Transformation of ordinary differential equations into an equivalent polynomial form

Jan Chaloupka (xchalo08)

October 26, 2013

Abstract

Complex dynamical systems can be described by a system of differential equations. These systems are usually hard to solve analytically so numerical approach has to be taken. Partial differential equations (PDEs) can be transformed into ordinary differential equations (ODEs), so it is adequate to concern with latter ones. These equations can be solved in many ways by sophisticated methods, but all of them are sensitive to errors. So we have to solve them with given precision. This cannot be avoided. However, we can transform them to simpler equivalent systems that can be solved by less sensitive methods. Aim of this presentation is concerned with a transformation of ODEs to equivalent ODEs in polynomial form using predefined grammar that simulates derivation of polynomial functions. At the beginning of the presentation, basic mathematical definitions are going to be defined, followed by definition of a transformation problem. Simple rules for derivation and its corresponding grammar will guide the listener through the process. Simple example will follow to show the principle.

So far, this transformation has been applied to so called "technical problems", in which differential equations consist only of elemenentary functions. Elementary functions are those functions that can be constructed from basic elementary functions using finite number of additions, substractions, multiplication, division and composition.

Nevertheless, to attract the listener, let us give more detailed information. Only a few physical problems are solved using constant tools. In real, there is no such thing as a constant temperature in a room, there is no such thing as a constant air flow in an open space. The space we perceive is 3-dimensional. Theoretically, there is always a function describing actual temperature on given coordinates x, y, z. Only in rare situation we can suppose that the function is constant everywhere. However, to deduce more precise information about a given problem, we need to work with variable functions. In railway construction, emphasis has to be given on forces working on the train riding through curvature. And so on. All these problems have to be described in words of dynamic system changing in time. Differential equations suggest themselves as a powerful tool. Let us take a look at a short example. More aware reader knows that if we derive $\sin(t)$, we will get $\cos(t)$, so if we write $y = \sin(t)$, after deriving we have $y' = \cos(t)$. If we repeat the process, we get $y'' = -\sin(t)$. At this stage, we can write $y'' = -y$. For complete information, we need initial condition. We set $y(0) = \sin(0) = 0$. The same works for $\cos(t)$. Similar equations can be derived for every elementary function. So the following question arises: if every elementary function can be transformed into one or more ODEs with constant coefficients, is there a class of differential equations consisting of elementary functions that can be transformed into equivalent systems of ODEs with only constant coefficients that are much easier to be solve numerically?