

Improved Results on the Descriptive Complexity of Scattered Context Grammars

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1 Definitions

- Descriptive Complexity

2 Main Result

3 Proof

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- Basic Idea
- Theorem Proof
- Corollary Proof

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Number of Elements in a SCG Production

Number of elements in a SCG production

- for a SCG production, $p, (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$, $\pi(p) = n$

If $\pi(p)$

$= 1$ then the production is context-free

≥ 2 then the production is context-sensitive

Example

$G_1 = (V_1, T_1, P_1, S)$, where $V_1 = \{a, b, c, A, B, C, S\}$, $T_1 = \{a, b, c\}$

$P_1 = \{(S) \rightarrow (ABC),$

$(A, B, C) \rightarrow (aA, bB, cC),$

$(A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon)\}$

$\pi((S) \rightarrow (ABC)) = 1$ – context-free production

$\pi((A, B, C) \rightarrow (aA, bB, cC)) = 3$ – context-sensitive production

Notation

Let $G = (V, T, P, S)$ be a **SCG**, $p_1, \dots, p_n \in P$

Nonterminal complexity: number of nonterminals in G

Degree of context-sensitivity (δ -CS(G)): number of context-sensitive productions

Maximum context-sensitivity (\max -CS(G)):

$$\max(\{\pi(p_i) - 1 \mid 1 \leq i \leq |P|\})$$

Overall context-sensitivity ($\text{sum-CS}(G)$): $\Sigma(\{\pi(p_i) - 1 \mid 1 \leq i \leq |P|\})$

Example

$G_1 = (\{a, b, c, A, B, C, S\}, \{a, b, c\}, \{(S) \rightarrow (ABC),$
 $(A, B, C) \rightarrow (aA, bB, cC), (A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon)\}, S)$

G_1 's nonterminal complexity = 4 δ -CS(G_1) = 2

\max -CS(G_1) = 2

$\text{sum-CS}(G_1) = 0 + 2 + 2 = 4$

$\mathcal{L}(\mathbf{SCG}, k, l, m, n)$

$L \in \mathcal{L}(\mathbf{SCG}, k, l, m, n)$ iff there exists a **SCG**, G , $L(G) = L$ and

- G 's nonterminal complexity $\leq k$
- $\delta\text{-CS}(G) \leq l$
- $\max\text{-CS}(G) \leq m$
- $\text{sum-CS}(G) \leq n$

Recent results

- 1 $\mathcal{L}(\mathbf{SCG}, 3, \infty, \infty, \infty) = \mathcal{L}_{\mathbf{RE}}$ (Meduna, 2000, [2])
- 2 $\mathcal{L}(\mathbf{SCG}, \infty, 2, 3, 6) = \mathcal{L}_{\mathbf{RE}}$ (Meduna, Fernau, 2003, [3])
- 3 $\mathcal{L}(\mathbf{SCG}, 7, 5, 8, 29) = \mathcal{L}_{\mathbf{RE}}$ (Meduna, Fernau, 2003, [4])
- 4 $\mathcal{L}(\mathbf{SCG}, 6, 6, 12, 44) = \mathcal{L}_{\mathbf{RE}}$ (Meduna, Fernau, 2003, [4])

Theorem

$\mathcal{L}(\mathbf{SCG}, 5, 2, 3, 6)$ coincides with the family of recursively enumerable languages.

- context-sensitive productions of the form
 - $(0, \$, \$, 0) \rightarrow (\$, \varepsilon, \varepsilon, \$)$
 - $(1, \$, \$, 1) \rightarrow (\$, \varepsilon, \varepsilon, \$)$
- improves results 2, 3, 4 from previous slide

Corollary

$\mathcal{L}(\mathbf{SCG}, 8, 6, 1, 6)$ coincides with the family of recursively enumerable languages.

Post Correspondence Problem

Post correspondence problem (**PCP**)

- alphabet $\Sigma = \{0, 1\}$
- **PCP**, $P = \{(u_1, v_1), (u_2, v_2), \dots, (u_r, v_r)\}$, $u_i, v_i \in \Sigma^*$
- solution is a sequence of indexes $s_1, s_2, \dots, s_l \in \{1, \dots, r\}$ such that $u_{s_1} u_{s_2} \dots u_{s_l} = v_{s_1} v_{s_2} \dots v_{s_l}$
- undecidable problem

Example

$$P_1 = \{(1, 111), (10111, 10), (10, 0)\}$$

Solution: 2, 1, 1, 3

$$101111110 = 101111110$$

$$P_2 = \{(01, 011), (1, 10), (1, 11)\}$$

No solution ($|u_i| < |v_i|$)

Extended Post Correspondence Problem

Extended Post correspondence problem (**EPC**)

- $L \in \mathcal{L}_{\text{RE}}$ over $T = \{a_1, \dots, a_n\}$
- alphabet $\Sigma = \{0, 1\}$
- **EPC**, $E = \{ \{(u_1, v_1), (u_2, v_2), \dots, (u_r, v_r)\}, (z_{a_1}, \dots, z_{a_n}) \}$,
 $u_i, v_i, z_{a_j} \in \Sigma^*$
- $L(E)$ – language defined by **EPC**, E
- $L(E) = \{ b_1 \dots b_k \in T^* \mid \text{exists } s_1, \dots, s_l \in \{1, \dots, r\},$
 $v_{s_1} v_{s_2} \dots v_{s_l} = u_{s_1} u_{s_2} \dots u_{s_l} z_{b_1} \dots z_{b_k} \}$

Example

$E_1 = \{ \{(01, 011), (1, 10), (1, 11)\}, (1(a), 01(b)) \}$, $T = \{a, b\}$

$0111011 = 0111011 (123ba)$

$ba \in L(E_1)$

Theorem

*Every recursively enumerable language can be represented by **EPC** (see [1]).*

Basic idea

- 1 represent $L \in \mathcal{L}_{\text{RE}}$ by **EPC**
- 2 use context-free productions to generate a possible solution of **EPC**
- 3 verify this solution by context-sensitive productions

Note

Similar idea used by Geffert in [1]

Theorem Proof I

- let $L \in \mathcal{L}_{\text{RE}}$ over $T = \{a_1, \dots, a_n\}$
- **EPC**, $E = (\{(u_1, v_1), \dots, (u_r, v_r)\}, (z_{a_1}, \dots, z_{a_n}))$, $L(E) = L$

Define the **SCG**, $G = (\{S, A, 0, 1, \$\} \cup T, T, P, S)$

Context-free productions

- 1 For every $a \in T$, add
 - 1 $(S) \rightarrow (\text{rev}(z_a)Sa)$, and
 - 2 $(S) \rightarrow (\text{rev}(z_a)Aa)$ to P
- 2 For every $(u_i, v_i) \in E$, add $(A) \rightarrow (\text{rev}(u_i)Av_i)$ to P
- 3 Add $(A) \rightarrow (\$\$)$ to P

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Theorem Proof II

Context-sensitive productions

4 Add

$$1 \quad (0, \$, \$, 0) \rightarrow (\$, \varepsilon, \varepsilon, \$),$$

$$2 \quad (1, \$, \$, 1) \rightarrow (\$, \varepsilon, \varepsilon, \$) \text{ to } P$$

One more context-free production

5 Add $(\$) \rightarrow (\varepsilon)$ to P

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$$E_1 = \{ \{ (01, 011), (1, 10), (1, 11) \}, (1, 01) \}, T = \{ a, b \}$$

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$$ba \in L(E_1)$$

$$00ab \notin L(E_1)$$

Idea

- simulate every context-sensitive production, p , with $\pi(p) = 4$ by 3 productions p_1, p_2, p_3 with $\pi(p_1) = \pi(p_2) = \pi(p_3) = 2$ in 3 derivation steps

Define the **SCG**, $\bar{G} = (\{S, A, 0, 1, \$_L, \$_R, \$_0, \$_1\} \cup T, T, \bar{P}, S)$ with \bar{P} initialized with productions from steps (1) and (2).

3 Add $(A) \rightarrow (\$_L \$_R)$ to \bar{P}

4 Add

1 $(0, \$_L) \rightarrow (\$_0, \varepsilon), (\$_R, 0) \rightarrow (\varepsilon, \$_0), (\$_0, \$_0) \rightarrow (\$_L, \$_R),$

2 $(1, \$_L) \rightarrow (\$_1, \varepsilon), (\$_R, 1) \rightarrow (\varepsilon, \$_1), (\$_1, \$_1) \rightarrow (\$_L, \$_R)$ to \bar{P}

5 Add $(\$_L) \rightarrow (\varepsilon)$, and $(\$_R) \rightarrow (\varepsilon)$ to \bar{P}

We have proved that

- $\mathcal{L}(\mathbf{SCG}, 5, 2, 3, 6) = \mathcal{L}_{\text{RE}}$

Open problems

- We know that
 - $\mathcal{L}(\mathbf{SCG}, 1, \infty, \infty, \infty) \subset \mathcal{L}_{\text{RE}}$
 - $\mathcal{L}(\mathbf{SCG}, 3, \infty, \infty, \infty) = \mathcal{L}_{\text{RE}}$

What is the power of $\mathcal{L}(\mathbf{SCG}, 2, \infty, \infty, \infty)$?

- Can we generate any recursively enumerable language with only 1 context-sensitive production?



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