Generation of Sentences with Their Parses: the Case of Scattered Context Grammars

Scattered Context Grammar (*SCG*)

Scattered context grammar G = (V, P, S, T)

V is a finite alphabet

T is a set of terminals, $T \subset V$

S is a starting symbol, $S \in (V - T)$

P is a finite set of productions of the form: $(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$; $A_1, \ldots, A_n \in (V - T)$; $x_1, \ldots, x_n \in V^*$

Propagating scattered context grammar (*PSCG*)

- special case of *SCG*
- every $(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$ satisfies $x_1, \ldots, x_n \in V^+$

Generated Language

Derivation step

If: $(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n) \in P$

$\quad u = u_1 A_1 \ldots u_n A_n u_{n+1}$

$\quad v = u_1 x_1 \ldots u_n x_n u_{n+1}$

then $u \Rightarrow v \, [(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)]$

Generated language

$L(G) = \{x \mid x \in T^*, S \Rightarrow^* x\}$

Generative power

- $\mathcal{L}_{SCG} = \mathcal{L}_{RE}$
- $\mathcal{L}_{CF} \subset \mathcal{L}_{PSCG} \subseteq \mathcal{L}_{CS}$

Production Labels

- for every grammar $G = (V, P, S, T)$, there is a set of production labels
- we denote them $lab(G)$
- every $p \in lab(G)$ uniquely identifies one production
- we write $p : (A_1, \ldots, A_n) \to (x_1, \ldots, x_n)$

Example

$G_1 = (\{S, A, B, C, a, b, c\}, P_1, S, \{a, b, c\})$
$lab(G_1) = \{1, 2, 3\}$

$P_1 :$ $1 : (S) \to (ABC)$
       $2 : (A, B, C) \to (aA, bB, cC)$
       $3 : (A, B, C) \to (\epsilon, \epsilon, \epsilon)$

$L(G_1) = \{a^n b^n c^n \mid n \geq 0\}$

Production Labels (cont.)

- to express that $x \Rightarrow y$ by $p : (A_1, \ldots, A_n) \to (x_1, \ldots, x_n)$, we write $x \Rightarrow y\,[p]$

Example

$S \Rightarrow ABC\,[1] \Rightarrow aAbBcC\,[2] \Rightarrow aaAbbBccC\,[2] \Rightarrow aabbcc\,[3]$ in G_1

- to express that $x \Rightarrow^* y$ by productions labeled with $p_1, \ldots, p_n$, we write $x \Rightarrow^* y\,[p_1 \ldots p_n]$
- $p_1 \ldots p_n \in lab(G)^*$

Example

$S \Rightarrow^* aabbcc\,[1223]$ in G_1
$1223 \in lab(G_1)^*$

Proper Generator of its Sentences With Their Parses

Parse

If $S \Rightarrow^* x[\rho], x \in T^*, \rho \in lab(G)^*$, then x is a sentence generated by G according to *parse* ρ

Example

aabbcc is a sentence generated according to parse 1223 in G_1

Proper generator of its sentences with their parses

G is a proper generator of its sentences with their parses if
$L(G) = \{x \mid x = y\rho, y \in (T - lab(G))^*, \rho \in lab(G)^*, S \Rightarrow^* x[\rho]\}$

Example

$G_2 = (\{S, A, B, C, a, b, c, 1, 2, 3, \$\}, P_2, S, \{a, b, c, 1, 2, 3\})$
$lab(G_2) = \{1, 2, 3\}$

$P_2 : 1 : (S) \rightarrow (ABC1\$)$
$\qquad 2 : (A, B, C, \$) \rightarrow (aA, bB, cC, 2\$)$
$\qquad 3 : (A, B, C, \$) \rightarrow (\epsilon, \epsilon, \epsilon, 3)$

$S \Rightarrow ABC1\$\ [1] \Rightarrow aAbBcC12\$\ [2] \Rightarrow aaAbbBccC122\$\ [2] \Rightarrow$
$\qquad aabbcc1223\ [3]$
$S \Rightarrow^* aabbcc1223\ [1223]$
$L(G_2) = \{a^n b^n c^n \rho \mid n \geq 0, S \Rightarrow^* a^n b^n c^n \rho\ [\rho], \rho = 12^n 3\}$
G_2 is a proper generator of its sentences with their parses

Main Result

- let $G = (V, P, S, T)$ be a proper generator of its sentences with their parses
- we define the weak identity π from V^* to $(V - lab(G))^*$ as
    - $\pi(a) = a$ for every $a \in (V - lab(G))$
    - $\pi(p) = \epsilon$ for every $p \in lab(G)$

Example

$\pi(aabbcc1223) = aabbcc$ in G_2

Theorem

For every recursively enumerable language, L, there exists a PSCG, G, such that G is a proper generator of its sentences with their parses and $L = \pi(L(G))$.

Problem Sketch

- **idea:** simulation of SCG by $PSCG$ and adding labels of used productions
- **problem:** productions of $PSCG$ do not contain ϵ on their right hand sides
- instead of ϵ, we can use some fill-in symbol (y)

Example

$G_3 = (\{S, A, B, C, a, b, c, y, 1, 2, 3, \$\}, P_3, S, \{a, b, c, y, 1, 2, 3\})$
$lab(G_3) = \{1, 2, 3\}$

$P_3 : 1 : (S) \rightarrow (ABC1\$)$
$\quad\quad 2 : (A, B, C, \$) \rightarrow (aA, bB, cC, 2\$)$
$\quad\quad 3 : (A, B, C, \$) \rightarrow (y, y, y, 3)$

$S \Rightarrow ABC1\$\,[1] \Rightarrow aAbBcC12\$\,[2] \Rightarrow aaAbbBccC122\$\,[2] \Rightarrow$
$\quad aaybbyccy1223\,[3]$

Problem Solution

Basic idea

1. generate the parse before the sentence in the sentential form
2. move the symbols of generated sentence before the parse
3. replace fill-in symbols with the parse

Demonstration

1. $S \Rightarrow^* 1223aaybbyccy$
2. $1223aaybbyccy \Rightarrow^* aabbcc1223\ldots yyyyyyyyy$
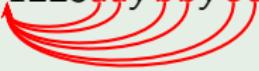
3. $aabbcc1223\ldots yyyyyyyyy \Rightarrow^* aabbcc1223\ldots$

Proof

- original SCG $\bar{G} = (\bar{V}, \bar{P}, \bar{S}, \bar{T})$
- $\Phi = \{\langle a \rangle \mid a \in \bar{T}\}$
- $\Gamma = \{\$_1, \$_2, \$_3\}$

Constructed $PSCG$ $G = (V, P, S, T)$

$G = (\{S, X, Y, Z\} \cup \bar{V} \cup lab(G) \cup \Phi \cup \Gamma, P, S, \bar{T} \cup lab(G))$

Example

$\bar{G}_4 (\approx G_1) = (\{\bar{S}, \bar{A}, \bar{B}, \bar{C}, a, b, c\}, \bar{P}_4, \bar{S}, \{a, b, c\}); \; lab(\bar{G}_4) = \{\bar{1}, \bar{2}, \bar{3}\}$

$\bar{P}_4 : \; \bar{1} : (\bar{S}) \to (\bar{A}\bar{B}\bar{C})$
$\bar{2} : (\bar{A}, \bar{B}, \bar{C}) \to (a\bar{A}, b\bar{B}, c\bar{C})$
$\bar{3} : (\bar{A}, \bar{B}, \bar{C}) \to (\epsilon, \epsilon, \epsilon)$

$G_4 = (\; \{S, X, Y, Z\} \cup \{\bar{S}, \bar{A}, \bar{B}, \bar{C}, a, b, c\} \cup lab(G_4) \cup \{\langle a \rangle, \langle b \rangle, \langle c \rangle\} \cup$
$\{\$_1, \$_2, \$_3\}, P_4, S, \{a, b, c\} \cup lab(G_4))$

Productions

Step 1 (preparation)

Add $\lfloor 1 \rfloor : (S) \rightarrow (X\lfloor 1 \rfloor \$_1 Z \bar{S})$ to P

Example

Add $\lfloor 1 \rfloor : (S) \rightarrow (X\lfloor 1 \rfloor \$_1 Z \bar{S})$ to P_4

Productions (cont.)

- $\gamma(a) = \langle a \rangle$, $a \in \bar{T}$
  $\gamma(A) = A$, $A \in \bar{V} - \bar{T}$
  $\gamma(\epsilon) = Y$

Step 2 (simulation of the SCG)

For every $p : (A_1, \ldots, A_n) \to (x_1, \ldots, x_n) \in \bar{P}$ add
  $\lfloor p1 \rfloor : (\$_1, A_1, \ldots, A_n) \to (\lfloor p1 \rfloor \$_1, \gamma(x_1), \ldots, \gamma(x_n))$ to P;
add $\lfloor 2 \rfloor : (\$_1) \to (\lfloor 2 \rfloor \$_2)$ to P

Example

Add $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \to (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$ to P_4
  $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$ to P_4
  $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$ to P_4
Add $\lfloor 2 \rfloor : (\$_1) \to (\lfloor 2 \rfloor \$_2)$ to P_4

Productions (cont.)

Step 3 (moving symbols before the parse)

For every $a \in \bar{T}$, add

$\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$ to P;

$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$ to P

Example

Add $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$ to P_4

$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$ to P_4

$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$ to P_4

$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$ to P_4

$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$ to P_4

$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$ to P_4

Step 4 (replacing Y with the parse)

Add $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$ to P

Step 5 (removing $\$_3$)

Add $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$ to P

Example

Add $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$ to P_4
$\qquad \lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$ to P_4

S

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

S

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$

$\qquad \lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$

$\qquad \lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$

$\qquad \lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$

$\qquad \lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$

$\qquad \lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$

$\qquad \lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$

$\qquad \lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$

$\qquad \lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

Example

$X \lfloor 1 \rfloor \$_1 Z \bar{S}$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$X\lfloor 1\rfloor \$_1 Z\bar{S}$

Step 1 $\lfloor 1\rfloor : (S) \to (X\lfloor 1\rfloor \$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \to (\lfloor \bar{1}1\rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\quad\quad\ \lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{2}1\rfloor \$_1, \langle a\rangle \bar{A}, \langle b\rangle \bar{B}, \langle c\rangle \bar{C})$
$\quad\quad\ \lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{3}1\rfloor \$_1, Y, Y, Y)$
$\quad\quad\ \lfloor 2\rfloor : (\$_1) \to (\lfloor 2\rfloor \$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \to (aX, \lfloor a2\rfloor \$_2, Y, Z)$
$\quad\quad\ \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \to (bX, \lfloor b2\rfloor \$_2, Y, Z)$
$\quad\quad\ \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \to (cX, \lfloor c2\rfloor \$_2, Y, Z)$
$\quad\quad\ \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \to (a, \lfloor a3\rfloor \$_3, Y, Y)$
$\quad\quad\ \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \to (b, \lfloor b3\rfloor \$_3, Y, Y)$
$\quad\quad\ \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \to (c, \lfloor c3\rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \to (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \to (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\$_1 Z\bar{A}\bar{B}\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\$_1 Z\bar{A}\bar{B}\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\$_1 Z\langle a\rangle\bar{A}\langle b\rangle\bar{B}\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\$_1 Z\langle a\rangle\bar{A}\langle b\rangle\bar{B}\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\$_1 Z\langle a\rangle\langle a\rangle\bar{A}\langle b\rangle\langle b\rangle\bar{B}\langle c\rangle\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\$_1 Z\langle a\rangle\langle a\rangle\bar{A}\langle b\rangle\langle b\rangle\bar{B}\langle c\rangle\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \to (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \to (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\quad\quad\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\quad\quad\textcolor{red}{\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)}$
$\quad\quad\lfloor 2\rfloor : (\$_1) \to (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \to (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\quad\quad\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \to (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\quad\quad\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \to (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\quad\quad\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \to (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\quad\quad\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \to (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\quad\quad\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \to (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \to (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \to (\lfloor 4\rfloor)$

Example

$X \lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \$_1 Z \langle a \rangle \langle a \rangle Y \langle b \rangle \langle b \rangle Y \langle c \rangle \langle c \rangle Y$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
         $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
         $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
         $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
         $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
         $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
         $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
         $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
         $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

Example

$X \lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \$_1 Z \langle a \rangle \langle a \rangle Y \langle b \rangle \langle b \rangle Y \langle c \rangle \langle c \rangle Y$

Step 1 $\lfloor 1 \rfloor : (S) \to (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \to (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\lfloor 2 \rfloor : (\$_1) \to (\lfloor 2 \rfloor \$_2) \text{ to } P_4$

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \to (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \to (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \to (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \to (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \to (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \to (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \to (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \to (\lfloor 4 \rfloor)$

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\$_2 Z\langle a\rangle\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\qquad \lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\qquad \lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\qquad \lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\qquad \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\qquad \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\qquad \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\qquad \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\qquad \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$X \lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \$_2 Z \langle a \rangle \langle a \rangle Y \langle b \rangle \langle b \rangle Y \langle c \rangle \langle c \rangle Y$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
  $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
  $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
  $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
  $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
  $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
  $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
  $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
  $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$aX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\$_2 YZ\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\quad\quad\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\quad\quad\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\quad\quad\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\quad\quad\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\quad\quad\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\quad\quad\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\quad\quad\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\quad\quad\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$$aX\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\$_2\,YZ\langle a\rangle\,Y\langle b\rangle\langle b\rangle\,Y\langle c\rangle\langle c\rangle\,Y$$

Step 1 $\lfloor 1\rfloor : (S) \to (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1,\bar{S}) \to (\lfloor \bar{1}1\rfloor\$_1,\bar{A}\bar{B}\bar{C})$
  $\lfloor \bar{2}1\rfloor : (\$_1,\bar{A},\bar{B},\bar{C}) \to (\lfloor \bar{2}1\rfloor\$_1,\langle a\rangle\bar{A},\langle b\rangle\bar{B},\langle c\rangle\bar{C})$
  $\lfloor \bar{3}1\rfloor : (\$_1,\bar{A},\bar{B},\bar{C}) \to (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
  $\lfloor 2\rfloor : (\$_1) \to (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X,\$_2,Z,\langle a\rangle) \to (aX,\lfloor a2\rfloor\$_2,Y,Z)$
  $\lfloor b2\rfloor : (X,\$_2,Z,\langle b\rangle) \to (bX,\lfloor b2\rfloor\$_2,Y,Z)$
  $\lfloor c2\rfloor : (X,\$_2,Z,\langle c\rangle) \to (cX,\lfloor c2\rfloor\$_2,Y,Z)$
  $\lfloor a3\rfloor : (X,\$_2,Z,\langle a\rangle) \to (a,\lfloor a3\rfloor\$_3,Y,Y)$
  $\lfloor b3\rfloor : (X,\$_2,Z,\langle b\rangle) \to (b,\lfloor b3\rfloor\$_3,Y,Y)$
  $\lfloor c3\rfloor : (X,\$_2,Z,\langle c\rangle) \to (c,\lfloor c3\rfloor\$_3,Y,Y)$

Step 4 $\lfloor 3\rfloor : (\$_3,Y) \to (\lfloor 3\rfloor,\$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \to (\lfloor 4\rfloor)$

Example

$a\,a\,X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\$_2\,Y\,Y\,Z\,Y\langle b\rangle\langle b\rangle\,Y\langle c\rangle\langle c\rangle\,Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
       $\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
       $\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
       $\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
       $\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
       $\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
       $\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
       $\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
       $\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$aa X \lfloor 1 \rfloor \lfloor \bar{1} 1 \rfloor \lfloor \bar{2} 1 \rfloor \lfloor \bar{2} 1 \rfloor \lfloor \bar{3} 1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \$_2 Y Y Z Y \langle b \rangle \langle b \rangle Y \langle c \rangle \langle c \rangle Y$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1} 1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1} 1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
$\lfloor \bar{2} 1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2} 1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\lfloor \bar{3} 1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3} 1 \rfloor \$_1, Y, Y, Y)$
$\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$aabX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\$_2 YYYYZ\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$aabX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\$_2\,YYYY\,Z\langle b\rangle\,Y\langle c\rangle\langle c\rangle\,Y$

Step 1 $\lfloor 1\rfloor : (S) \to (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \to (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \to (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \to (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \to (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \to (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \to (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \to (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \to (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \to (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \to (\lfloor 4\rfloor)$

Example

$aab\textcolor{red}{b}X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\textcolor{red}{\lfloor b2\rfloor}\$_2\,YYYY\,Y\,Z\,Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S)\rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1,\bar{S})\rightarrow(\lfloor\bar{1}1\rfloor\$_1,\bar{A}\bar{B}\bar{C})$
        $\lfloor\bar{2}1\rfloor : (\$_1,\bar{A},\bar{B},\bar{C})\rightarrow(\lfloor\bar{2}1\rfloor\$_1,\langle a\rangle\bar{A},\langle b\rangle\bar{B},\langle c\rangle\bar{C})$
        $\lfloor\bar{3}1\rfloor : (\$_1,\bar{A},\bar{B},\bar{C})\rightarrow(\lfloor\bar{3}1\rfloor\$_1,Y,Y,Y)$
        $\lfloor 2\rfloor : (\$_1)\rightarrow(\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X,\$_2,Z,\langle a\rangle)\rightarrow(aX,\lfloor a2\rfloor\$_2,Y,Z)$
        $\textcolor{red}{\lfloor b2\rfloor : (X,\$_2,Z,\langle b\rangle)\rightarrow(bX,\lfloor b2\rfloor\$_2,Y,Z)}$
        $\lfloor c2\rfloor : (X,\$_2,Z,\langle c\rangle)\rightarrow(cX,\lfloor c2\rfloor\$_2,Y,Z)$
        $\lfloor a3\rfloor : (X,\$_2,Z,\langle a\rangle)\rightarrow(a,\lfloor a3\rfloor\$_3,Y,Y)$
        $\lfloor b3\rfloor : (X,\$_2,Z,\langle b\rangle)\rightarrow(b,\lfloor b3\rfloor\$_3,Y,Y)$
        $\lfloor c3\rfloor : (X,\$_2,Z,\langle c\rangle)\rightarrow(c,\lfloor c3\rfloor\$_3,Y,Y)$

Step 4 $\lfloor 3\rfloor : (\$_3,Y)\rightarrow(\lfloor 3\rfloor,\$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3)\rightarrow(\lfloor 4\rfloor)$

Example

$aabbX \lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \$_2 \, YYYYY \, ZY \langle c \rangle \langle c \rangle Y$

Step 1 $\lfloor 1 \rfloor : (S) \to (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \to (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\qquad \lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\qquad \lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\qquad \lfloor 2 \rfloor : (\$_1) \to (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \to (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\qquad \lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \to (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\qquad \lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \to (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\qquad \lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \to (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\qquad \lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \to (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\qquad \lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \to (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \to (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \to (\lfloor 4 \rfloor)$

Example

$aabbcX\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \$_2 YYYYY\, Y\, Y\, Z$
$\langle c \rangle Y$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X\lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
   $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
   $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
   $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
   $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
   $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
   $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
   $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
   $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

Example

$aabbcX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\$_2 YYYYYYY Z\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \to (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \to (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\qquad \lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\qquad \lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\qquad \lfloor 2\rfloor : (\$_1) \to (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \to (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\qquad \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \to (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\qquad \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \to (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\qquad \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \to (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\qquad \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \to (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\qquad \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \to (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \to (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \to (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\$_3 YYYYYYY$ YYY

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
    $\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
    $\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
    $\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
    $\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
    $\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
    $\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
    $\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
    $\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc \lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \$_3 Y Y Y Y Y Y Y Y Y$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\$_3 YYYYY YYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\quad\quad \lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\quad\quad \lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\quad\quad \lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\quad\quad \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\quad\quad \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\quad\quad \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\quad\quad \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\quad\quad \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\$_3\,Y\,YYY$ $YYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
  $\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
  $\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
  $\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
  $\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
  $\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
  $\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
  $\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
  $\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \$_3 YYY$
$YYYYY$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\qquad \lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\qquad \lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\qquad \lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\qquad \lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\qquad \lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\qquad \lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\qquad \lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\qquad \lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$$\$_3$$Y\,Y$
$YYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
  $\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
  $\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
  $\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
  $\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
  $\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
  $\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
  $\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
  $\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \$_3 Y$ $YYYYY$

Step 1 $\lfloor 1 \rfloor : (S) \to (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \to (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
  $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
  $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \to (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
  $\lfloor 2 \rfloor : (\$_1) \to (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \to (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
  $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \to (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
  $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \to (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
  $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \to (a, \lfloor a3 \rfloor \$_3, Y, Y)$
  $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \to (b, \lfloor b3 \rfloor \$_3, Y, Y)$
  $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \to (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \to (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \to (\lfloor 4 \rfloor)$

Example

$aabbcc\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \$_3$
$Y\,YYYYYY$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
       $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
       $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
       $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
       $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
       $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
       $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
       $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
       $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

Example

$aabbcc\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor$
$\$_3 YYYYYY$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$aabbcc\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor$
$\textcolor{red}{\$_3 Y}\,YYYYY$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
 $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
 $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
 $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
 $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
 $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
 $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
 $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
 $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\textcolor{red}{\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)}$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$aabbcc\lfloor1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor3\rfloor\lfloor3\rfloor\lfloor3\rfloor\lfloor3\rfloor$
$\lfloor3\rfloor\$_3 YYYYY$

Step 1 $\lfloor1\rfloor : (S) \rightarrow (X\lfloor1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\quad\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\quad\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\quad\lfloor2\rfloor : (\$_1) \rightarrow (\lfloor2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\quad\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\quad\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\quad\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\quad\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\quad\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor3\rfloor : (\$_3, Y) \rightarrow (\lfloor3\rfloor, \$_3)$

Step 5 $\lfloor4\rfloor : (\$_3) \rightarrow (\lfloor4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\$_3 Y\,YYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\qquad \lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\qquad \lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\qquad \lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\qquad \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\qquad \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\qquad \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\qquad \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\qquad \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \$_3 YYYY$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
       $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
       $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
       $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
       $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
       $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
       $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
       $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
       $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$ $\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y YYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\qquad \lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\qquad \lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\qquad \lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\qquad \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\qquad \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\qquad \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\qquad \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\qquad \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$ $\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 YYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
  $\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
  $\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
  $\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
  $\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
  $\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
  $\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
  $\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
  $\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$ \$$_3$ YYY

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 YY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\qquad \lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\qquad \lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\qquad \lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\qquad \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\qquad \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\qquad \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\qquad \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\qquad \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1 \rfloor \lfloor \bar{1}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{2}1 \rfloor \lfloor \bar{3}1 \rfloor \lfloor 2 \rfloor \lfloor a2 \rfloor \lfloor a2 \rfloor \lfloor b2 \rfloor \lfloor b2 \rfloor \lfloor c2 \rfloor \lfloor c3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor \lfloor 3 \rfloor$ $\$_3 Y$

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A} \bar{B} \bar{C})$
   $\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
   $\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
   $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
   $\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
   $\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
   $\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$
   $\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
   $\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$ $\$_3$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\textcolor{red}{\lfloor 4\rfloor}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\textcolor{red}{\lfloor 4\rfloor} : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 4\rfloor$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\quad\quad \lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\quad\quad \lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\quad\quad \lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\quad\quad \lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\quad\quad \lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\quad\quad \lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\quad\quad \lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\quad\quad \lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Additional Productions

- 2 additional productions for generation of sentence which corresponds to ϵ in $\bar{G}$

Step 1 (without symbols X, Z)

Add $\lfloor 1_\epsilon \rfloor : (S) \to (\lfloor 1_\epsilon \rfloor \$_1 \bar{S})$ to P

Step 2 (skipping step 3)

Add $\lfloor 2_\epsilon \rfloor : (\$_1) \to (\lfloor 2_\epsilon \rfloor \$_3)$ to P

Example

Add $\lfloor 1_\epsilon \rfloor : (S) \to (\lfloor 1_\epsilon \rfloor \$_1 \bar{S})$ to P_4
   $\lfloor 2_\epsilon \rfloor : (\$_1) \to (\lfloor 2_\epsilon \rfloor \$_3)$ to P_4

Future Investigation

- can the same result be proven if we use the leftmost derivation? [YES]
- can we reduce the number of nonterminals? [YES]
- which other grammars can be proper generators of their sentences with their parses?