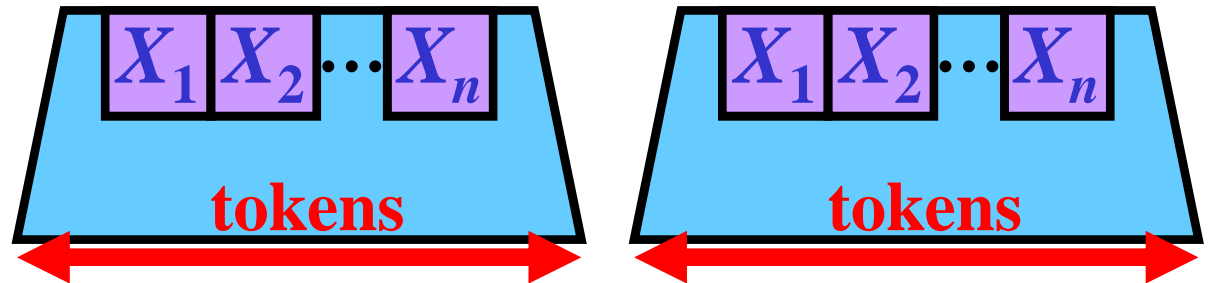


Part VIII.

Bottom-Up Parsing

Bottom-Up Parsing: Problems

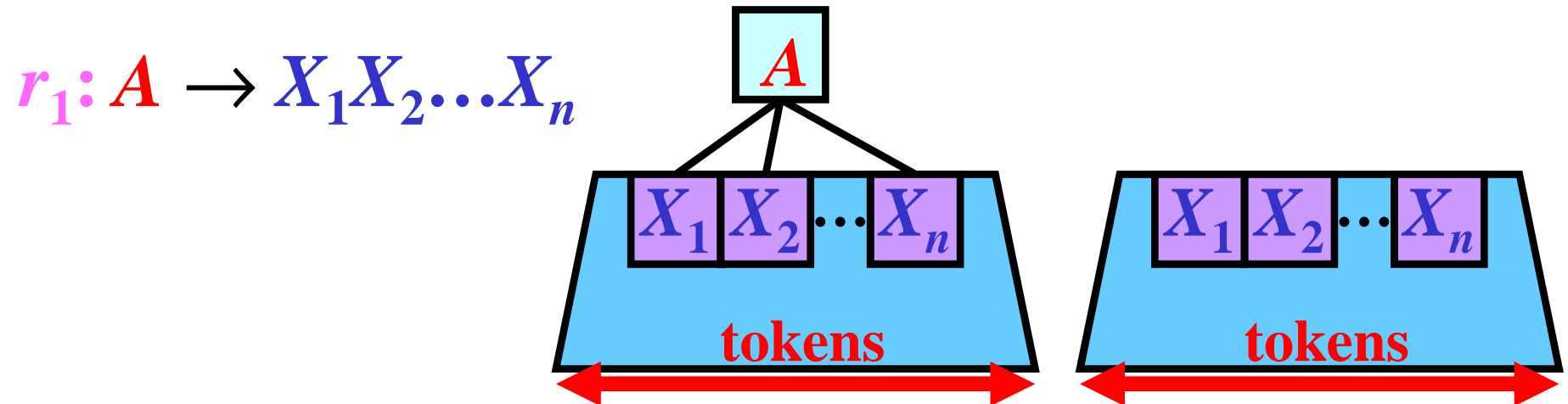
1) Two or more rules have the same *handle*



Note: A *handle* is the right-hand side of a rule.

Bottom-Up Parsing: Problems

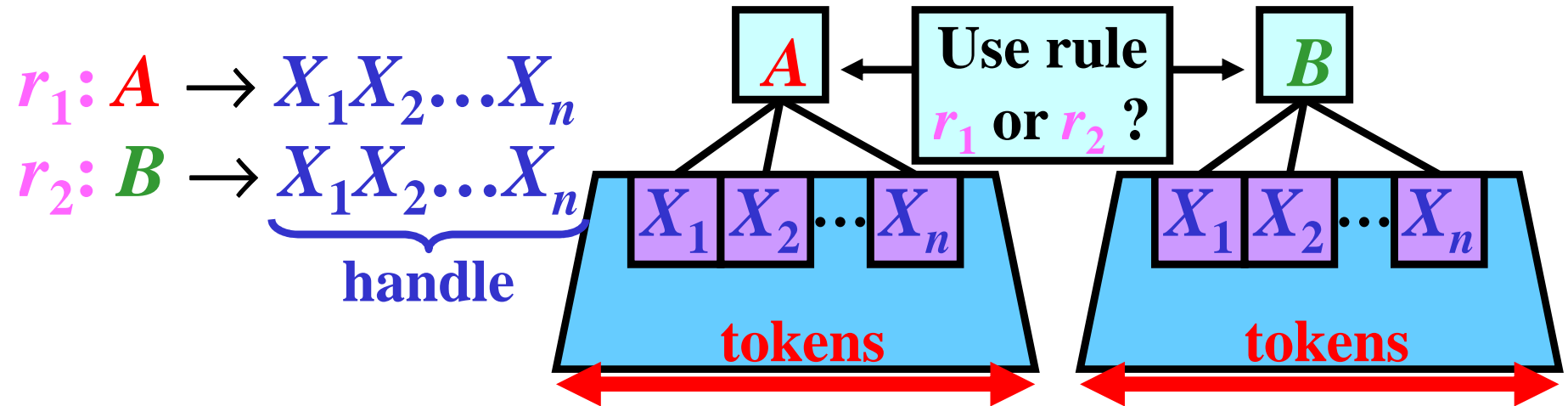
1) Two or more rules have the same *handle*



Note: A *handle* is the right-hand side of a rule.

Bottom-Up Parsing: Problems

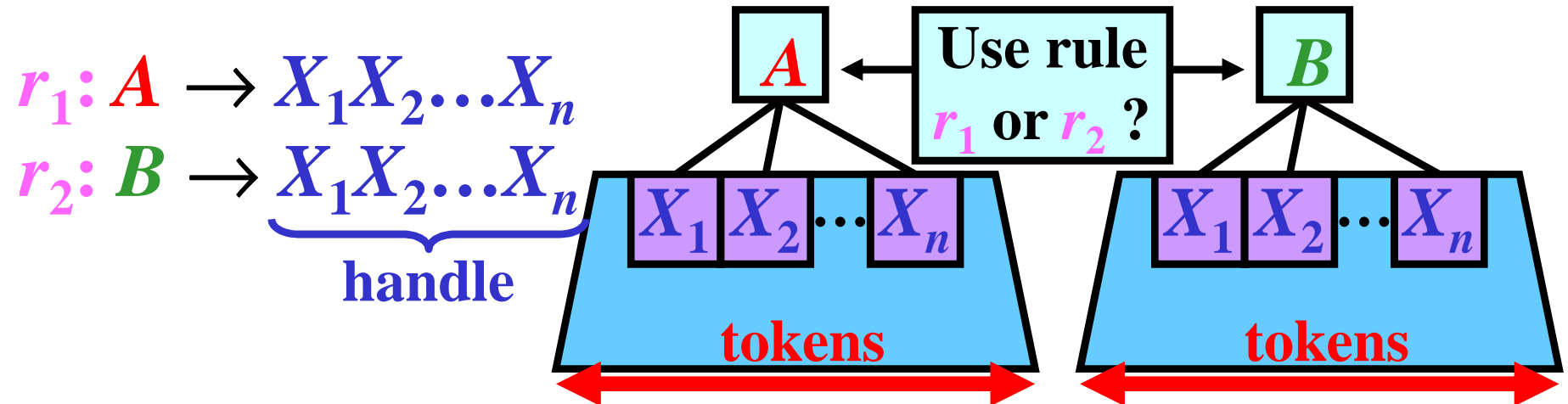
1) Two or more rules have the same *handle*



Note: A *handle* is the right-hand side of a rule.

Bottom-Up Parsing: Problems

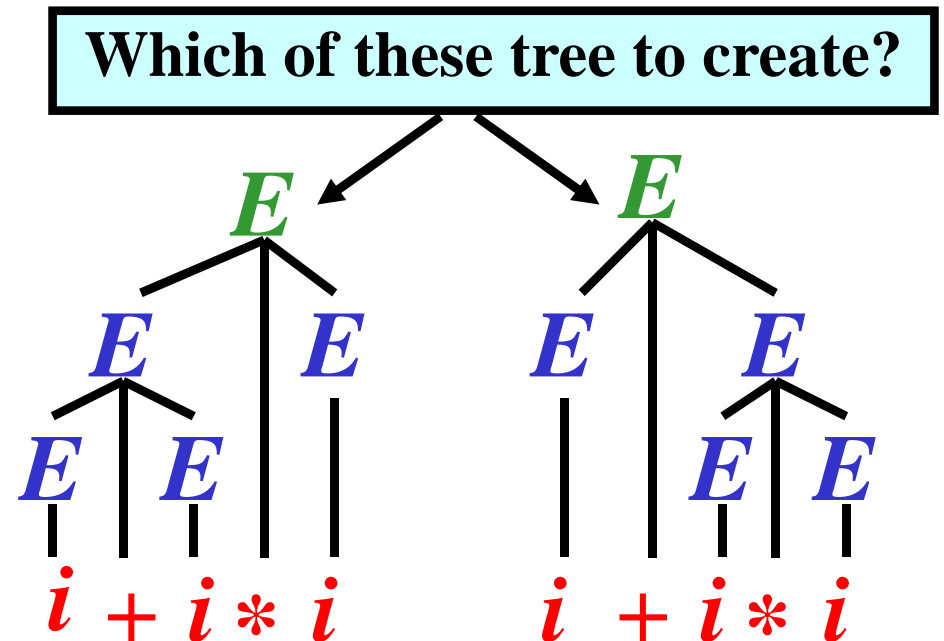
1) Two or more rules have the same *handle*



Note: A *handle* is the right-hand side of a rule.

2) Ambiguous grammars

$G_{expr2} = (N, T, P, E)$, where
 $N = \{E\}$, $T = \{i, +, *, (,)\}$,
 $P = \{$
 $\quad 1: E \rightarrow E + E, \quad 2: E \rightarrow E * E,$
 $\quad 3: E \rightarrow (E), \quad 4: E \rightarrow i \}$



Bottom-Up Parsers

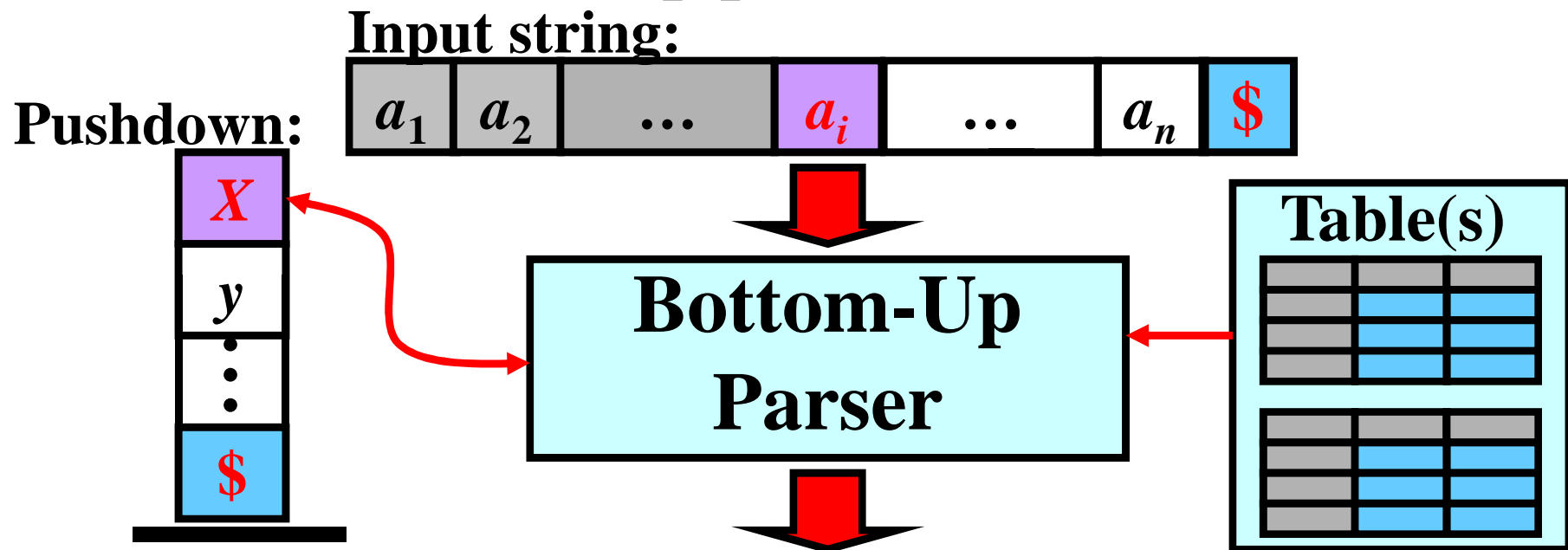
1) Operator-precedence parser

- the least powerful, but simple & easy-to-make

2) LR parser

- the most powerful

• Model of Bottom-Up parser:



Right parse = **reverse** sequence of rules used in the **rightmost derivation** of the tokenized source program

Operator-Precedence Parser

- No two distinct nonterminals have the same handle
 - No ϵ -rules.
-
- Let $G = (N, T, P, S)$ be CFG, where $T = \{a_1, a_2, \dots, a_n\}$

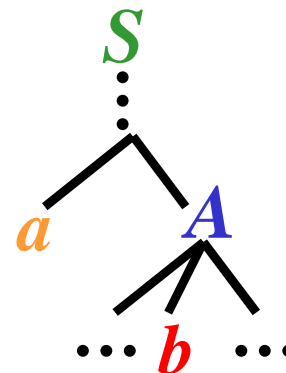
Precedence-table:

	a_1	...	a_j	...	a_n	\$
a_1						
...						
a_i						
...						
a_n						
\$						

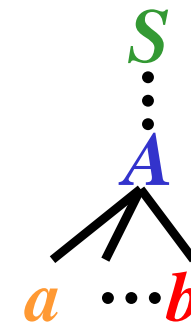
Table[a_i, a_j] $\in \{<, =, >, blank\}$

Illustration of meaning of $<, =, >$:

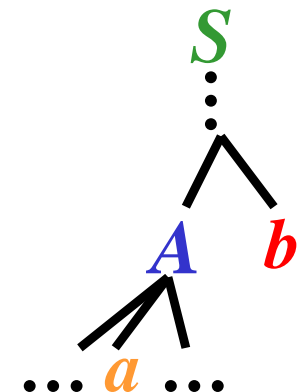
$a < b$



$a = b$



$a > b$



Operator-Precedence Parser: Algorithm

- **Input:** Precedence-table for $G = (N, T, P, S)$; $x \in T^*$
 - **Output:** Right parse of x if $x \in L(G)$; otherwise, **error**
-
- **Method:**
 - Push **\$** onto the pushdown;
 - **repeat**
 - let **a** = the topmost **terminal** on the pushdown and
b = the current token
 - **case** Table[**a**, **b**] **of:**
 - **=** : push(**b**) & read next **b** from input string
 - **<** : replace **a** with **a<** on the pushdown &
push(**b**) & read next **b** from input string
 - **>** : **if** **<y** is the pushdown top string **and** **r: A** \rightarrow **y** $\in P$
then replace **<y** with **A** & write **r** to output
else error
 - **blank** : **if** **a** = **\$** **and** **b** = **\$** **then success**
else error
- until success or error**

Operator-Precedence Parser: Example

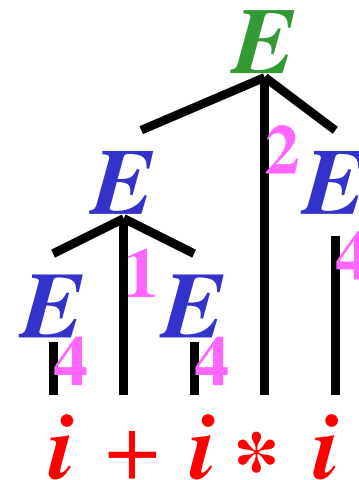
$G_{expr2} = (N, T, P, E)$, where $N = \{E\}$, $T = \{i, +, *, (,)\}$,
 $P = \{ 1: E \rightarrow E+E, 2: E \rightarrow E*E, 3: E \rightarrow (E), 4: E \rightarrow i \}$

Precedence-table for G_{expr2} :

Input token		+	*	()	i	\$
Pushdown topmost terminal	+	>	<	<	>	<	>
	*	>	>	<	>	<	>
	(<	<	<	=	<	
)	>	>		>		>
	i	>	>		>		>
	\$	<	<	<		<	

Note: Operator associativity and precedence rules underlie the precedence table:

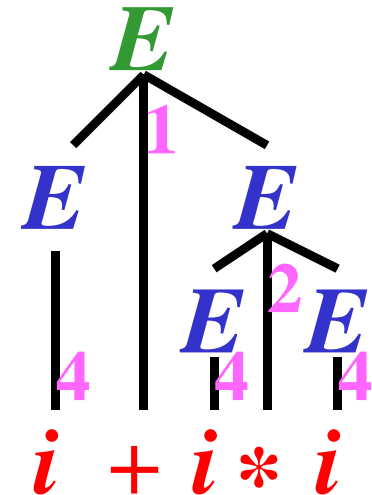
☹ Wrong tree:



Right parse:

44142

☺ Right tree:



Right parse:

44421

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: $i + i * i \$$

Pushdown	Op	Input	Rule

Rules:

$$1: E \rightarrow E + E$$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Pushdown	Op	Input	Rule
\$	<	<i>i + i * i \$</i>	

Rules:

1: $E \rightarrow E + E$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i* \$

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: <i>E</i> → <i>i</i>

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	4: <i>E</i> → <i>i</i>
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	4: <i>E</i> → <i>i</i>
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i* + *i* * *i* \$

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: <i>E</i> → <i>i</i>
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	
\$< <i>E</i> ++< <i>i</i>	>	* <i>i</i> \$	4: <i>E</i> → <i>i</i>

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i* + *i* * *i* \$

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: <i>E</i> → <i>i</i>
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	>	* <i>i</i> \$	4: <i>E</i> → <i>i</i>
\$< <i>E</i> + <i>E</i>	<	* <i>i</i> \$	

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: <i>E</i> → <i>i</i>
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	>	* <i>i</i> \$	4: <i>E</i> → <i>i</i>
\$< <i>E</i> + <i>E</i>	<	* <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> \$	

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Rules:

1: $E \rightarrow E + E$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: $E \rightarrow i$
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	>	* <i>i</i> \$	4: $E \rightarrow i$
\$< <i>E</i> + <i>E</i>	<	* <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> \$	
\$< <i>E</i> +	>	\$	4: $E \rightarrow i$

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Rules:

1: $E \rightarrow E + E$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: $E \rightarrow i$
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	>	* <i>i</i> \$	4: $E \rightarrow i$
\$< <i>E</i> + <i>E</i>	<	* <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> \$	
\$< <i>E</i> +	>	\$	4: $E \rightarrow i$
\$< <i>E</i> +	>	\$	2: $E \rightarrow E * E$

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Rules:

1: $E \rightarrow E + E$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: $E \rightarrow i$
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	>	* <i>i</i> \$	4: $E \rightarrow i$
\$< <i>E</i> + <i>E</i>	<	* <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> \$	
\$< <i>E</i> +	>	\$	4: $E \rightarrow i$
\$< <i>E</i> +	>	\$	2: $E \rightarrow E * E$
\$< <i>E</i> +	>	\$	1: $E \rightarrow E + E$

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Input string: *i + i * i \$*

Rules:

1: $E \rightarrow E + E$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Pushdown	Op	Input	Rule
\$	<	<i>i</i> + <i>i</i> * <i>i</i> \$	
\$< <i>i</i>	>	+ <i>i</i> * <i>i</i> \$	4: $E \rightarrow i$
\$ <i>E</i>	<	+ <i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> * <i>i</i> \$	
\$< <i>E</i> +	>	* <i>i</i> \$	4: $E \rightarrow i$
\$< <i>E</i> + <i>E</i>	<	* <i>i</i> \$	
\$< <i>E</i> +	<	<i>i</i> \$	
\$< <i>E</i> +	>	\$	4: $E \rightarrow i$
\$< <i>E</i> +	>	\$	2: $E \rightarrow E * E$
\$< <i>E</i> + <i>E</i>	>	\$	1: $E \rightarrow E + E$
\$ <i>E</i>		\$	

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

Rules:

1: $E \rightarrow E + E$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Input string: $i + i * i \$$

Pushdown	Op	Input	Rule
\$	<	$i + i * i \$$	
$\$ < i$	>	$+ i * i \$$	4: $E \rightarrow i$
$\$ E$	<	$+ i * i \$$	
$\$ < E +$	<	$i * i \$$	
$\$ < E + < i$	>	$* i \$$	4: $E \rightarrow i$
$\$ < E + E$	<	$* i \$$	
$\$ < E + < E *$	<	$i \$$	
$\$ < E + < E * < i$	>	$\$$	4: $E \rightarrow i$
$\$ < E + < E * E$	>	$\$$	2: $E \rightarrow E * E$
$\$ < E + E$	>	$\$$	1: $E \rightarrow E + E$
$\$ E$		$\$$	

Success

Right parse: 44421

Construction of Precedence Table 1/5

- Let $G_{expr} = (N, T, P, E)$, where $N = \{E\}$,
 $T = \{ (,), id_1, id_2, \dots, id_m, op_1, op_2, \dots, op_n \}$,
 $P = \{ E \rightarrow (E), E \rightarrow id_1, E \rightarrow id_2, \dots, E \rightarrow id_m,$
 $E \rightarrow E op_1 E, E \rightarrow E op_2 E, \dots, E \rightarrow E op_n E \}$

Note: id_1, id_2, \dots, id_m are identifiers,

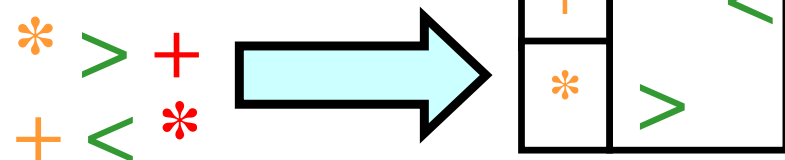
op_1, op_2, \dots, op_n are different operators

1) Precedence of operators:

- If op_i has higher precedence than op_j then

$$op_i > op_j \text{ and } op_j < op_i$$

Example: Precedence-table part derived from the precedence of operators in G_{expr2}



Construction of Precedence Table 2/5

2) Associativity:

Note:

- op_i is left-associative $\Leftrightarrow a \text{ op}_i b \text{ op}_i c = (a \text{ op}_i b) \text{ op}_i c$
- op_i is right-associative $\Leftrightarrow a \text{ op}_i b \text{ op}_i c = a \text{ op}_i (b \text{ op}_i c)$

- Let op_i and op_j have equal precedence

- If op_i and op_j are left associative then

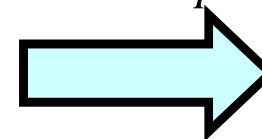
$$\text{op}_i > \text{op}_j \text{ and } \text{op}_j > \text{op}_i$$

- If op_i and op_j are right associative then

$$\text{op}_i < \text{op}_j \text{ and } \text{op}_j < \text{op}_i$$

Example: Precedence-table part derived from the associativity of operators in G_{expr2}

$+$ is left-associative
 $*$ is left-associative



	+	*
+	>	
*		>

Construction of Precedence Table 3/5

3) Identifiers:

- If $a \in T$ may precede id_i , then
- If $a \in T$ may follow id_i , then

$$a < id_i$$

$$id_i > a$$

Example: Precedence-table part for identifiers

$\$i * (i + i) * i$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $\$, (, +, *$ may precede i

$i * (i + i) * i \$$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $*, +,), \$$ may follow i

	+	*	()	i	\$
+					<	
*					<	
(<	
)						
i	>	>		>		>
\$					<	

Construction of Precedence Table 4/5

4) Parentheses:

- A pair of parentheses:
- Let $a \in T - \{), \$\}$. Then,
- Let $a \in T - \{ (, \$\}$. Then,
- Let $a \in T$ and a may precede (. Then,
- Let $a \in T$ and a may follow). Then,

(=)
(< a
a >)

a < (
) > a

Example: Precedence-table
part for parentheses.

$\$(i + ((i * (i + (i + i))))))$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $\$, \quad (, \quad *, \quad +$ may precede (

$(((((i + i) + i) * i) + i)\$)$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $+, \quad *, \quad), \quad \$$ may follow)

	+	*	()	i	\$
+			<	>		
*			<	>		
(<	<	<	=	<	
)	>	>		>		>
i				>		
\$			<			

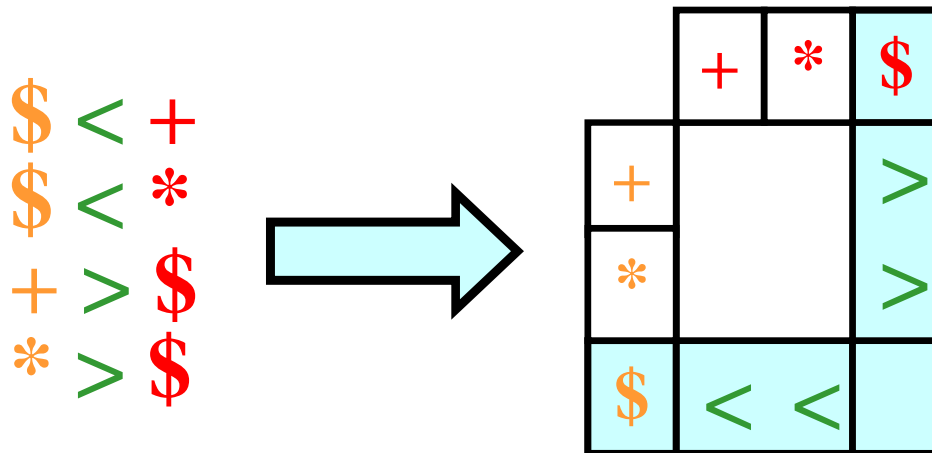
Construction of Precedence Table 5/5

5) End Marker \$

- Let op_i be any operator. Then:

$$\text{\textcolor{teal}{\$}} < \text{\textcolor{red}{op}_i} \text{ and } \text{\textcolor{red}{op}_i} > \text{\textcolor{teal}{\$}}$$

Example: Precedence-table part for end-markers.



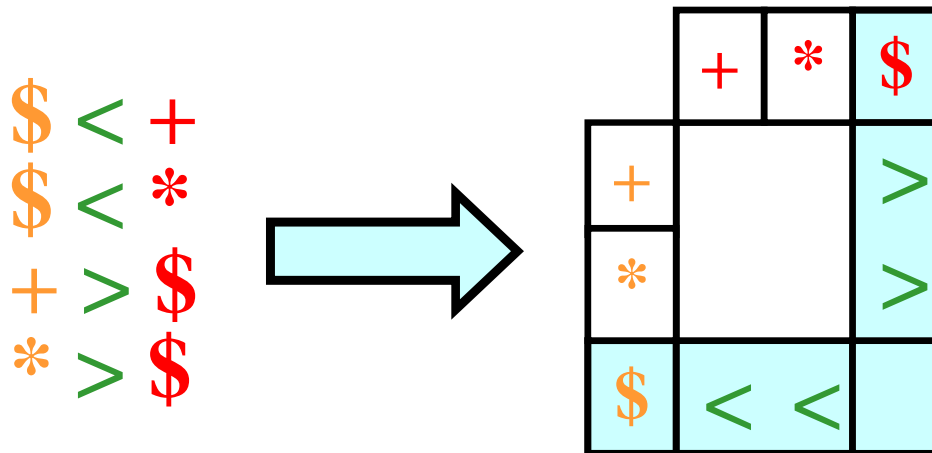
Construction of Precedence Table 5/5

5) End Marker \$

- Let op_i be any operator. Then:

$$\text{\textcolor{teal}{\$}} < \text{\textcolor{red}{op}_i} \text{ and } \text{\textcolor{red}{op}_i} > \text{\textcolor{teal}{\$}}$$

Example: Precedence-table part for end-markers.



Summary:

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
<i>i</i>	>	>		>		>
\$	<	<	<		<	

LR-Parser

- Let $G = (N, T, P, S)$ be a CFG,
where $N = \{A_1, A_2, \dots, A_n\}$, $T = \{a_1, a_2, \dots, a_m\}$
- LR-parser is a EPDA, M , with states
 $Q = \{q_0, q_1, \dots, q_k\}$, where q_0 is the start state.
- M is based on LR table that has these two parts
 - 1) **Action part**
 - 2) **Go-to part**

Action Part & Go-to Part

Action Part:

α	a_1	...	a_j	...	a_m	\$
q_0						
...						
q_i						
...						
q_k						

$\alpha[q_i, a_j] = 1 \text{ or } 2 \text{ or } 3 \text{ or } 4$

- 1) **sq**: **s** = shift, $q \in Q$
- 2) **rp**: **r** = reduce, $p \in P$
- 3) 😊 : success
- 4) **blank**: error

Go-to Part:

β	A_1	...	A_j	...	A_n
q_0					
...					
q_i					
...					
q_k					

$\beta[q_i, A_j] = 1 \text{ or } 2$

- 1) **q**: $q \in Q$
- 2) **blank**

LR-Parser: Algorithm

- **Input:** LR-table for $G = (N, T, P, S)$; $x \in T^*$
 - **Output:** Right parse of x if $x \in L(G)$; otherwise, **error**
-
- **Method:**
 - push($\langle \$, q_0 \rangle$) onto pushdown; $state := q_0$;
 - **repeat**
 - let a = the current token
 - case** $\alpha[state, a]$ **of:**
 - **sq:** push($\langle a, q \rangle$) & read next a from input string & $state := q$;
 - **rp:** if $p: A \rightarrow X_1 X_2 \dots X_n \in P$ and $\langle ?, q \rangle \langle X_1, ? \rangle \langle X_2, ? \rangle \dots \langle X_n, ? \rangle$ is pushdown top then $state := \beta[q, A]$ & replace $\langle X_1, ? \rangle \langle X_2, ? \rangle \dots \langle X_n, ? \rangle$ with $\langle A, state \rangle$ on the pushdown & write r to output **else error**
 - 😊: **success**
 - **blank:** **error**
- until success or error**



LR-Parser: Example 1/2

$K = (N, T, P, \mathbf{S})$, where $N = \{\mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$

LR-table for K :

α	\mathbf{i}	\mathbf{o}	$($	$)$	$\mathbf{\$}$
0	s3		s4		
1		s6			☺
2		r2		r2	r2
3		r3		r3	r3
4	s3		s4		
5		s6		s8	
6	s3		s4		
7		r1		r1	r1
8		r4		r4	r4

Action part
for K

Go-to part
for K

β	\mathbf{S}	\mathbf{A}
0	1	2
1		
2		
3		
4	5	2
5		
6		7
7		
8		

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	$3: A \rightarrow i$
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$ $\beta[0, A] = 2$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle$	6	$i\$$	$\alpha[6, i] = s3$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle i, 3 \rangle$	3	$\$$	$\alpha[3, \$] = r3$	3: $A \rightarrow i$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$ $\beta[0, A] = 2$	3: $A \rightarrow i$
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$ $\beta[0, S] = 1$	2: $S \rightarrow A$
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle i, 3 \rangle$	3	$\$$	$\alpha[3, \$] = r3$ $\beta[6, A] = 7$	3: $A \rightarrow i$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle i, 3 \rangle$	3	$\$$	$\alpha[3, \$] = r3$	3: $A \rightarrow i$
			$\beta[6, A] = 7$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle A, 7 \rangle$	7	$\$$	$\alpha[7, \$] = r1$	1: $S \rightarrow SoA$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle i, 3 \rangle$	3	$\$$	$\alpha[3, \$] = r3$	3: $A \rightarrow i$
			$\beta[6, A] = 7$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle A, 7 \rangle$	7	$\$$	$\alpha[7, \$] = r1$	1: $S \rightarrow SoA$
			$\beta[0, S] = 1$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle i, 3 \rangle$	3	$\$$	$\alpha[3, \$] = r3$	3: $A \rightarrow i$
			$\beta[6, A] = 7$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle A, 7 \rangle$	7	$\$$	$\alpha[7, \$] = r1$	1: $S \rightarrow SoA$
			$\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$\$$	$\alpha[1, \$] = \text{☺}$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i o i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	$3: A \rightarrow i$
$\langle \$, 0 \rangle \langle i, 3 \rangle$	3	$oi\$$	$\alpha[3, o] = r3$ $\beta[0, A] = 2$	
$\langle \$, 0 \rangle \langle A, 2 \rangle$	2	$oi\$$	$\alpha[2, o] = r2$ $\beta[0, S] = 1$	
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$oi\$$	$\alpha[1, o] = s6$	$2: S \rightarrow A$
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle i, 3 \rangle$	3	$\$$	$\alpha[3, \$] = r3$ $\beta[6, A] = 7$	
$\langle \$, 0 \rangle \langle S, 1 \rangle \langle o, 6 \rangle \langle A, 7 \rangle$	7	$\$$	$\alpha[7, \$] = r1$ $\beta[0, S] = 1$	$1: S \rightarrow SoA$
$\langle \$, 0 \rangle \langle S, 1 \rangle$	1	$\$$	$\alpha[1, \$] = \text{☺}$	
				Success
				Right parse: 3231

Construction of LR Table: Introduction

- **One parsing algorithm but many algorithms for the construction of LR table.**
-

Basic algorithms for the construction of LR table:

- 1) **Simple LR (SLR)**: the least powerful, but simple and few states
 - 2) **Canonical LR**: more powerful, but many states
 - 3) **Lookahead LR (LALR)**: the best because the most powerful and the same number of states as SLR
-

Extended Grammar with a “Dummy Rule”

Gist: Grammar with special “starting rule”

Definition: Let $G = (N, T, P, S)$ be a CFG, $S' \notin N$. *Extended grammar* for G is grammar $G' = (N \cup \{S'\}, T, P \cup \{S' \rightarrow S\}, S')$.

Why a dummy rule? When $S' \rightarrow S$ is used and the input token is endmarker, then **syntax analysis is successfully completed**.

Example:

$K = (N, T, P, S)$, where $N = \{S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$

Extended grammar for K :

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$

Construction of LR Table: Items

Gist: Item is a rule of CFG with \bullet in the right-hand side of rule.

Definition: Let $G = (N, T, P, S)$ be a CFG, $A \rightarrow x \in P$, $x = yz$. Then, $A \rightarrow y\bullet z$ is an *item*.

Example: Consider $S \rightarrow SoA$

All items for $S \rightarrow SoA$ are:

$S \rightarrow \bullet SoA$, $S \rightarrow S\bullet oA$, $S \rightarrow So\bullet A$, $S \rightarrow SoA\bullet$

Meaning: $A \rightarrow y\bullet z$ means that if y appears on the pushdown top and a prefix of the input is eventually reduced to z , then yz ($= x$) as a handle can be reduced to A according to $A \rightarrow x$.

Closure of Item: Algorithm

Note: $\text{Closure}(I)$ is the set of items defined by the following algorithm:

- **Input:** $G = (N, T, P, S)$; item I
 - **Output:** $\text{Closure}(I)$
-

- **Method:**
- $\text{Closure}(I) := \{I\};$
- **Apply the following rule until $\text{Closure}(I)$ cannot be changed:**
 - if $A \rightarrow y \bullet B z \in \text{Closure}(I)$ and $B \rightarrow x \in P$ then add $B \rightarrow \bullet x$ to $\text{Closure}(I)$

Closure of Item: Example 1/2

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$

Task: $Closure(I)$ for $I = S' \rightarrow \bullet S$

$Closure(I) := \{S' \rightarrow \bullet S\}$

1) $S' \rightarrow \bullet S \in Closure(I)$ & $S \rightarrow SoA \in P$:
 add $S \rightarrow \bullet SoA$ to $Closure(I)$

$Closure(I) = \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA\}$

2) $S' \rightarrow \bullet S \in Closure(I)$ & $S \rightarrow A \in P$:
 add $S \rightarrow \bullet A$ to $Closure(I)$

$Closure(I) = \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A\}$

Closure of Item: Example 2/2

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$

3) $S \rightarrow \bullet A \in \text{Closure}(I)$ & $A \rightarrow i \in P$:
 add $A \rightarrow \bullet i$ to $\text{Closure}(I)$

$\text{Closure}(I) = \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i\}$

4) $S \rightarrow \bullet A \in \text{Closure}(I)$ & $A \rightarrow (S) \in P$:
 add $A \rightarrow \bullet (S)$ to $\text{Closure}(I)$

Summary:

$\text{Closure}(I) = \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU \bullet z)$, where $A \rightarrow y \bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU \bullet z), A \rightarrow y \bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So \bullet A, S \rightarrow \bullet A, A \rightarrow \bullet (S)\}$

Task: $\Theta_A(I)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

$Closure(S \rightarrow SoA\bullet)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet (S)\}$

Task: $\Theta_A(I)$

$Closure(S \rightarrow SoA\bullet)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

$Closure(S \rightarrow SoA\bullet) \cup Closure(S \rightarrow A\bullet)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

$$Closure(S \rightarrow SoA\bullet) \cup Closure(S \rightarrow A\bullet) = \{S \rightarrow SoA\bullet, S \rightarrow A\bullet\}$$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

$$Closure(S \rightarrow SoA\bullet) \cup Closure(S \rightarrow A\bullet) = \{S \rightarrow SoA\bullet, S \rightarrow A\bullet\}$$

Task: $\Theta_{(}(I)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

$$Closure(S \rightarrow SoA\bullet) \cup Closure(S \rightarrow A\bullet) = \{S \rightarrow SoA\bullet, S \rightarrow A\bullet\}$$

Task: $\Theta_{(}(I)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $\text{Closure}(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in \text{Closure}(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

$$\text{Closure}(S \rightarrow SoA\bullet) \cup \text{Closure}(S \rightarrow A\bullet) = \{S \rightarrow SoA\bullet, S \rightarrow A\bullet\}$$

Task: $\Theta_{(}(I)$

$$\text{Closure}(A \rightarrow (\bullet S))$$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I , $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I .

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,

$$\Theta_U(I) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in I\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,
 $I = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(I)$

$$Closure(S \rightarrow SoA\bullet) \cup Closure(S \rightarrow A\bullet) = \{S \rightarrow SoA\bullet, S \rightarrow A\bullet\}$$

Task: $\Theta_{(}(I)$

$$Closure(A \rightarrow (\bullet S)) = \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet(S)\}$$

Set Θ_G for Grammar G

Note: Set Θ_G for grammar G is the set of sets of items defined by the following algorithm:

- **Input:** Extended $G = (N, T, P, S')$
 - **Output:** Θ_G for grammar G
-

- **Method:**

- $\Theta_G := \{ \text{Closure}(S' \rightarrow \bullet S) \};$
- for each $I \in \Theta_G$ and $U \in N \cup T$
 if $\Theta_U(I) \neq \emptyset$ then include set $\Theta_U(I)$ into Θ_G

Set Θ_G : Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
 $P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$

Initialization:

Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$

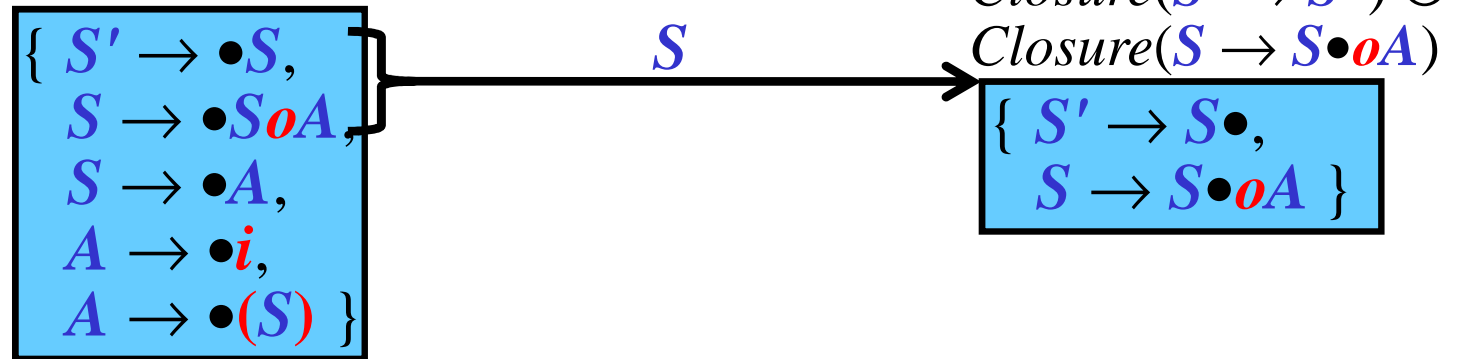
Initialization: $Closure(\mathbf{S}' \rightarrow \bullet \mathbf{S})$

$\{$ $\mathbf{S}' \rightarrow \bullet \mathbf{S},$
 $\mathbf{S} \rightarrow \bullet \mathbf{S}\mathbf{o}\mathbf{A},$
 $\mathbf{S} \rightarrow \bullet \mathbf{A},$
 $\mathbf{A} \rightarrow \bullet \mathbf{i},$
 $\mathbf{A} \rightarrow \bullet (\mathbf{S}) \}$

Set Θ_G : Example

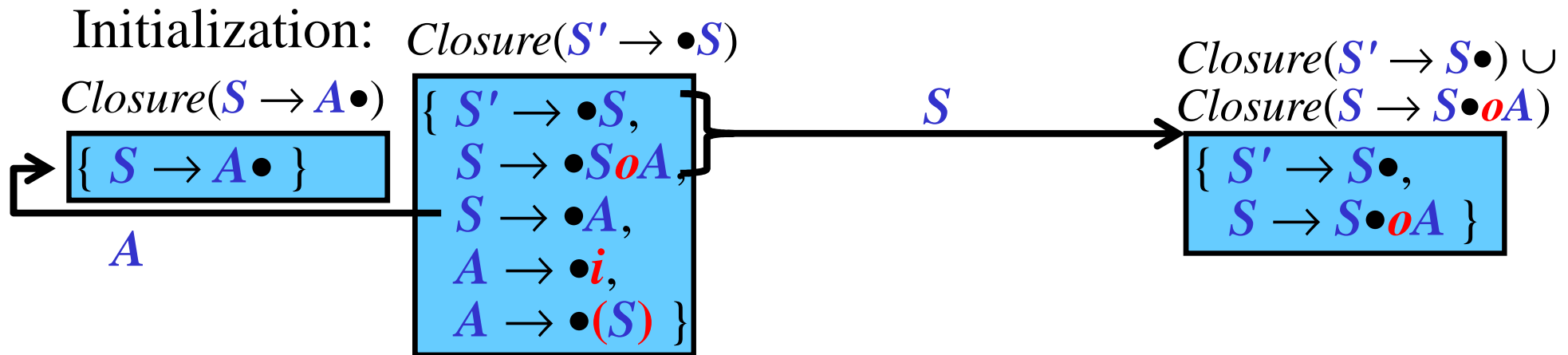
$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$

Initialization: $Closure(\mathbf{S}' \rightarrow \bullet \mathbf{S})$



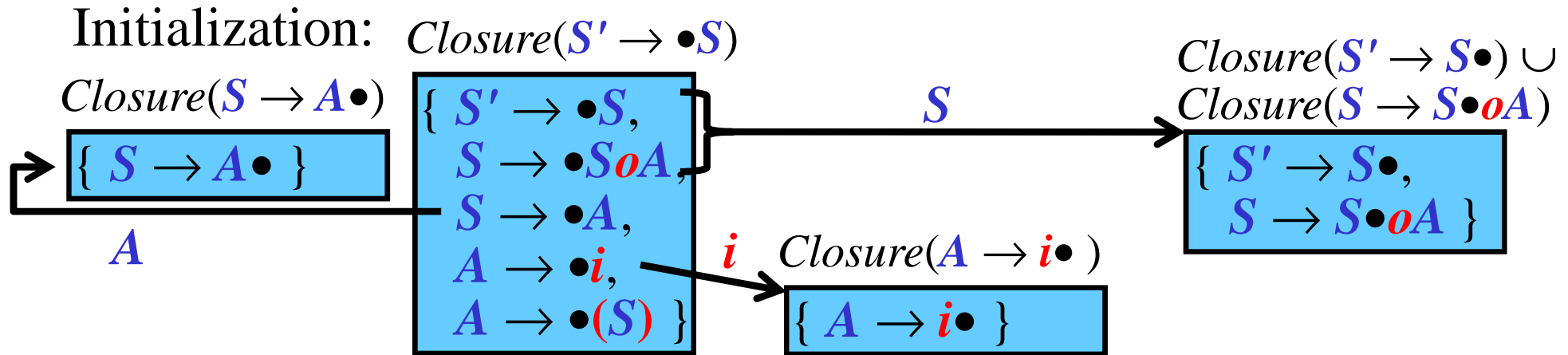
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



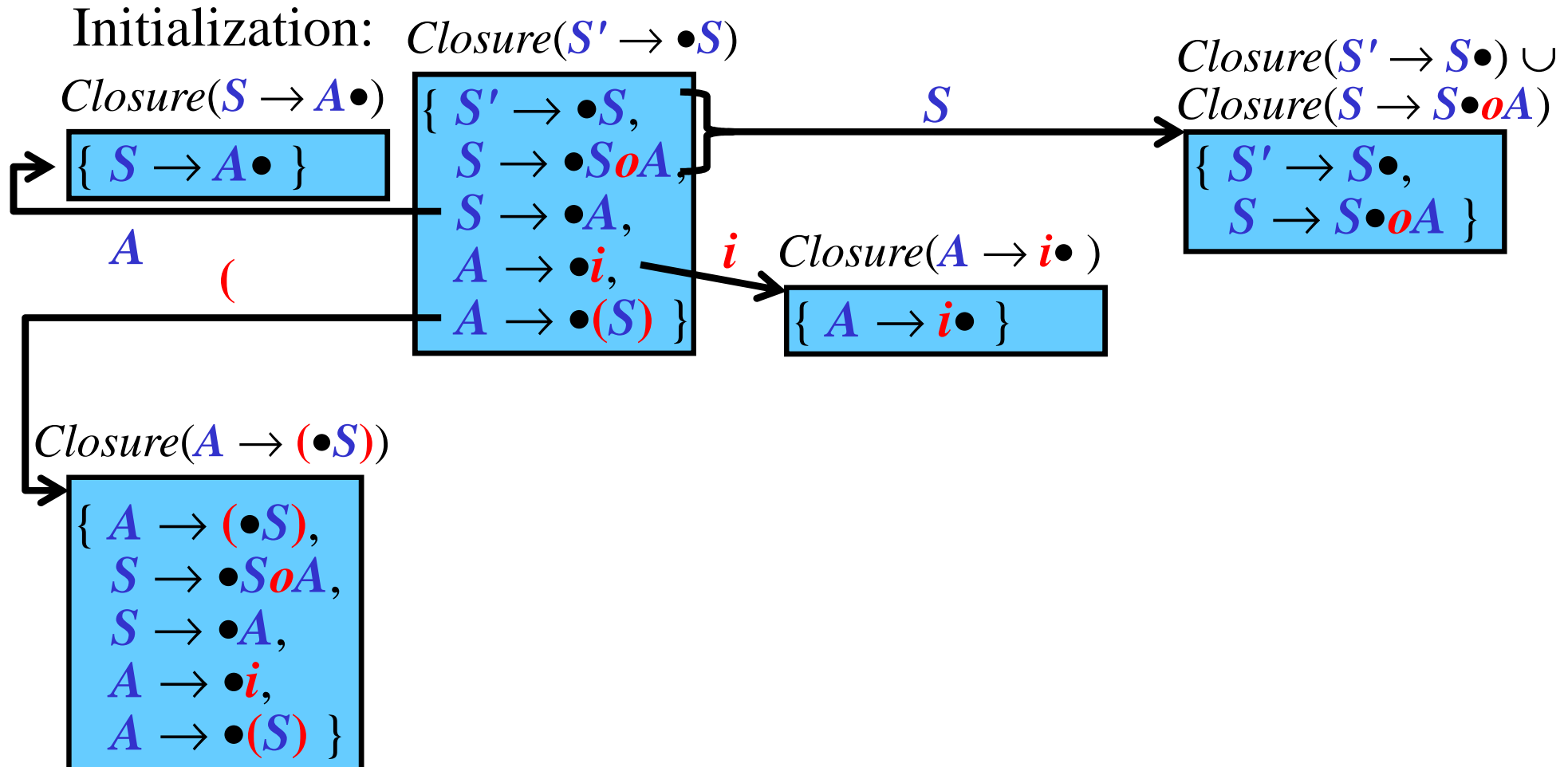
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



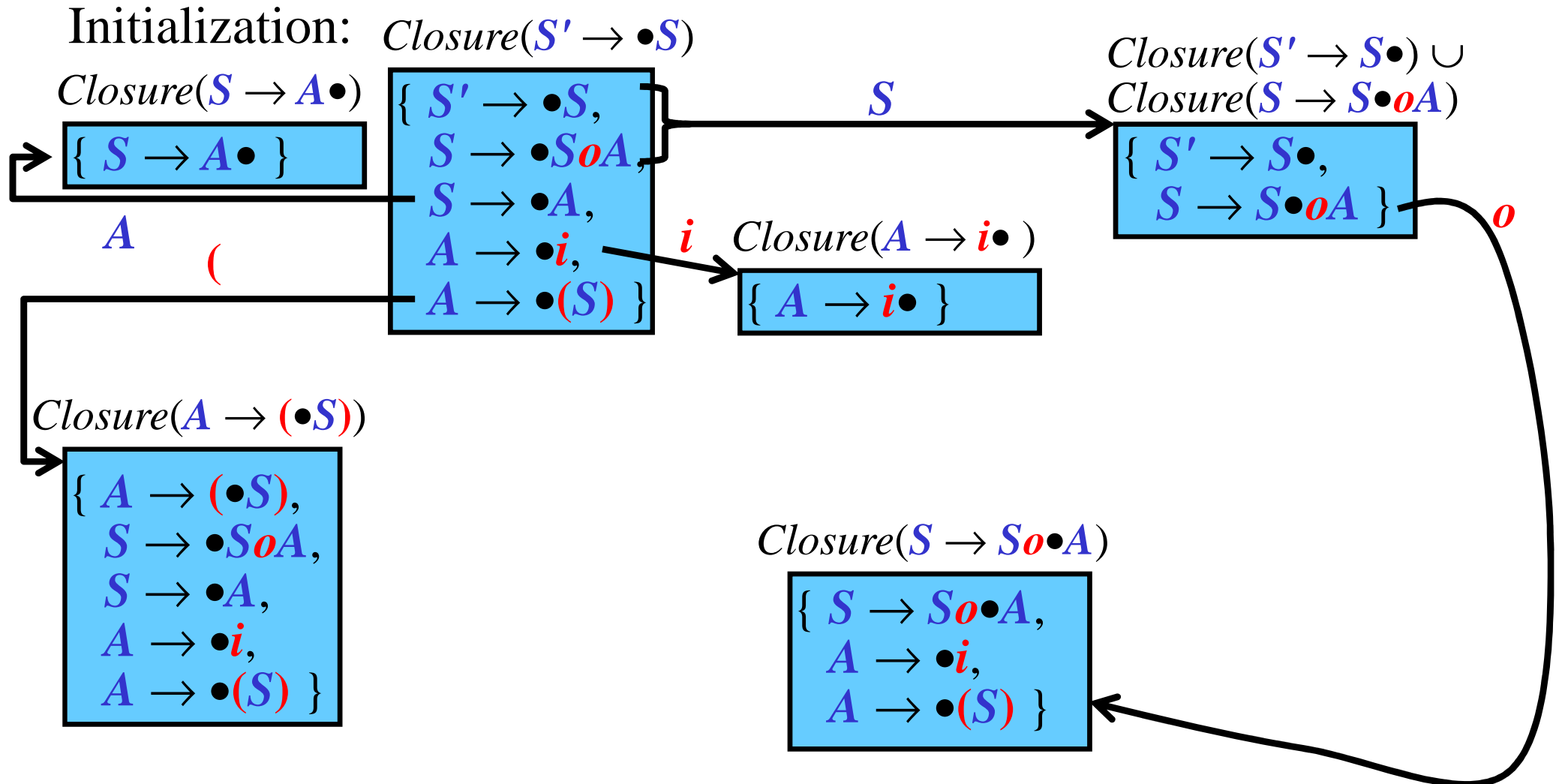
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



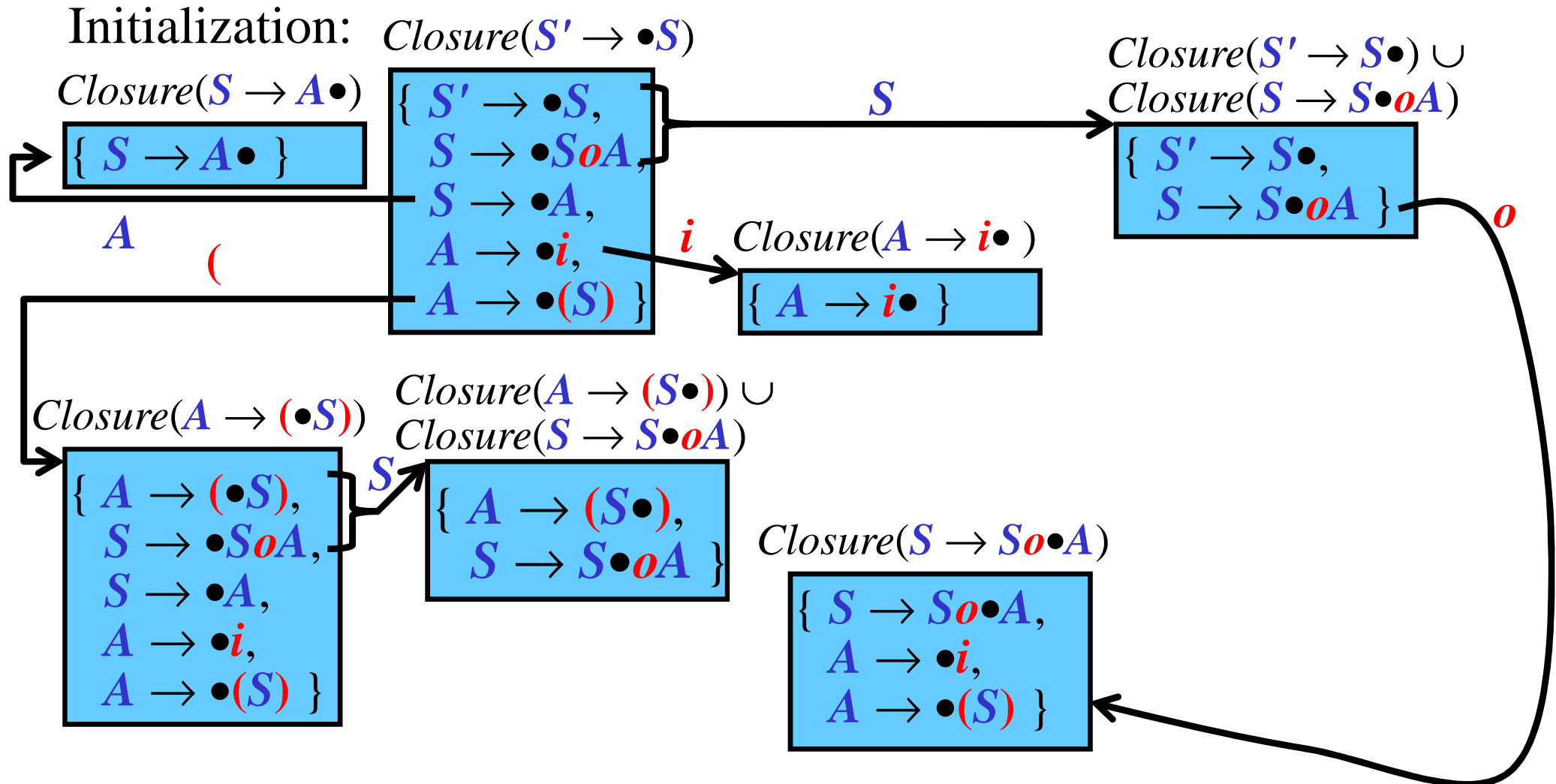
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



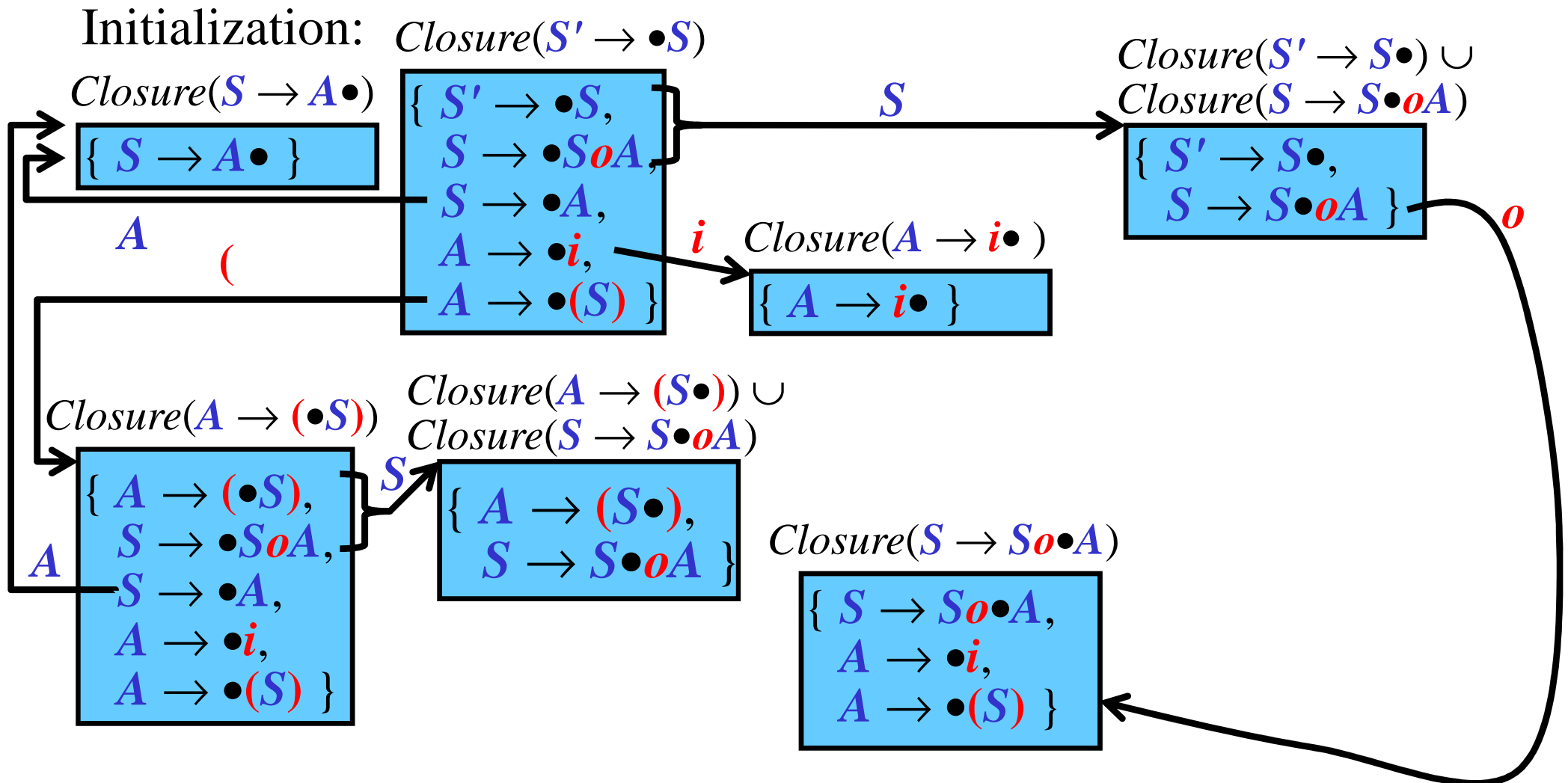
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



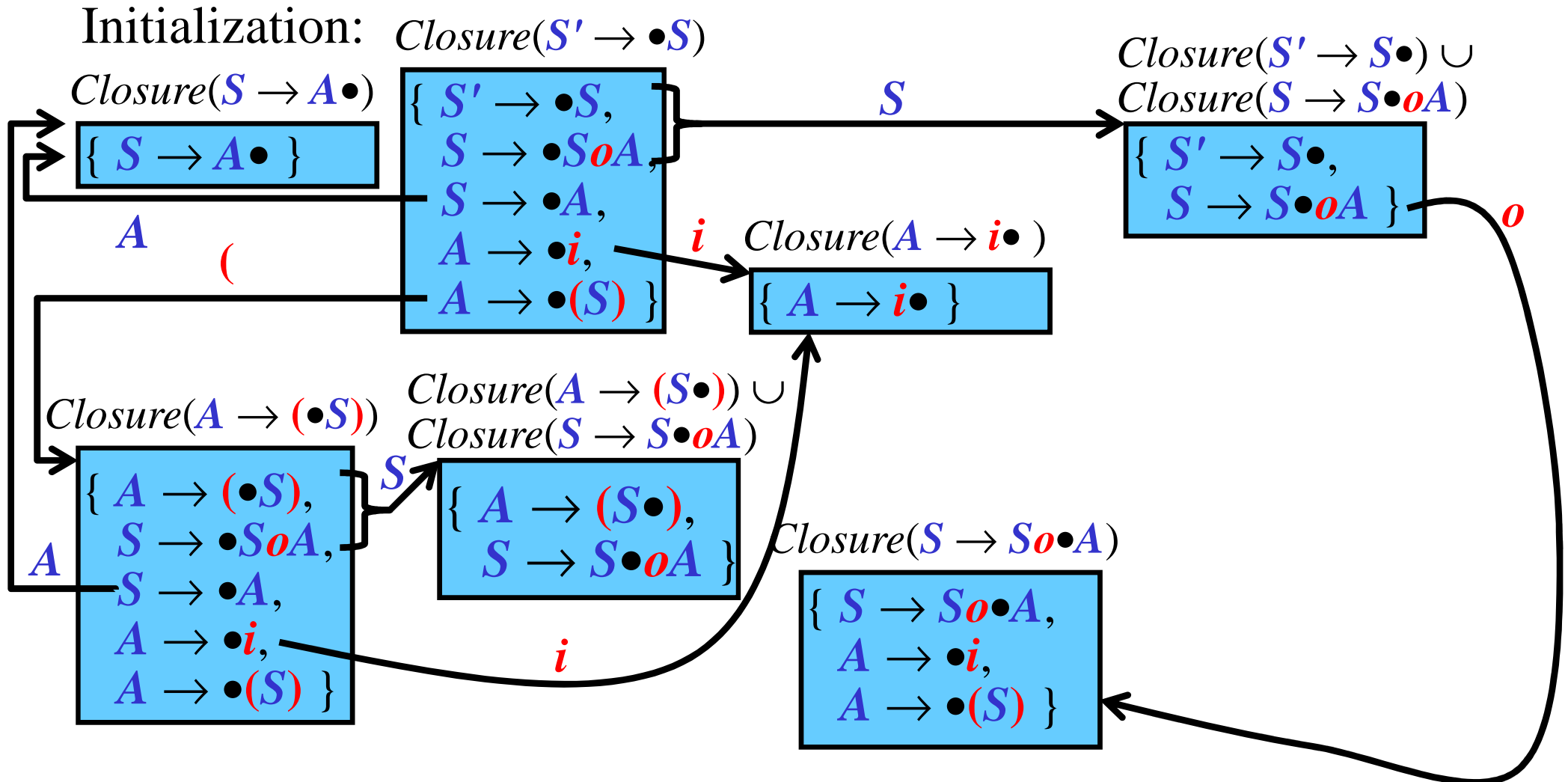
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



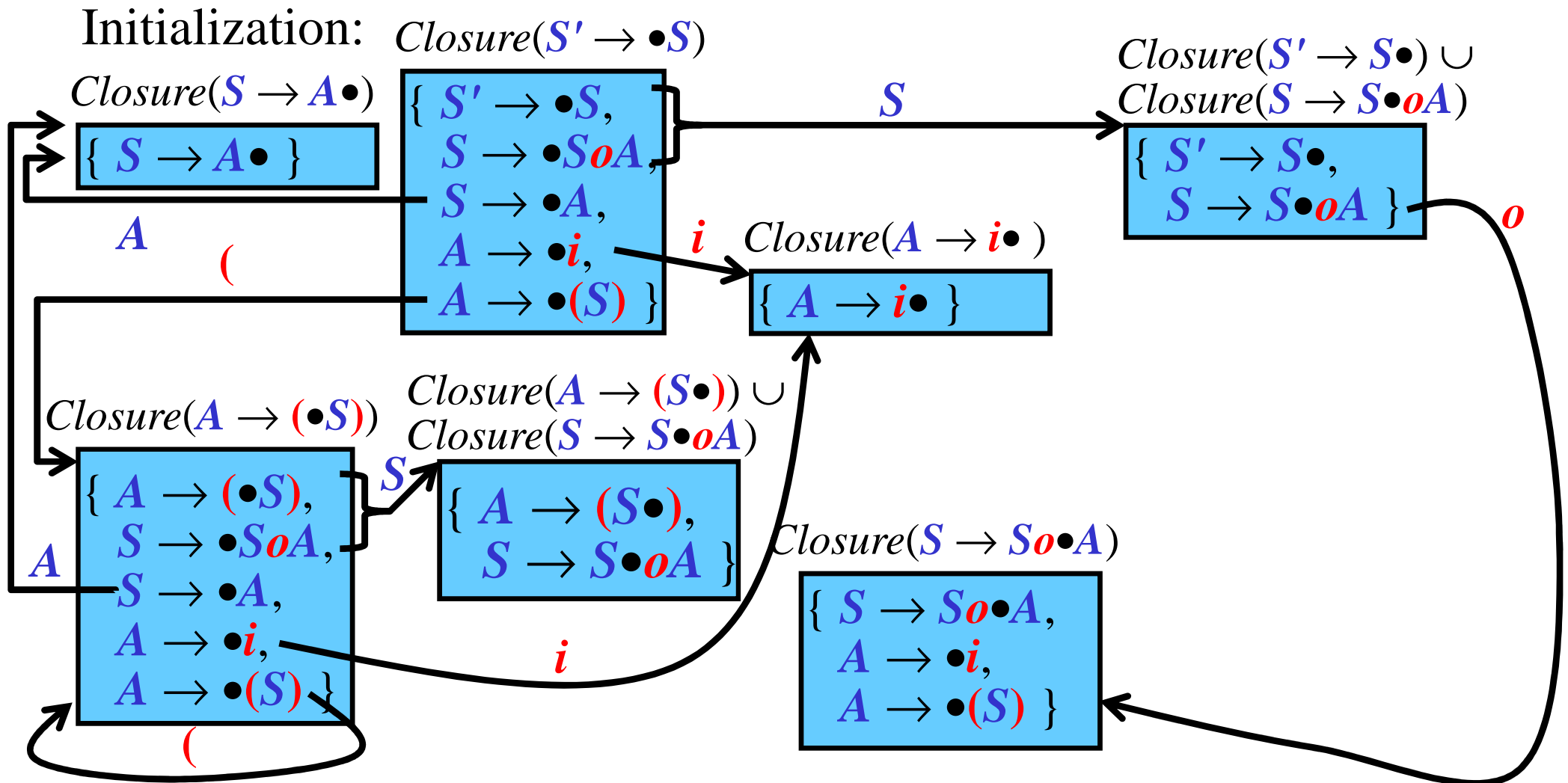
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



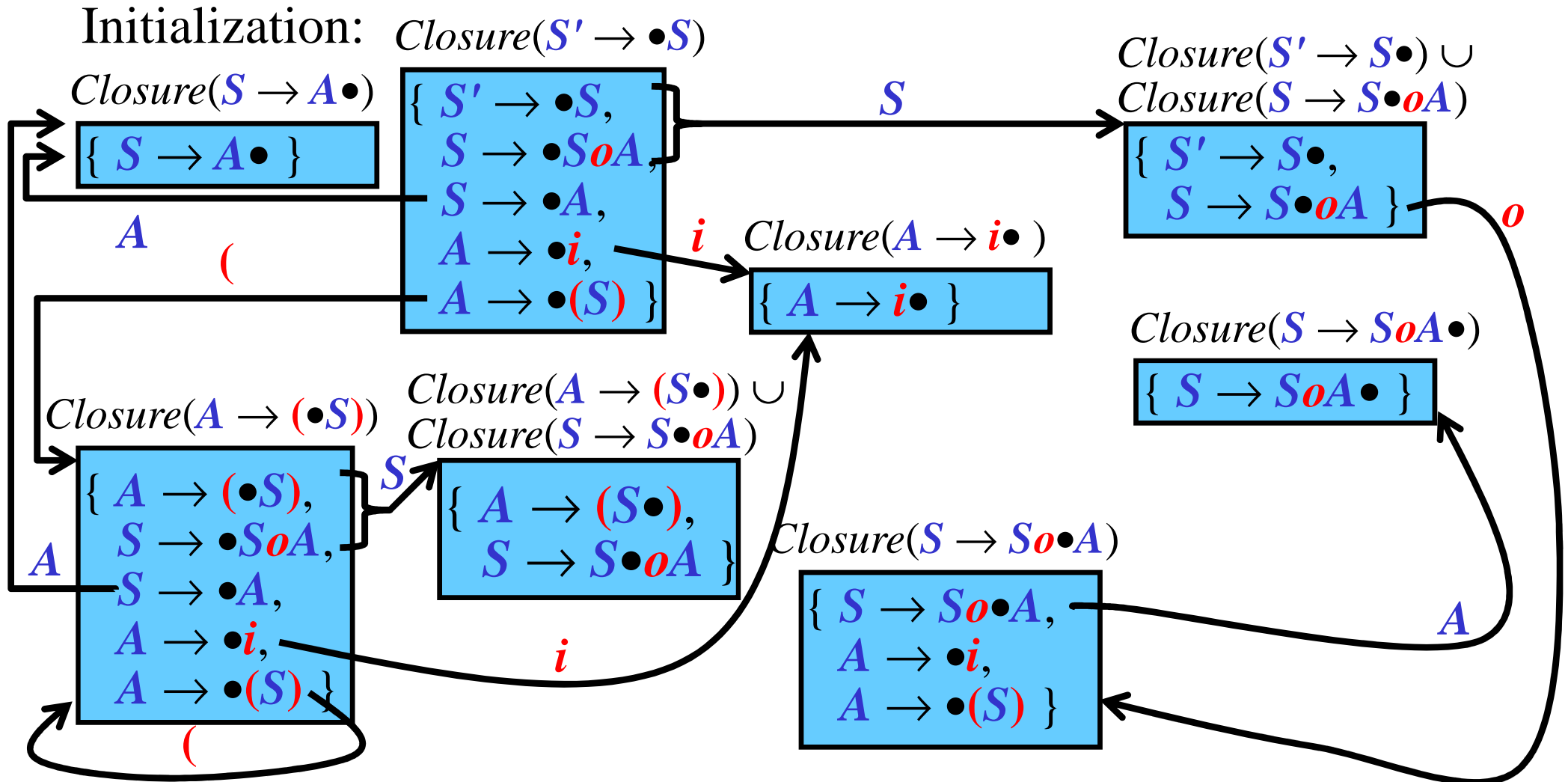
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



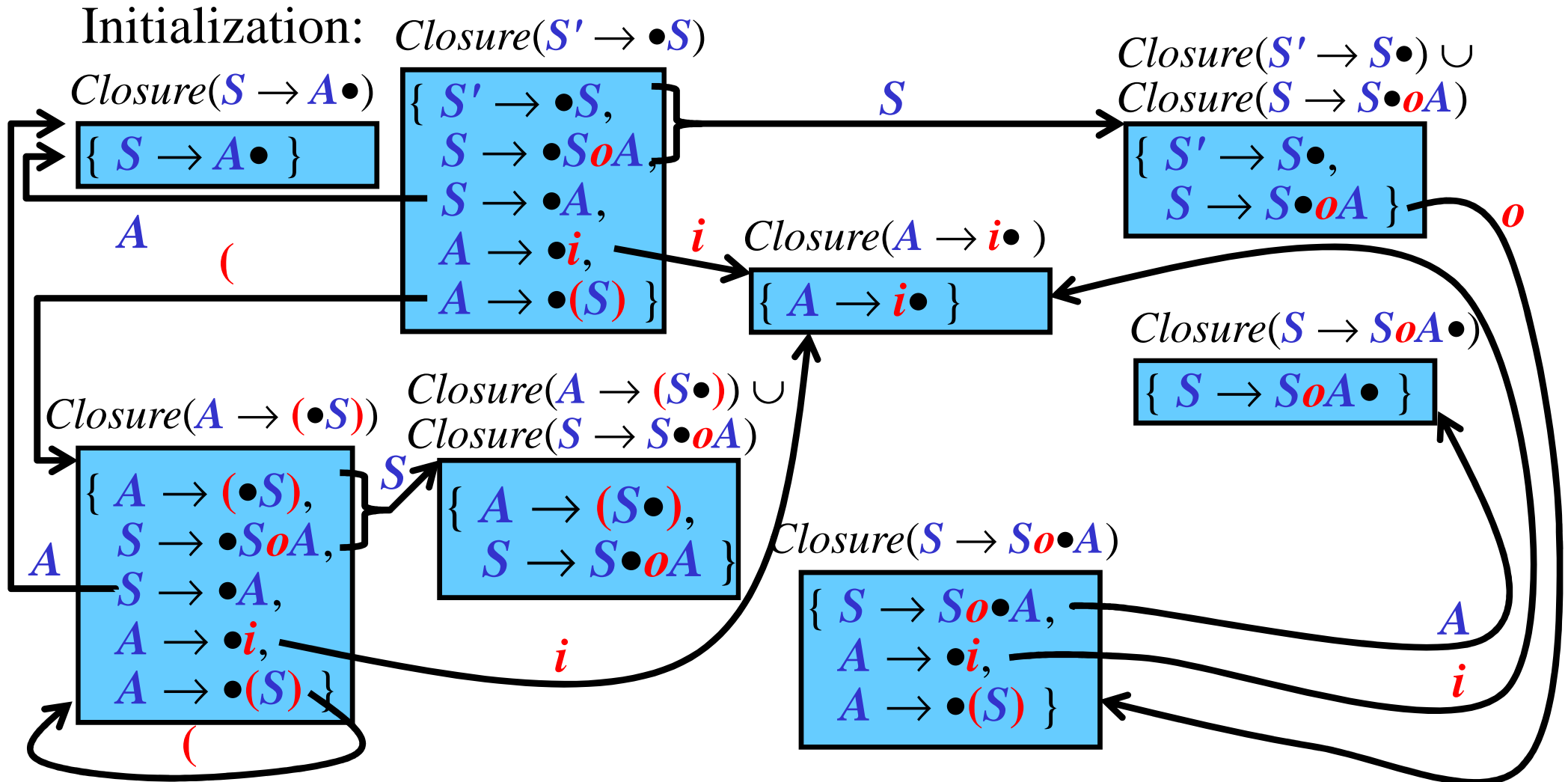
Set Θ_G : Example

$H = (N, T, P, \mathbf{S'})$, where $N = \{\mathbf{S'}, S, A\}$, $T = \{\mathbf{i, o, (,)}\}$,
 $P = \{\mathbf{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)}\}$



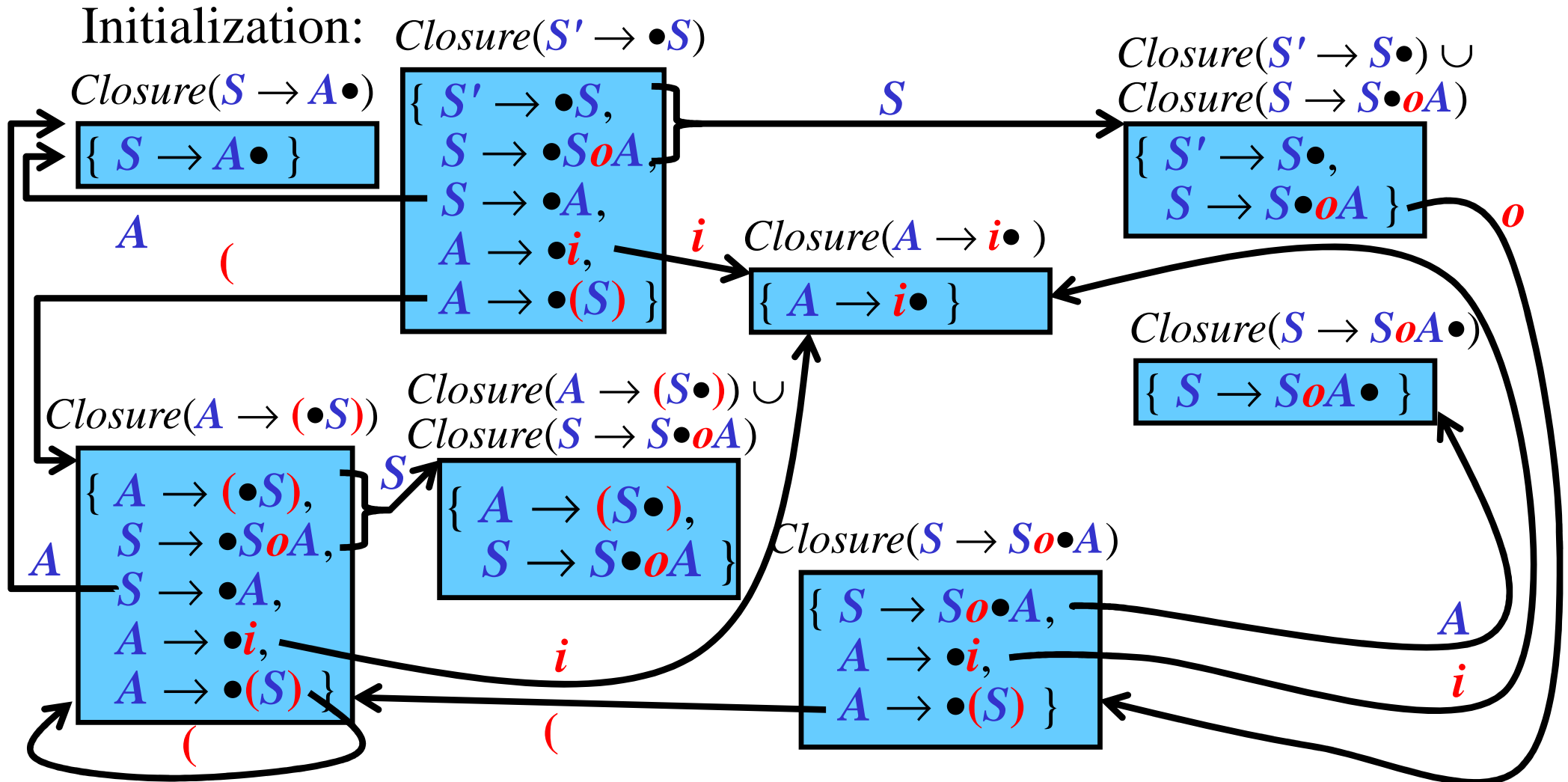
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



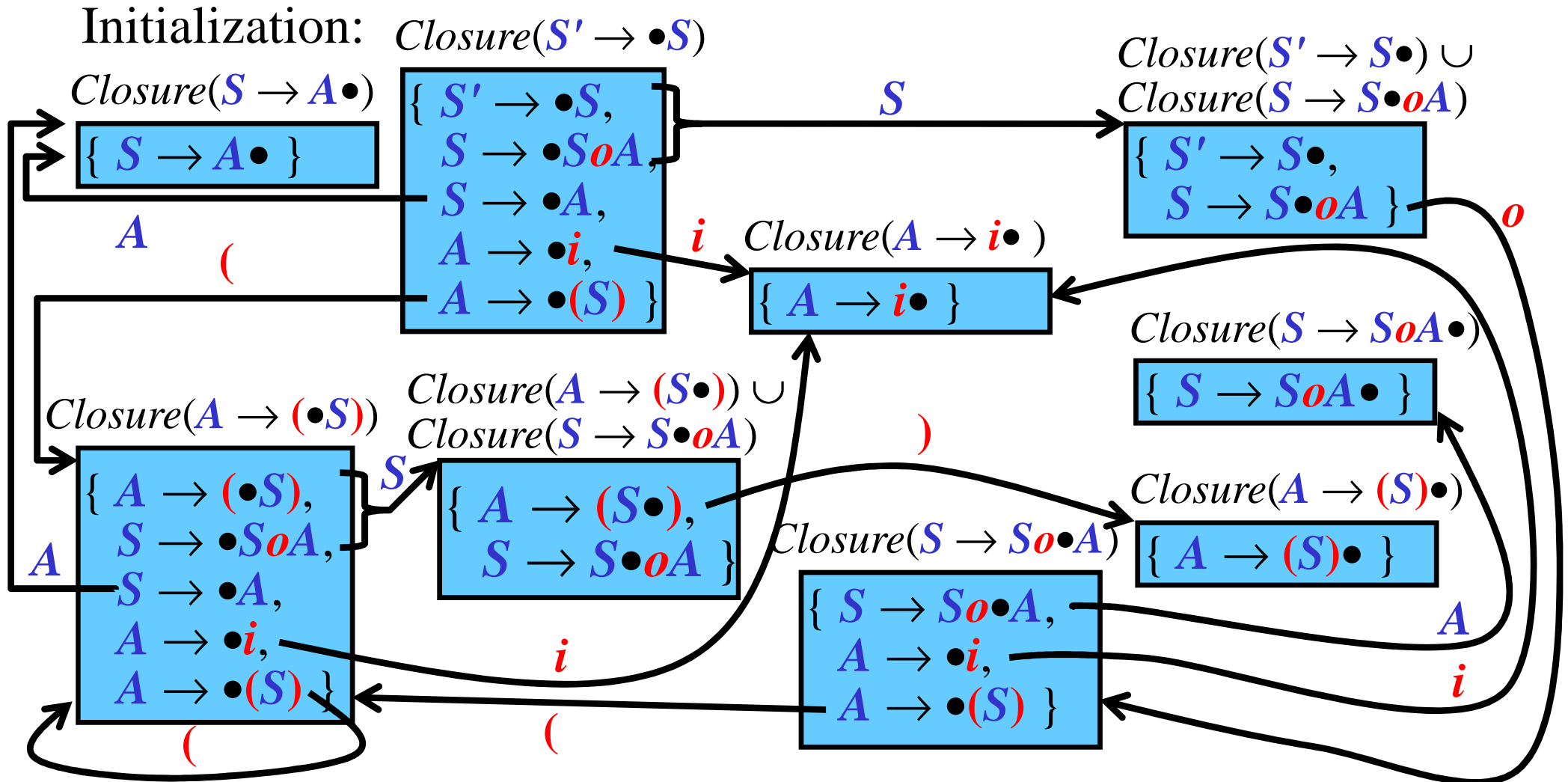
Set Θ_G : Example

$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



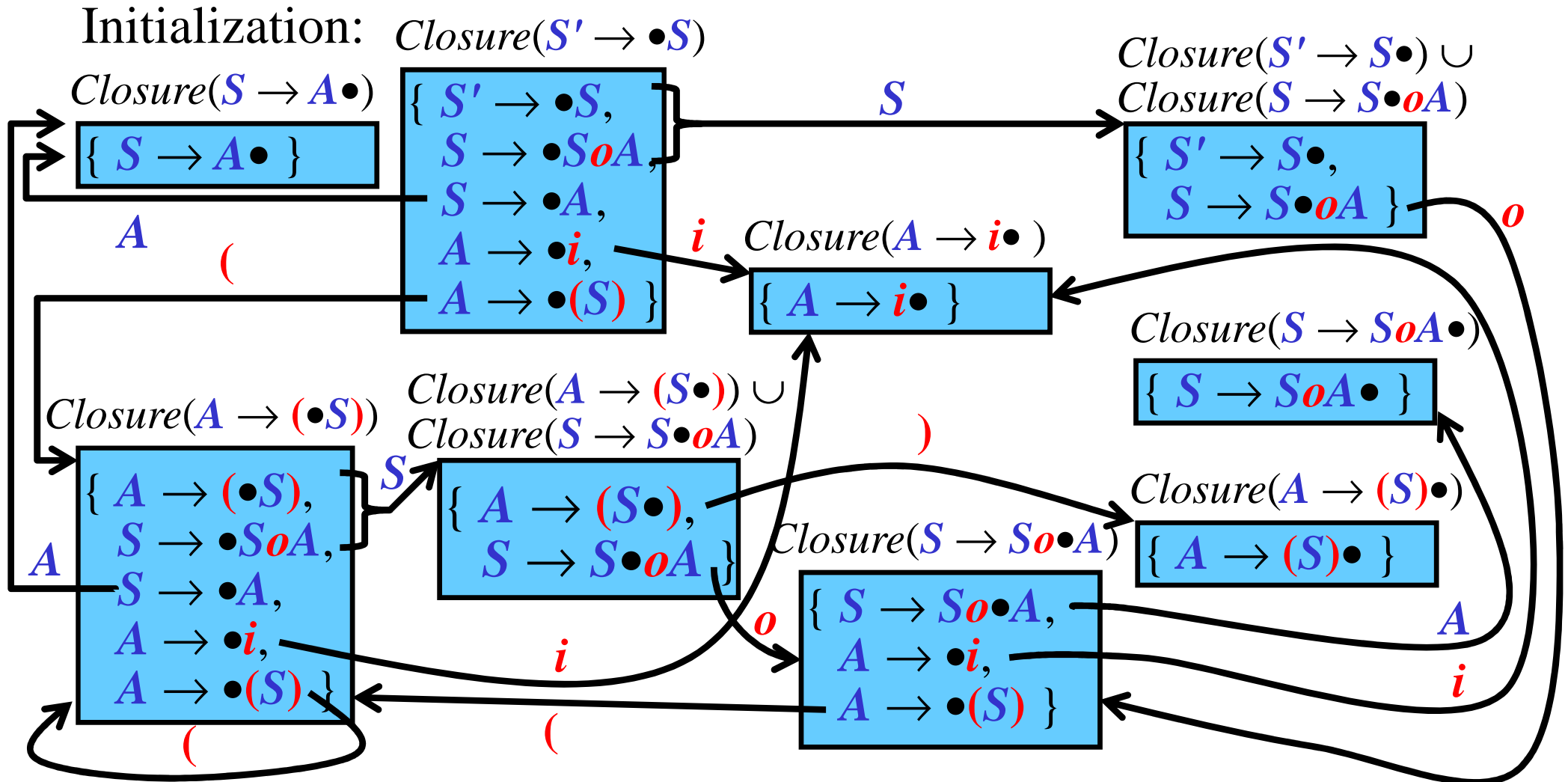
Set Θ_G : Example

$H = (N, T, P, \mathbf{S'})$, where $N = \{\mathbf{S'}, S, A\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S'} \rightarrow S, \mathbf{1}: S \rightarrow S\mathbf{o}A, \mathbf{2}: S \rightarrow A, \mathbf{3}: A \rightarrow \mathbf{i}, \mathbf{4}: A \rightarrow (S)\}$



Set Θ_G : Example

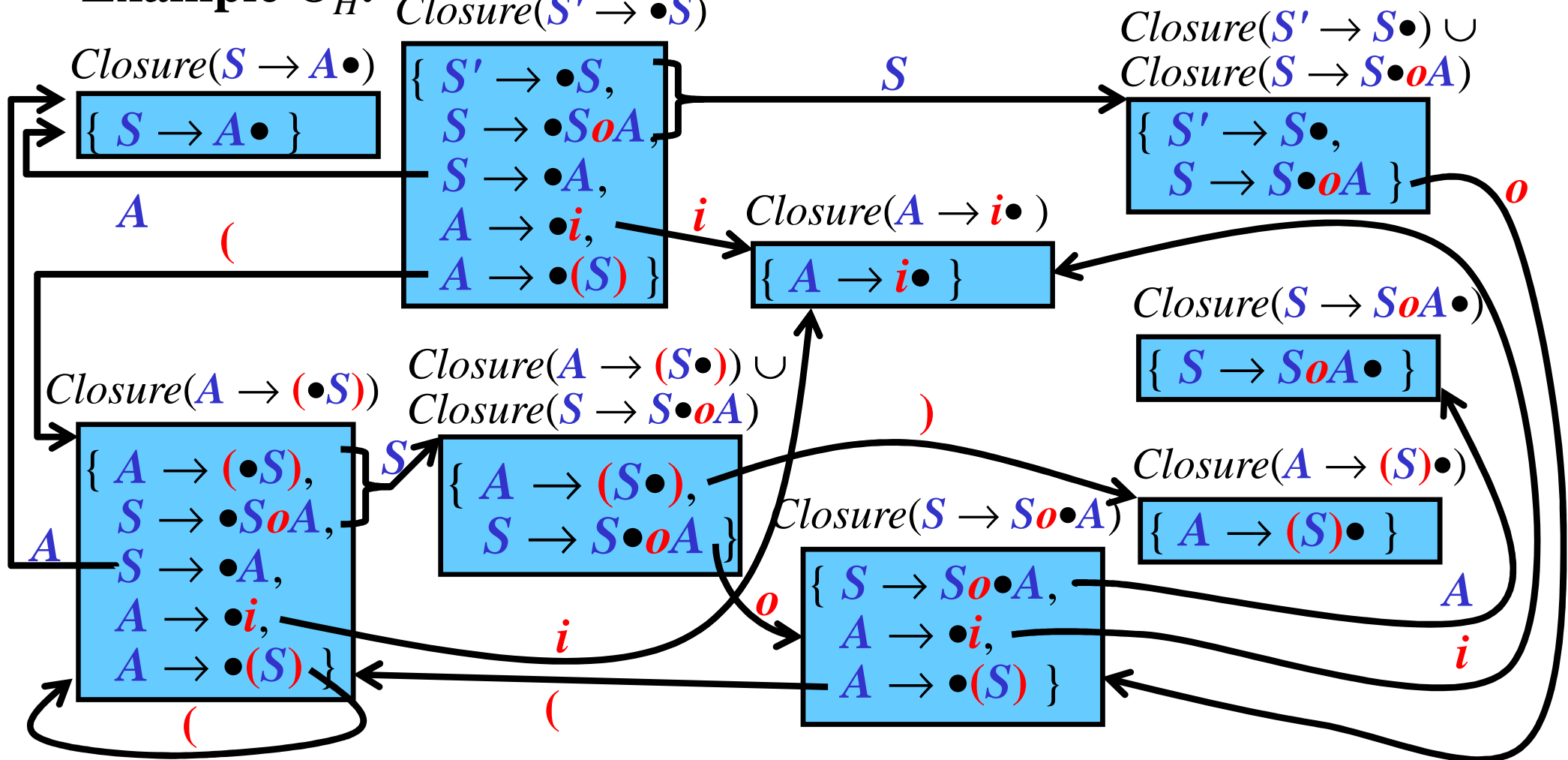
$H = (N, T, P, \mathbf{S}')$, where $N = \{\mathbf{S}', \mathbf{S}, \mathbf{A}\}$, $T = \{\mathbf{i}, \mathbf{o}, (,)\}$,
 $P = \{\mathbf{0}: \mathbf{S}' \rightarrow \mathbf{S}, \mathbf{1}: \mathbf{S} \rightarrow \mathbf{S}\mathbf{o}\mathbf{A}, \mathbf{2}: \mathbf{S} \rightarrow \mathbf{A}, \mathbf{3}: \mathbf{A} \rightarrow \mathbf{i}, \mathbf{4}: \mathbf{A} \rightarrow (\mathbf{S})\}$



Naming of members in set Θ_G

Name the elements of Θ_G as I_0 to I_n , where $n+1$ is number of elements in Θ_G . The member with $S' \rightarrow \bullet S$ is I_0 .

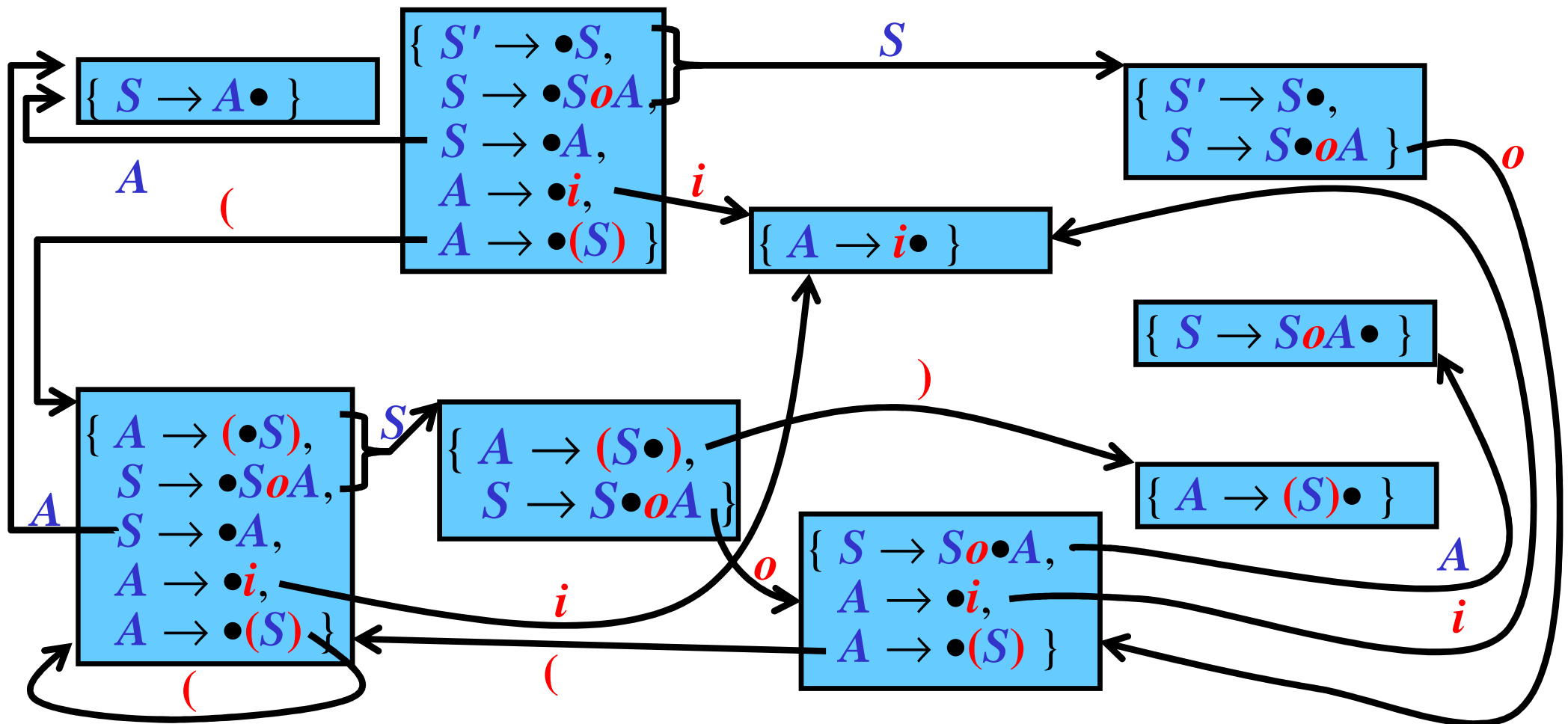
Example Θ_H : $\text{Closure}(S' \rightarrow \bullet S)$



Naming of members in set Θ_G

Name the elements of Θ_G as I_0 to I_n , where $n+1$ is number of elements in Θ_G . The member with $S' \rightarrow \bullet S$ is I_0 .

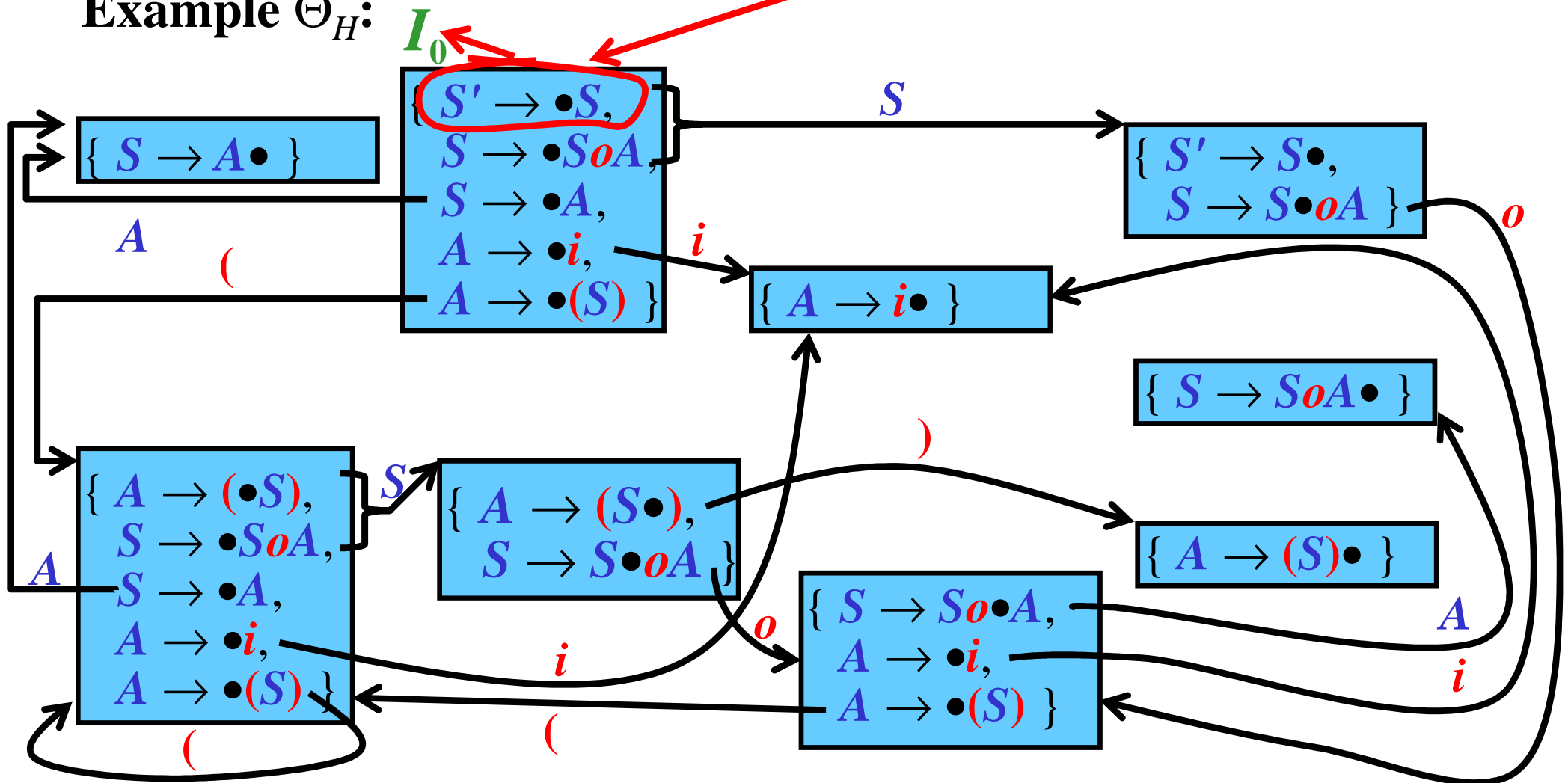
Example Θ_H :



Naming of members in set Θ_G

Name the elements of Θ_G as I_0 to I_n , where $n+1$ is number of elements in Θ_G . The member with $S' \rightarrow \bullet S$ is I_0 .

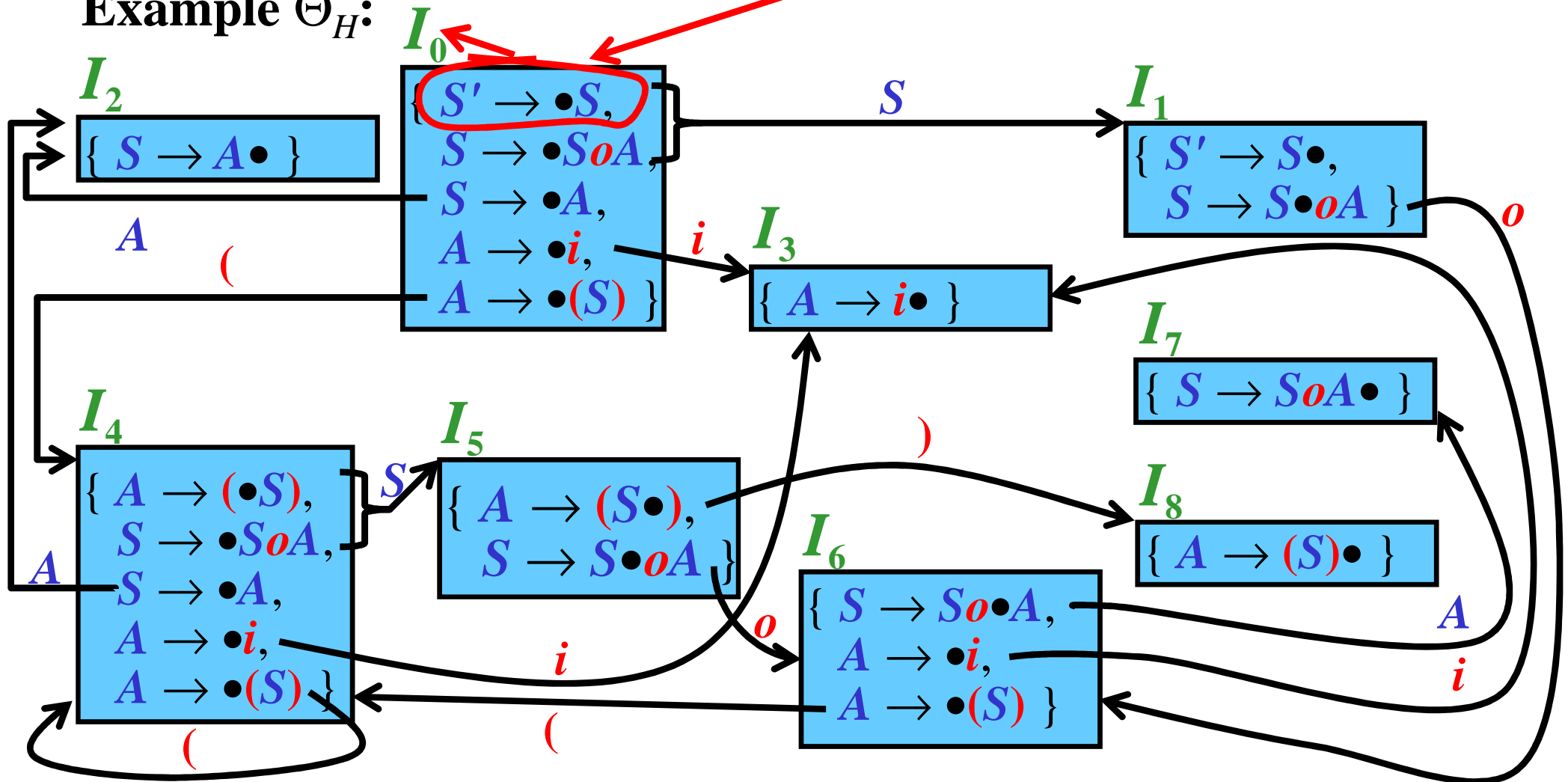
Example Θ_H :



Naming of members in set Θ_G

Name the elements of Θ_G as I_0 to I_n , where $n+1$ is number of elements in Θ_G . The member with $S' \rightarrow \bullet S$ is I_0 .

Example Θ_H :



Construction of LR-table: SLR Algorithm

- **Input:** Extended $G = (N, T, P, S')$; Θ_G ;
 $Follow(A)$ for all $A \in N$
- **Output:** LR-table for G (α = Action part, β = Go-to part)

• Method:

- $StatesOfTable := \Theta_G$; $StartState := Closure(S' \rightarrow \bullet S)$;
- for each $x \in \Theta_G$ do
- for each $I \in x$ do
 - case I of
 - $I = A \rightarrow y \bullet Xz$, where $X \in N$:
 $\beta[x, X] := \Theta_X(x)$
 - $I = A \rightarrow y \bullet Xz$, where $X \in T$:
 $\alpha[x, X] := s \Theta_X(x)$
 - $I = S' \rightarrow S \bullet$: $\alpha[x, \$] := \text{☺}$
 - $I = A \rightarrow y \bullet$ ($A \neq S'$):
for each $a \in Follow(A)$ do $\alpha[x, a] := rp$,
 where p is a label of rule $A \rightarrow y$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	<i>i</i>	<i>o</i>	()	\$	<i>S</i>	<i>A</i>
I_0							

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = \{ & I_0: \{ S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_1: \{ S' \rightarrow S \bullet, S \rightarrow S \bullet oA \}, I_2: \{ S \rightarrow A \bullet \}, I_3: \{ A \rightarrow i \bullet \}, \\ & I_4: \{ A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_5: \{ A \rightarrow (S \bullet), S \rightarrow S \bullet oA \}, I_6: \{ S \rightarrow So \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_7: \{ S \rightarrow SoA \bullet \}, I_8: \{ A \rightarrow (S) \bullet \} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0							

$\rightarrow S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = \{ & I_0: \{ S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_1: \{ S' \rightarrow S \bullet, S \rightarrow S \bullet oA \}, I_2: \{ S \rightarrow A \bullet \}, I_3: \{ A \rightarrow i \bullet \}, \\ & I_4: \{ A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_5: \{ A \rightarrow (S \bullet), S \rightarrow S \bullet oA \}, I_6: \{ S \rightarrow So \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_7: \{ S \rightarrow SoA \bullet \}, I_8: \{ A \rightarrow (S) \bullet \} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0						I_1	

$$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0						I_1	

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet SoA \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0						I_1	

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet SoA \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0						I_1	

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet SoA \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet A \in I_0, A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\},$
 $I_1: \{S' \rightarrow S \bullet, S \rightarrow S \bullet o A\}, I_2: \{S \rightarrow A \bullet\}, I_3: \{A \rightarrow i \bullet\},$
 $I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\},$
 $I_5: \{A \rightarrow (S \bullet), S \rightarrow S \bullet o A\}, I_6: \{S \rightarrow S o \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\},$
 $I_7: \{S \rightarrow S o A \bullet\}, I_8: \{A \rightarrow (S) \bullet\}\}$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0						I_1	I_2

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet S o A \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet A \in I_0, A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = \{ & I_0: \{ S' \rightarrow \bullet S, S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_1: \{ S' \rightarrow S \bullet, S \rightarrow S \bullet o A \}, I_2: \{ S \rightarrow A \bullet \}, I_3: \{ A \rightarrow i \bullet \}, \\ & I_4: \{ A \rightarrow (\bullet S), S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_5: \{ A \rightarrow (S \bullet), S \rightarrow S \bullet o A \}, I_6: \{ S \rightarrow S o \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_7: \{ S \rightarrow S o A \bullet \}, I_8: \{ A \rightarrow (S) \bullet \} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0						I_1	I_2

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet S o A \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet A \in I_0, A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$
 $A \rightarrow \bullet i \in I_0, i \in T: \alpha[I_0, i] := s\Theta_i(I_0) = sI_3$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3					I_1	I_2

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet SoA \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet A \in I_0, A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$
 $A \rightarrow \bullet i \in I_0, i \in T: \alpha[I_0, i] := s\Theta_i(I_0) = sI_3$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = \{ & I_0: \{ S' \rightarrow \bullet S, S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_1: \{ S' \rightarrow S \bullet, S \rightarrow S \bullet o A \}, I_2: \{ S \rightarrow A \bullet \}, I_3: \{ A \rightarrow i \bullet \}, \\ & I_4: \{ A \rightarrow (\bullet S), S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_5: \{ A \rightarrow (S \bullet), S \rightarrow S \bullet o A \}, I_6: \{ S \rightarrow S o \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_7: \{ S \rightarrow S o A \bullet \}, I_8: \{ A \rightarrow (S) \bullet \} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3					I_1	I_2

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet S o A \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet A \in I_0, A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$
 $A \rightarrow \bullet i \in I_0, i \in T: \alpha[I_0, i] := s\Theta_i(I_0) = sI_3$
 $A \rightarrow \bullet (S) \in I_0, (\in T: \alpha[I_0, (] := s\Theta_{(}(I_0) = sI_4$

Construction of LR-table: Example 1/5

$$\begin{aligned} \Theta_H = \{ & I_0: \{ S' \rightarrow \bullet S, S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_1: \{ S' \rightarrow S \bullet, S \rightarrow S \bullet o A \}, I_2: \{ S \rightarrow A \bullet \}, I_3: \{ A \rightarrow i \bullet \}, \\ & I_4: \{ A \rightarrow (\bullet S), S \rightarrow \bullet S o A, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_5: \{ A \rightarrow (S \bullet), S \rightarrow S \bullet o A \}, I_6: \{ S \rightarrow S o \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S) \}, \\ & I_7: \{ S \rightarrow S o A \bullet \}, I_8: \{ A \rightarrow (S) \bullet \} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet S o A \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$
 $S \rightarrow \bullet A \in I_0, A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$
 $A \rightarrow \bullet i \in I_0, i \in T: \alpha[I_0, i] := s\Theta_i(I_0) = sI_3$
 $A \rightarrow \bullet (S) \in I_0, (\in T: \alpha[I_0, (] := s\Theta_(I_0) = sI_4$

Construction of LR-table: Example 2/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1							

Construction of LR-table: Example 2/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	$S I_3$		$S I_4$			I_1	I_2
I_1							

$S' \rightarrow S\bullet \in I_1: \alpha[I_1, \$] := \text{☺}$

Construction of LR-table: Example 2/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	$S I_3$		$S I_4$			I_1	I_2
I_1					☺		

$S' \rightarrow S\bullet \in I_1: \alpha[I_1, \$] := \text{☺}$

Construction of LR-table: Example 2/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1					☺		

$$S' \rightarrow S\bullet \in I_1: \alpha[I_1, \$] := \text{☺}$$

$$S \rightarrow S\bullet oA \in I_1, o \in T: \alpha[I_1, o] := s\Theta_o(I_1) = sI_6$$

Construction of LR-table: Example 2/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		

$S' \rightarrow S\bullet \in I_1: \alpha[I_1, \$] := \text{☺}$

$S \rightarrow S\bullet oA \in I_1, o \in T: \alpha[I_1, o] := s\Theta_o(I_1) = sI_6$

Construction of LR-table: Example 3/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2							

Construction of LR-table: Example 3/5

$$\begin{aligned} \Theta_H = \{ & I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\} \} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2							

$S \rightarrow A\bullet \in I_2, Follow(S) = \{o,), \$\}$:
 $\alpha[I_2, o] = \alpha[I_2,)] = \alpha[I_2, \$] := r2$

Construction of LR-table: Example 3/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		$r2$		$r2$	$r2$		

$S \rightarrow A\bullet \in I_2, Follow(S) = \{o,), \$\}$:
 $\alpha[I_2, o] = \alpha[I_2,)] = \alpha[I_2, \$] := r2$

Construction of LR-table: Example 4/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		$r2$		$r2$	$r2$		
I_3							

Construction of LR-table: Example 4/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		$r2$		$r2$	$r2$		
I_3							

$A \rightarrow i\bullet \in I_3, \text{Follow}(A) = \{o,), \$\}:$
 $\alpha[I_3, o] = \alpha[I_3,)] = \alpha[I_3, \$] := r3$

Construction of LR-table: Example 4/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		$r2$		$r2$	$r2$		
I_3		$r3$		$r3$	$r3$		

$A \rightarrow i\bullet \in I_3, Follow(A) = \{o,), \$\}:$
 $\alpha[I_3, o] = \alpha[I_3,)] = \alpha[I_3, \$] := r3$

Construction of LR-table: Example 4/5

$$\begin{aligned} \Theta_H = & \{I_0: \{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_1: \{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2: \{S \rightarrow A\bullet\}, I_3: \{A \rightarrow i\bullet\}, \\ & I_4: \{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_5: \{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6: \{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\}, \\ & I_7: \{S \rightarrow SoA\bullet\}, I_8: \{A \rightarrow (S)\bullet\}\} \end{aligned}$$

Task: LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		$r2$		$r2$	$r2$		
I_3		$r3$		$r3$	$r3$		

Construct the rest analogically.

$A \rightarrow i\bullet \in I_3, \text{Follow}(A) = \{o,), \$\}:$
 $\alpha[I_3, o] = \alpha[I_3,)] = \alpha[I_3, \$] := r3$

Construction of LR-table: Example 5/5

Final LR-table for K

	α					β	
	i	o	$($	$)$	$\$$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		$r2$		$r2$	$r2$		
I_3		$r3$		$r3$	$r3$		
I_4	sI_3		sI_4			I_5	I_2
I_5		sI_6		sI_8			
I_6	sI_3		sI_4				I_7
I_7		$r1$		$r1$	$r1$		
I_8		$r4$		$r4$	$r4$		

Renaming the states

**Rename
the states:**

Old	New
I_0	0
I_1	1
I_2	2
I_3	3
I_4	4
I_5	5
I_6	6
I_7	7
I_8	8

LR-table for K with the renamed states:

α	i	o	()	\$
0	s3		s4		
1		s6			☺
2		r2		r2	r2
3		r3		r3	r3
4	s3		s4		
5		s6		s8	
6	s3		s4		
7		r1		r1	r1
8		r4		r4	r4

β	S	A
0	1	2
1		
2		
3		
4	5	2
5		
6		7
7		
8		