Formal Languages and Compilers

Alexander Meduna

&

Roman Lukáš

- These lecture notes are based on *Automata and Languages* by Alexander Meduna, Springer, 2000

Part I.
Alphabets, Strings, and Languages

Alphabets and symbols

Definition: An *alphabet* is a finite, nonempty set of elements, which are called *symbols*.

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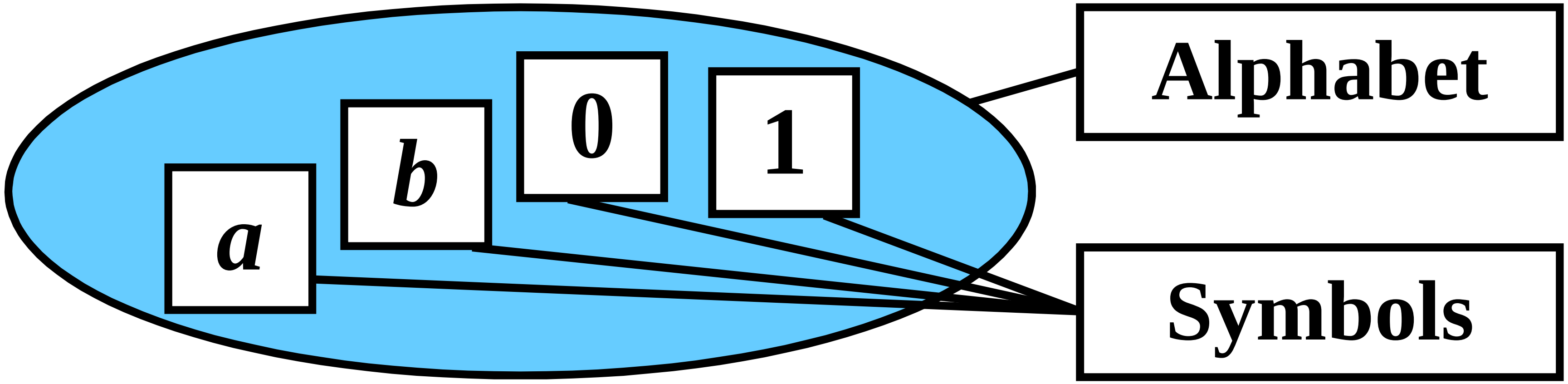
Example:



If we denote this alphabet as Σ, then **Σ = {*a*, *b*, 0, 1}**

String

Gist: $x = a_1a_2\ldots a_n$

Definition: Let Σ be an alphabet.

1) ε is a string over Σ

2) if x is a string over Σ and $a \in \Sigma$ then xa is a string over Σ

Note: ε denotes ***the empty string*** that contains no symbols.

Example: Consider $\Sigma = \{\mathbf{0}, \mathbf{1}\}$**:**

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Example: Consider Σ = {**0**, **1**}:

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Is string over Σ.

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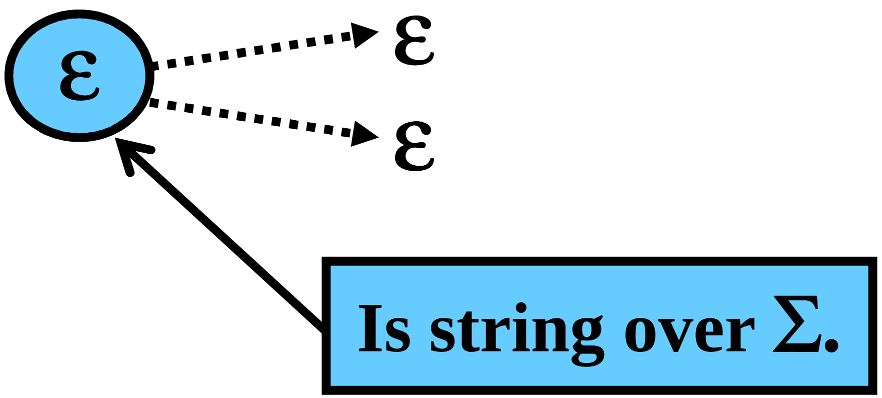

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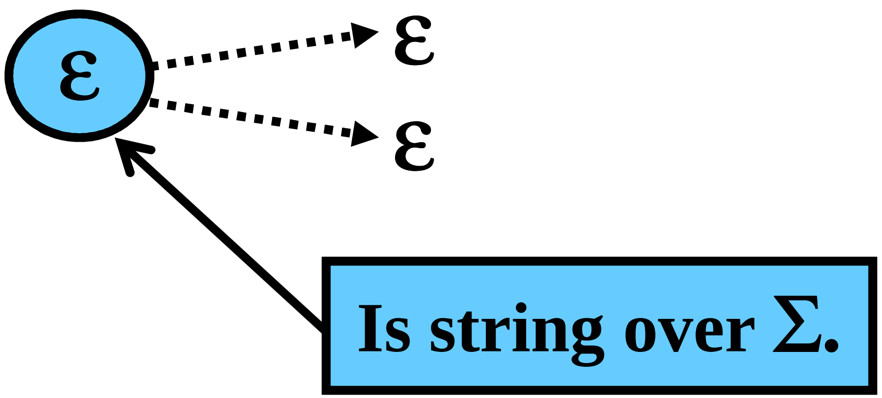

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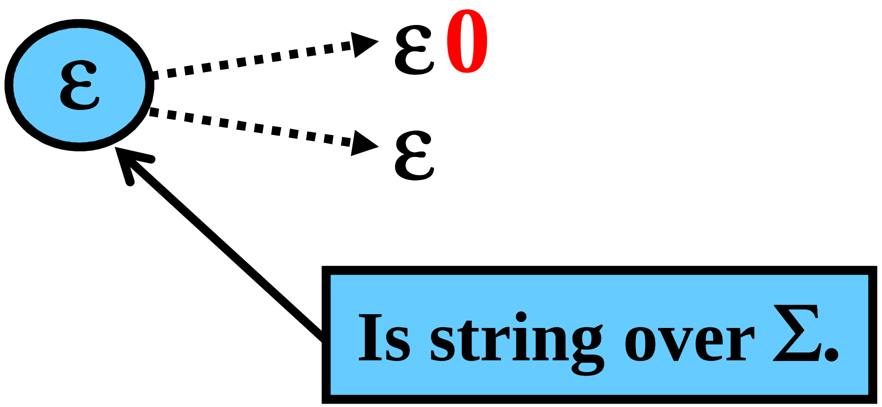

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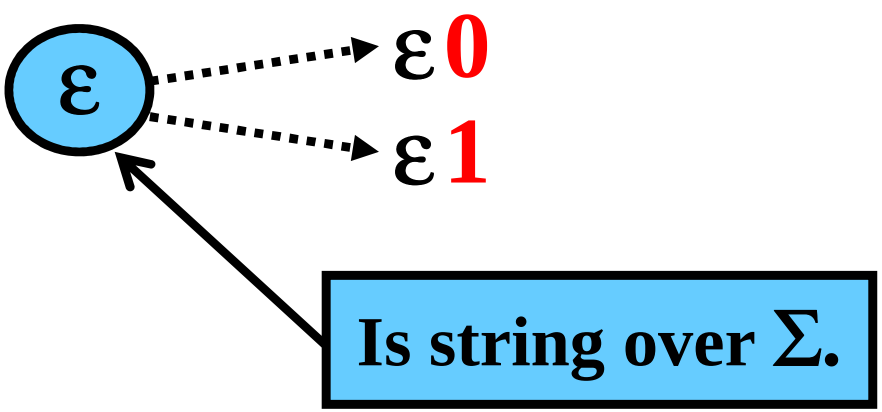

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ε

$\varepsilon\mathbf{0} = \mathbf{0}$

$\varepsilon\mathbf{1} = \mathbf{1}$

Is string over Σ.

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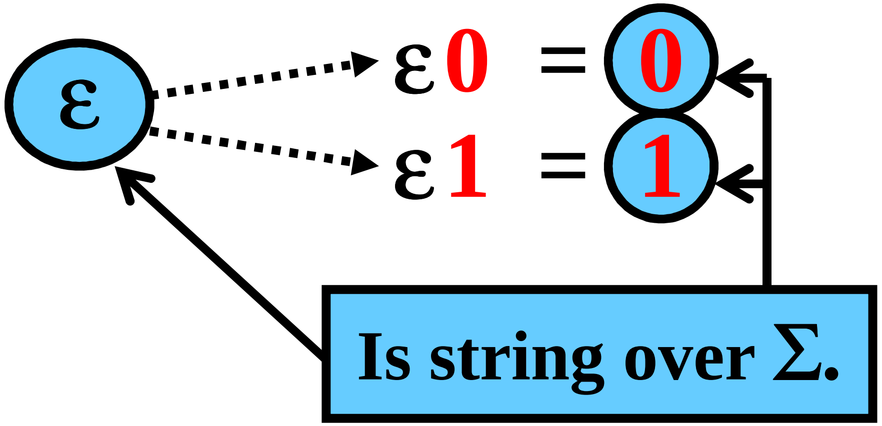

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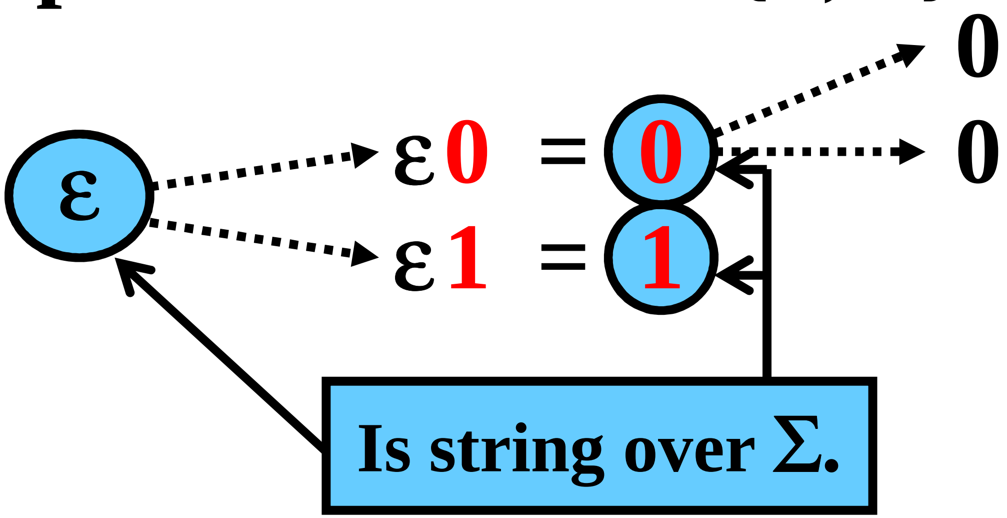

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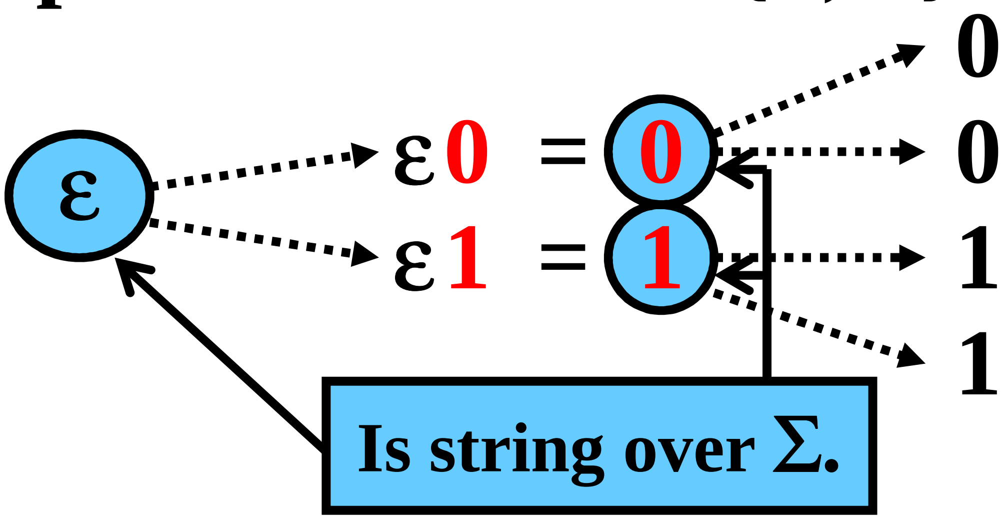

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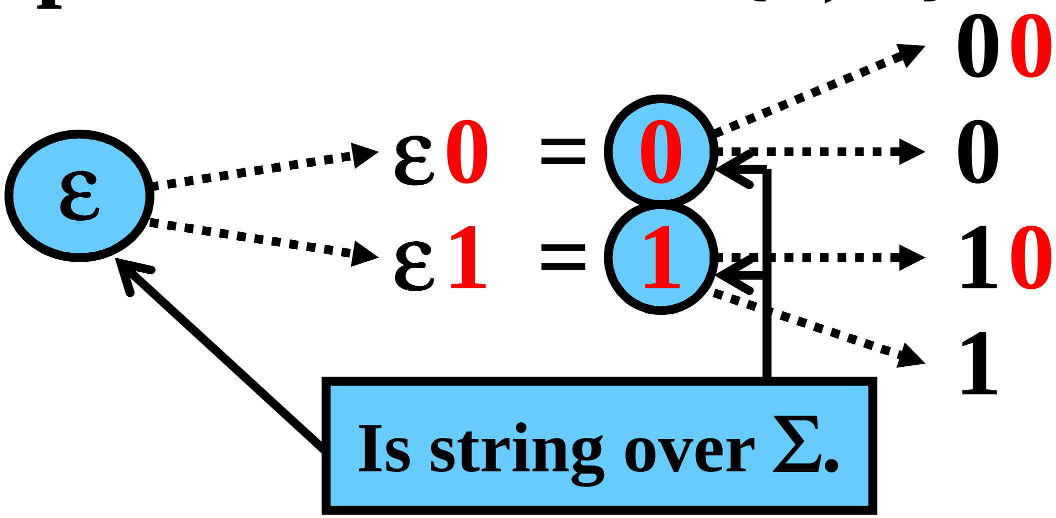

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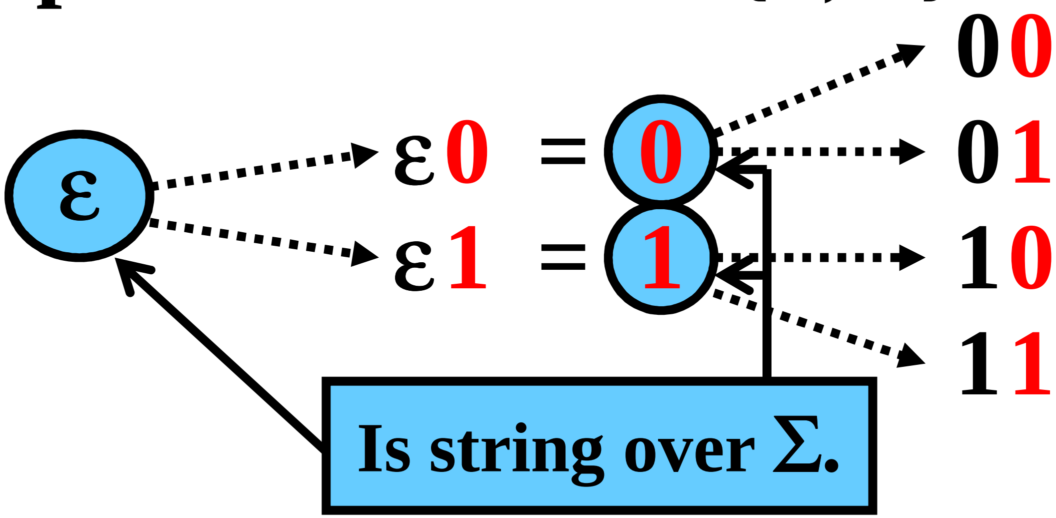

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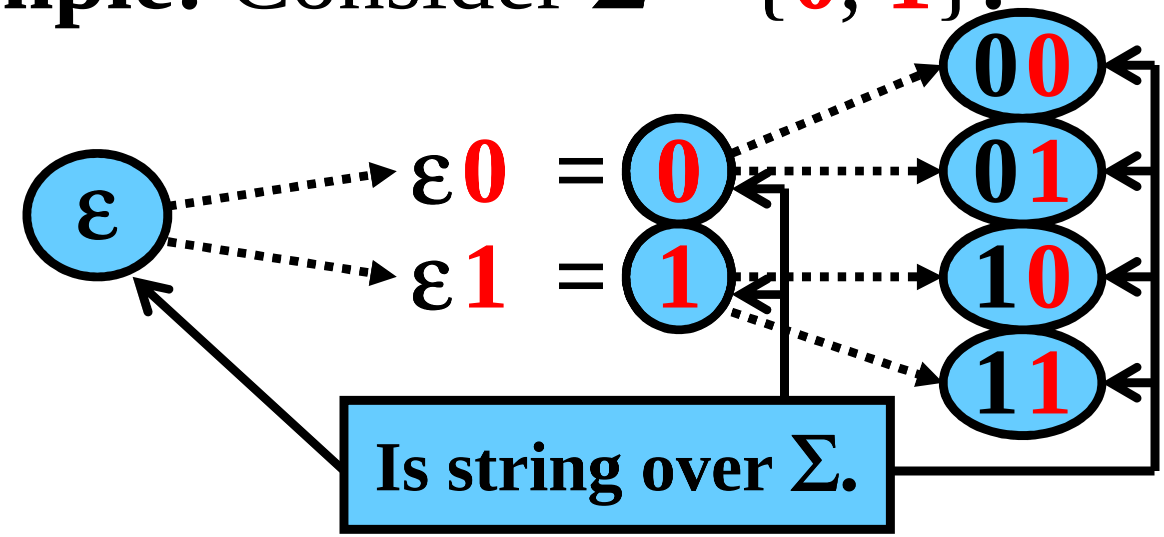

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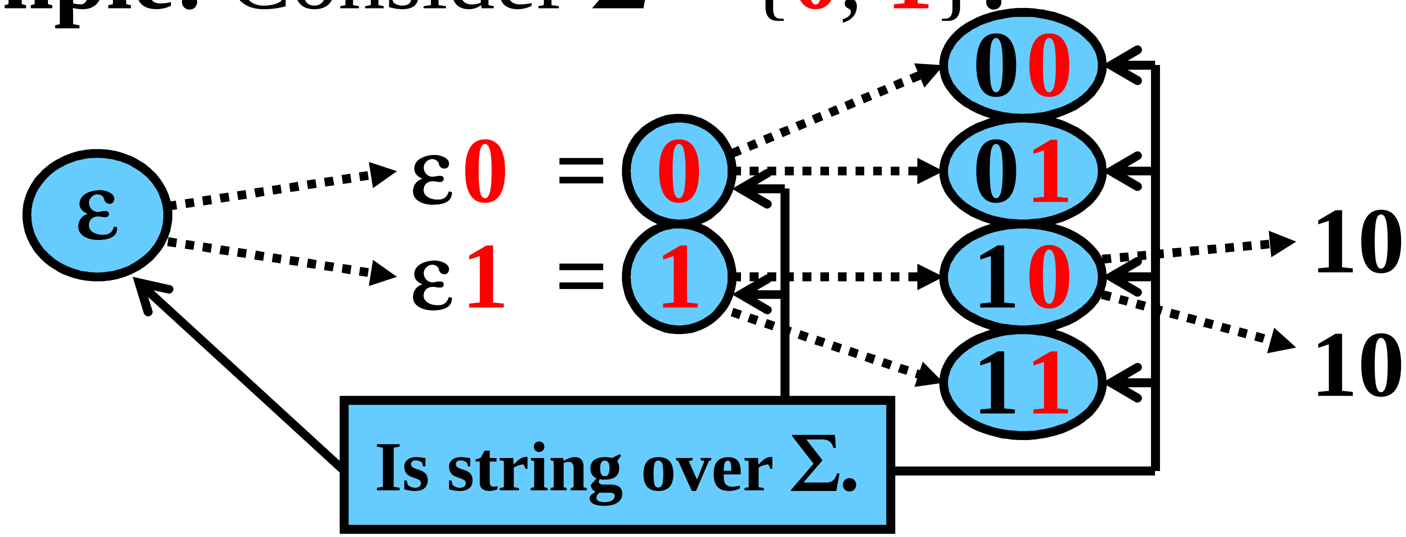

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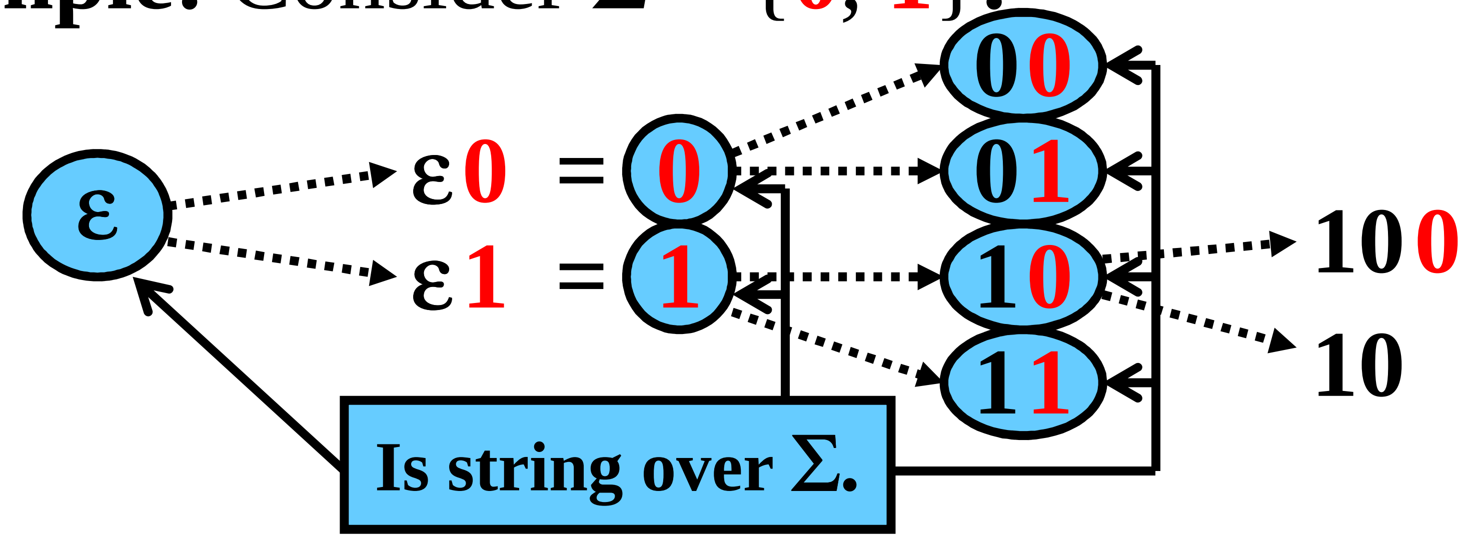

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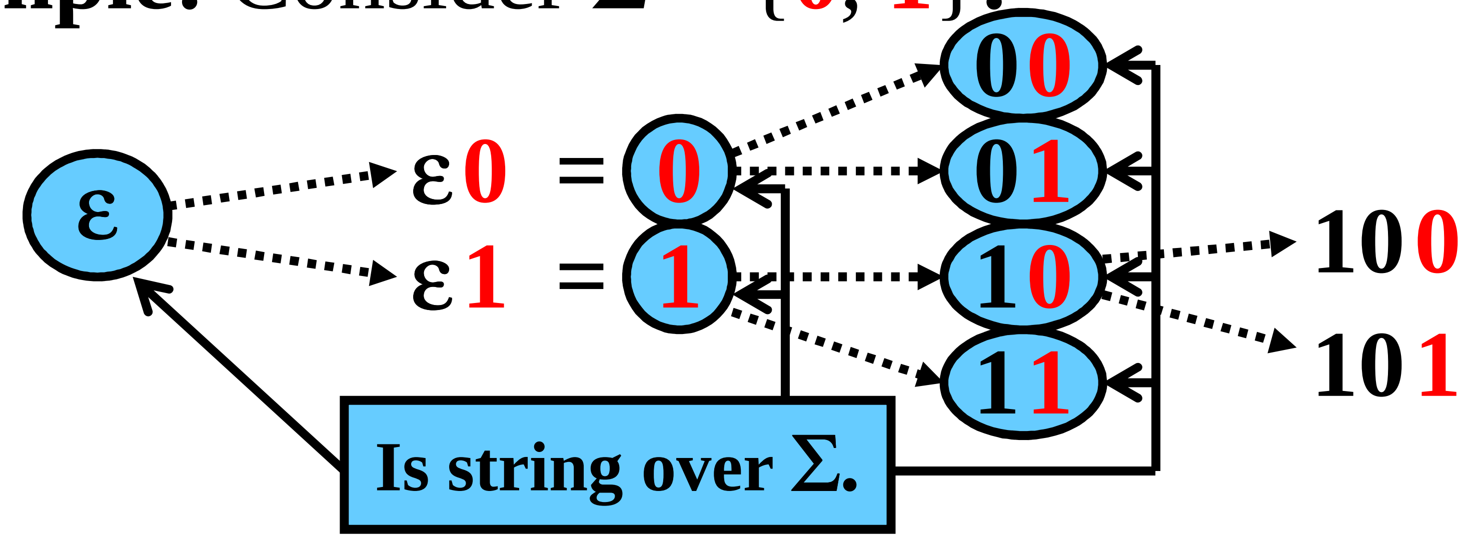

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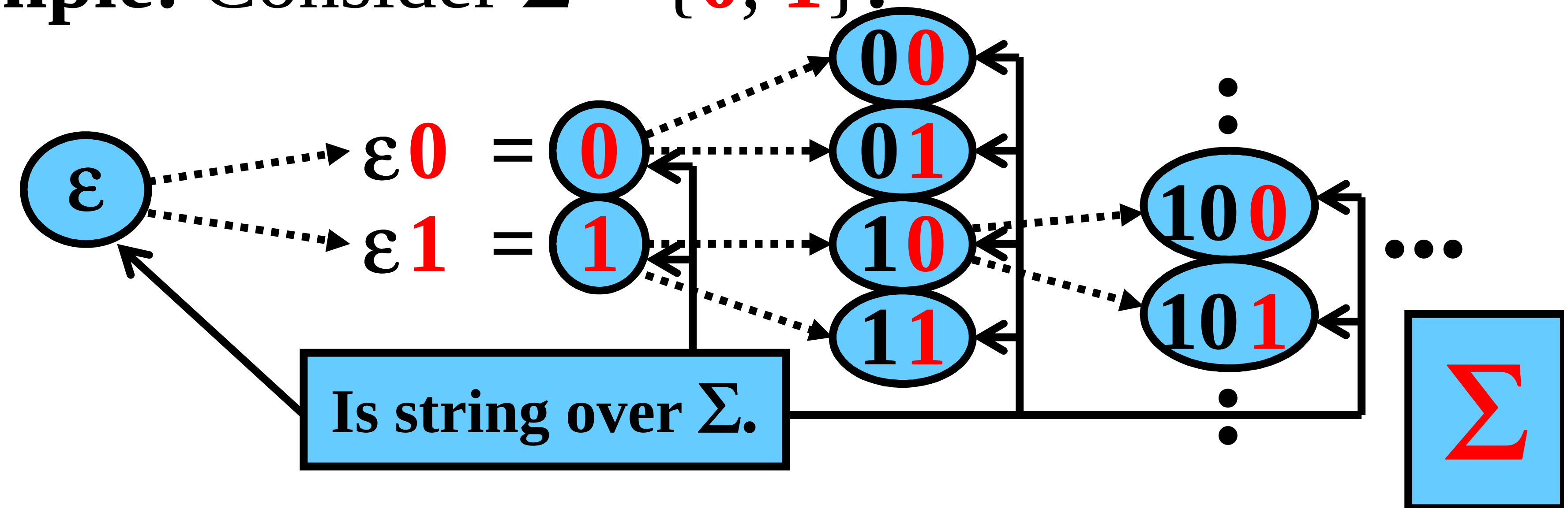

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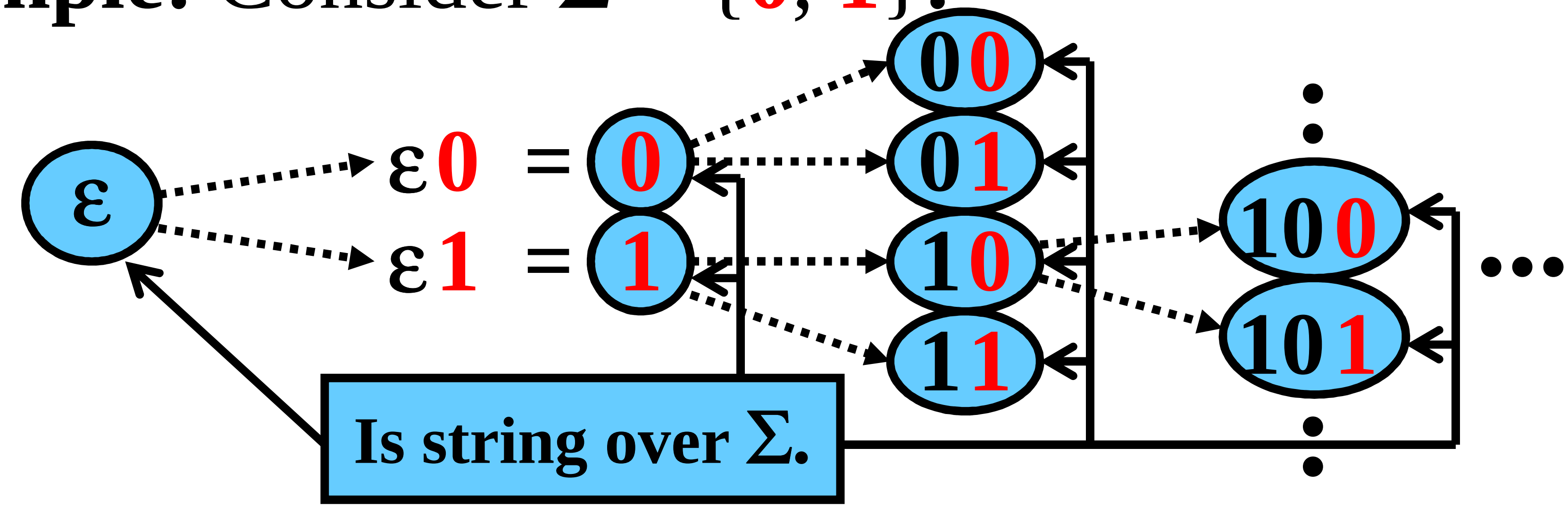

Length of String

Gist: $|a_1a_2\ldots a_n| = n$

Definition: Let x be a string over Σ.
The *length* of x, $|x|$, is defined as follows:
1) if $x = \varepsilon$, then $|x| = 0$
2) if $x = a_1\ldots a_n$, then $|x| = n$
for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1,\ldots,n$

Note: The length of x is the number of all symbols in x.

Example: Consider $x = 1010$

Task: $|x|$

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Task: $|x|$

$$x = 1\ 0\ 1\ 0$$

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$$x = \underbrace{1\ 0\ 1\ 0}_{a_1a_2a_3a_4}$$

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Example: Consider x =1010

Task: $|x|$

$$x = \mathbf{1\ 0\ 1\ 0}$$

$$a_1a_2a_3a_4 \longrightarrow n = 4, \text{ thus } |x| = 4$$

Concatenation of Strings

Gist: ***xy***

Definition: Let x and y be two strings over Σ. The *concatenation* of x and y is xy.

Note: $x\varepsilon = \varepsilon x = x$

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Examples:

Concatenation of **101** and **001** is **101001**

Concatenation of ε and **001** is ε**001** = **001**

Power of String

Gist: $x^i = \underbrace{xx\ldots x}_{i\text{-times}}$

Definition: Let x be a string over Σ.

For $i \geq 0$, the i-th *power* of x, x^i, is defined as

1) $x^0 = \varepsilon$ **2)** if $i \geq 1$ then $x^i = xx^{i-1}$

Note: $x^i x^j = x^j x^i = x^{i+j}$, where $i, j \geq 0$

Example: Consider $x = 10$

Task: x^3

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$$x^3 = xx^2 = \mathbf{10}x^2$$

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Example: Consider $x = 10$

Task: x^3

$$x^3 = xx^2 = \mathbf{10}x^2$$
$$x^2 = xx^1 = \mathbf{10}x^1$$
$$x^1 = xx^0 = \mathbf{10}x^0$$
$$x^0 = \varepsilon$$

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$x^3 = xx^2 = \mathbf{10}x^2$

$x^2 = xx^1 = \mathbf{10}x^1$

$x^1 = xx^0 = \mathbf{10}x^0 \longrightarrow x^1 = \mathbf{10}\varepsilon = \mathbf{10}$

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$x^1 = xx^0 = \mathbf{10}x^0$ $\rightarrow$ $x^1 = \mathbf{10}\varepsilon = \mathbf{10}$

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$x^3 = xx^2 = \mathbf{10}x^2$

$x^2 = xx^1 = \mathbf{10}x^1 \rightarrow x^2 = \mathbf{1010}$

$x^1 = xx^0 = \mathbf{10}x^0 \rightarrow x^1 = \mathbf{10}\varepsilon = \mathbf{10}$

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Example: Consider $x = 10$

Task: x^3

$x^3 = xx^2 = \mathbf{10}x^2$

$x^2 = xx^1 = \mathbf{10}x^1$ $\quad x^2 = \mathbf{1010}$

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$x^2 = xx^1 = \mathbf{10}x^1$ $x^2 = \mathbf{1010}$

$x^1 = xx^0 = \mathbf{10}x^0$ $x^1 = \mathbf{10}\varepsilon = \mathbf{10}$

$x^0 = \varepsilon$

Power of String

Gist: $x^i = \underbrace{xx\ldots x}_{i\text{-times}}$

Definition: Let x be a string over Σ.
For $i \geq 0$, the i-th *power* of x, x^i, is defined as

1) $x^0 = \varepsilon$ **2)** if $i \geq 1$ then $x^i = xx^{i-1}$

Note: $x^i x^j = x^j x^i = x^{i+j}$, where $i, j \geq 0$

Example: Consider $x = 10$

Task: x^3

$x^3 = xx^2 = \mathbf{10}x^2$

$x^2 = xx^1 = \mathbf{10}x^1$

$x^1 = xx^0 = \mathbf{10}x^0$

$x^0 = \varepsilon$

$x^1 = \mathbf{10}\varepsilon = \mathbf{10}$

$x^2 = \mathbf{1010}$

$\boxed{x^3 = \mathbf{101010}}$

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.

The *reversal* of x, *reversal*(x), is defined as:

1) if $x = \varepsilon$ then reversal(ε) = ε

2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$ for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:

1) if $x = \varepsilon$ then reversal(ε) = ε

2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$
for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

reversal( ) = , so

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.

The *reversal* of x, *reversal*(x), is defined as:

1) if $x = \varepsilon$ then reversal(ε) = ε

2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$ for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1}$) = $\boldsymbol{a_1}$, so

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:
1) if $x = \varepsilon$ then reversal(ε) = ε
2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$ for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1,\ldots,n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1 a_2}$) = $\boldsymbol{a_2 a_1}$, so

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:
1) if $x = \varepsilon$ then reversal(ε) = ε
2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$
for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010
Task: reversal(x)

reversal($\boldsymbol{a_1 a_2 a_3}$) = $\boldsymbol{a_3 a_2 a_1}$, so

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:
1) if $x = \varepsilon$ then reversal(ε) = ε
2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$
for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1 a_2 a_3 a_4}$) = $\boldsymbol{a_4 a_3 a_2 a_1}$, so

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.

The *reversal* of x, *reversal*(x), is defined as:

1) if $x = \varepsilon$ then reversal(ε) = ε

2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$ for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1a_2a_3a_4}$) = $\boldsymbol{a_4a_3a_2a_1}$, so

reversal(   ) =

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:
1) if $x = \varepsilon$ then reversal(ε) = ε
2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$
for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1 a_2 a_3 a_4}$) = $\boldsymbol{a_4 a_3 a_2 a_1}$, so

reversal(**1**) = **1**

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:

1) if $x = \varepsilon$ then reversal(ε) = ε

2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$ for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1a_2a_3a_4}$) = $\boldsymbol{a_4a_3a_2a_1}$, so

reversal(**1 0**) = **0 1**

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:
1) if $x = \varepsilon$ then reversal(ε) = ε
2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$
for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1,\ldots,n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1a_2a_3a_4}$) = $\boldsymbol{a_4a_3a_2a_1}$, so

reversal(**1 0 1**) = **1 0 1**

Reversal of String

Gist: reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$

Definition: Let x be a string over Σ.
The *reversal* of x, *reversal*(x), is defined as:

1) if $x = \varepsilon$ then reversal(ε) = ε

2) if $x = a_1 \ldots a_n$ then reversal($a_1 \ldots a_n$) = $a_n \ldots a_1$ for some $n \geq 1$, and $a_i \in \Sigma$ for all $i = 1, \ldots, n$

Example: Consider x =1010

Task: reversal(x)

reversal($\boldsymbol{a_1 a_2 a_3 a_4}$) = $\boldsymbol{a_4 a_3 a_2 a_1}$, so

reversal(**1 0 1 0**) = **0 1 0 1**

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$xz = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$xz = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

ε

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$xz = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

ε

1

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$xz = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

ε
1
10

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$xz = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

ε
1
10
101

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$\boldsymbol{xz = y}$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

ε
1
10
101
1010

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$xz = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

$$\textbf{Prefixes of 1010} \left\{ \begin{array}{l} \varepsilon \\ 1 \\ 10 \\ 101 \\ 1010 \end{array} \right.$$

Prefix of String

Gist: *x* is a prefix of *xz*

Definition: Let x and y be two strings over Σ; x is *prefix* of y if there is a string z over Σ so

$$\boldsymbol{xz = y}$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper prefix*** of y.

Example: Consider 1010

Task: All prefixes of **1010**

Prefixes of 1010

ε
1
10
101
1010

Proper prefixes of 1010

Suffix of String

Gist: x is a suffix of zx

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$zx = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

Suffix of String

Gist: *x* is a suffix of *zx*

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$zx = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

ε

Suffix of String

Gist: *x* is a suffix of *zx*

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$zx = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

ε

0

Suffix of String

Gist: *x* is a suffix of *zx*

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$\boldsymbol{zx = y}$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

ε

0

10

Suffix of String

Gist: *x* is a suffix of *zx*

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$zx = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

ε
0
10
010

Suffix of String

Gist: *x* is a suffix of *zx*

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$zx = y$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

ε
0
10
010
1010

Suffix of String

Gist: *x* is a suffix of *zx*

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$\boldsymbol{zx = y}$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

$$\textbf{Suffixes of 1010} \begin{cases} \varepsilon \\ 0 \\ 10 \\ 010 \\ 1010 \end{cases}$$

Suffix of String

Gist: *x* is a suffix of *zx*

Definition: Let x and y be two strings over Σ; x is *suffix* of y if there is a string z over Σ so

$$\boldsymbol{zx = y}$$

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper suffix*** of y.

Example: Consider 1010

Task: All suffixes of **1010**

$$\text{Suffixes of 1010} \left\{ \begin{array}{r} \varepsilon \\ \left.\begin{array}{r} 0 \\ 10 \\ 010 \end{array}\right\} \text{Proper suffixes of 1010} \\ 1010 \end{array} \right.$$

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{\mathbf{1}}\,\mathbf{0\ 1\ 0}$

ε

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{\mathbf{1}}$ **0 1 0**

ε

1

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{1}\,\boxed{0}$ **1 0**

ε

1

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 $\boxed{\mathbf{0}}$ 1 0**

ε

1

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 $\boxed{\mathbf{0}}$ 1 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 $\boxed{0}\boxed{1}$ 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 $\boxed{\mathbf{1}}$ 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 $\boxed{\mathbf{1}}$ 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 $\boxed{0}$**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let *x* and *y* be two strings over Σ; *x* is *substring* of *y* if there are two string *z*, *z'* over Σ so ***zxz' = y*.**

Note: if $x \notin \{\varepsilon, y\}$ then *x* is ***proper substring*** of *y*.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{\mathbf{1\,0}}\,\mathbf{1}\,\boxed{\mathbf{0}}$

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{\mathbf{1\ 0}}\,\mathbf{1\ 0}$

ε

1, 0

10

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

10

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let *x* and *y* be two strings over Σ; *x* is *substring* of *y* if there are two string *z*, *z'* over Σ so ***zxz' = y*.**

Note: if $x \notin \{\varepsilon, y\}$ then *x* is ***proper substring*** of *y*.

Example: Consider 1010

Task: All substrings of **1 $\boxed{\textbf{0 1}}$ 0**

ε

1, 0

10

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 $\boxed{\textbf{0 1}}$ 0**

ε

1, 0

10, 01

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 $\boxed{0}\,\boxed{1\ 0}$**

ε

1, 0

10, 01

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0** $\boxed{\mathbf{1\ 0}}$

ε

1, 0

10, 01

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let *x* and *y* be two strings over Σ; *x* is *substring* of *y* if there are two string *z*, *z'* over Σ so ***zxz' = y*.**

Note: if $x \notin \{\varepsilon, y\}$ then *x* is ***proper substring*** of *y*.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

10, 01

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

10, 01

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of **1 0 1 0**

ε

1, 0

10, 01

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{\mathbf{1\,0\,1}}\,\mathbf{0}$

ε

1, 0

10, 01

Substring

Gist: *x* is a substring of *zxz'*

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{\mathbf{1\ 0\ 1}}\,\mathbf{0}$

ε

1, 0
10, 01
101

Substring

Gist: x is a substring of zxz'

Definition: Let x and y be two strings over Σ; x is *substring* of y if there are two string z, z' over Σ so $\boldsymbol{zxz' = y}$.

Note: if $x \notin \{\varepsilon, y\}$ then x is ***proper substring*** of y.

Example: Consider 1010

Task: All substrings of $\boxed{1}\boxed{0\ 1\ 0}$

ε

1, 0

10, 01

101

Substring

Gist: *x* is a substring of *zxz'*

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Gist: $L \subseteq \Sigma^*$

Definition: Let Σ^* denote the set of all strings over Σ. Every subset $L \subseteq \Sigma^*$ is a *language* over Σ.

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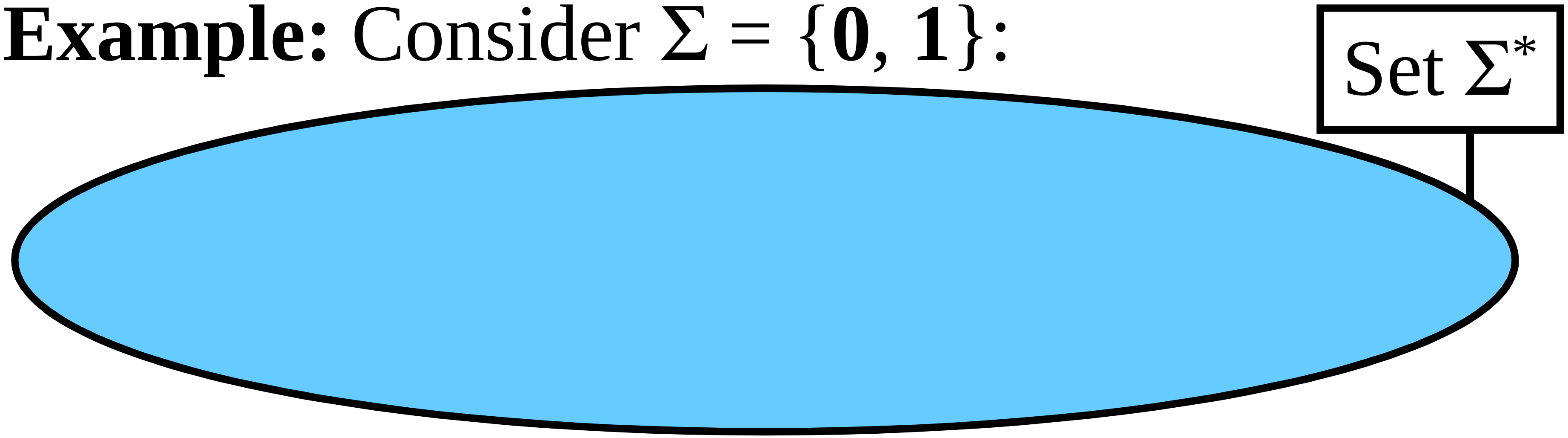

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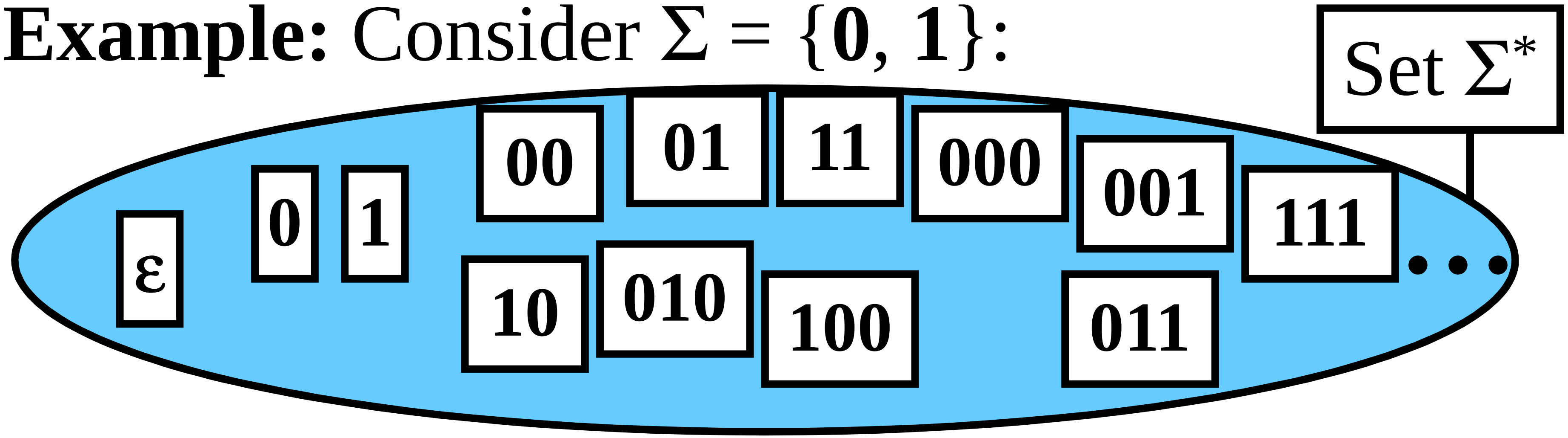

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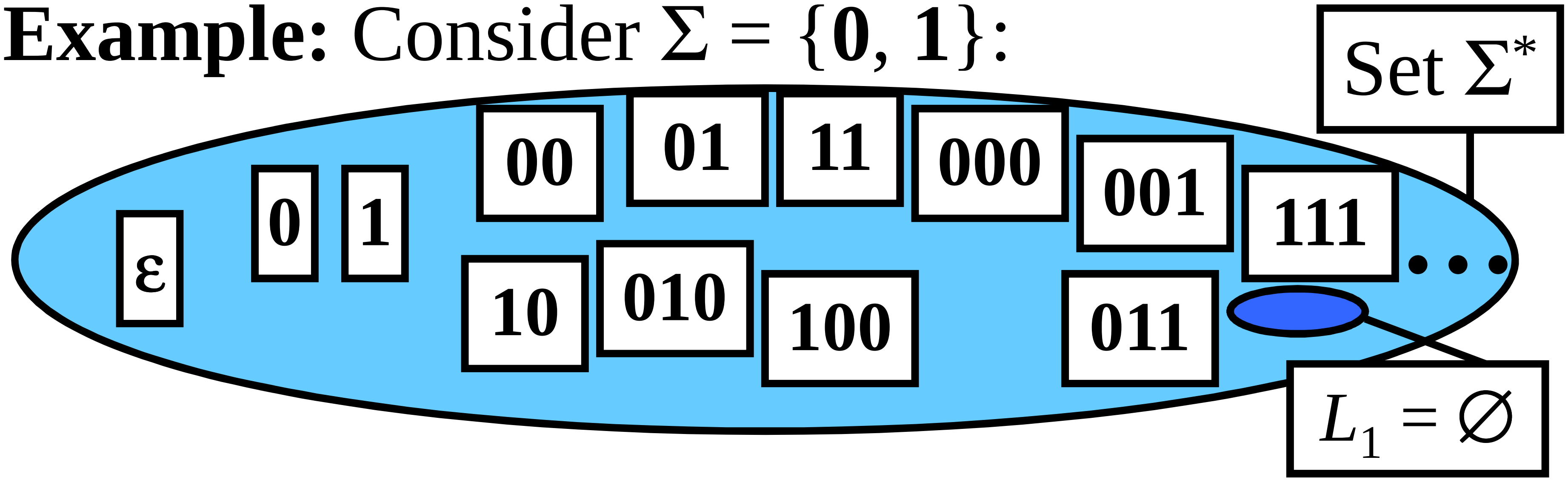

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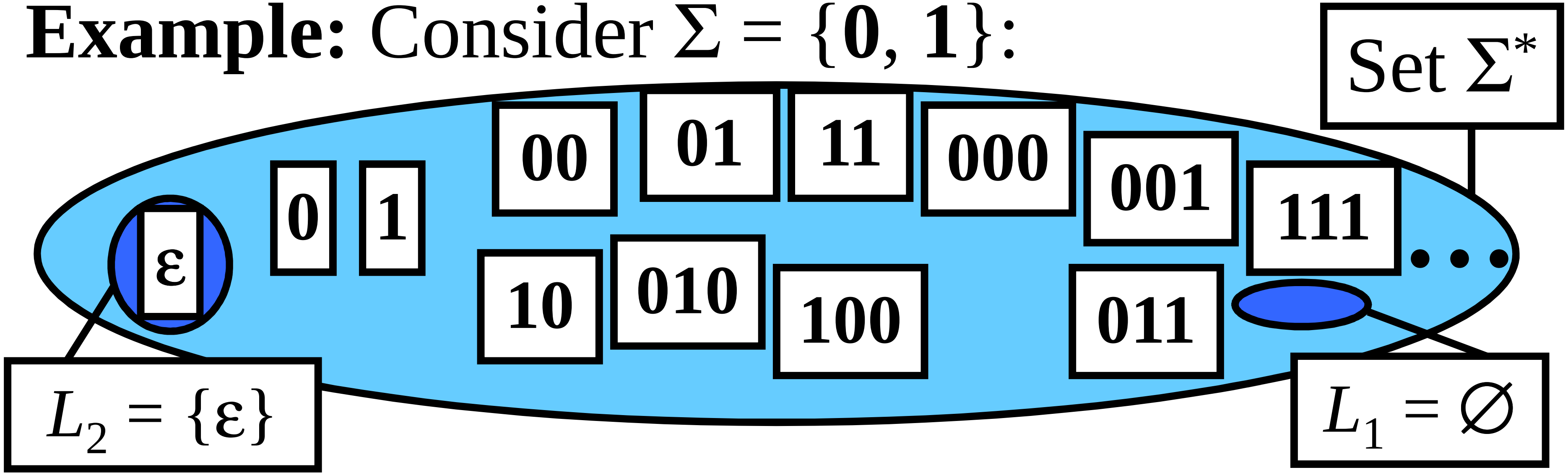

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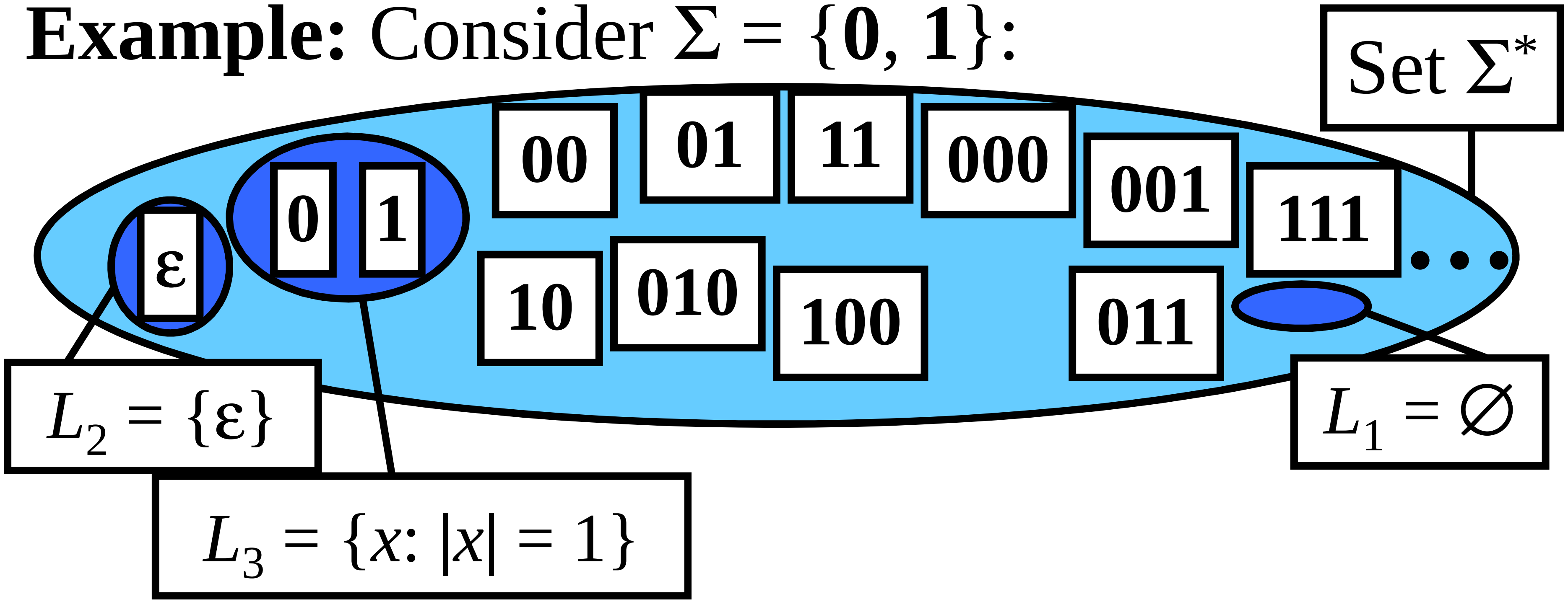

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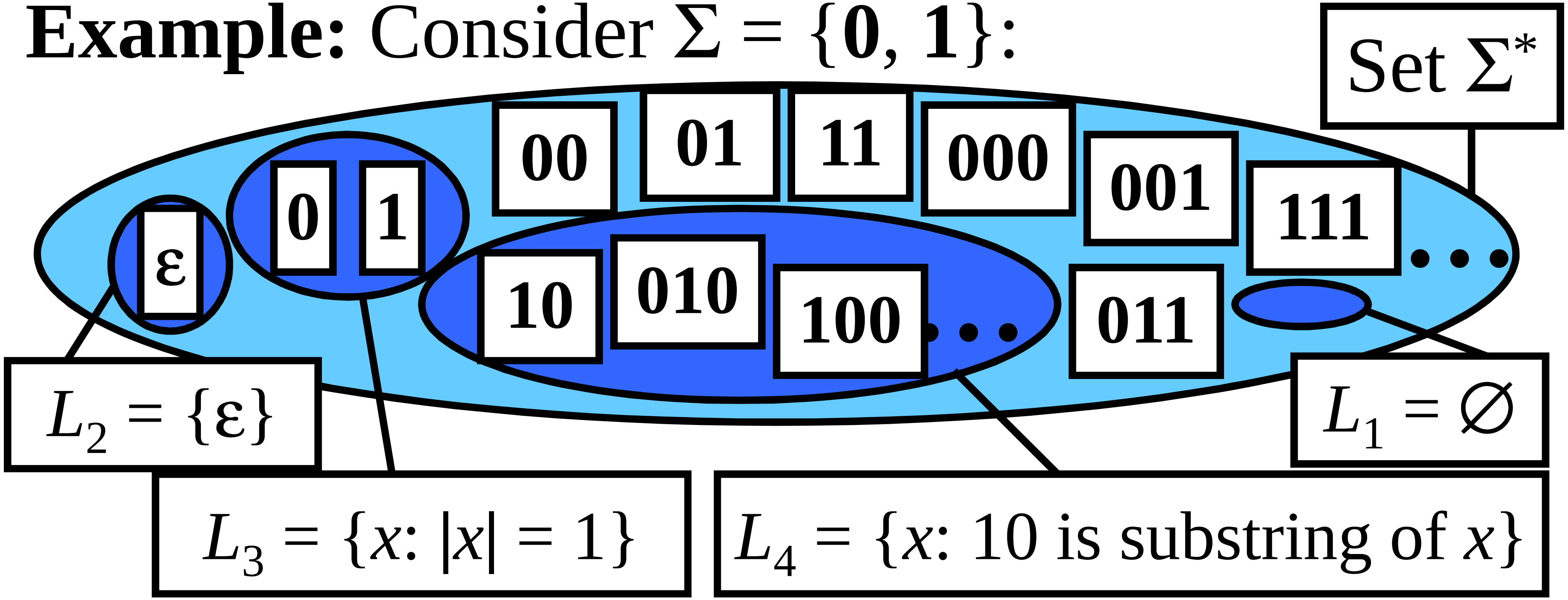

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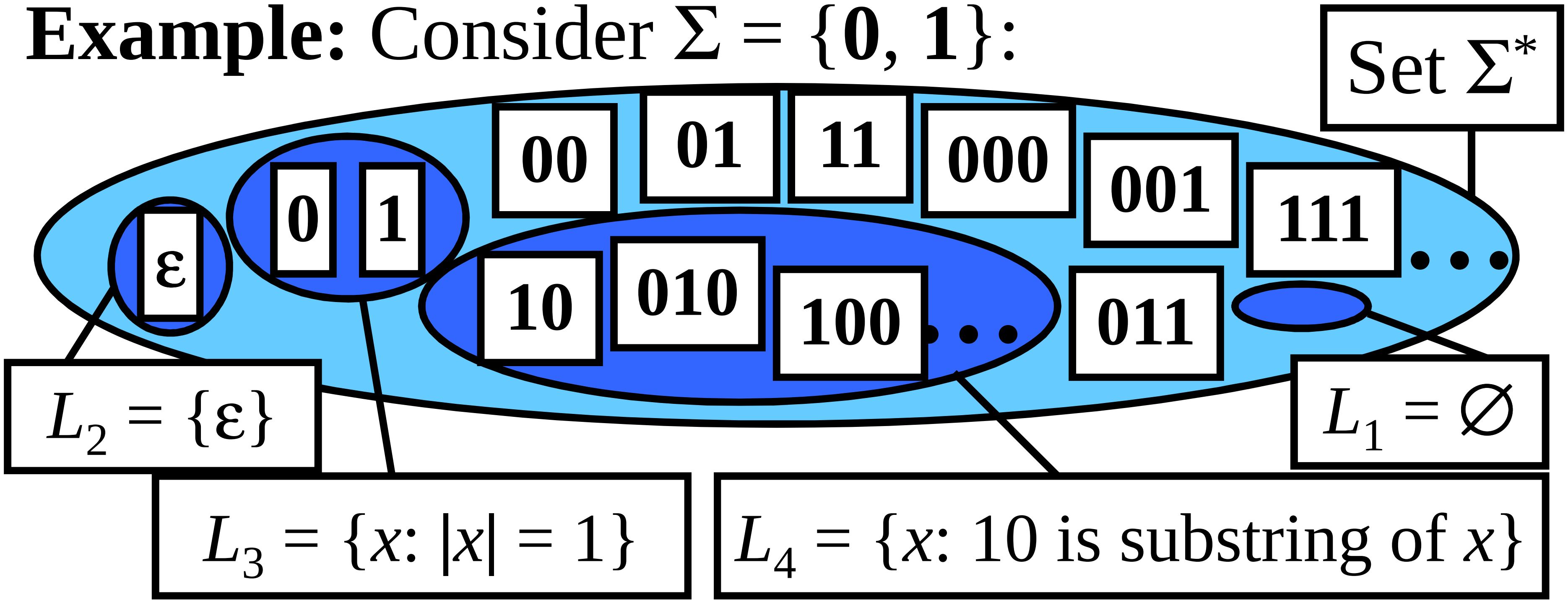

L_1, L_2, L_3, L_4 are languages over Σ

Finite and Infinite Languages

Gist: finite language contains a finite number of strings

Definition: A language, *L*, is *finite* if *L* contains a finite number of strings; otherwise, *L* is *infinite*.

Note: Let *S* be a set; card(*S*) is the number of its members.

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Examples:

- $L_1 = \emptyset$ is **finite** because card(L_1) = **0**
- $L_2 = \{\varepsilon\}$ is **finite** because card(L_2) = **1**
- $L_3 = \{x: |x| = 1\} = \{0, 1\}$ is **finite** because card(L_3) = **2**
- $L_4 = \{x: 10 \text{ is substring of } x\} = \{10, 010, 100, \dots\}$ is **infinite**

Union of Languages

Gist: Union of L_1 and L_2 is $L_1 \cup L_2$

Definition: Let L_1 and L_2 be two languages over Σ. The *union* of L_1 and L_2, $L_1 \cup L_2$, is defined as

$$L_1 \cup L_2 = \{x : x \in L_1 \text{ or } x \in L_2\}$$

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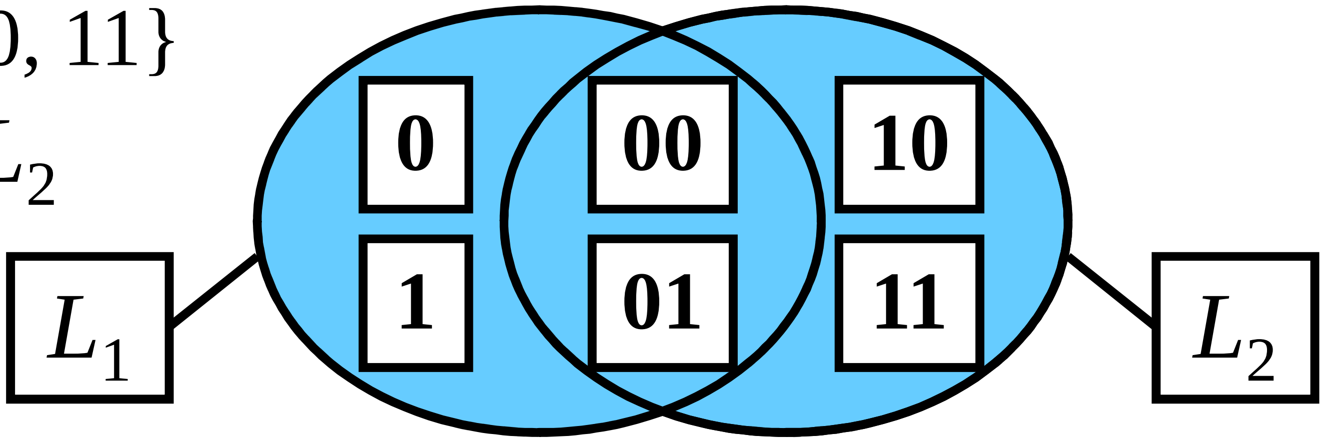

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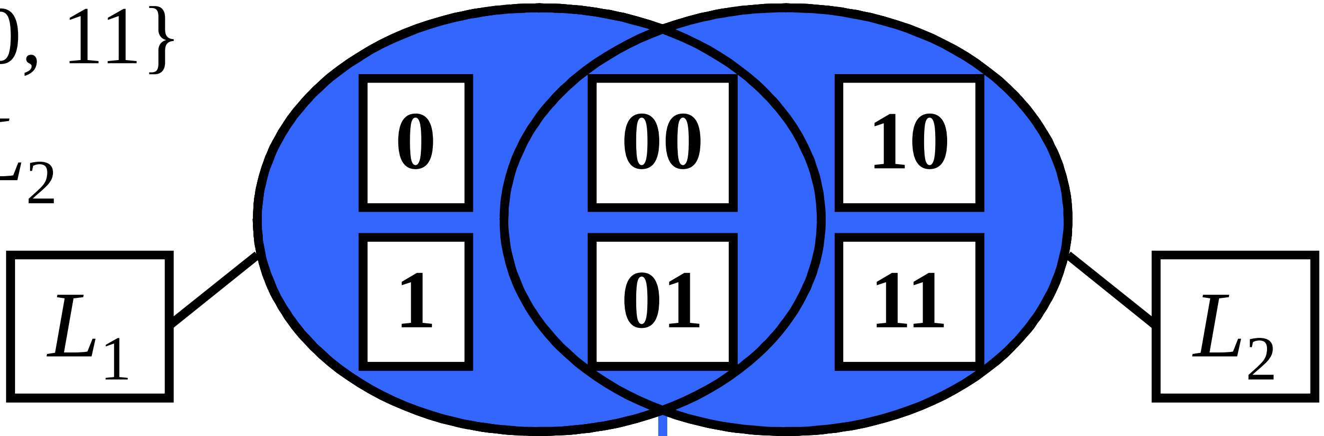

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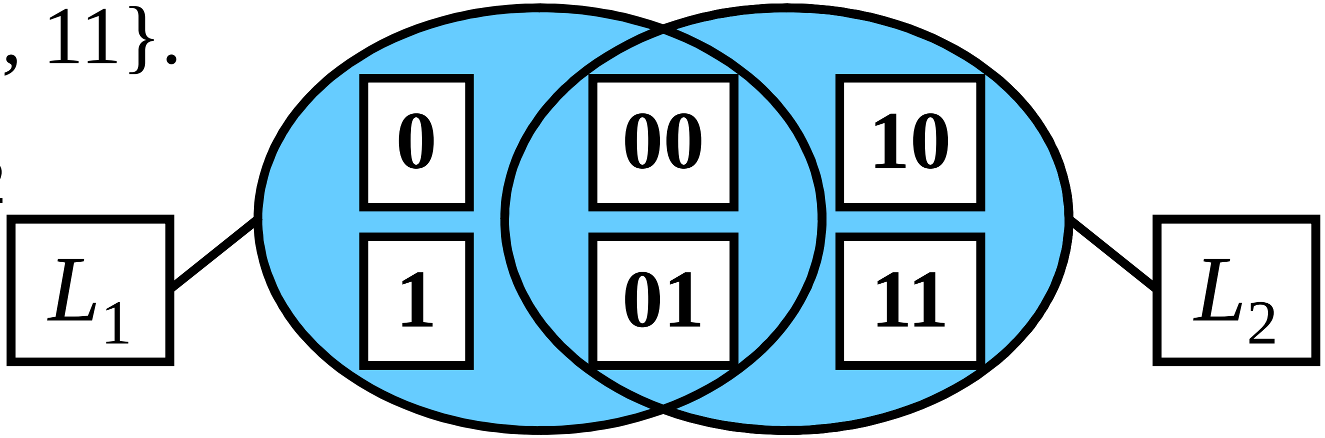

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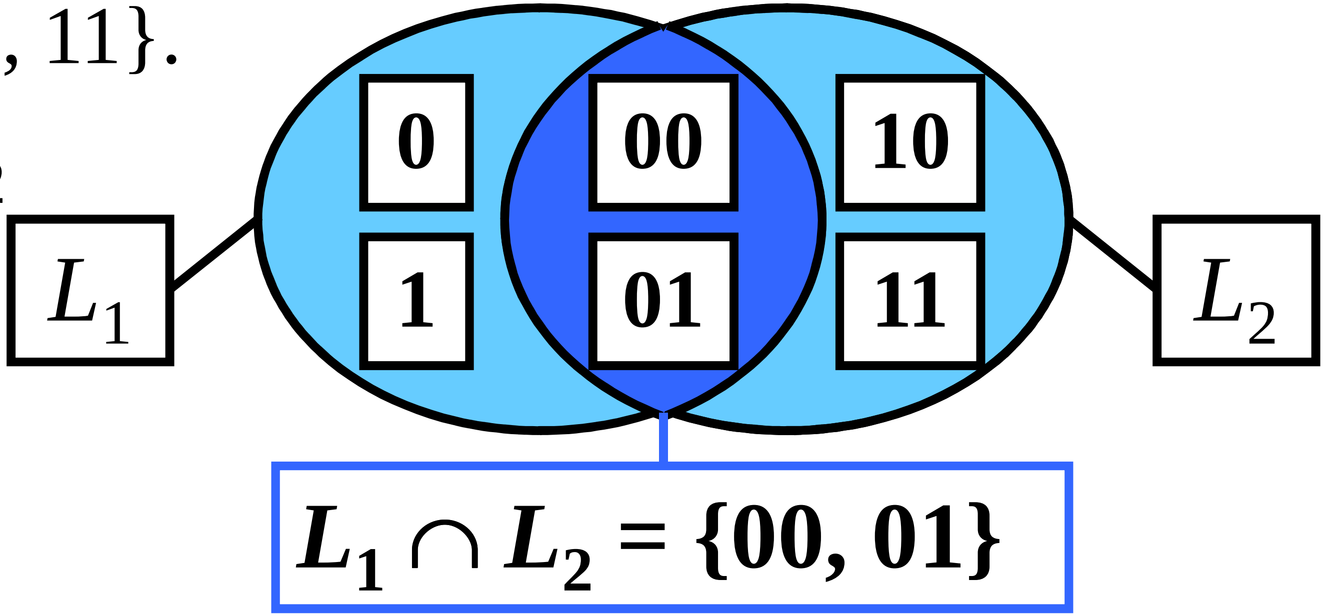

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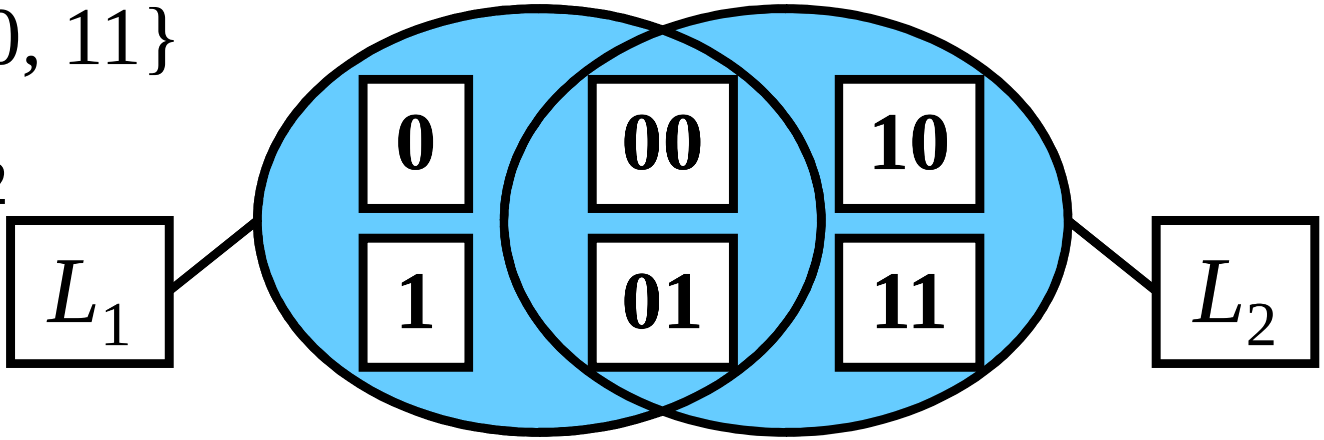

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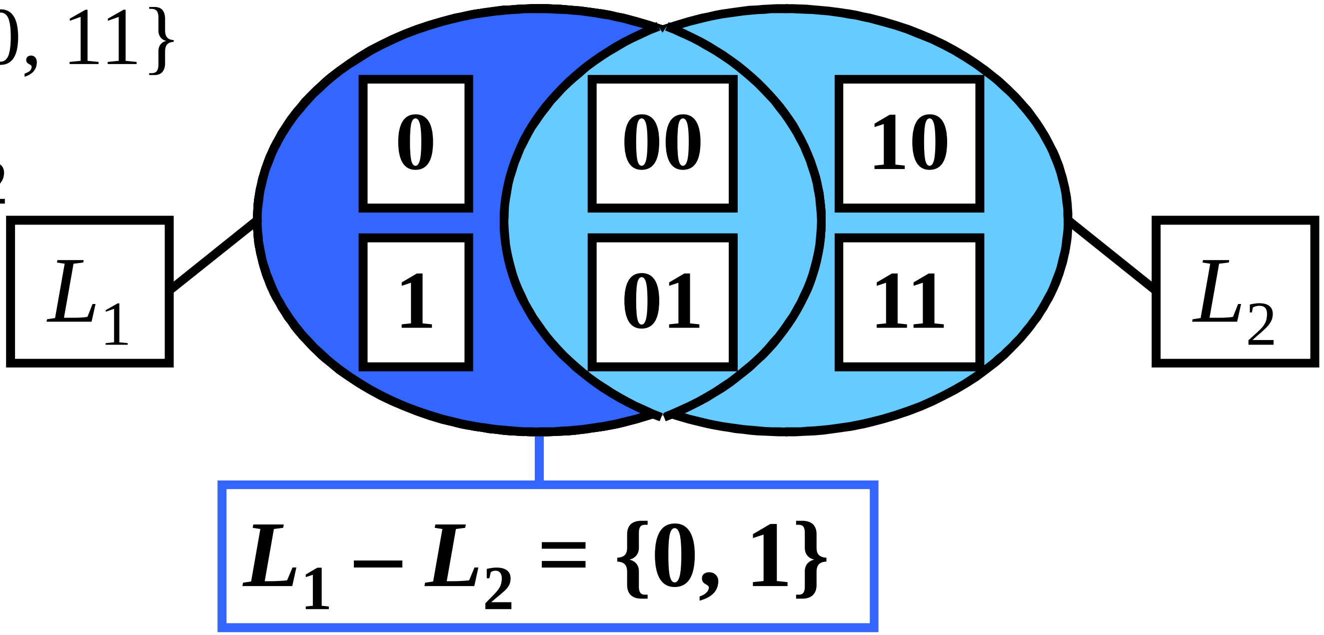

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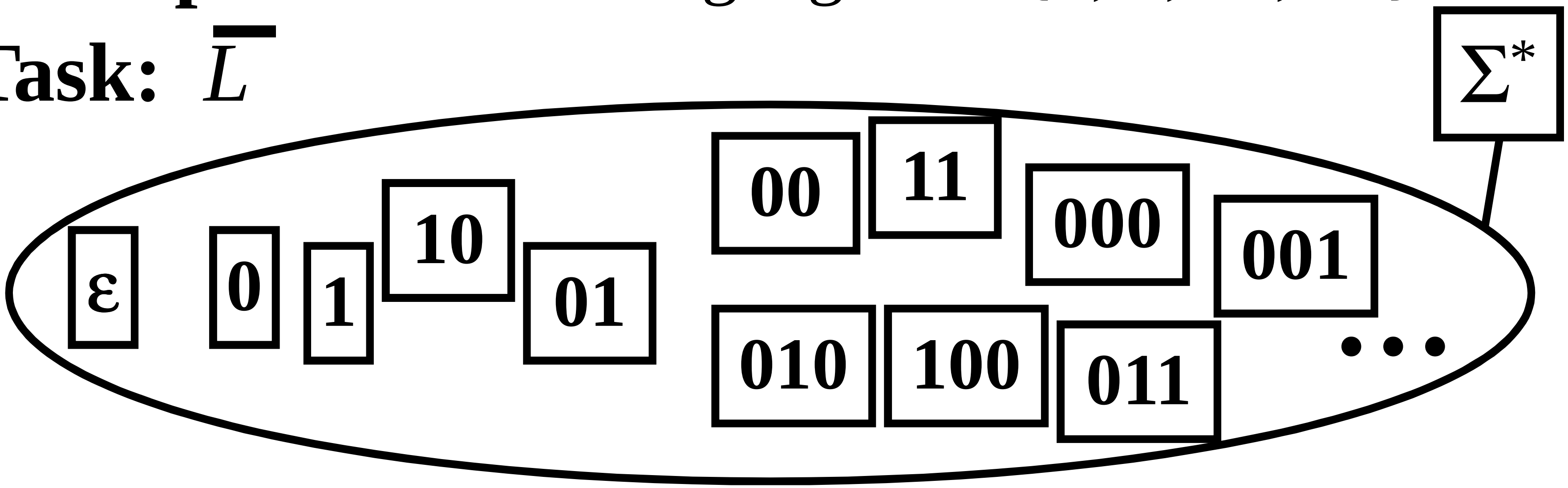

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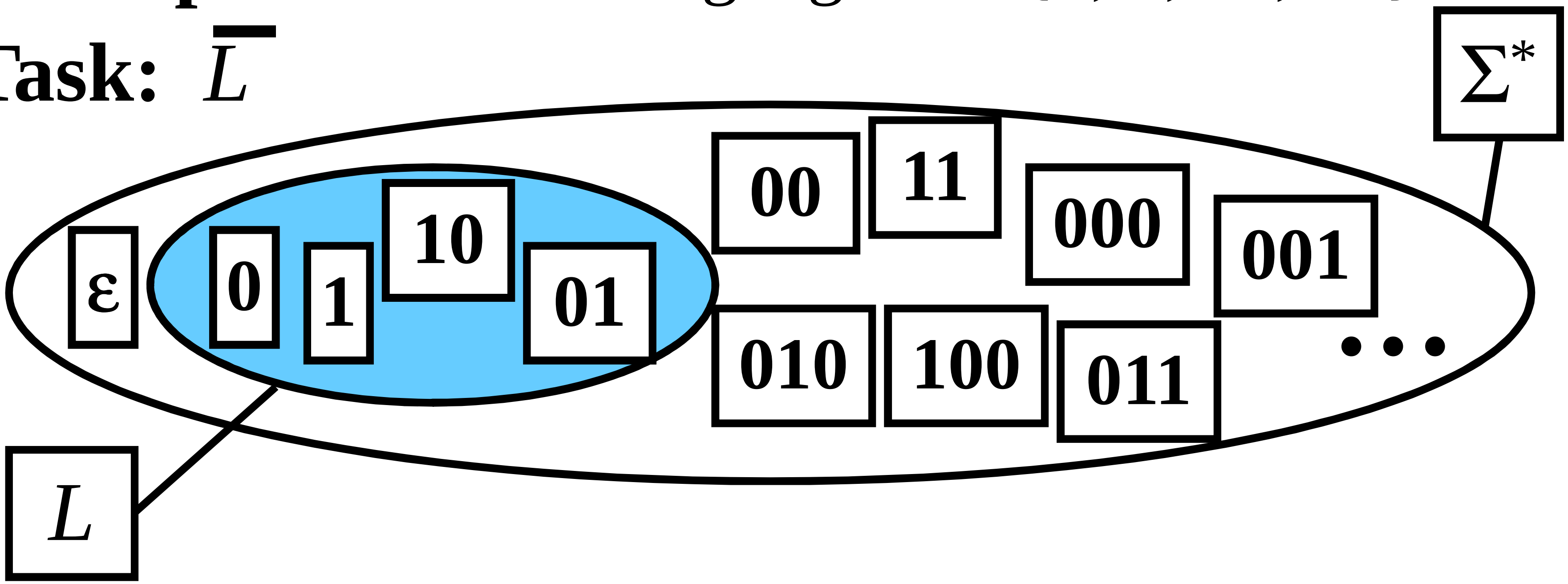

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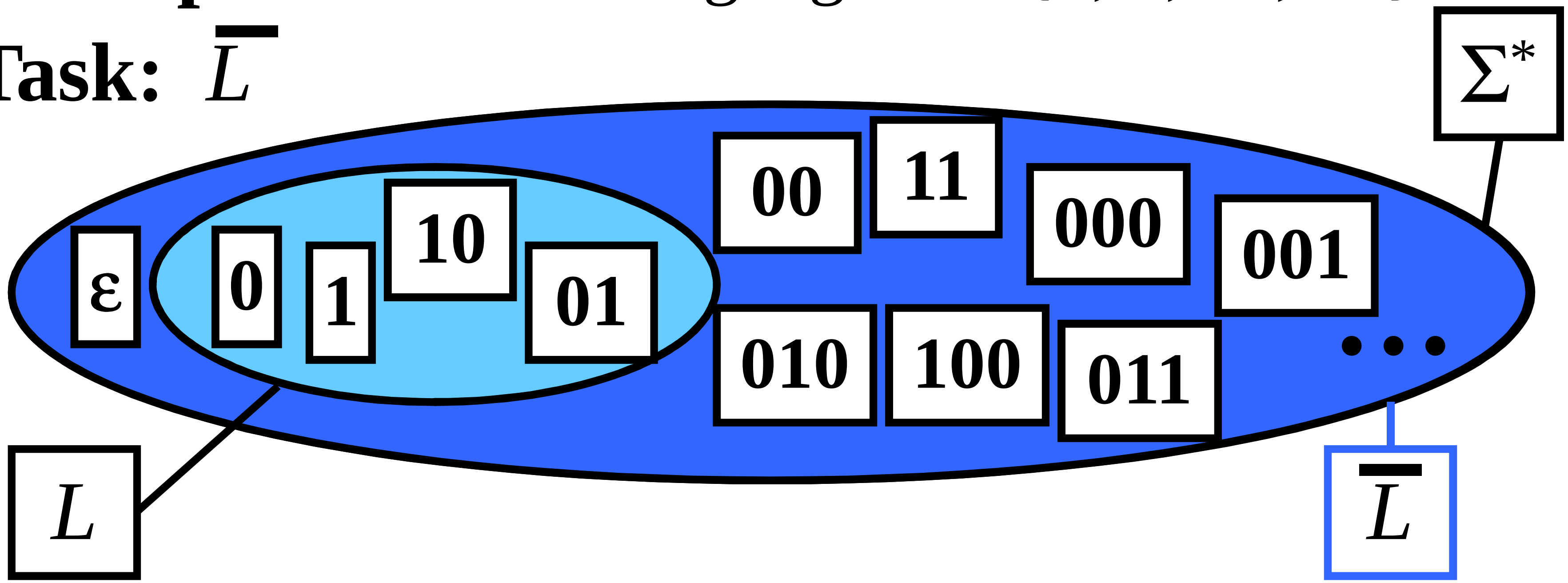

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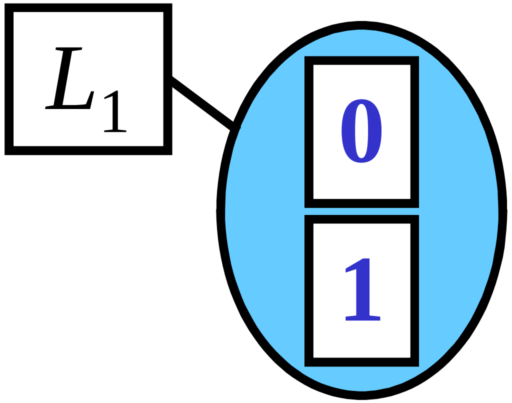

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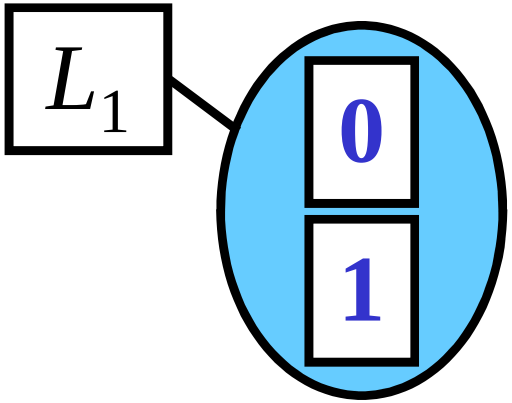

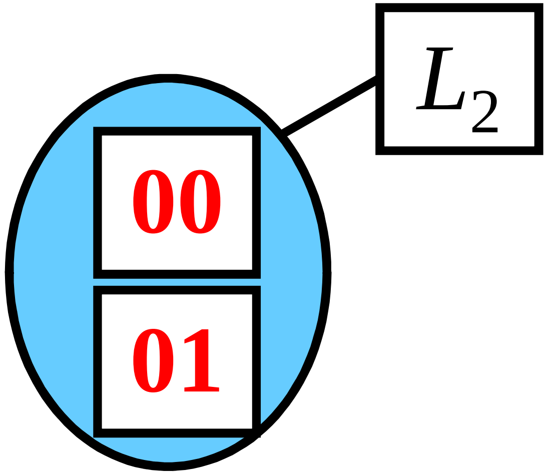

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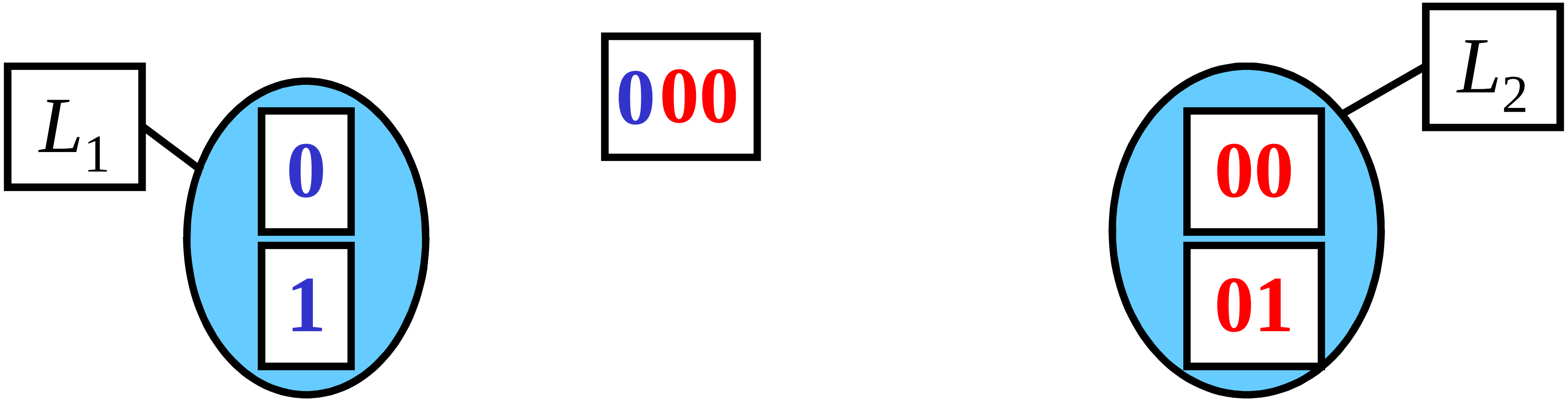

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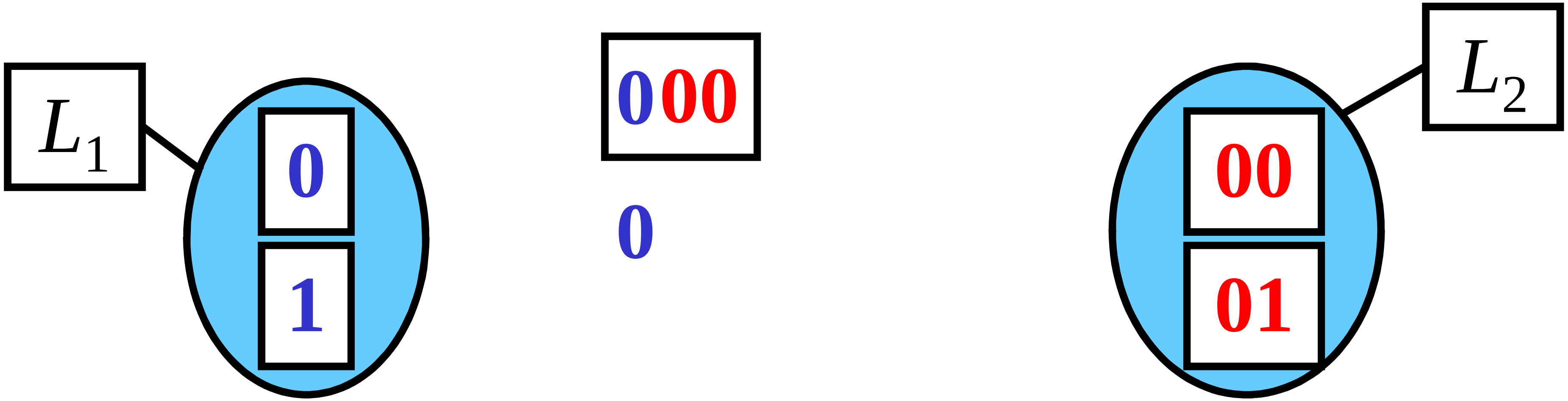

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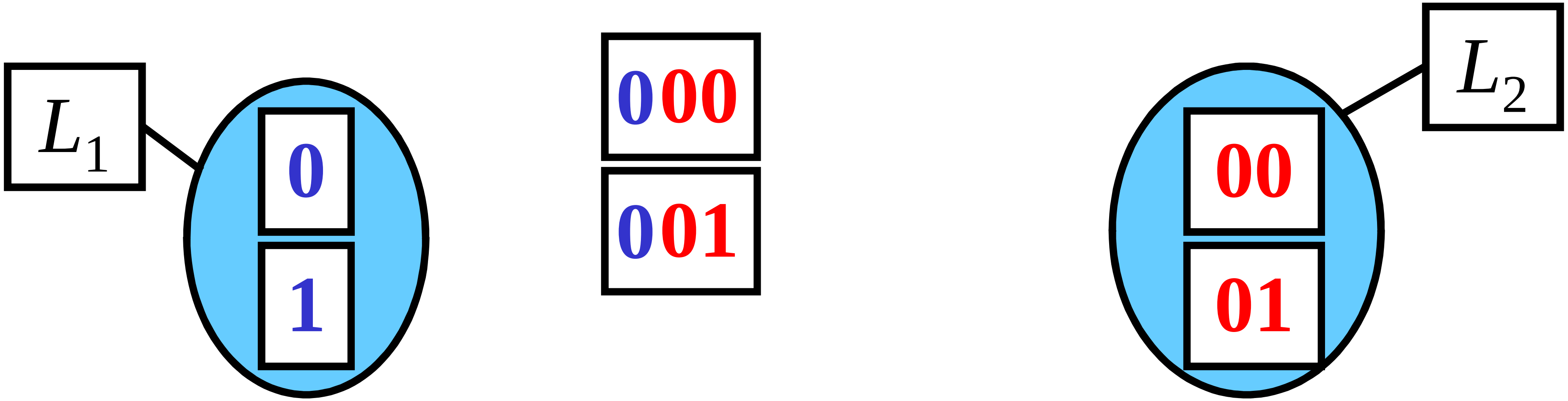

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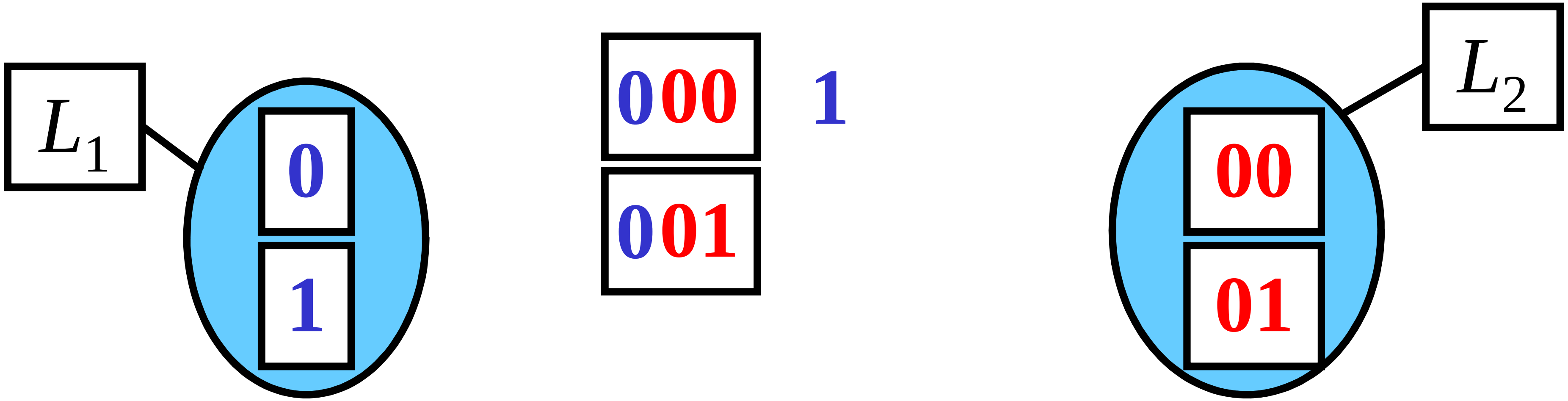

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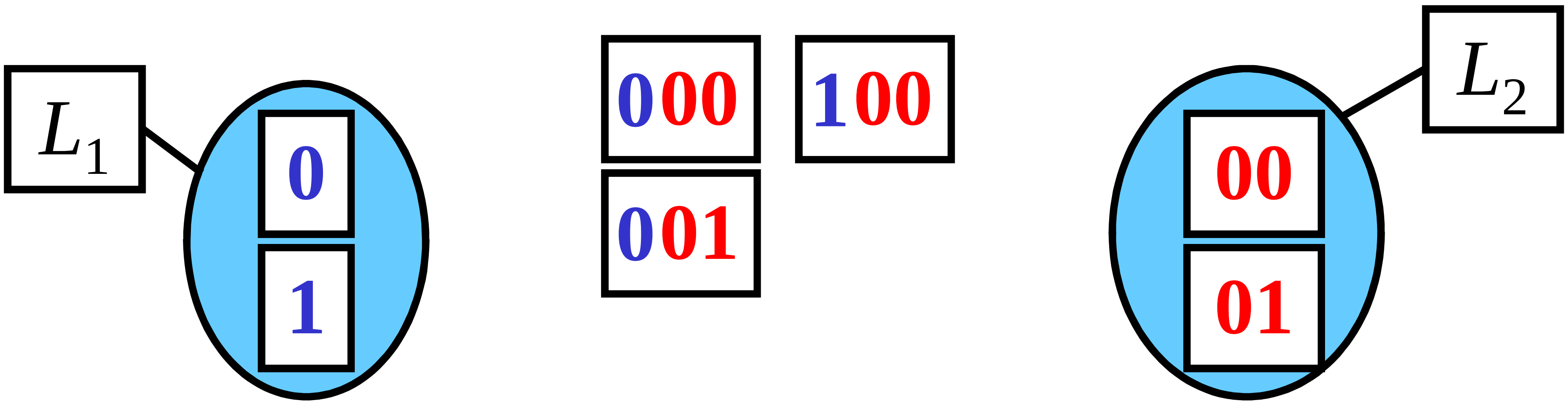

Concatenation of Languages

Gist: $L_1L_2 = \{xy: x \in L_1 \text{ and } y \in L_2\}$

Definition: Let L_1 and L_2 be two languages over Σ. The *concatenation* of L_1 and L_2, L_1L_2, is defined as

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Note: 1) $L\{\varepsilon\} = \{\varepsilon\}L = L$ **2)** $L\emptyset = \emptyset L = \emptyset$

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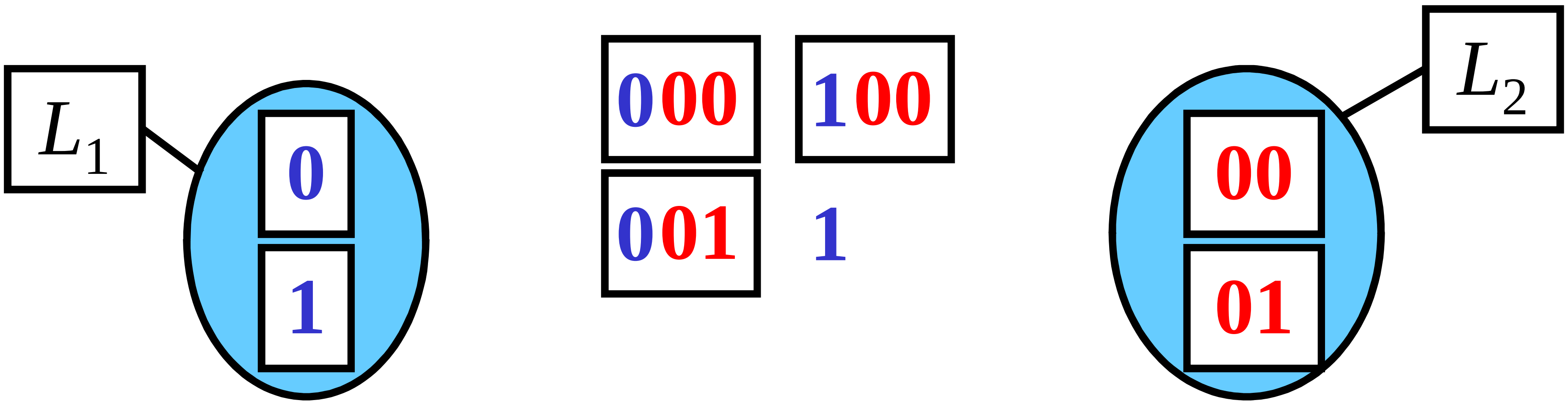

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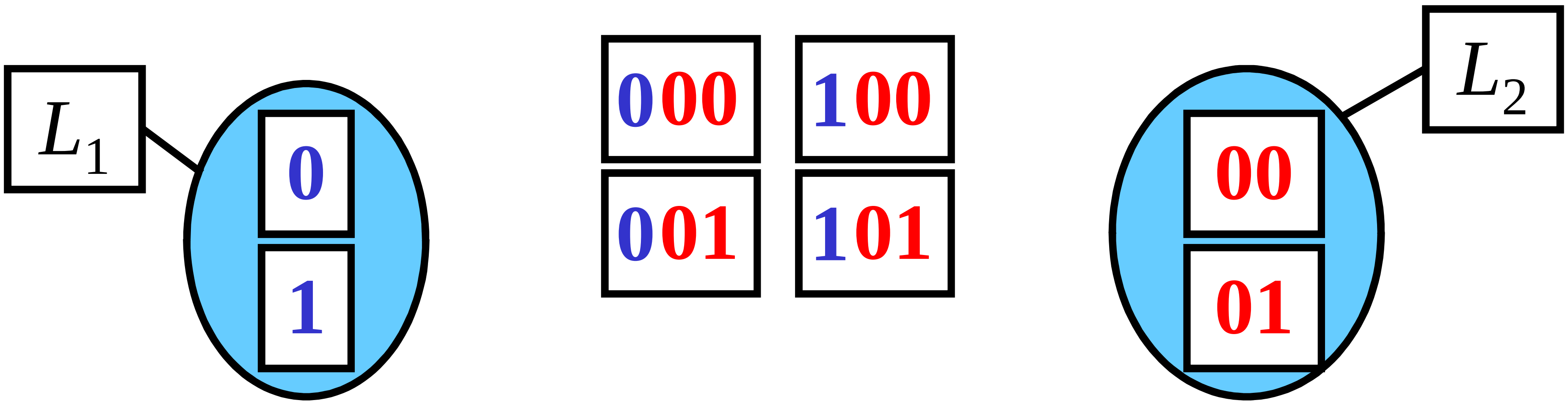

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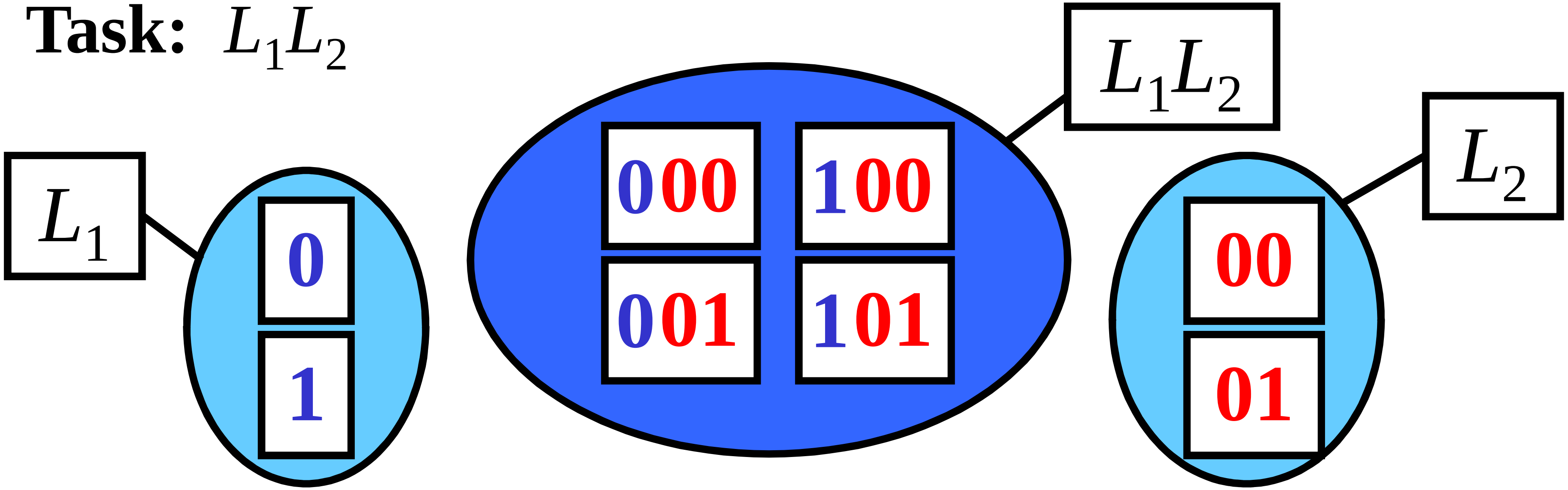

Reversal of Language

Gist: *reversal*(*L*) = {*reversal*(*x*): $x \in L$}

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$$reversal(L) = \{reversal(x) : x \in L\}$$

Example: Consider $L = \{01, 011\}$

Task: ***reversal***(L)

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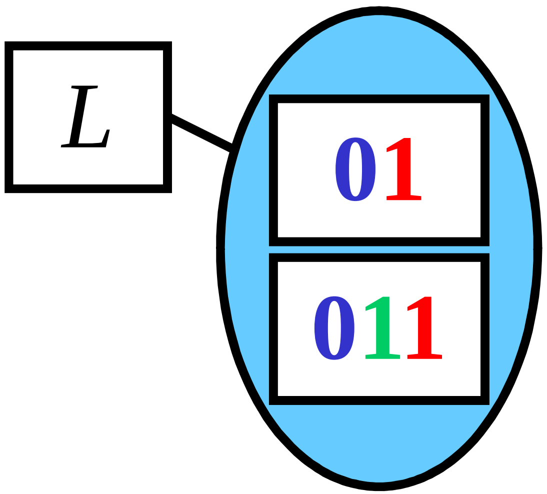

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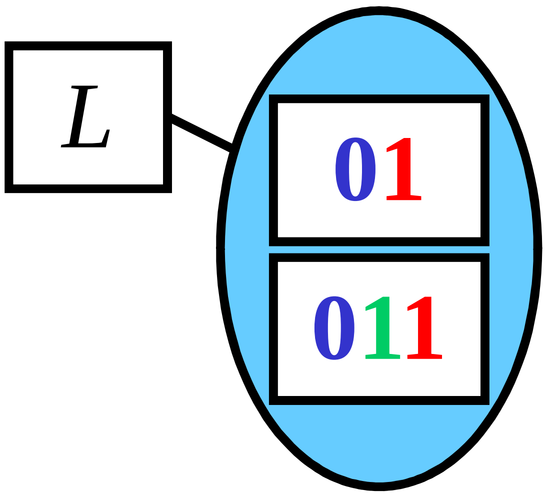

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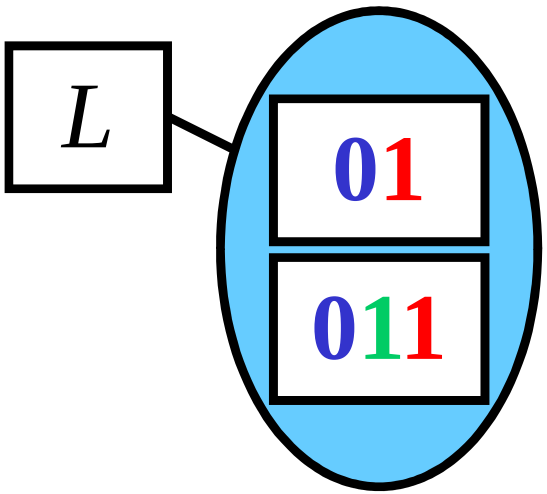

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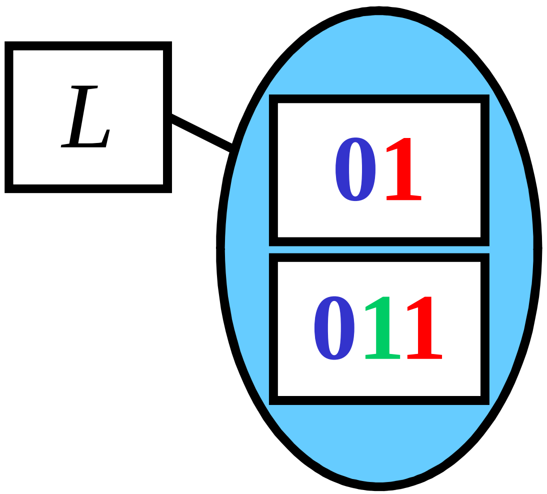

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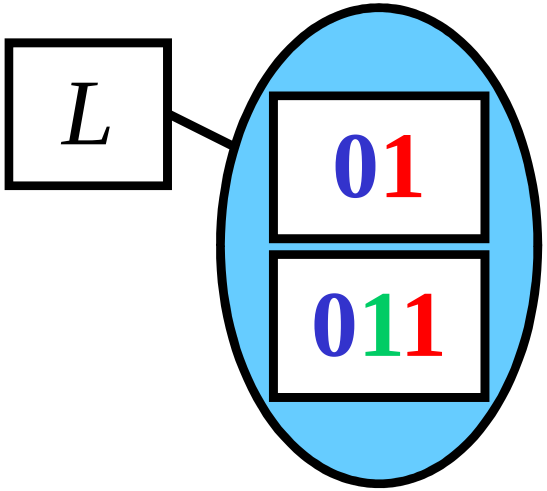

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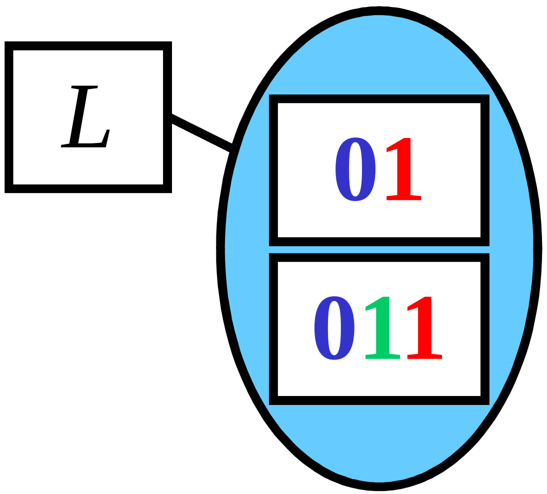

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10

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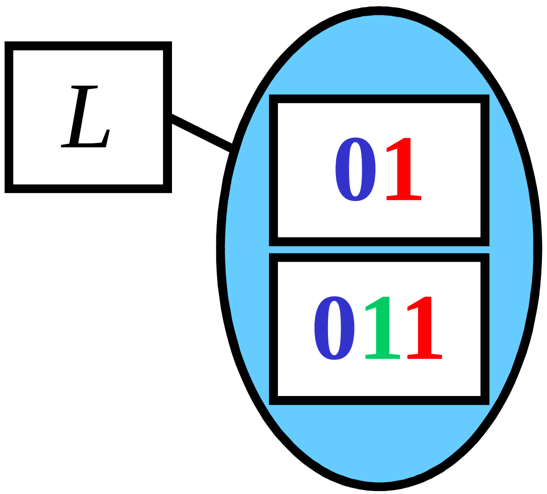

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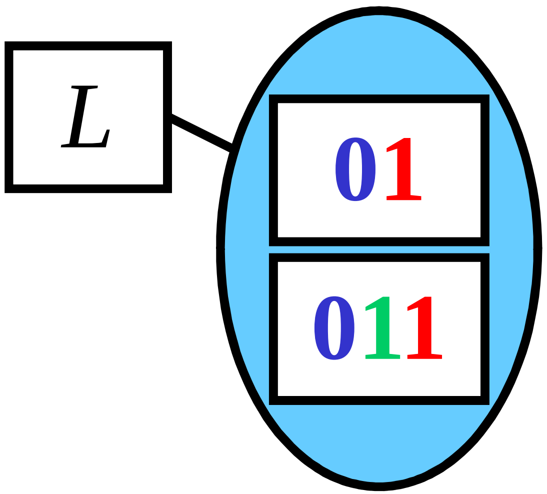

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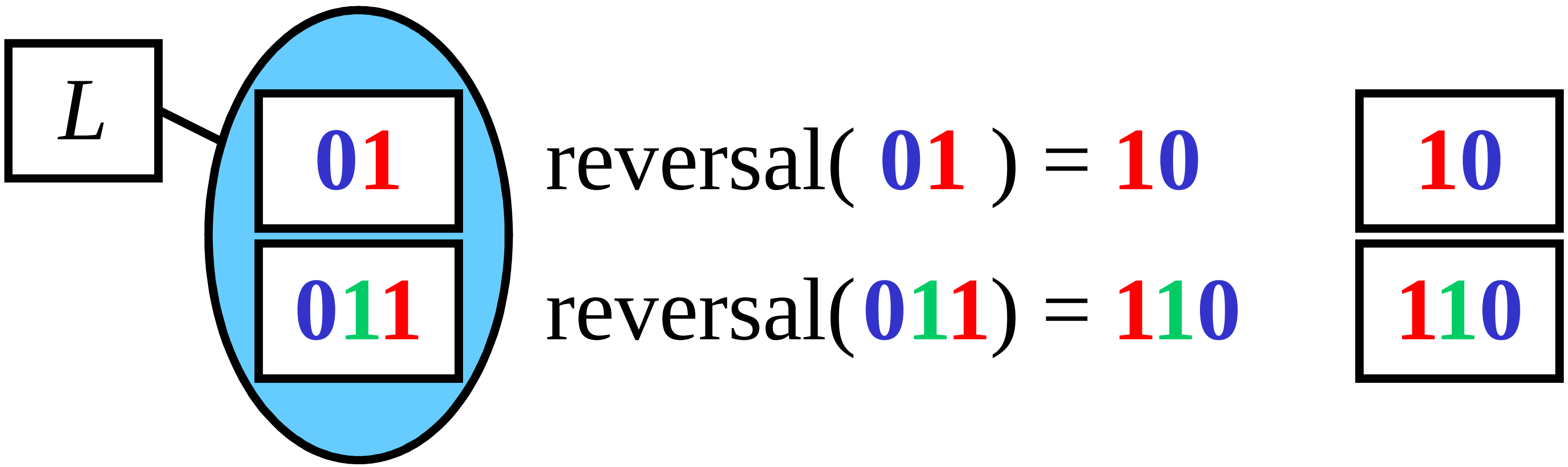

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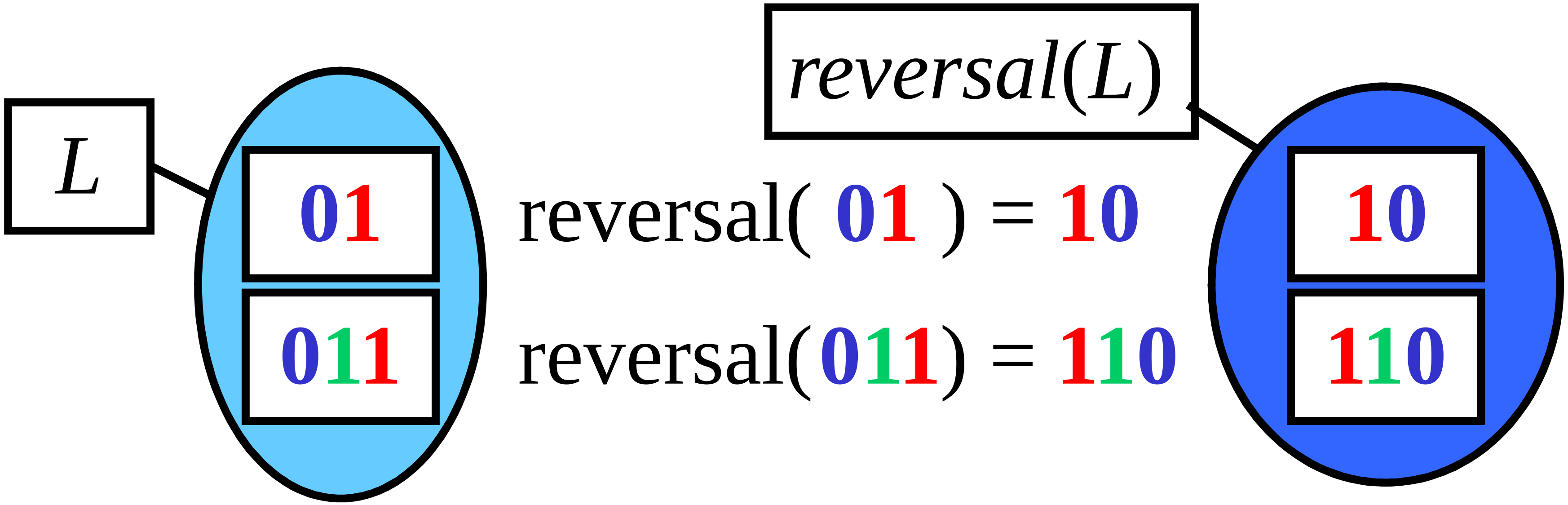

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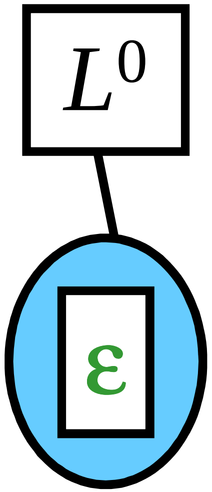

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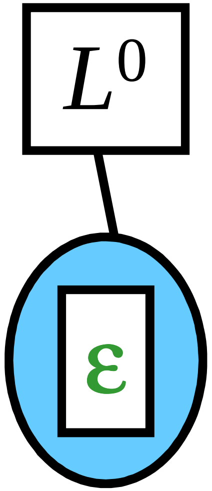

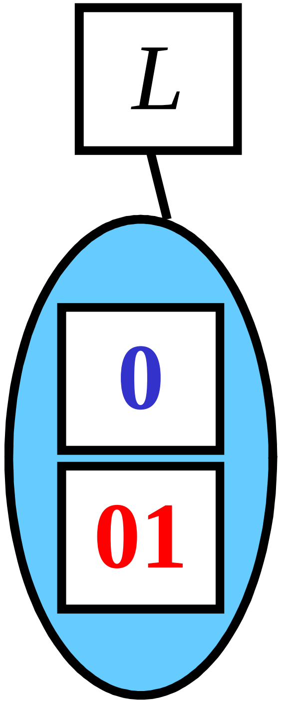

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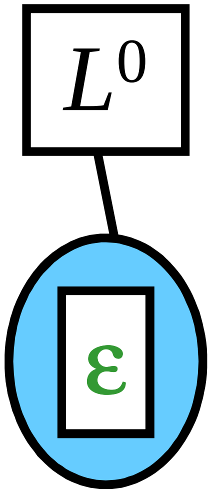

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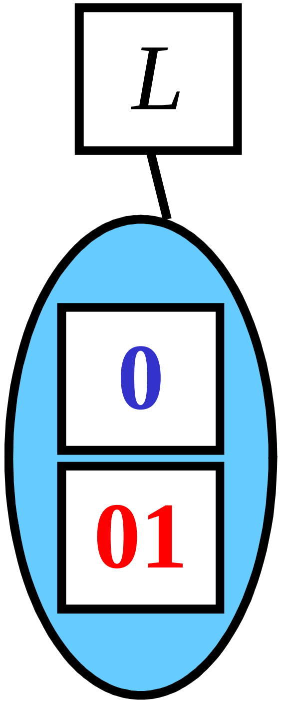

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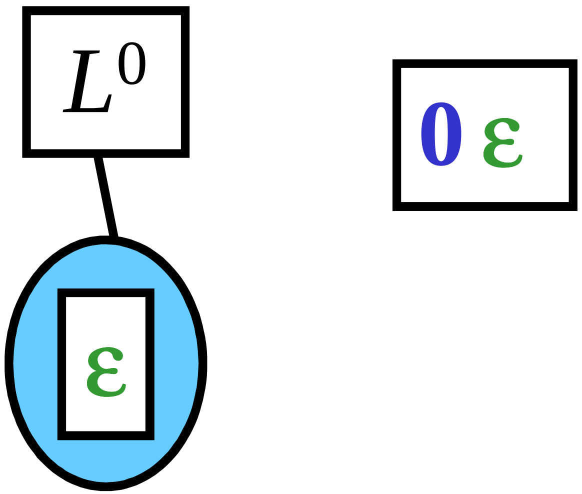

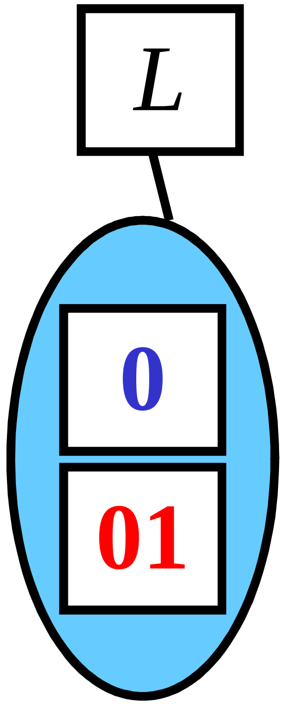

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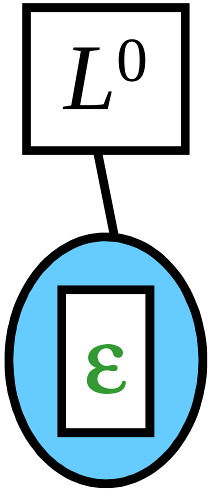

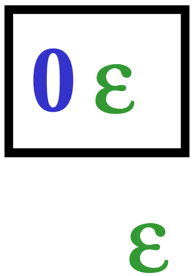

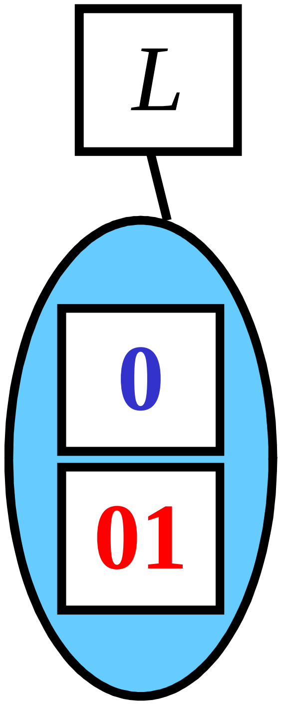

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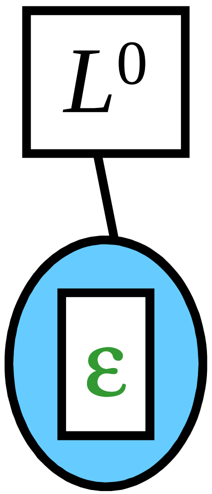

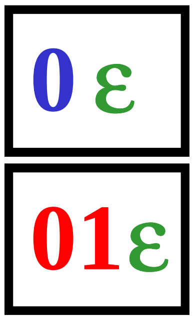

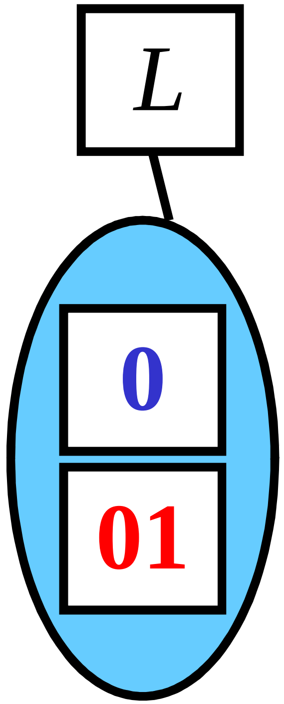

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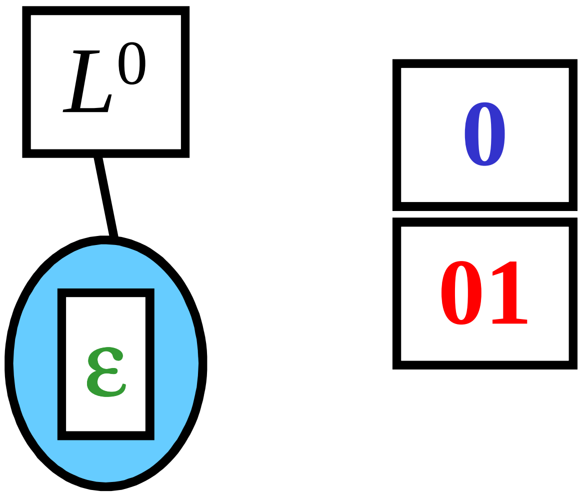

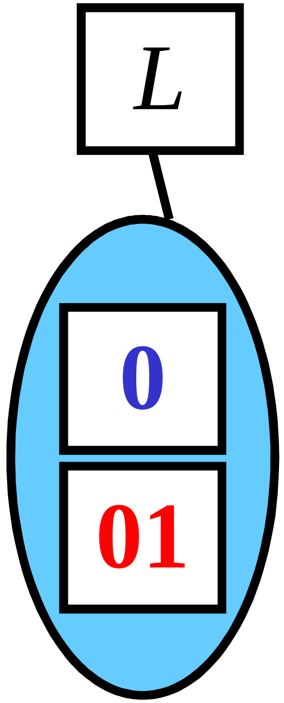

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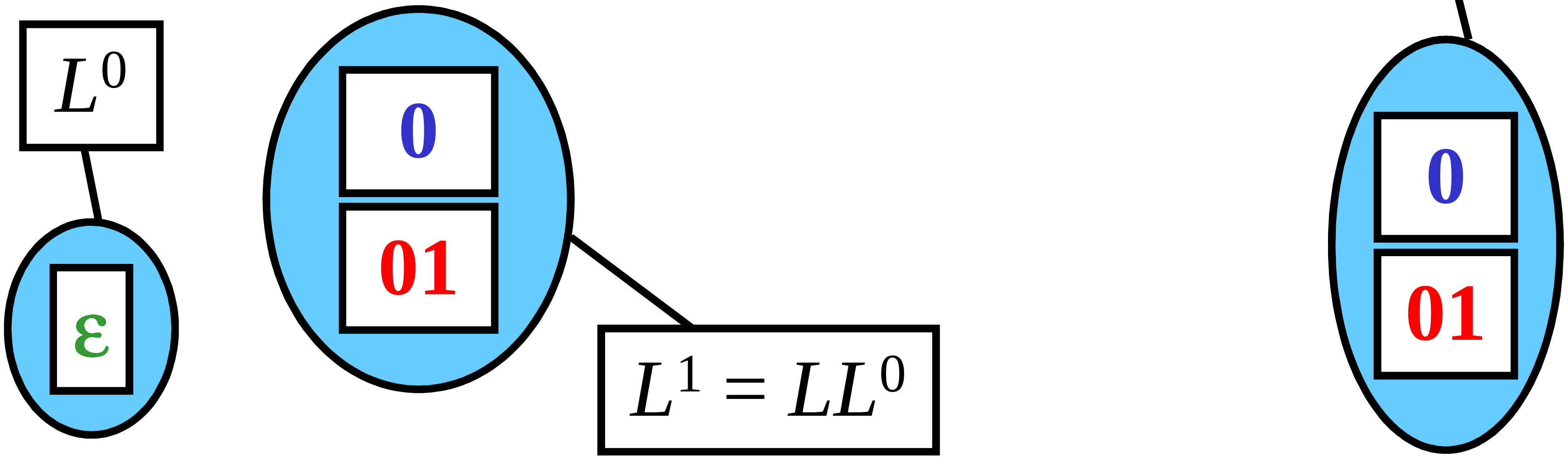

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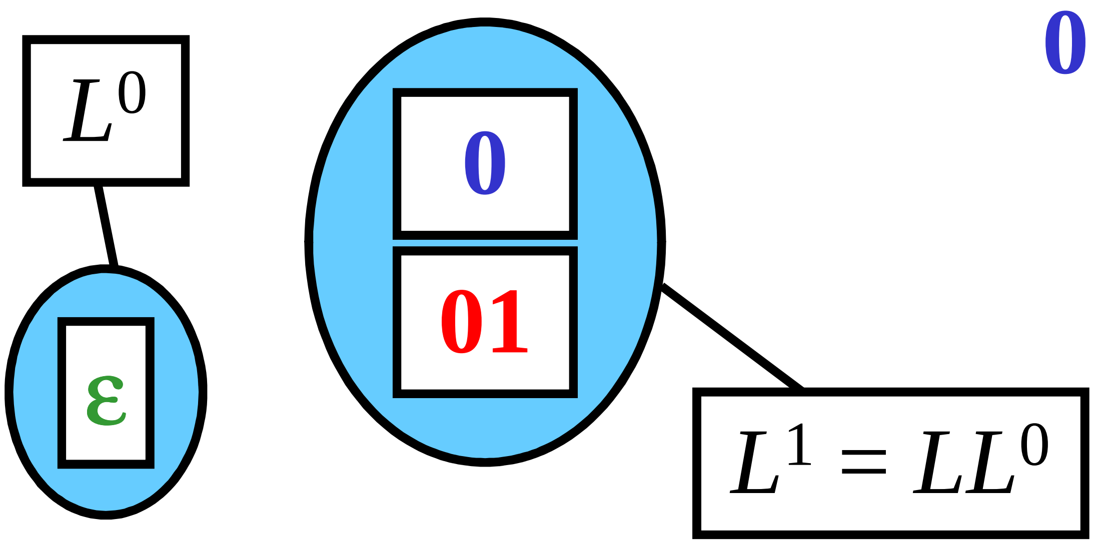

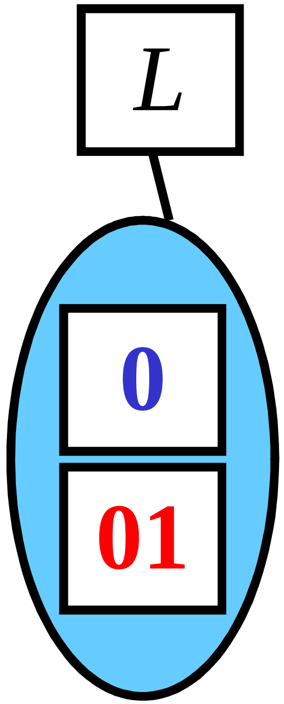

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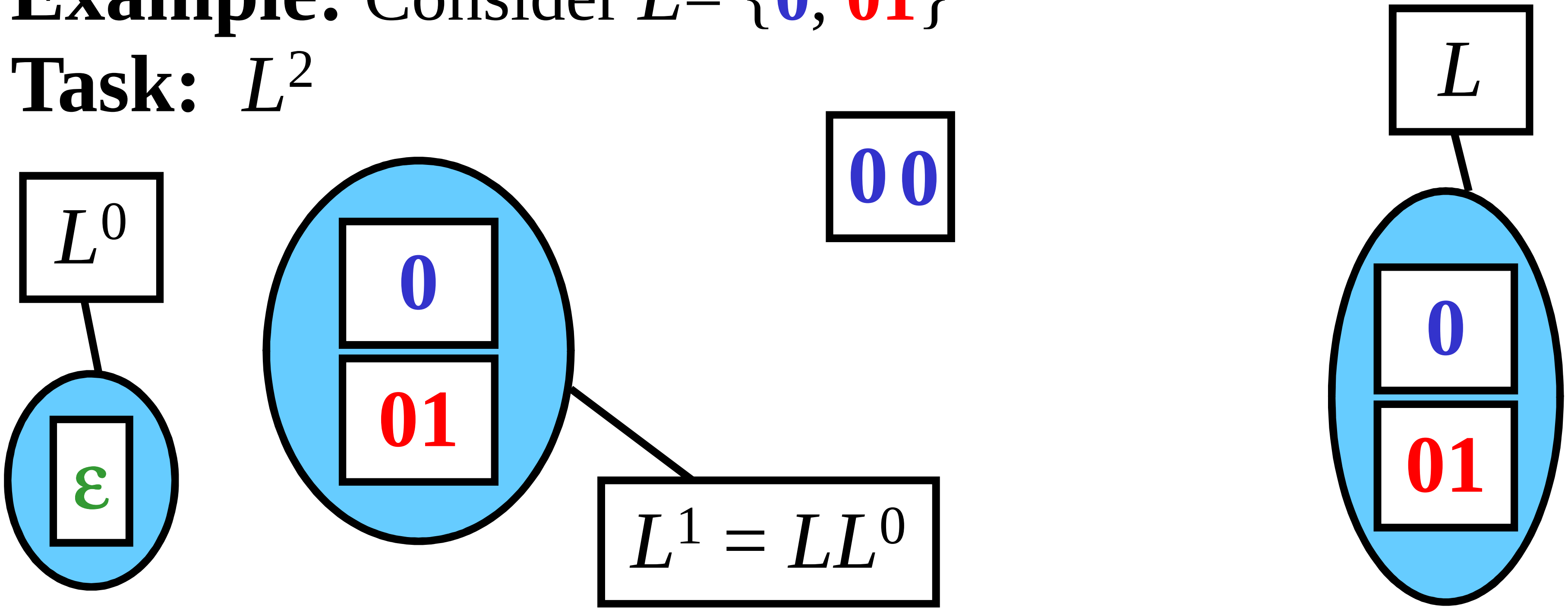

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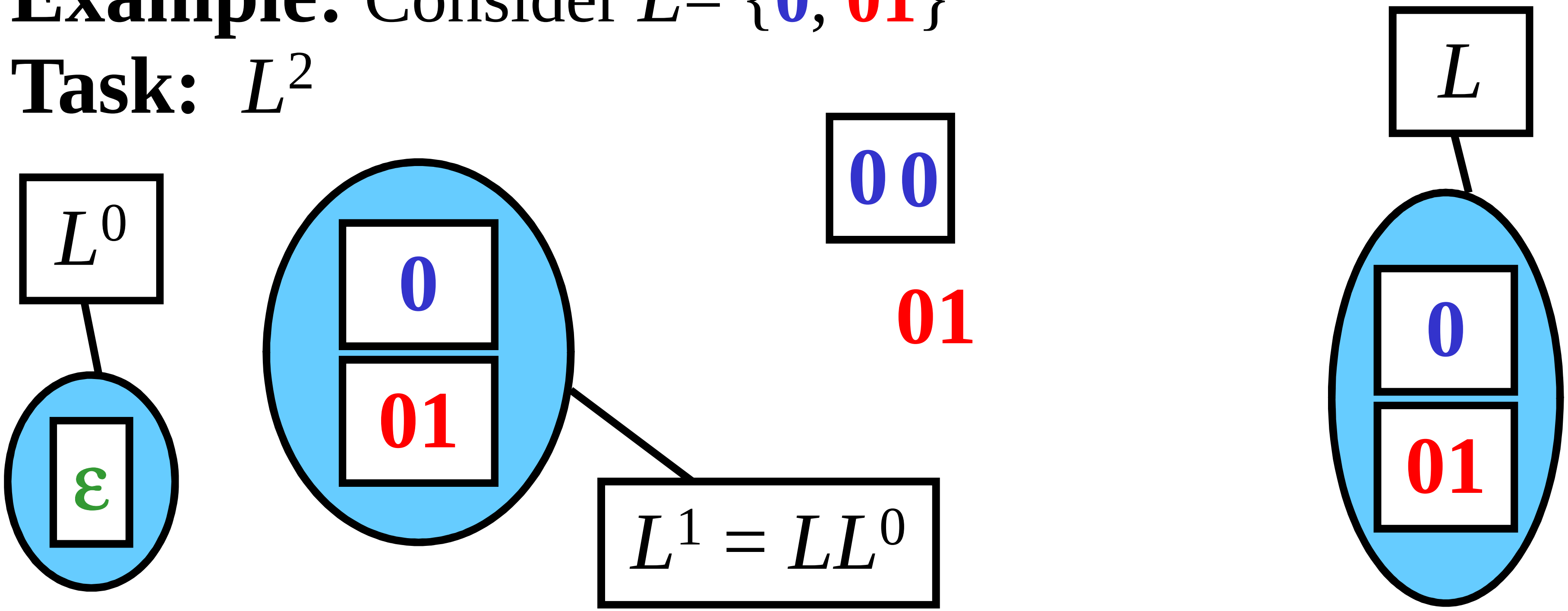

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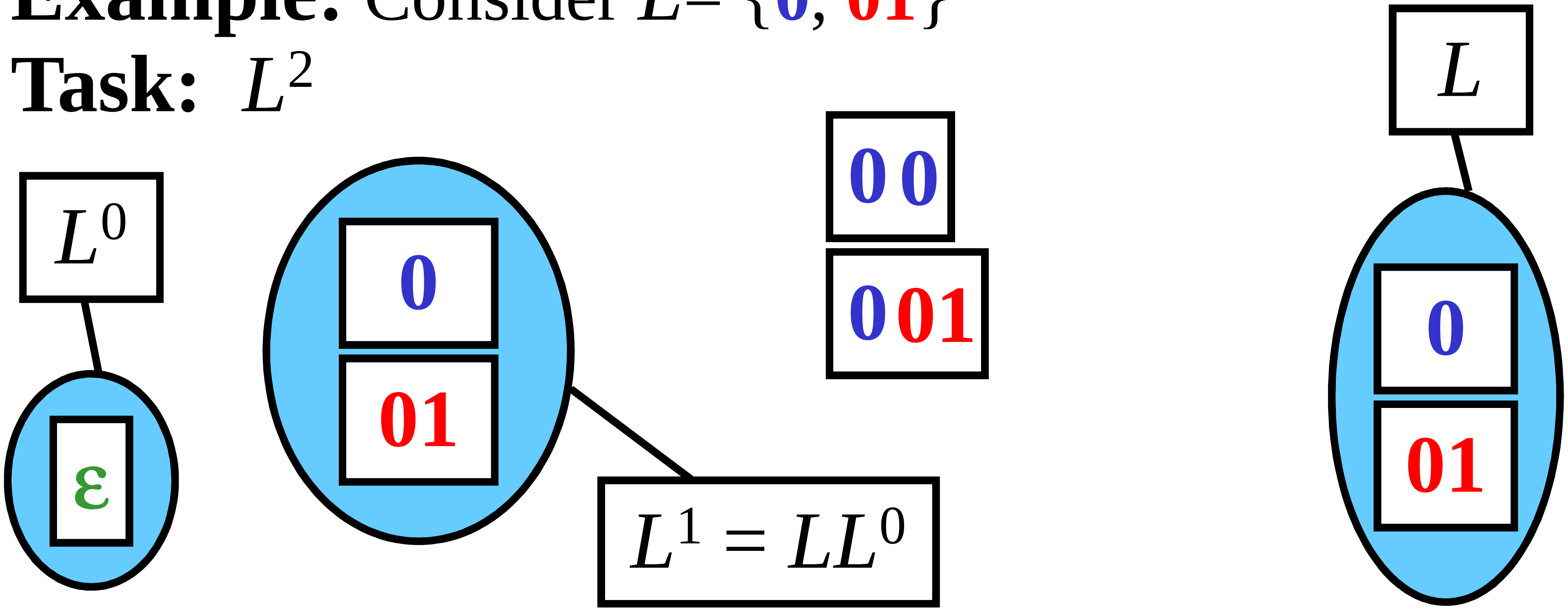

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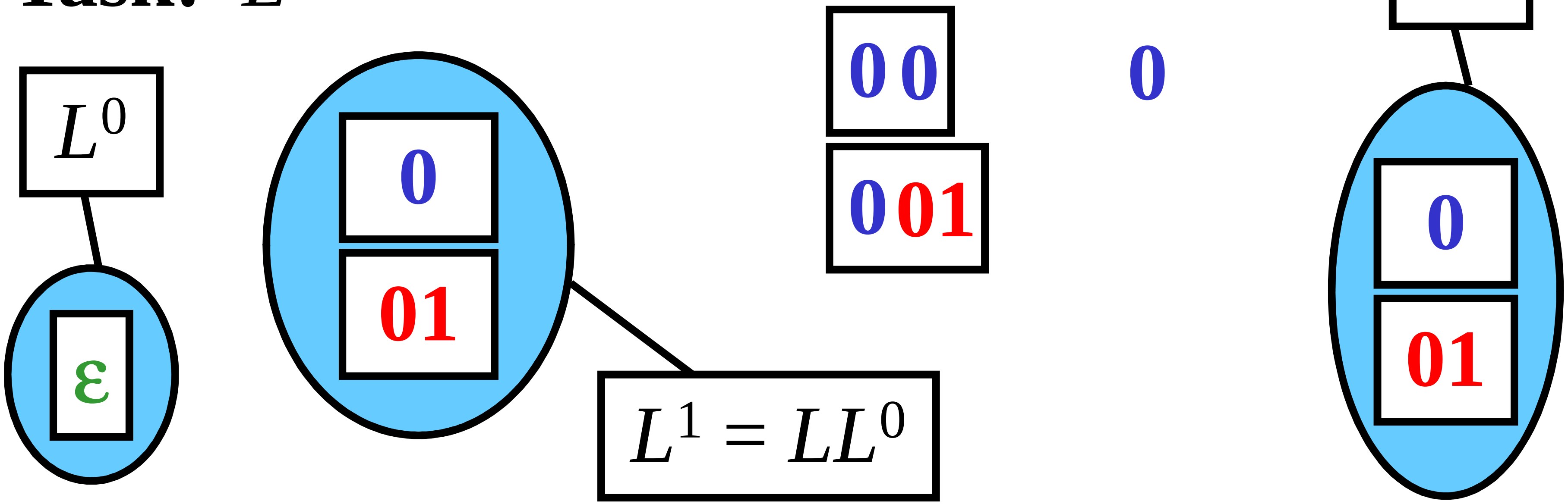

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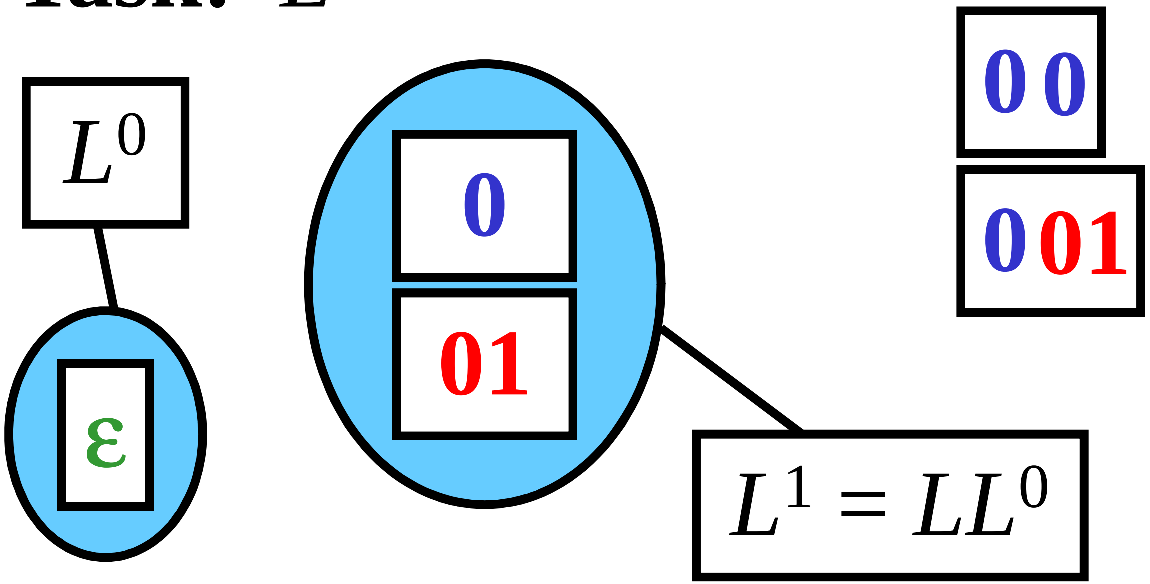

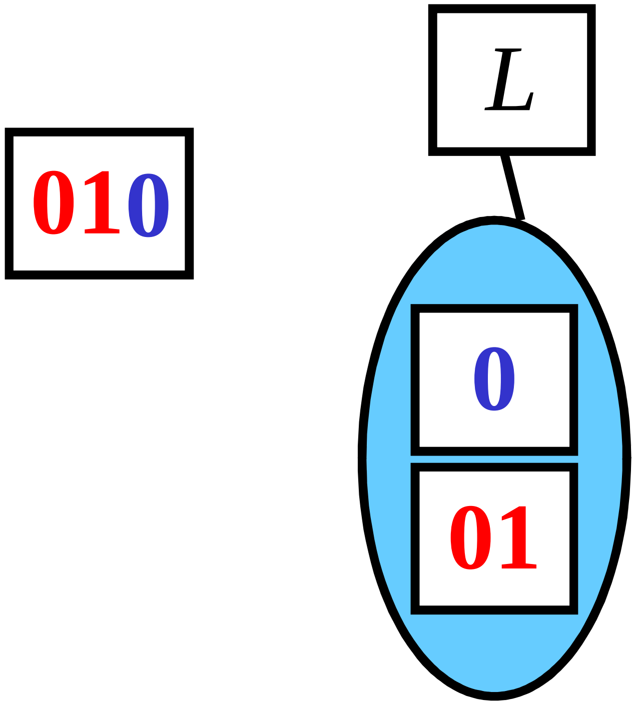

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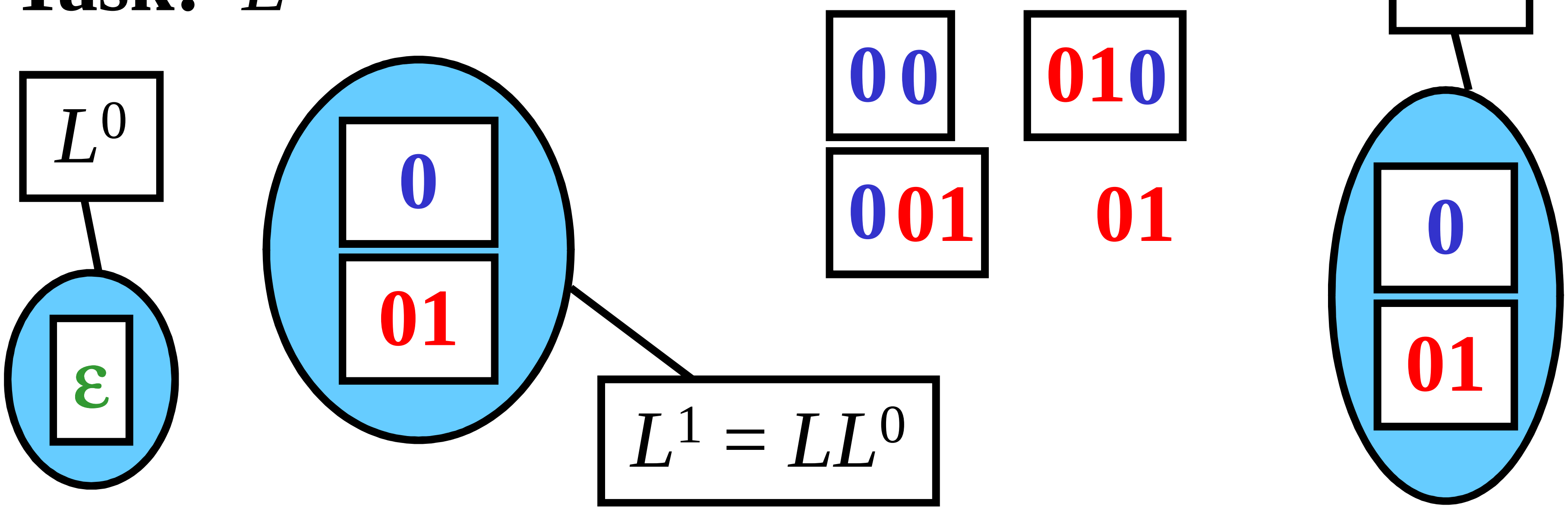

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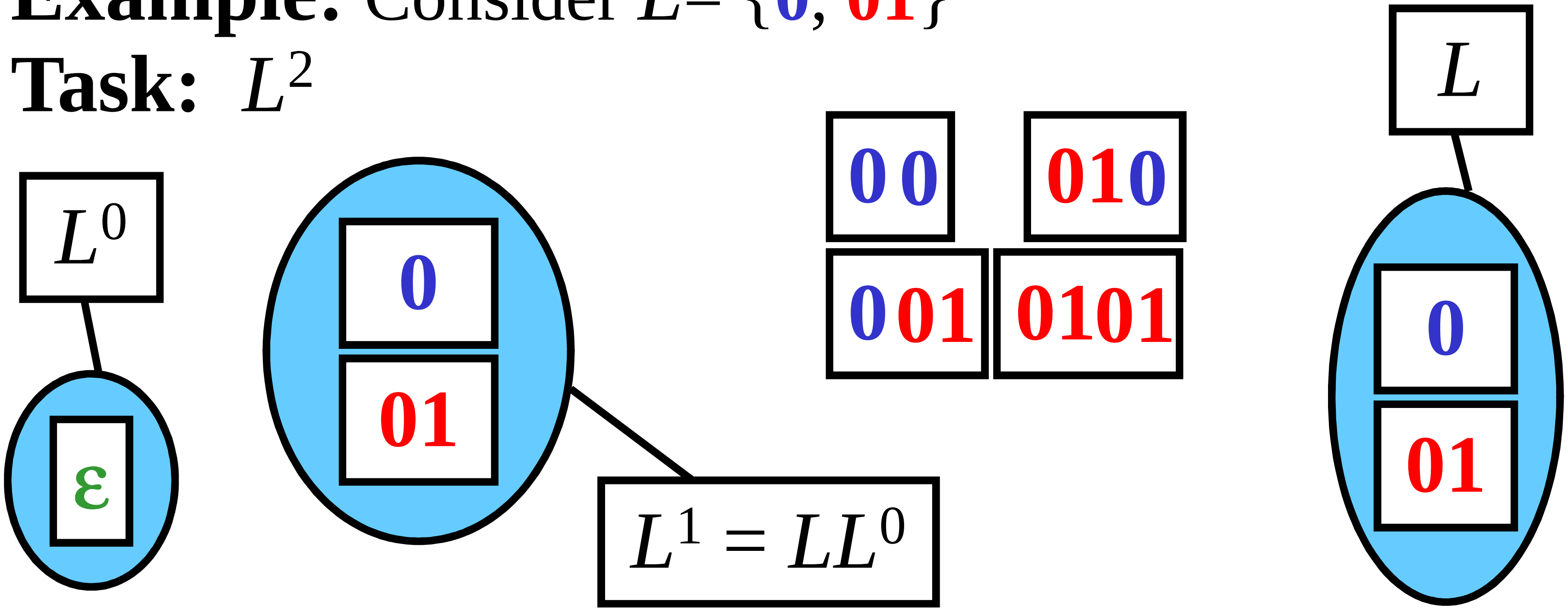
Example: Consider $L = \{0, 01\}$

Task: L^2



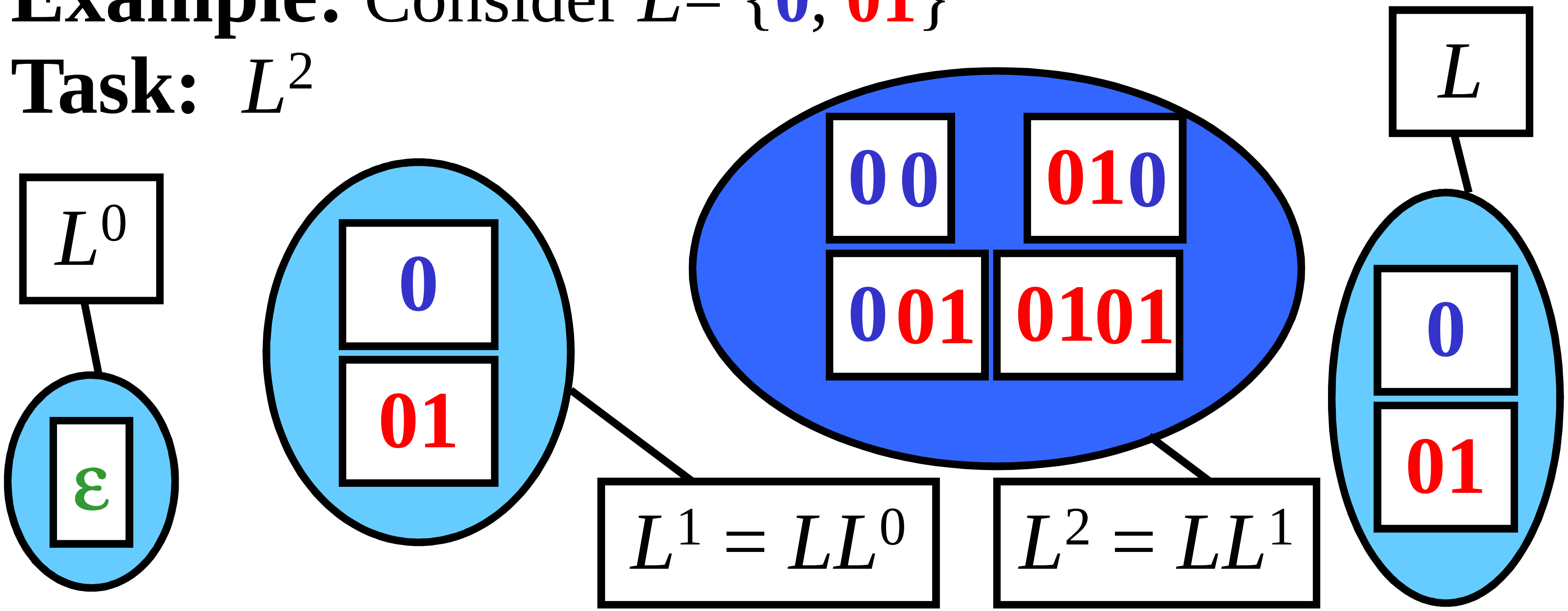
Power of Language

Gist: $L^i = \underbrace{LL\ldots L}_{i\text{-times}}$

Definition: Let L be a language over Σ.
For $i \geq 0$, the i-th *power* of L, L^i, is defined as:
1) $L^0 = \{\varepsilon\}$ **2)** if $i \geq 1$ then $L^i = LL^{i-1}$

Example: Consider $L = \{0, 01\}$

Task: L^2



Iteration of Language

Gist: $L^* = L^0 \cup L^1 \cup L^2 \cup \ldots \cup L^i \cup \ldots$

$L^+ = L^1 \cup L^2 \cup \ldots \cup L^i \cup \ldots$

Definition: Let L be a language over Σ. The *iteration* of L, L^*, and the *positive iteration* of L, L^+, are defined as $L^* = \bigcup_{i=0}^{\infty} L^i$, $L^+ = \bigcup_{i=1}^{\infty} L^i$

Note: 1) $L^+ = LL^* = L^*L$ **2)** $L^* = L^+ \cup \{\varepsilon\}$

Example:

Consider language $L = \{0, 01\}$ over $\Sigma = \{0, 1\}$.

Task: L^* **and** L^+

Iteration of Language

Gist: $L^* = L^0 \cup L^1 \cup L^2 \cup \ldots \cup L^i \cup \ldots$

$L^+ = L^1 \cup L^2 \cup \ldots \cup L^i \cup \ldots$

Definition: Let L be a language over Σ. The *iteration* of L, L^*, and the *positive iteration* of L, L^+, are defined as $L^* = \bigcup_{i=0}^{\infty} L^i$, $L^+ = \bigcup_{i=1}^{\infty} L^i$

Note: **1)** $L^+ = LL^* = L^*L$ **2)** $L^* = L^+ \cup \{\varepsilon\}$

Example:

Consider language $L = \{0, 01\}$ over $\Sigma = \{0, 1\}$.

Task: L^* **and** L^+

$L^0 = \{\varepsilon\}$, $L^1 = \{0, 01\}$, $L^2 = \{00, 001, 010, 0101\}, \ldots$

$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots = \{\varepsilon, 0, 01, 00, 001, 010, 0101, \ldots\}$

$L^+ = L^1 \cup L^2 \cup \ldots = \{0, 01, 00, 001, 010, 0101, \ldots\}$