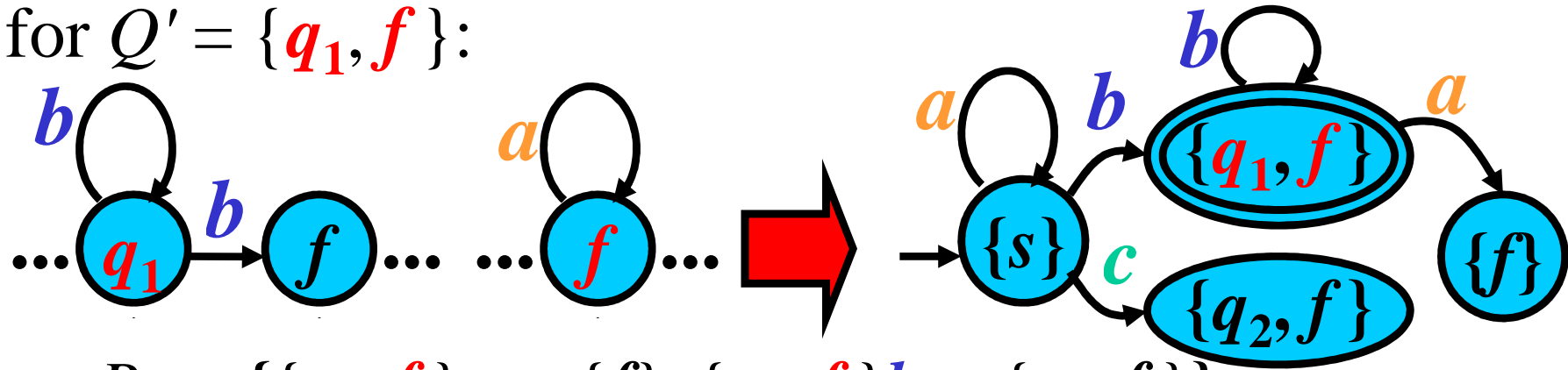
for  $Q' = \{q_2, f\}$ :





# $\epsilon$ -free FA to DFA: Example 2/3

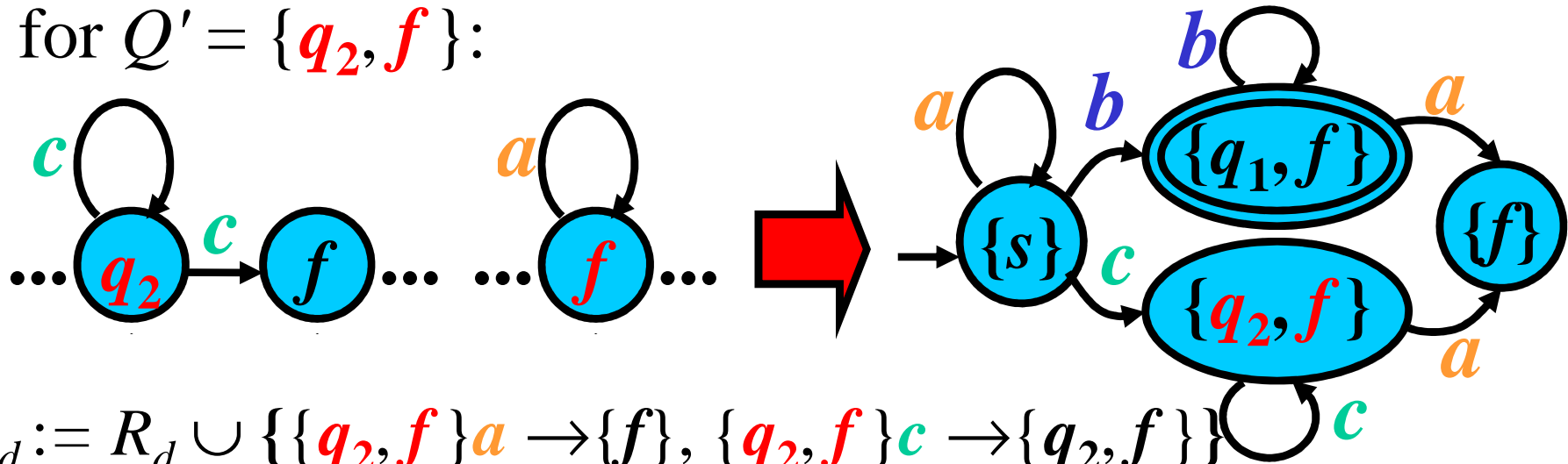
for  $Q' = \{q_1, f\}$ :



$$R_d := R_d \cup \{ \{q_1, f\} a \rightarrow \{f\}, \{q_1, f\} b \rightarrow \{q_1, f\} \}$$

$$Q_{new} = \{ \{q_2, f\}, \{f\} \}, Q_d = Q_d \cup \{ \{q_1, f\} \}, F_d := \emptyset \cup \{ \{q_1, f\} \}$$

for  $Q' = \{q_2, f\}$ :

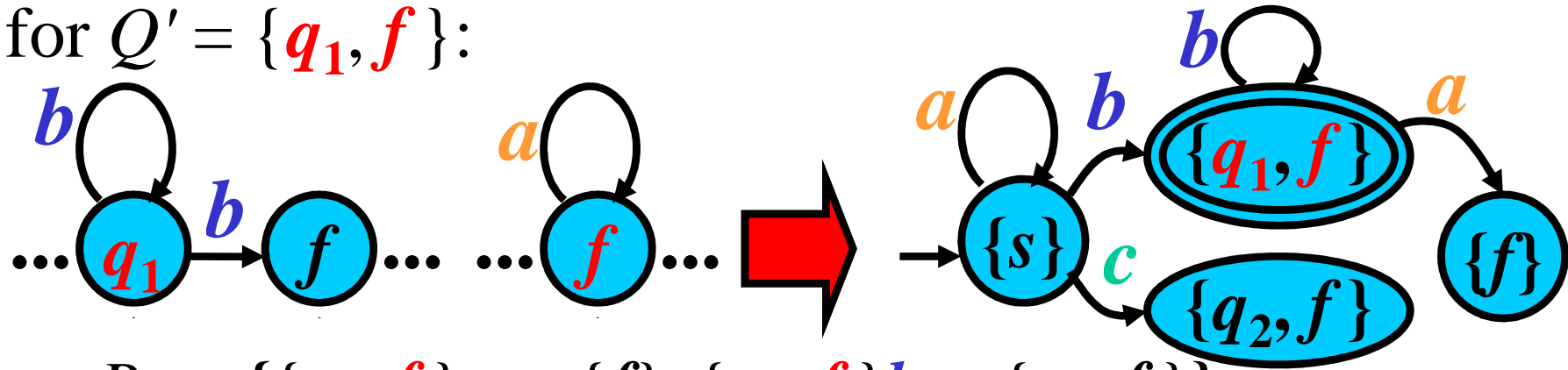


$$R_d := R_d \cup \{ \{q_2, f\} a \rightarrow \{f\}, \{q_2, f\} c \rightarrow \{q_2, f\} \}$$

$$Q_{new} = \{ \{f\} \}, Q_d = Q_d \cup \{ \{q_2, f\} \},$$

# $\epsilon$ -free FA to DFA: Example 2/3

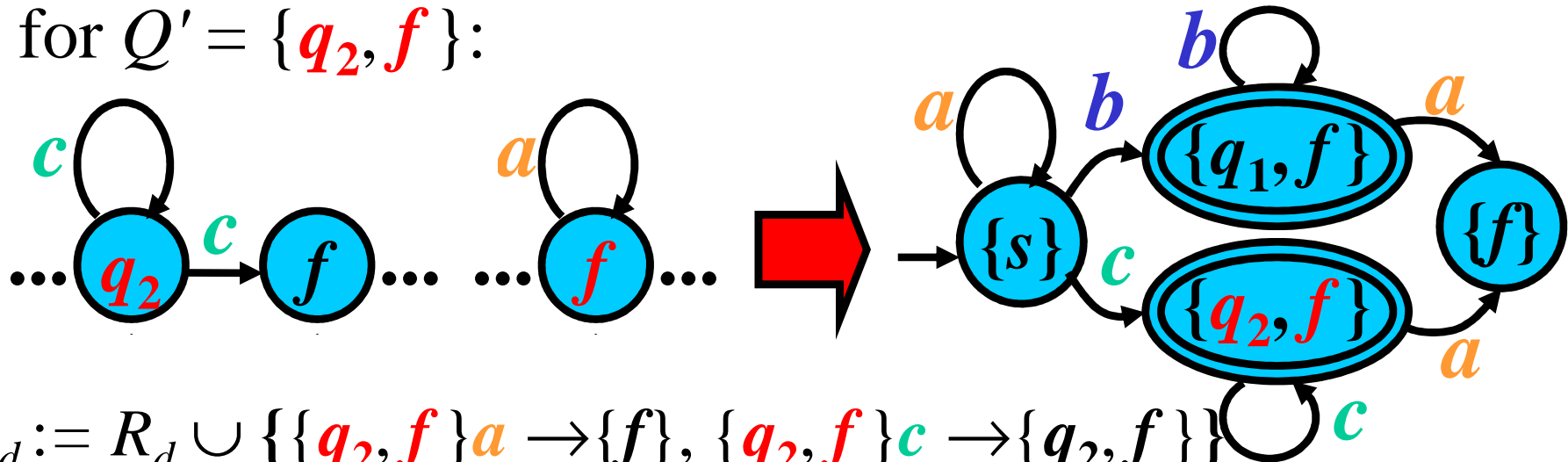
for  $Q' = \{q_1, f\}$ :



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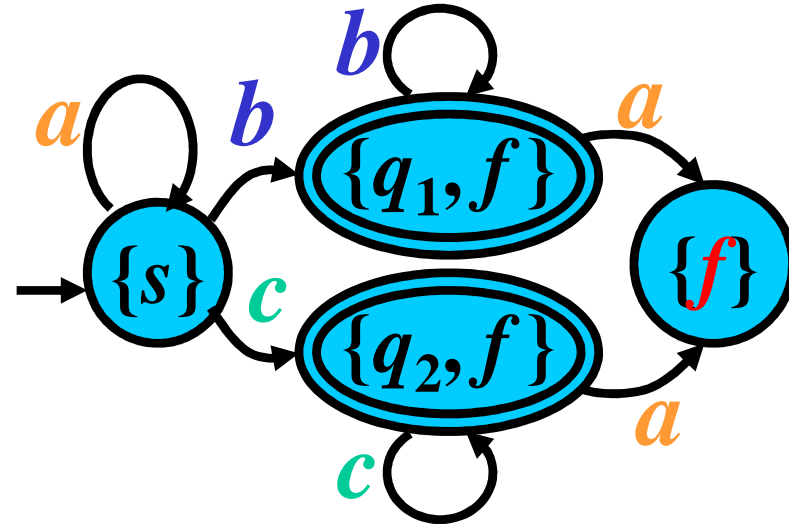
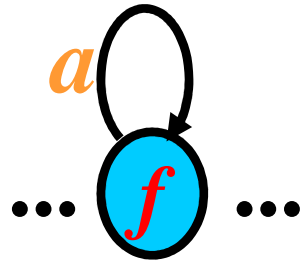


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$$Q_{new} = \{ \{f\} \}, Q_d = Q_d \cup \{ \{q_2, f\} \}, F_d := F_d \cup \{ \{q_2, f\} \}$$

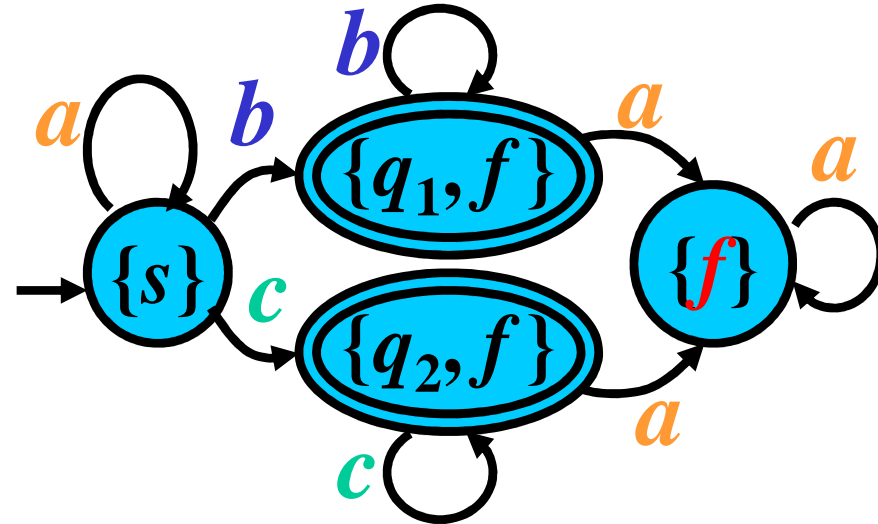
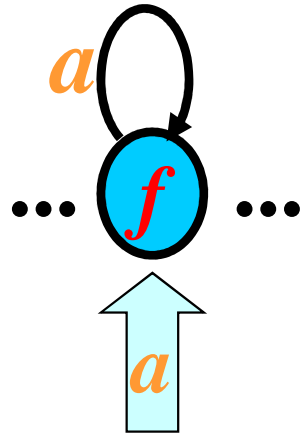
# $\epsilon$ -free FA to DFA: Example 3/3

for  $Q' = \{f\}$ :



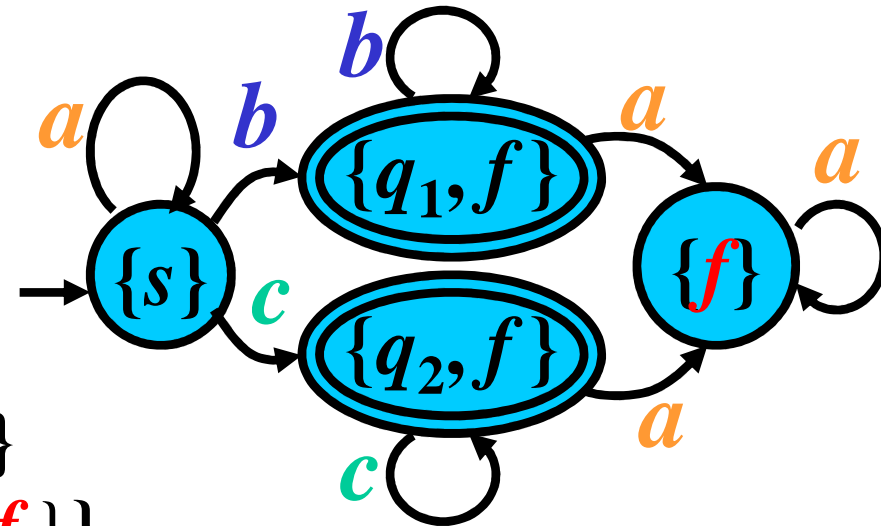
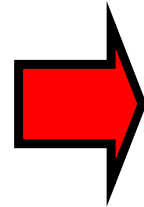
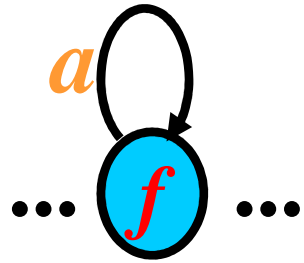
# $\epsilon$ -free FA to DFA: Example 3/3

for  $Q' = \{f\}$ :



# $\epsilon$ -free FA to DFA: Example 3/3

for  $Q' = \{f\}$ :

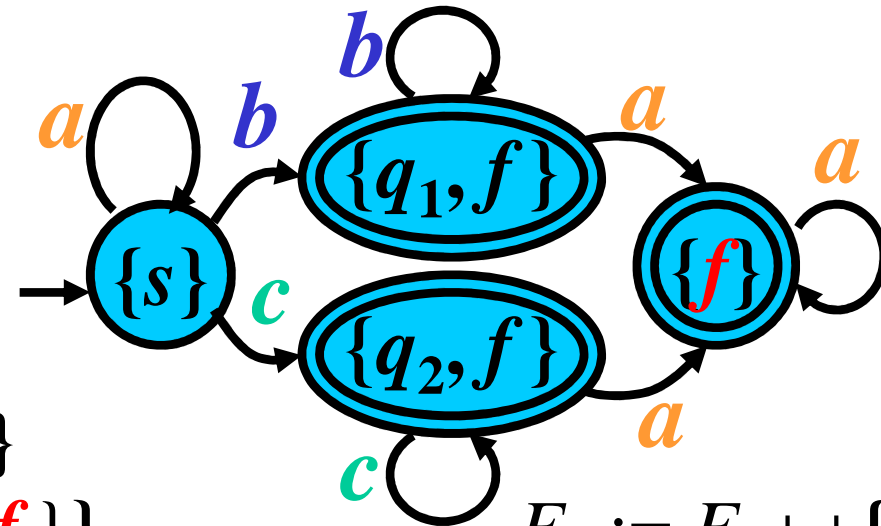
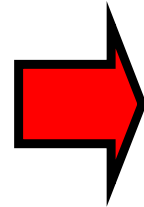
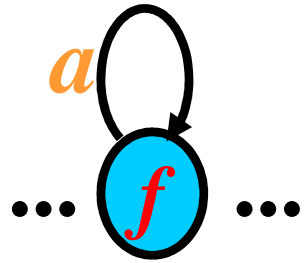


$$R_d := R_d \cup \{\{f\}a \rightarrow \{f\}\}$$

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# $\epsilon$ -free FA to DFA: Example 3/3

for  $Q' = \{f\}$ :



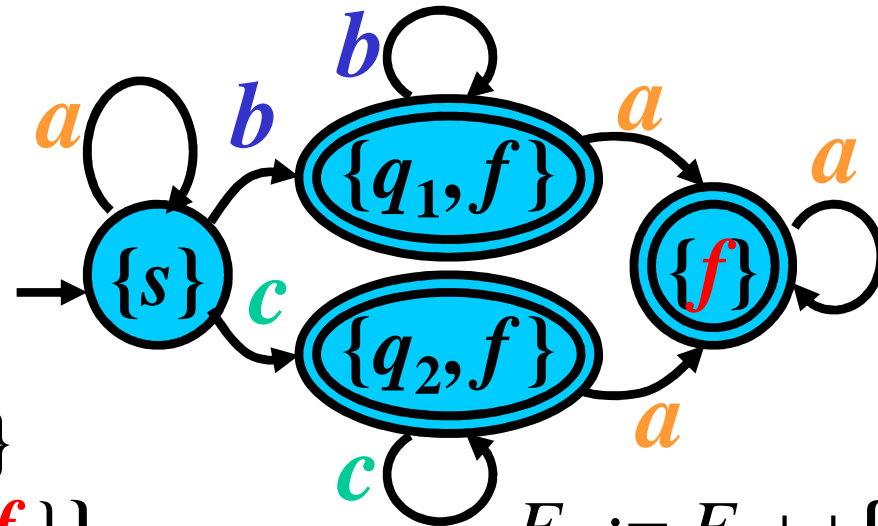
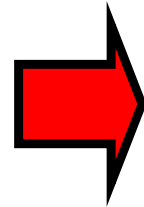
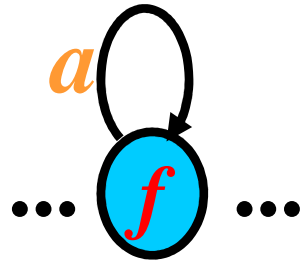
$$R_d := R_d \cup \{\{f\}a \rightarrow \{f\}\}$$

$$Q_{new} = \emptyset, Q_d = Q_d \cup \{\{f\}\},$$

$$F_d := F_d \cup \{\{f\}\}$$

# $\epsilon$ -free FA to DFA: Example 3/3

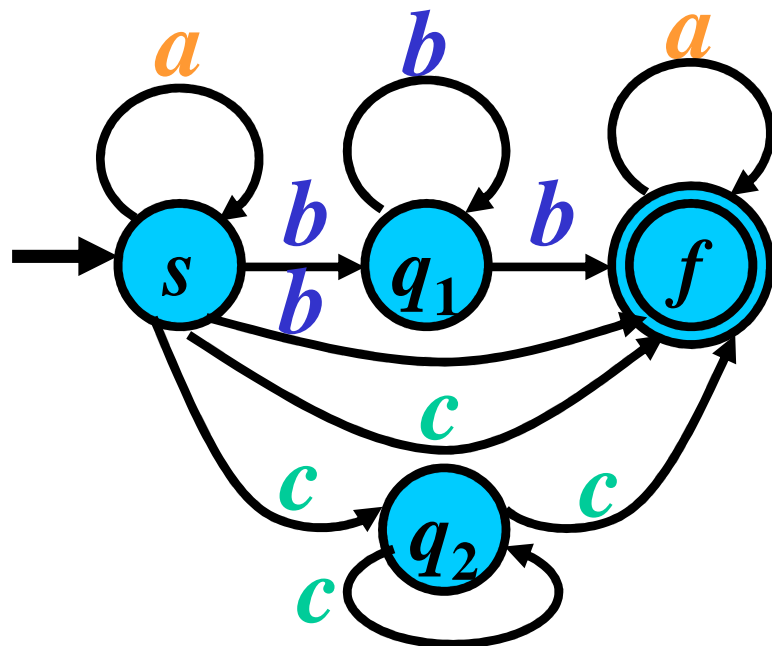
for  $Q' = \{f\}$ :



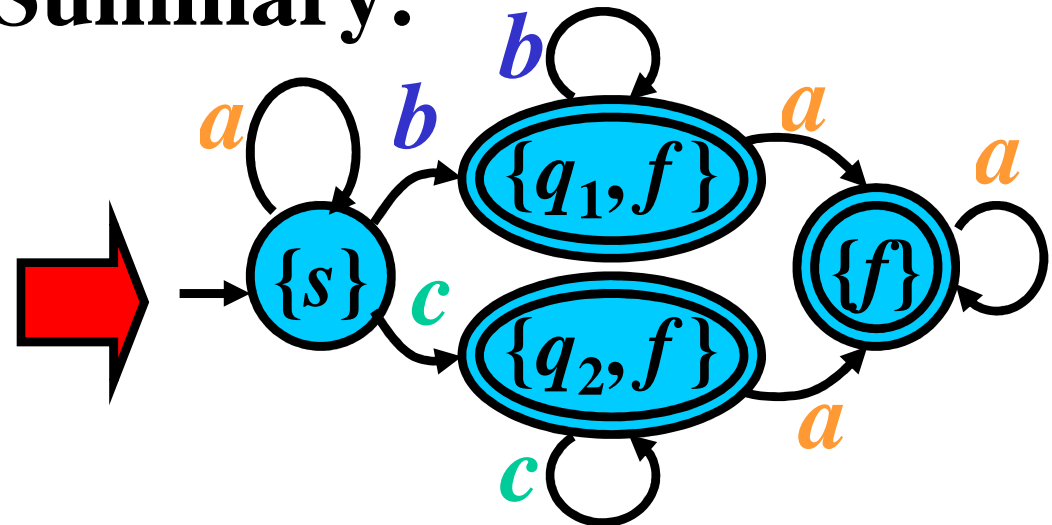
$$R_d := R_d \cup \{\{f\}a \rightarrow \{f\}\}$$

$$Q_{new} = \emptyset, Q_d = Q_d \cup \{\{f\}\},$$

$$F_d := F_d \cup \{\{f\}\}$$



Summary:



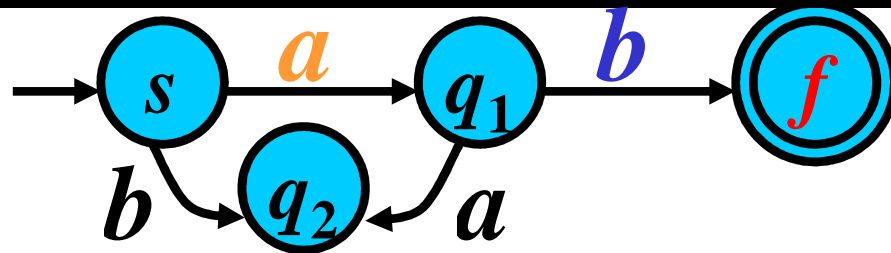
# Terminating States

**Gist:** State  $q$  is *terminating* if a string takes DFA from  $q$  to a final state.

**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be a DFA. A state  $q \in Q$  is *terminating* if there exists  $w \in \Sigma^*$  such that  $qw \vdash^* f$  with  $f \in F$ ; otherwise,  $q$  is *nonterminating*.

**Note:** Each nonterminating state can be removed from DFA

**Example:**



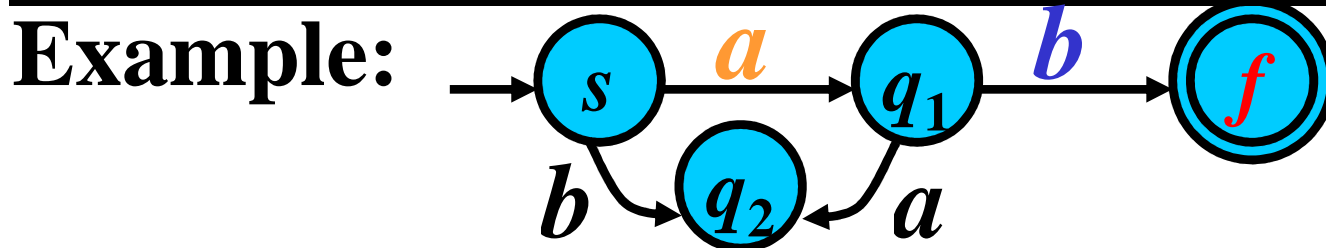


# Terminating States

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**Note:** Each nonterminating state can be removed from DFA



State  $s$  - terminating:  $w = ab$  :  $sab \vdash q_1b \vdash f$

State  $q_1$  - terminating:  $w = b$  :  $q_1b \vdash f$

State  $f$  - terminating:  $w = \varepsilon$  :  $f \vdash^0 f$

State  $q_2$  - **nonterminating** (there is no  $w \in \Sigma^*$  such that  $q_2w \vdash^* q, q \in F$ )

# Algorithm: Removal of nont. states

- **Input:** DFA:  $M = (Q, \Sigma, R, s, F)$
  - **Output:** DFA:  $M_t = (Q_t, \Sigma, R_t, s, F)$
- 

- **Method:**

- $Q_0 := F; i := 0;$
- **repeat**
  - $i := i + 1;$
  - $Q_i := Q_{i-1} \cup \{q: qa \rightarrow p \in R, a \in \Sigma, p \in Q_{i-1}\};$
- until**  $Q_i = Q_{i-1};$
- $Q_t := Q_i;$
- $R_t := \{qa \rightarrow p: qa \rightarrow p \in R, p, q \in Q_t, a \in \Sigma\}.$

# Nonterminating States: Example

$M = (Q, \Sigma, R, s, F)$ , where:  $Q = \{s, q_1, q_2, f\}$ ,  $\Sigma = \{a, b\}$ ,  
 $R = \{sa \rightarrow q_1, sb \rightarrow q_2, q_1a \rightarrow q_2, q_1b \rightarrow f\}$ ,  $F = \{f\}$

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$$1) qd \rightarrow f; q \in Q; d \in \Sigma: \quad q_1b \rightarrow f$$

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$$Q_1 = \{f\} \cup \{q_1\} = \{f, q_1\}$$


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$$2) \quad \begin{array}{ll} qd \rightarrow f; q \in Q; d \in \Sigma: & q_1b \rightarrow f \\ qd \rightarrow q_1; q \in Q; d \in \Sigma: & sa \rightarrow q_1 \end{array}$$

$$Q_2 = \{f, q_1\} \cup \{q_1, s\} = \{f, q_1, s\}$$


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# Nonterminating States: Example

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$$2) qd \rightarrow f; q \in Q; d \in \Sigma: \quad q_1b \rightarrow f$$

$$qd \rightarrow q_1; q \in Q; d \in \Sigma: \quad sa \rightarrow q_1$$

$$Q_2 = \{f, q_1\} \cup \{q_1, s\} = \{f, q_1, s\}$$


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$$3) qd \rightarrow f; q \in Q; d \in \Sigma: \quad q_1b \rightarrow f$$

$$qd \rightarrow q_1; q \in Q; d \in \Sigma: \quad sa \rightarrow q_1$$

$$qd \rightarrow s; q \in Q; d \in \Sigma: \quad \text{none}$$

$$Q_3 = \{f, q_1, s\} \cup \{q_1, s\} = \{f, q_1, s\} = Q_2 = Q_t$$


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# Nonterminating States: Example

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$$3) \begin{array}{ll} qd \rightarrow f; q \in Q; d \in \Sigma: & q_1b \rightarrow f \\ qd \rightarrow q_1; q \in Q; d \in \Sigma: & sa \rightarrow q_1 \\ qd \rightarrow s; q \in Q; d \in \Sigma: & \text{none} \end{array}$$

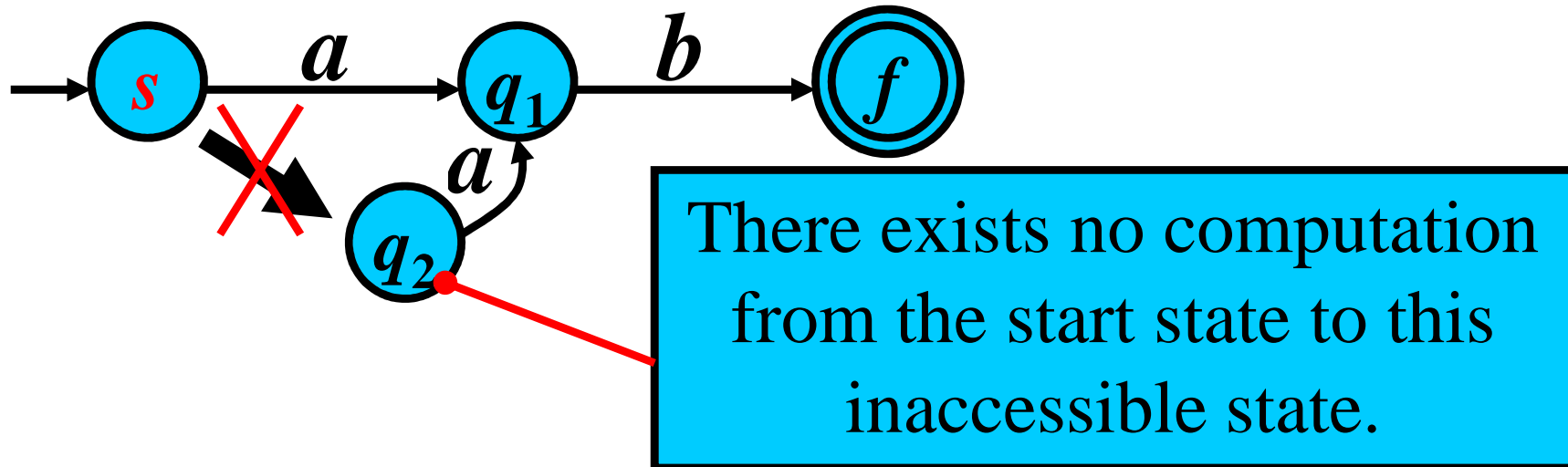
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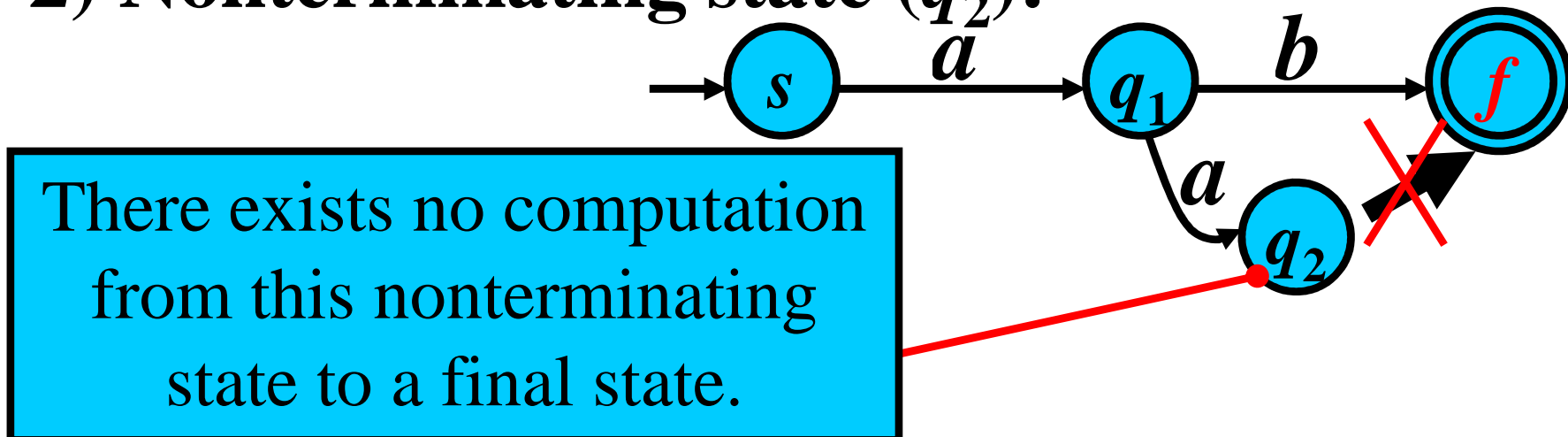
$$R_t = \{sa \rightarrow q_1, sb \not\rightarrow q_2, q_1a \not\rightarrow q_2, q_1b \rightarrow f\}$$

# Summary: States to Remove

## 1) Inaccessible state ( $q_2$ ):



## 2) Nonterminating state ( $q_2$ ):

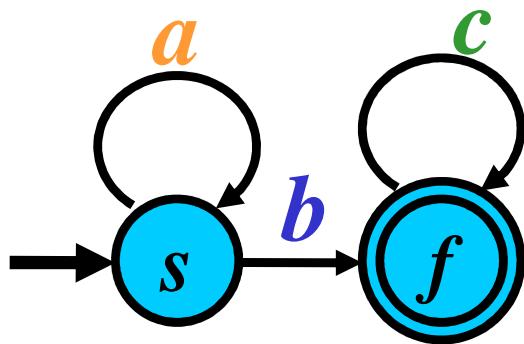


# Complete DFA

**Gist: Complete DFA cannot get stuck.**

**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be a **DFA**.  $M$  is *complete*, if for any  $p \in Q$ ,  $a \in \Sigma$  there is exactly one rule of the form  $pa \rightarrow q \in R$  for some  $q \in Q$ ; otherwise,  $M$  is *incomplete*

**Conversion:** Incomplete DFA



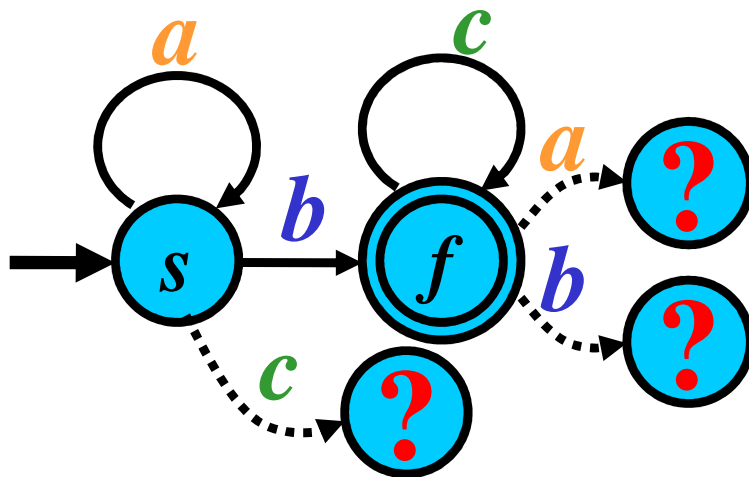
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$$\Sigma = \{a, b, c\}$$

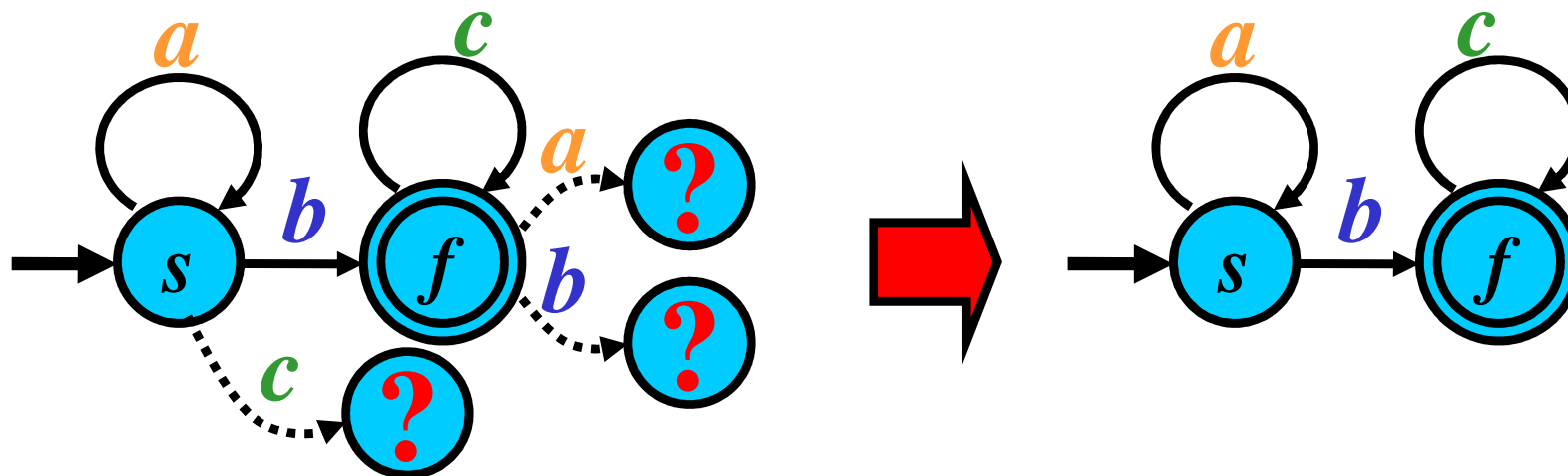
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**Conversion:** Incomplete DFA

to Complete DFA



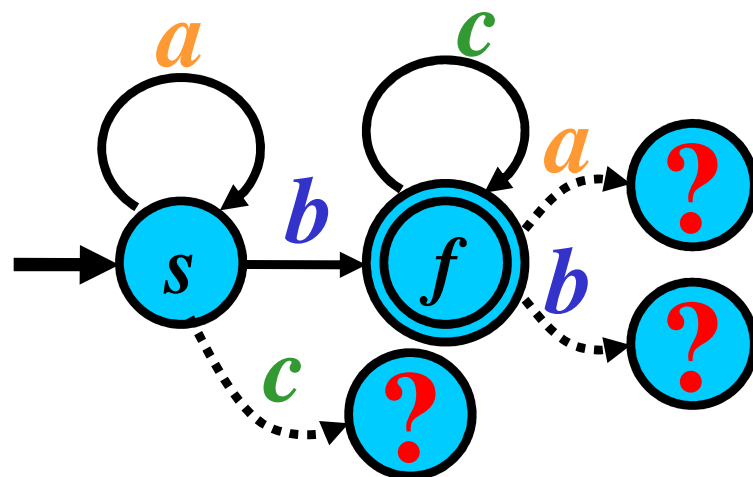
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# Complete DFA

**Gist: Complete DFA cannot get stuck.**

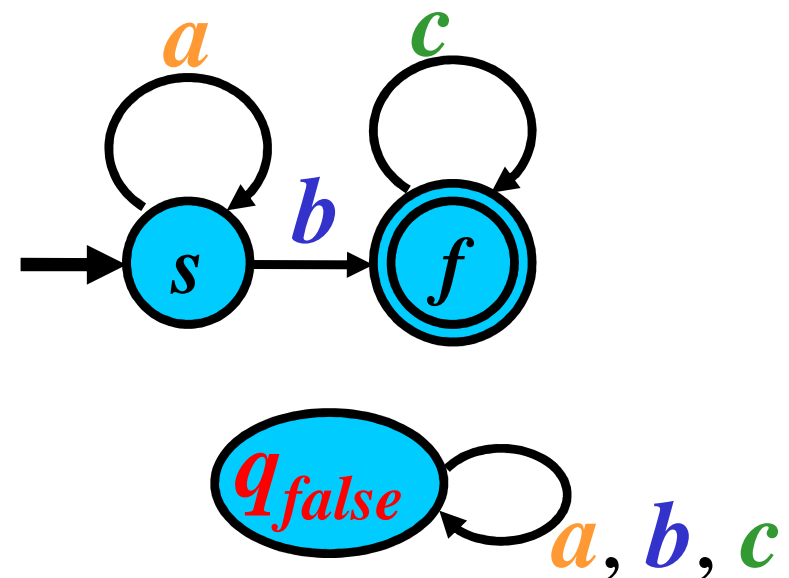
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$\Sigma = \{a, b, c\}$

to Complete DFA

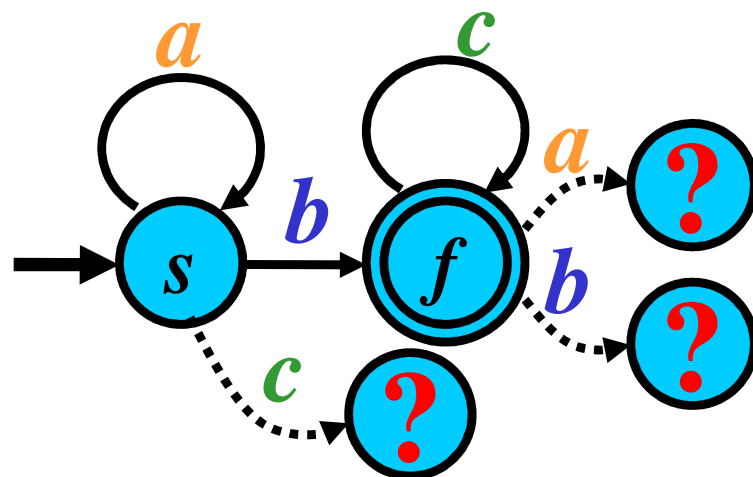


# Complete DFA

**Gist: Complete DFA cannot get stuck.**

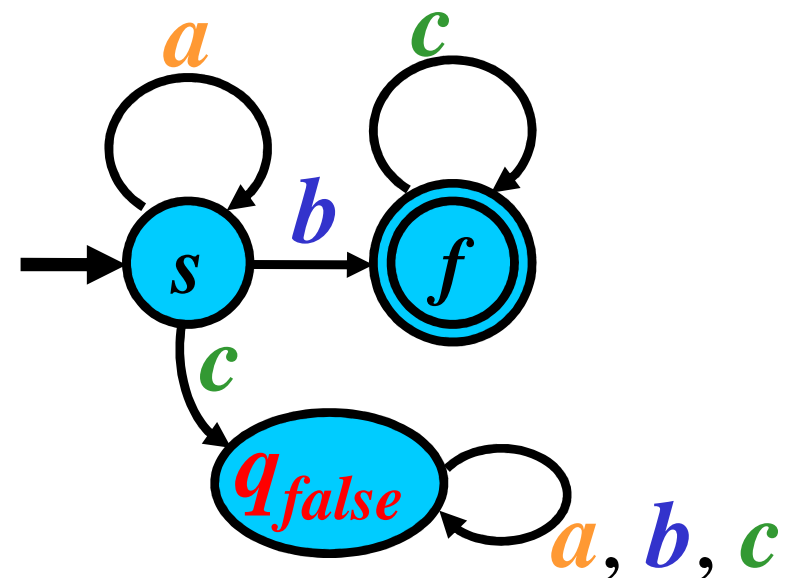
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**Conversion:** Incomplete DFA



$\Sigma = \{a, b, c\}$

to Complete DFA



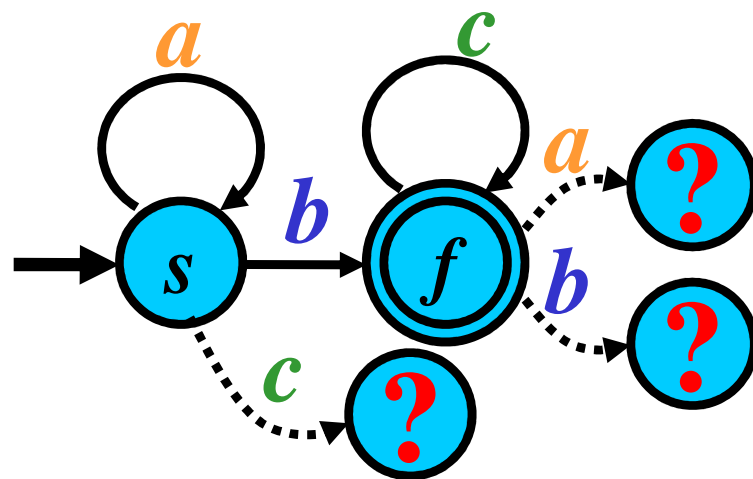


# Complete DFA

**Gist: Complete DFA cannot get stuck.**

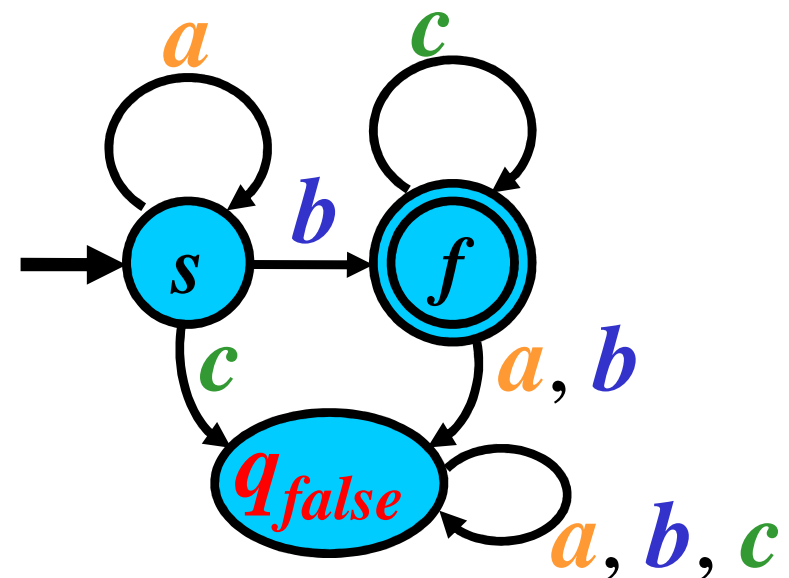
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**Conversion:** Incomplete DFA



$\Sigma = \{a, b, c\}$

to Complete DFA



# Algorithm: DFA to Complete DFA

**Gist: Add a “trap” state**

---

- **Input:** Incomplete DFA  $M = (Q, \Sigma, R, s, F)$
  - **Output:** Complete DFA  $M_c = (Q_c, \Sigma, R_c, s, F)$
- 

- **Method:**

- $Q_c := Q \cup \{q_{false}\};$
- $R_c := R \cup \{qa \rightarrow q_{false} : a \in \Sigma, q \in Q_c,$   
 $qa \rightarrow p \notin R, p \in Q\}.$

## Well-Specified FA

**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be a **complete DFA**. Then,  $M$  is *well-specified FA* (WSFA) if:

- 1)  $Q$  has no inaccessible state
- 2)  $Q$  has no more than one nonterminating state

---

**Note:** If well-specified FA has one nonterminating state, then it is  $q_{false}$  from the previous algorithm.

---

**Theorem:** For every FA  $M$ , there is an equivalent WSFA  $M_{ws}$ .

**Proof:** Use the next algorithm.

# Algorithm: FA to WSFA

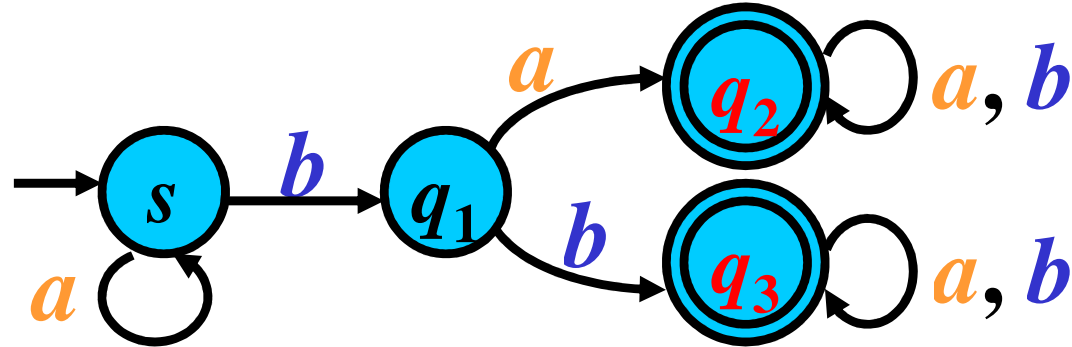
- **Input:** FA  $M$
  - **Output:** WSFA  $M_{ws}$
- 
- **Method:**
    - convert a FA  $M$  to an equivalent  $\varepsilon$ -free FA  $M'$
    - convert a  $M'$  to an equivalent DFA  $M_d$  without any inaccessible state
    - convert  $M_d$  to an equivalent DFA  $M_t$  without any nonterminating state
    - convert  $M_t$  to an equivalent complete FA  $M_c$
    - $M_{ws} := M_c$
- Note:** No more than one nonterminating state in  $M_{ws}$  —  $q_{false}$

## Distinguishable States

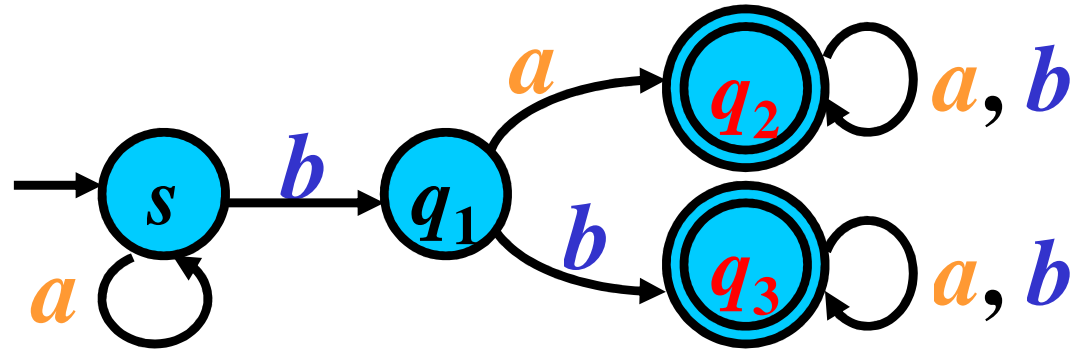
**Gist:** String  $w$  *distinguishes* states  $p$  and  $q$  if WSFA reaches a final state from precisely one of configurations  $pw$  and  $qw$ .

**Definition:** Let  $M = (Q, \Sigma, R, s, F)$  be a WSFA, and let  $p, q \in Q, p \neq q$ . States  $p$  and  $q$  are *distinguishable* if there exists  $w \in \Sigma^*$  such that:  $pw \vdash^* p'$  and  $qw \vdash^* q'$ , where  $p', q' \in Q$  and  $((p' \in F \text{ and } q' \notin F) \text{ or } (p' \notin F \text{ and } q' \in F))$ ; otherwise, states  $p$  and  $q$  are *indistinguishable*.

# Distinguishable States: Example



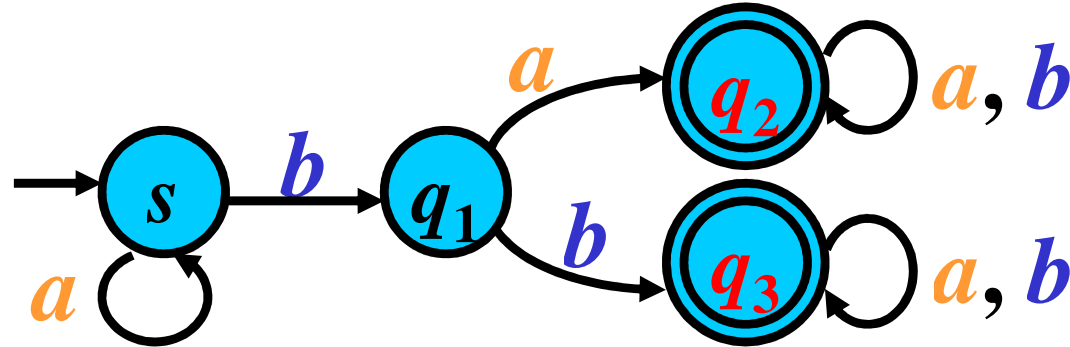
# Distinguishable States: Example



- $s$  and  $q_1$  are **distinguishable**, because for  $w = a$ :

$$\begin{aligned} sa &\vdash s, s \notin F \\ q_1 a &\vdash q_2, q_2 \in F \end{aligned}$$

# Distinguishable States: Example



- $s$  and  $q_1$  are **distinguishable**, because for  $w = a$ :

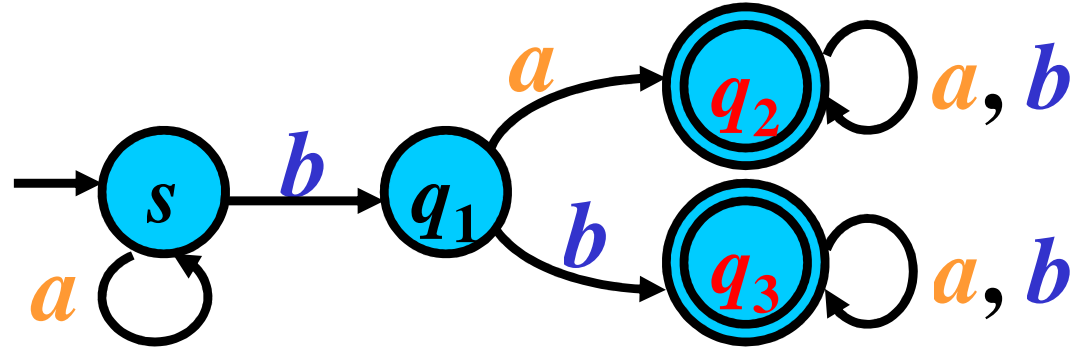
$$\begin{aligned} sa &\vdash s, s \notin F \\ q_1 a &\vdash q_2, q_2 \in F \end{aligned}$$

- $q_2$  and  $q_3$  are **indistinguishable**, because for each  $w \in \Sigma^*$ :

$$\begin{aligned} q_2 w &\vdash^* q_2, q_2 \in F \\ q_3 w &\vdash^* q_3, q_3 \in F \end{aligned}$$



# Distinguishable States: Example



- $s$  and  $q_1$  are **distinguishable**, because for  $w = a$ :

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$$\begin{aligned} q_2 w &\vdash^* q_2, q_2 \in F \\ q_3 w &\vdash^* q_3, q_3 \in F \end{aligned}$$

- Other pairs of states are trivially **distinguishable** for  $w = \varepsilon$ .

## Minimum-State FA

**Definition:** Let  $M$  be a WSFA. Then,  $M$  is *minimum-state FA* if  $M$  contains only distinguishable states.

**Theorem:** For every WSFA  $M$ , there is an equivalent minimum-state FA  $M_m$

**Proof:** Use the next algorithm.

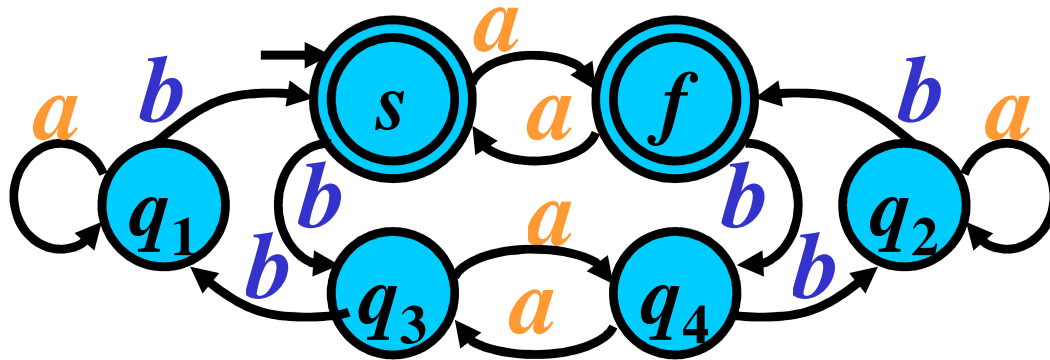
# Algorithm: WSFA to Min-State FA

- **Input:** WSFA  $M = (Q, \Sigma, R, s, F)$
- **Output:** Minimum-State FA  $M_m = (Q_m, \Sigma, R_m, s_m, F_m)$
- **Method:**
  - $Q_m = \{\{p: p \in F\}, \{q: q \in Q - F\}\};$
  - **repeat**
    - if there exist**  $X \in Q_m, d \in \Sigma, X_1, X_2 \subset X$  such that
 
$$X = X_1 \cup X_2, X_1 \cap X_2 = \emptyset \text{ and}$$

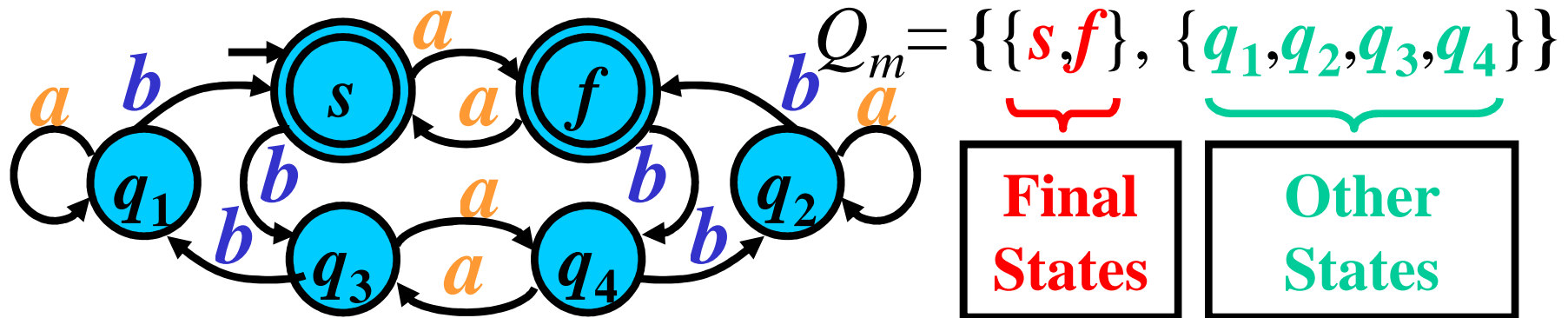
$$\{q_1: p_1 \in X_1, p_1 d \rightarrow q_1 \in R\} \subseteq Q_1, Q_1 \in Q_m,$$

$$\{q_2: p_2 \in X_2, p_2 d \rightarrow q_2 \in R\} \cap Q_1 = \emptyset$$
    - then** divide  $X$  into  $X_1$  and  $X_2$  in  $Q_m$
  - until** no division is possible;
  - $R_m = \{Xa \rightarrow Y: X, Y \in Q_m, pa \rightarrow q \in R, p \in X, q \in Y, a \in \Sigma\};$
  - $s_m = X$  with  $s \in X; F_m := \{X: X \in Q_m, X \cap F \neq \emptyset\}.$

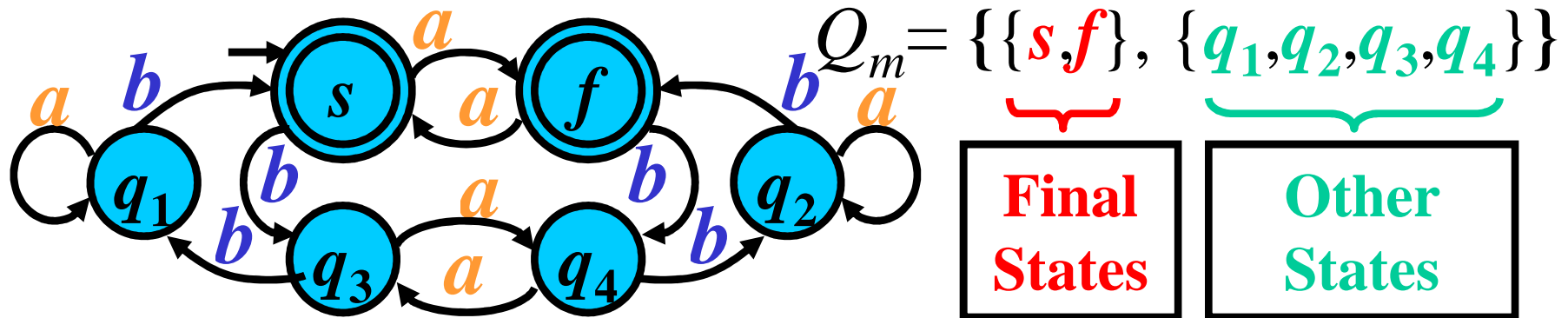
# Minimization: Example 1/4



# Minimization: Example 1/4



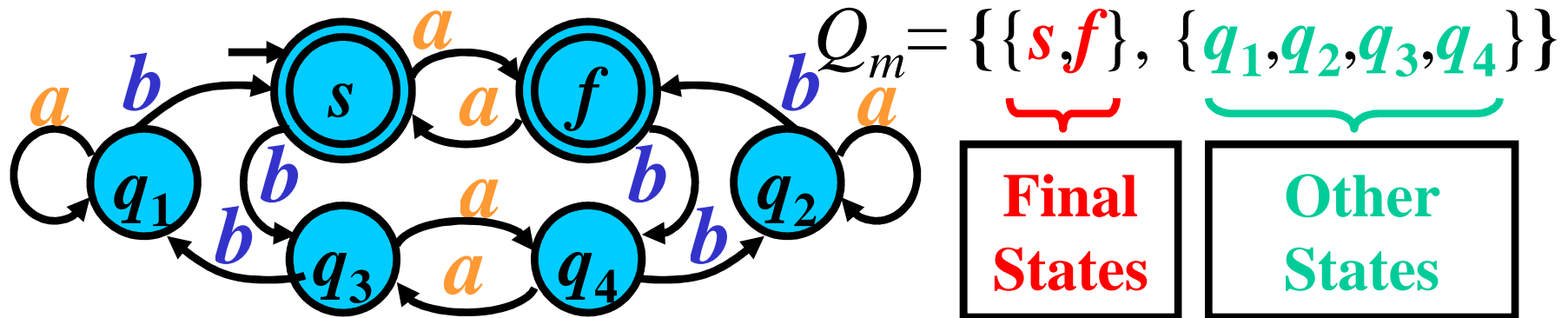
# Minimization: Example 1/4



1)  $X = \{s, f\}$ :

$d = a$ :  $sa \rightarrow f$   
 $fa \rightarrow s$

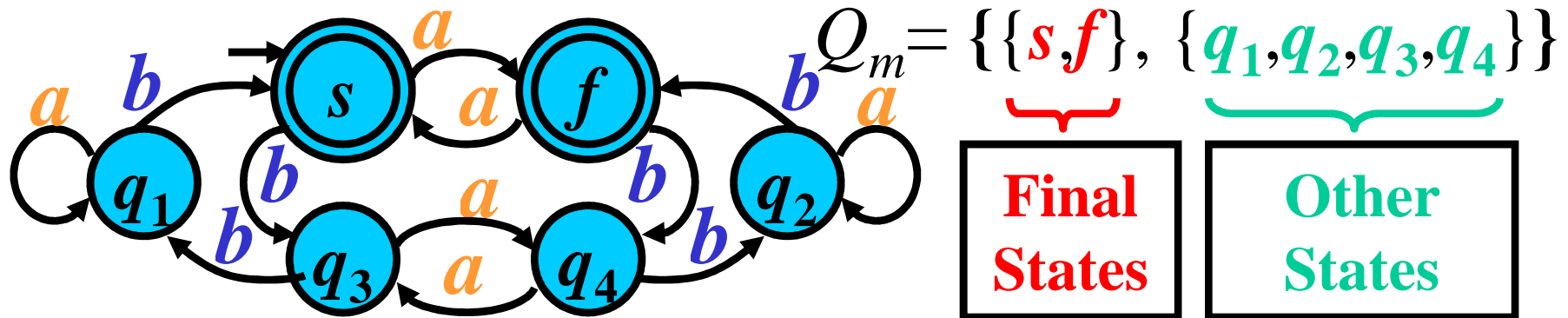
# Minimization: Example 1/4



1)  $X = \{s, f\}$ : From one set

$d = a$ :  $sa \rightarrow f$   
 $fa \rightarrow s$

# Minimization: Example 1/4



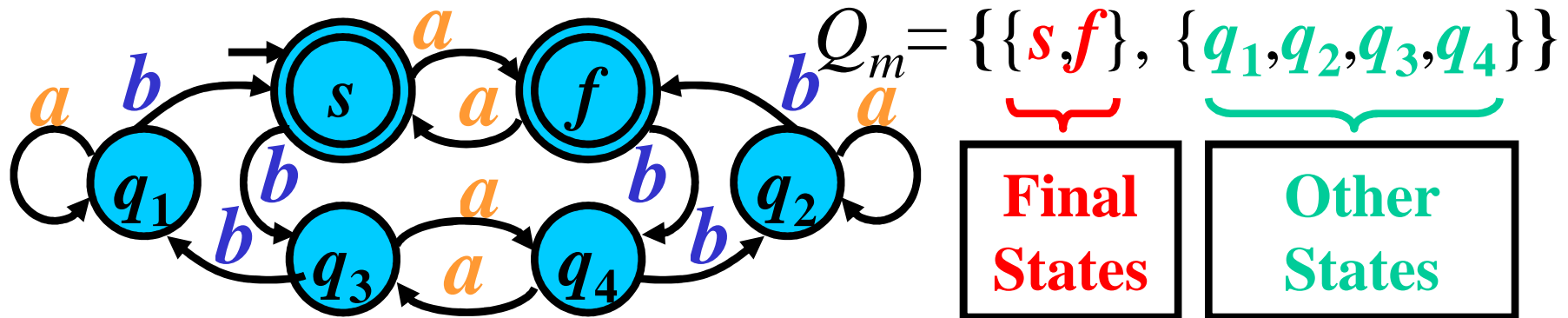
1)  $X = \{s, f\}$ : From one set

$d = a$ :  $sa \rightarrow f$   
 $fa \rightarrow s$

$d = b$ :  $sb \rightarrow q_3$   
 $fb \rightarrow q_4$



# Minimization: Example 1/4



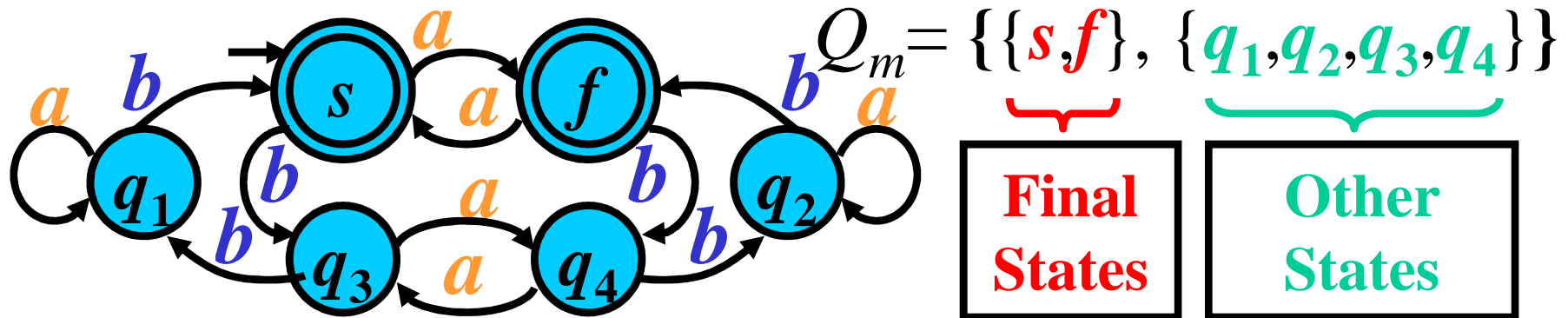
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 $fa \rightarrow s$

From one set

$d = b$ :  $sb \rightarrow q_3$   
 $fb \rightarrow q_4$

# Minimization: Example 1/4



1)  $X = \{s, f\}$ : **From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

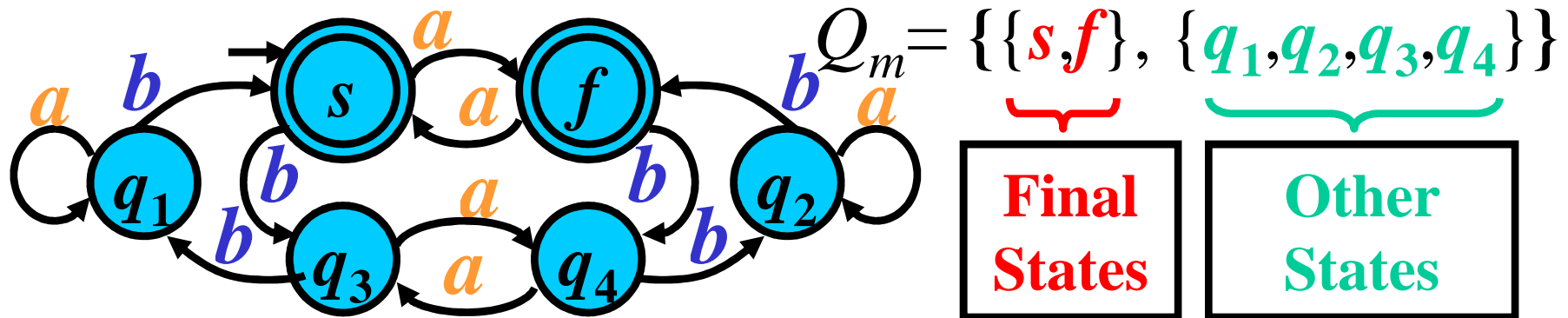
**From one set**

2)  $X = \{q_1, q_2, q_3, q_4\}$ :

$d = a$ :

- $q_1 a \rightarrow q_1$
- $q_2 a \rightarrow q_2$
- $q_3 a \rightarrow q_4$
- $q_4 a \rightarrow q_3$

# Minimization: Example 1/4



1)  $X = \{s, f\}$ : From one set

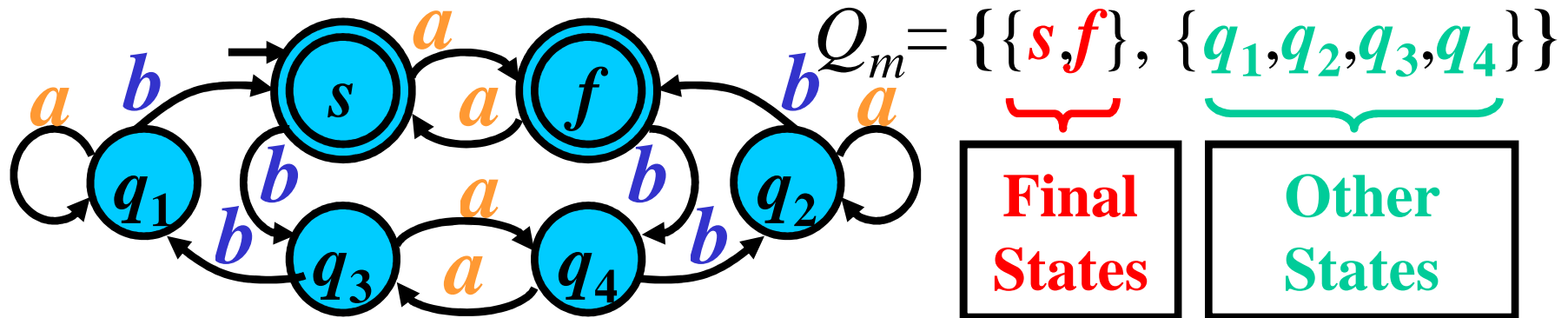
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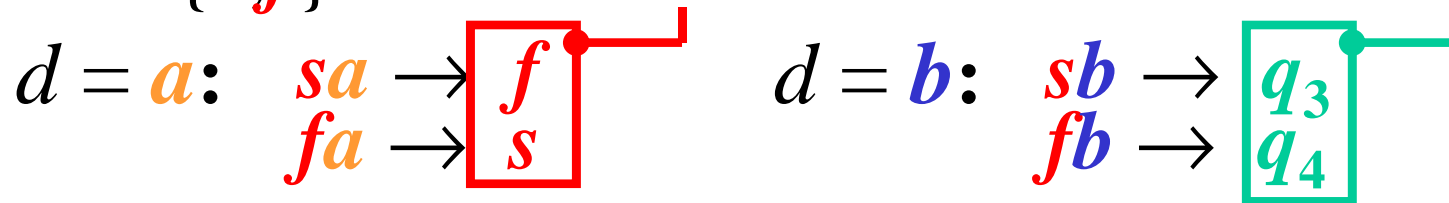
2)  $X = \{q_1, q_2, q_3, q_4\}$ : From one set

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$ ,  $q_3a \rightarrow q_4$ ,  $q_4a \rightarrow q_3$

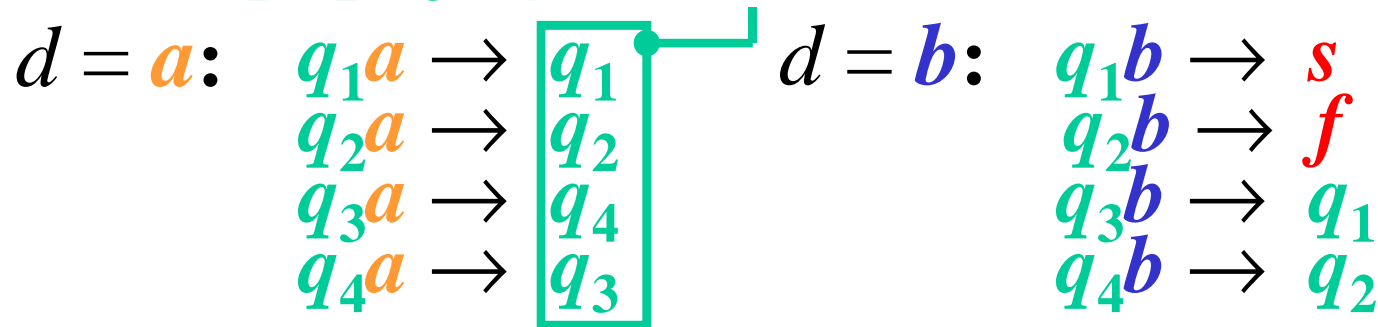
# Minimization: Example 1/4



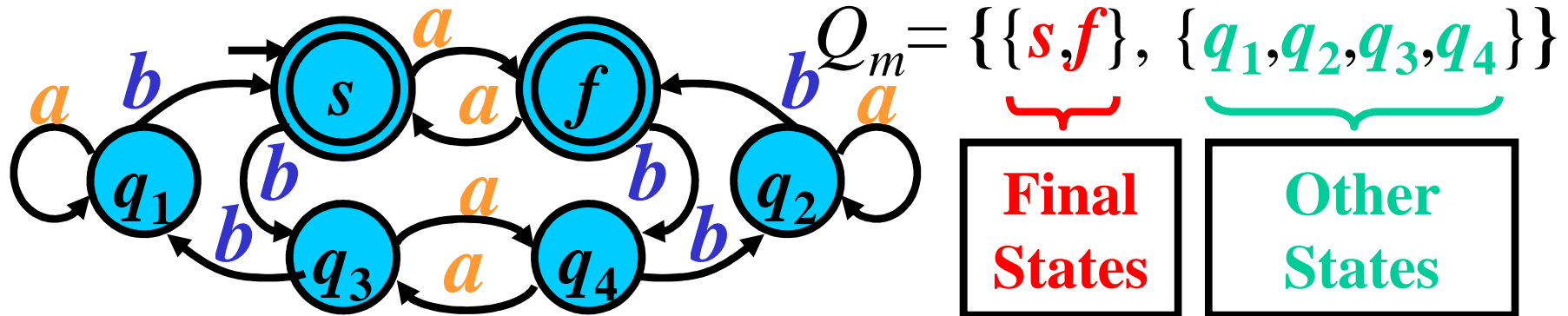
1)  $X = \{s, f\}$ :      **From one set**      **From one set**



2)  $X = \{q_1, q_2, q_3, q_4\}$ :      **From one set**



# Minimization: Example 1/4



1)  $X = \{s, f\}$ :      **From one set**      **From one set**

$d = a$ :  $sa \rightarrow f$   
 $fa \rightarrow s$

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 $fb \rightarrow q_4$

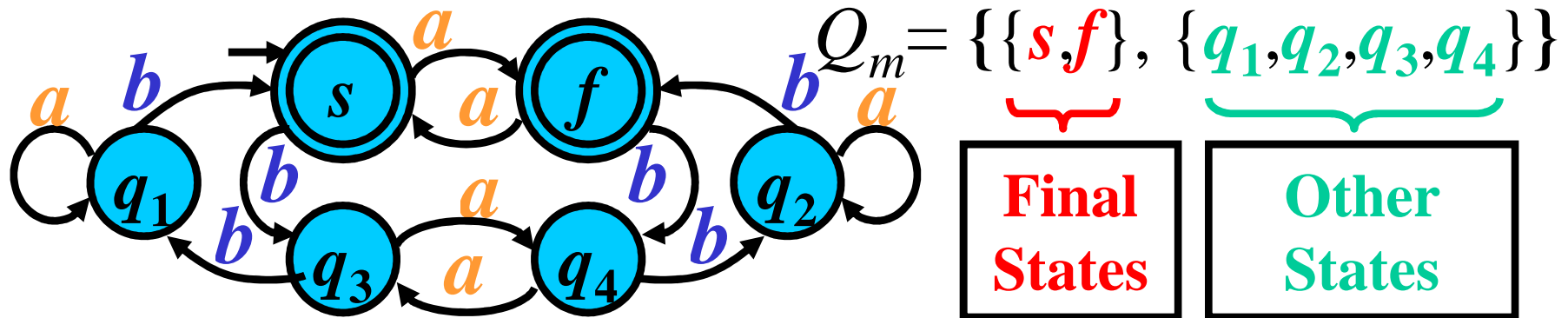
2)  $X = \{q_1, q_2, q_3, q_4\}$ :      **From one set**

$d = a$ :  $q_1 a \rightarrow q_1$   
 $q_2 a \rightarrow q_2$   
 $q_3 a \rightarrow q_4$   
 $q_4 a \rightarrow q_3$

$d = b$ :  $q_1 b \rightarrow s$   
 $q_2 b \rightarrow f$   
 $q_3 b \rightarrow q_1$   
 $q_4 b \rightarrow q_2$

$\subseteq Q_1 = \{s, f\}$

# Minimization: Example 1/4



1)  $X = \{s, f\}$ :      **From one set**      **From one set**

$d = a$ :  $sa \rightarrow f$   
 $fa \rightarrow s$

$d = b$ :  $sb \rightarrow q_3$   
 $fb \rightarrow q_4$

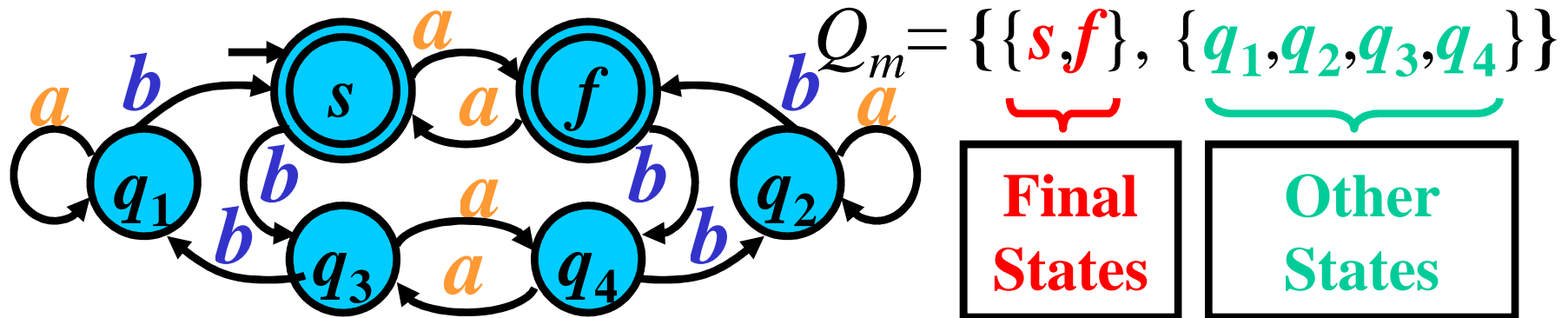
2)  $X = \{q_1, q_2, q_3, q_4\}$ :      **From one set**

$d = a$ :  $q_1 a \rightarrow q_1$   
 $q_2 a \rightarrow q_2$   
 $q_3 a \rightarrow q_4$   
 $q_4 a \rightarrow q_3$

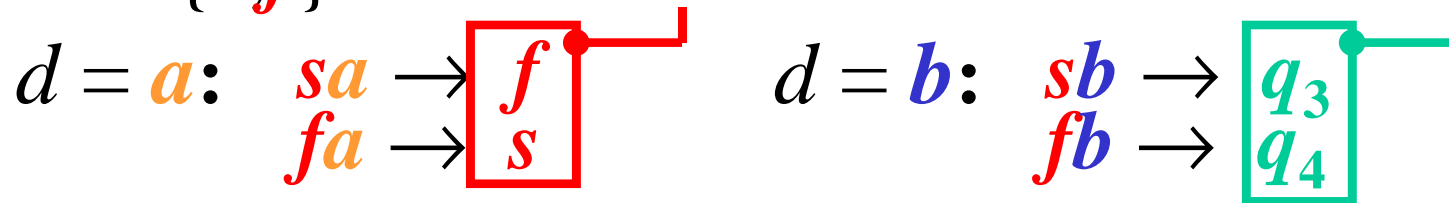
$d = b$ :  $q_1 b \rightarrow s$   
 $q_2 b \rightarrow f$   
 $q_3 b \rightarrow q_1$   
 $q_4 b \rightarrow q_2$

$$\{q_1, q_2\} \cap Q_1 = \emptyset$$

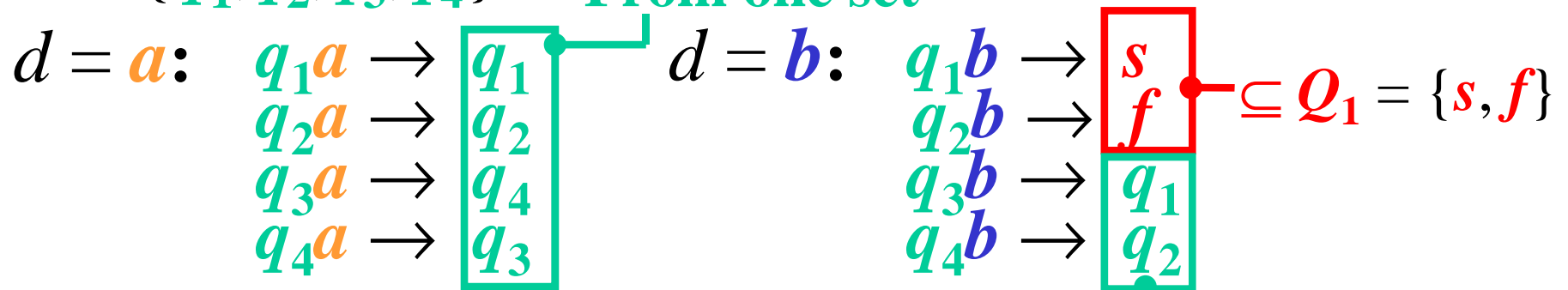
# Minimization: Example 1/4



1)  $X = \{s, f\}$ :      **From one set**      **From one set**



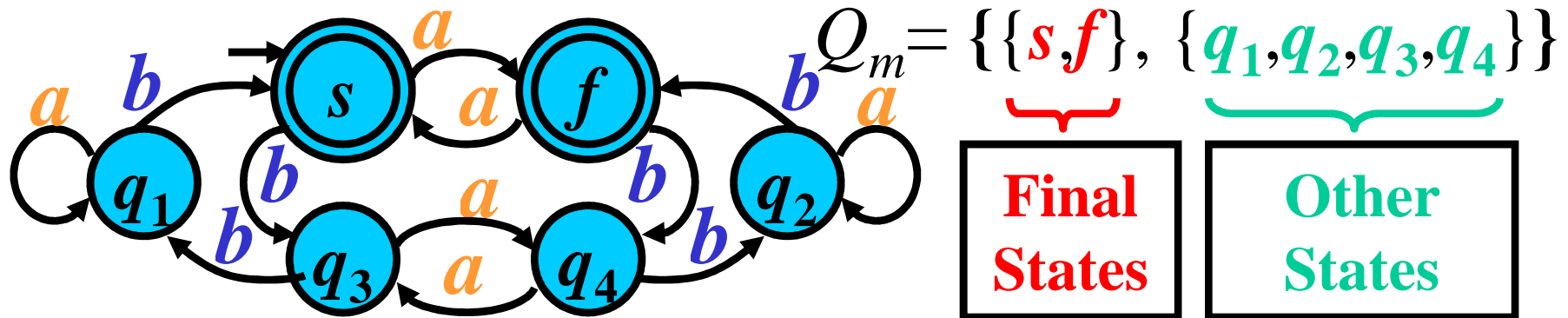
2)  $X = \{q_1, q_2, q_3, q_4\}$ :      **From one set**



**Division:**  $\{q_1, q_2, q_3, q_4\} \Rightarrow$

$$\{q_1, q_2\} \cap Q_1 = \emptyset$$

# Minimization: Example 1/4



1)  $X = \{s, f\}$ :      **From one set**      **From one set**

$d = a$ :  $sa \rightarrow f$   
 $fa \rightarrow s$

$d = b$ :  $sb \rightarrow q_3$   
 $fb \rightarrow q_4$

2)  $X = \{q_1, q_2, q_3, q_4\}$ :      **From one set**

$d = a$ :  $q_1a \rightarrow q_1$   
 $q_2a \rightarrow q_2$   
 $q_3a \rightarrow q_4$   
 $q_4a \rightarrow q_3$

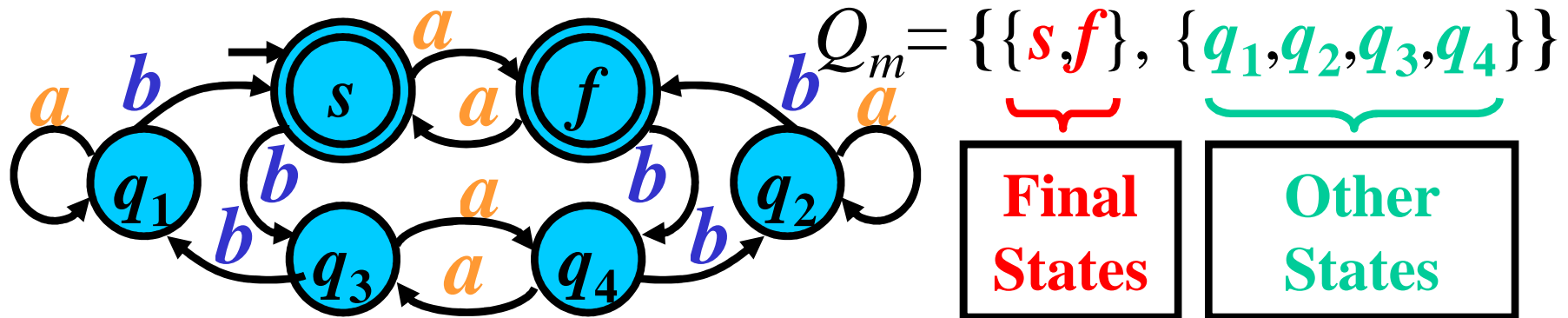
$d = b$ :  $q_1b \rightarrow s$   
 $q_2b \rightarrow f$   
 $q_3b \rightarrow q_1$   
 $q_4b \rightarrow q_2$

**Division:**  $\{q_1, q_2, q_3, q_4\} \Rightarrow \underbrace{\{q_1, q_2\}}_{X_1},$

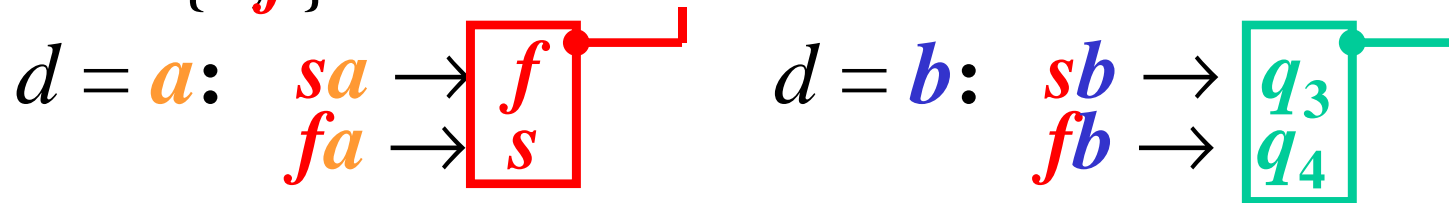
$\{q_1, q_2\} \cap Q_1 = \emptyset$



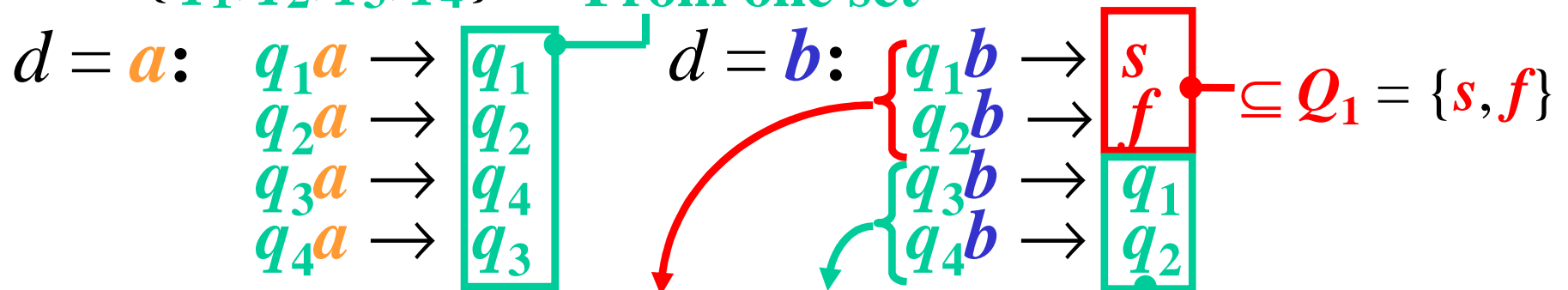
# Minimization: Example 1/4



1)  $X = \{s, f\}$ :      **From one set**      **From one set**



2)  $X = \{q_1, q_2, q_3, q_4\}$ :      **From one set**



**Division:**  $\{q_1, q_2, q_3, q_4\} \Rightarrow \underbrace{\{q_1, q_2\}}_{X_1}, \underbrace{\{q_3, q_4\}}_{X_2}$

$\{q_1, q_2\} \cap Q_1 = \emptyset$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

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# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

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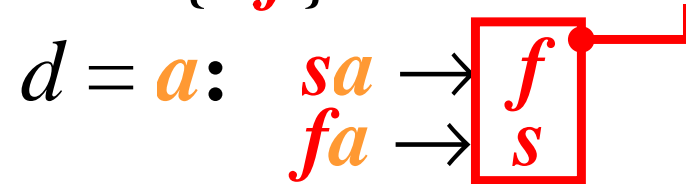
1)  $X = \{s, f\}$ :

$$d = a: \begin{array}{l} sa \rightarrow f \\ fa \rightarrow s \end{array}$$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

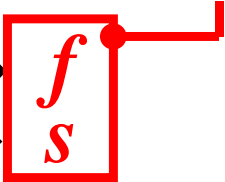
1)  $X = \{s, f\}$ : **From one set**



# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ : **From one set**

$$\begin{array}{ll}
 d = a: & \begin{array}{l} sa \rightarrow f \\ fa \rightarrow s \end{array} \\
 d = b: & \begin{array}{l} sb \rightarrow q_3 \\ fb \rightarrow q_4 \end{array}
 \end{array}$$


# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow \boxed{f}$   
 $fa \rightarrow \boxed{s}$

**From one set**

$d = b$ :  $sb \rightarrow \boxed{q_3}$   
 $fb \rightarrow \boxed{q_4}$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$   
 $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$   
 $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

$d = a$ :  $q_1 a \rightarrow q_1$   
 $q_2 a \rightarrow q_2$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ : **From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

**From one set**

2)  $X = \{q_1, q_2\}$ : **From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$



# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

**From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$

$d = b$ :  $q_1b \rightarrow s$ ,  $q_2b \rightarrow f$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

**From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$

**From one set**

$d = b$ :  $q_1b \rightarrow s$ ,  $q_2b \rightarrow f$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

**From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$

**From one set**

$d = b$ :  $q_1b \rightarrow s$ ,  $q_2b \rightarrow f$

3)  $X = \{q_3, q_4\}$ :

$d = a$ :  $q_3a \rightarrow q_3$ ,  $q_4a \rightarrow q_4$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

**From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$

**From one set**

$d = b$ :  $q_1b \rightarrow s$ ,  $q_2b \rightarrow f$

3)  $X = \{q_3, q_4\}$ :

**From one set**

$d = a$ :  $q_3a \rightarrow q_3$ ,  $q_4a \rightarrow q_4$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

**From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$

**From one set**

$d = b$ :  $q_1b \rightarrow s$ ,  $q_2b \rightarrow f$

3)  $X = \{q_3, q_4\}$ :

**From one set**

$d = a$ :  $q_3a \rightarrow q_3$ ,  $q_4a \rightarrow q_4$

$d = b$ :  $q_3b \rightarrow q_1$ ,  $q_4b \rightarrow q_2$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

**From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$

**From one set**

$d = b$ :  $q_1b \rightarrow s$ ,  $q_2b \rightarrow f$

3)  $X = \{q_3, q_4\}$ :

**From one set**

$d = a$ :  $q_3a \rightarrow q_3$ ,  $q_4a \rightarrow q_4$

**From one set**

$d = b$ :  $q_3b \rightarrow q_1$ ,  $q_4b \rightarrow q_2$

# Minimization: Example 2/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

1)  $X = \{s, f\}$ :

**From one set**

$d = a$ :  $sa \rightarrow f$ ,  $fa \rightarrow s$

**From one set**

$d = b$ :  $sb \rightarrow q_3$ ,  $fb \rightarrow q_4$

2)  $X = \{q_1, q_2\}$ :

**From one set**

$d = a$ :  $q_1a \rightarrow q_1$ ,  $q_2a \rightarrow q_2$

**From one set**

$d = b$ :  $q_1b \rightarrow s$ ,  $q_2b \rightarrow f$

3)  $X = \{q_3, q_4\}$ :

**From one set**

$d = a$ :  $q_3a \rightarrow q_3$ ,  $q_4a \rightarrow q_4$

**From one set**

$d = b$ :  $q_3b \rightarrow q_1$ ,  $q_4b \rightarrow q_2$

**No next divisions !!!**

# Minimization: Example 3/4

$$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$$

$$\begin{array}{l} sa \rightarrow f \in R: \\ fa \rightarrow s \in R: \end{array} \} \Rightarrow \{s, f\}a \rightarrow \{s, f\} \in R_m$$

$$\begin{array}{l} sb \rightarrow q_3 \in R: \\ fb \rightarrow q_4 \in R: \end{array} \} \Rightarrow \{s, f\}b \rightarrow \{q_3, q_4\} \in R_m$$

$$\begin{array}{l} q_1a \rightarrow q_1 \in R: \\ q_2a \rightarrow q_2 \in R: \end{array} \} \Rightarrow \{q_1, q_2\}a \rightarrow \{q_1, q_2\} \in R_m$$

$$\begin{array}{l} q_1b \rightarrow s \in R: \\ q_2b \rightarrow f \in R: \end{array} \} \Rightarrow \{q_1, q_2\}b \rightarrow \{s, f\} \in R_m$$

$$\begin{array}{l} q_3a \rightarrow q_3 \in R: \\ q_4a \rightarrow q_4 \in R: \end{array} \} \Rightarrow \{q_3, q_4\}a \rightarrow \{q_3, q_4\} \in R_m$$

$$\begin{array}{l} q_3b \rightarrow q_1 \in R: \\ q_4b \rightarrow q_2 \in R: \end{array} \} \Rightarrow \{q_3, q_4\}b \rightarrow \{q_1, q_2\} \in R_m$$



# Minimization: Example 4/4

$$s \in \{s, f\} \Rightarrow s_m := \{s, f\}$$


---

$$\begin{matrix} s \in F: \\ f \in F: \end{matrix} \Rightarrow \{s, f\} \in F_m$$


---

$M_m = (Q_m, \Sigma, R_m, s_m, F_m)$ , where:  $\Sigma = \{a, b\}$ ,  $s_m = \{s, f\}$

$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$ ,  $F_m = \{\{s, f\}\}$

$R_m = \{\{s, f\}a \rightarrow \{s, f\}, \{s, f\}b \rightarrow \{q_3, q_4\}, \{q_1, q_2\}a \rightarrow \{q_1, q_2\},$   
 $\{q_1, q_2\}b \rightarrow \{s, f\}, \{q_3, q_4\}a \rightarrow \{q_3, q_4\}, \{q_3, q_4\}b \rightarrow \{q_1, q_2\}\}$

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# Minimization: Example 4/4

$$s \in \{s, f\} \Longrightarrow s_m := \{s, f\}$$

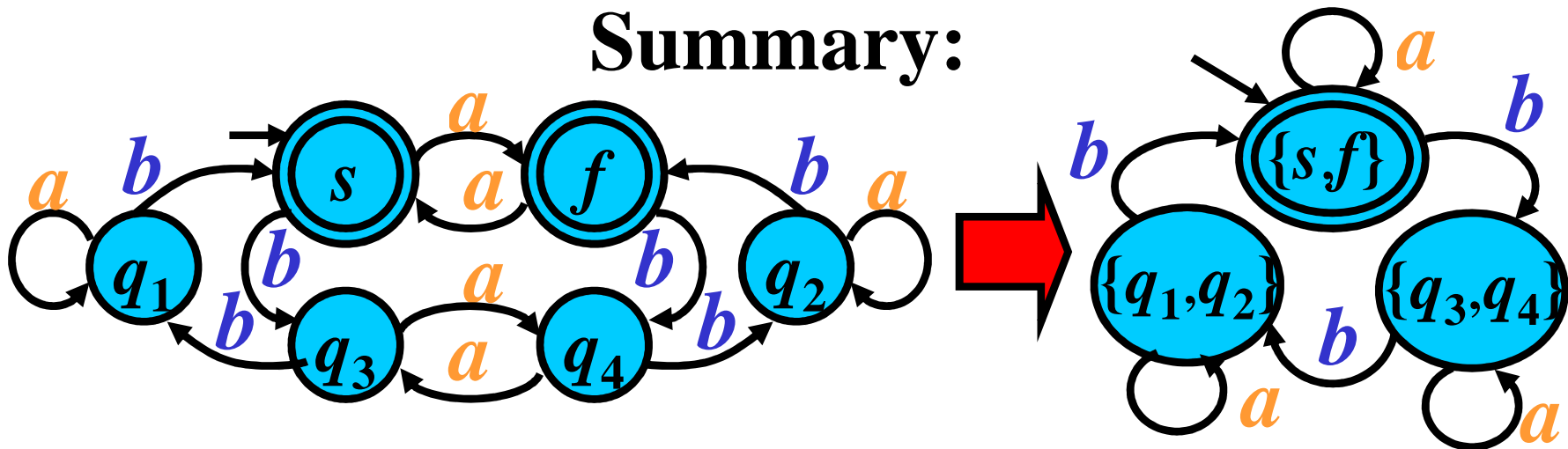
$$\left. \begin{array}{l} s \in F: \\ f \in F: \end{array} \right\} \Longrightarrow \{s, f\} \in F_m$$

$M_m = (Q_m, \Sigma, R_m, s_m, F_m)$ , where:  $\Sigma = \{a, b\}$ ,  $s_m = \{s, f\}$

$Q_m = \{\{s, f\}, \{q_1, q_2\}, \{q_3, q_4\}\}$ ,  $F_m = \{\{s, f\}\}$

$R_m = \{\{s, f\}a \rightarrow \{s, f\}, \{s, f\}b \rightarrow \{q_3, q_4\}, \{q_1, q_2\}a \rightarrow \{q_1, q_2\},$   
 $\{q_1, q_2\}b \rightarrow \{s, f\}, \{q_3, q_4\}a \rightarrow \{q_3, q_4\}, \{q_3, q_4\}b \rightarrow \{q_1, q_2\}\}$

Summary:



# Variants of FA: Summary

	FA	$\epsilon$ -free FA	DFA	Complete FA	WSFA	Min-State FA
Number of rules of the form $p \rightarrow q$ , where $p, q \in Q$	0- $n$	0	0	0	0	0
Number of rules of the form $pa \rightarrow q$ , for any $p \in Q, a \in \Sigma$	0- $n$	0- $n$	0-1	1	1	1
Number of inaccessible states	0- $n$	0- $n$	0- $n$	0- $n$	0	0
Number of nonterminating states	0- $n$	0- $n$	0- $n$	0- $n$	0-1	0-1
Number of this FAs for any regular language.	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	1