String-Partitioning Systems and An Infinite Hierarchy

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Advanced Lectures

Outline

Motivation

inspiration and special characteristics

Inspiration:

- another rewriting mechanisms
- models with properties of both automata and grammars
- generative power of such devices?
- different and common properties?

Definition

string partitioning system and it's configuration

SPS is a quadruple $M = (Q, \Sigma, s, R)$

- Q is a finite set of states
- Σ is an alphabet, $\# \in \Sigma$ called *bounder*
- $s \in Q$ is a start state
- R is finite set of rules of the form:
  $p_i\# \rightarrow qx \in R$, where $p, q \in Q, i \in I, x \in \Sigma^*$.

Configuration of SPS

- is string $c \in Q\Sigma^*$

Definition

derivation step and derived language

Derivation step from $pu\#v$ to $quxv$, where

- $p, q \in Q, u, v, x \in \Sigma^*$
- $occur(u, \#) = n - 1$
- by using $p_n\# \rightarrow qx \in R$

is $\boxed{pu\#v \Rightarrow quxv}$ $[p_n\# \rightarrow qx]$ in M

Language derived by M, $L(M)$:

- $L(M) = \{w \mid s\# \Rightarrow^* qw, q \in Q, w \in (\Sigma - \{\#\})^*\}$

Simple example of SPS

generation of language $a^n b^n c^n$

$M = (\{s, p, q, f\}, \{a, b, c, \#\}, s, R)$, where R contains:

1.	$s_1\# \rightarrow p\ \#\#$	4.	$p_1\# \rightarrow f\ ab$
2.	$p_1\# \rightarrow q\ a\#b$	5.	$f_1\# \rightarrow f\ c$
3.	$q_2\# \rightarrow p\ \#c$		

Example (derivation of string *aaabbbccc*)

$s\underline{\#} \Rightarrow p\underline{\#}\#[1] \Rightarrow qa\#b\underline{\#}\ [2] \Rightarrow pa\underline{\#}b\#c\ [3] \Rightarrow qaa\#bb\underline{\#}c\ [2] \Rightarrow paa\underline{\#}bb\#cc\ [3] \Rightarrow faaabbb\underline{\#}cc\ [4] \Rightarrow faaabbbccc\ [5]$.

$L(M) = \{a^n b^n c^n \mid n \geq 1\}$, with $Ind(M) = 2$

Definition
finite index of used formal models

Finite index of grammar?

- max. number of N's in sentential form w
- achievable sent. form - $S \Rightarrow^* w$
- leading to string x : $w \in x, x \in \Sigma^*$
- in the most economical derivation

Finite index of SPS?

- max.number of $\#$'s in sentential form

Definition
finite index of used formal models

Index of a language:

- equal to index of grammar/SPS

Family of languages of finite index *k*

- $\mathcal{L}_k(X)$

Family of all languages of finite index

- $\mathcal{L}_{fin}(X) = \bigcup_{i \geq 1} \mathcal{L}_k(X)$

Definition
programmed grammars

Programmed grammar (PG) – $G = (V, T, P, S)$

- V is a total alphabet
- $T \subseteq V$ is an alphabet of terminals
- $S \in (V - T)$ is the start symbol
- P is a finite set of rules of the form $p : A \rightarrow v, g(p)$
  - $p : A \rightarrow v$ is a context free rule labeled by p
  - $g(p)$ - set of rule labels associated with rule p (following set)
  - after p-application a rule labeled by a label from $g(p)$ has to be applied

Generative power
of programmed grammars

Programmed grammars:

- $\mathcal{L}(CF) \subset \mathcal{L}(PG) \subset \mathcal{L}(PG_{ac}) \subset \mathcal{L}(CS) \subset \mathcal{L}(\lambda PG_{ac}) = \mathcal{L}(RE)$
- $\mathcal{L}(CF) \subset \mathcal{L}(PG) \subset \mathcal{L}(\lambda PG) \subset \mathcal{L}(RE)$

Programmed grammars of index k:

- $\mathcal{L}_{fin}(PG) = \mathcal{L}_{fin}(\lambda PG) = \mathcal{L}_{fin}(PG_{ac}) = \mathcal{L}_{fin}(\lambda PG_{ac})$
- $\mathcal{L}(CF) - \mathcal{L}_{fin}(PG) \neq \emptyset$
  $\Rightarrow \mathcal{L}_{fin}(PG)$ is incomparable towards $\mathcal{L}(CF)$

Generative power and infinite hierarchy

of string partitioning systems of finite index

infinite hierarchy for SPS

$\mathcal{L}_k(SPS) \subset \mathcal{L}_{k+1}(SPS)$, for all $k \geq 1$

1. $\mathcal{L}_k(PG) \subset \mathcal{L}_{k+1}(PG)$, for all $k \geq 1$ (Gh. Păun, 1980)
2. $\mathcal{L}_k(SPS) = \mathcal{L}_k(PG)$

Theorem 2

$\mathcal{L}_k(SPS) = \mathcal{L}_k(PG)$, for every $k \geq 1$

Proof: 1) $\mathcal{L}_k(PG) \subseteq \mathcal{L}_k(SPS)$ 2) $\mathcal{L}_k(SPS) \subseteq \mathcal{L}_k(PG)$

Proof (basic idea)

first part: $PG_k \rightarrow SPS_k$

Conversion: $PG_k \rightarrow SPS_k$

- nonterminals represented by #s and information in state
- each state in SPS_k (2 components) of form:

$$\langle A_1 \dots A_j, q \rangle$$

$$A_1, \dots, A_j \in N_{PG_k},\ 0 \leq j \leq k,\ q \in g(p)$$

- one symbol in $A_1 \dots A_k$ is marked for following rewriting
- q represents next rule to use
- bounders mark positions for former nonterminals

$$x_0 A x_1 B x_2$$

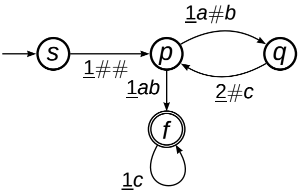
$$\Updownarrow$$

$$\langle AB, q \rangle x_0 \# x_1 \# x_2$$

Proof (demonstration)

second part: $SPS_k \rightarrow PG_k$

$$SPS_2\ M = (\{s, p, q, f\}, \{a, b, c, \#\}, s, R):$$



1. $s_1\# \rightarrow p\ \#\#$
2. $p_1\# \rightarrow q\ a\#b$
3. $q_2\# \rightarrow p\ \#c$
4. $p_1\# \rightarrow f\ ab$
5. $f_1\# \rightarrow f\ c$

How to construct PG's rule set based on SPS' rules?

basic idea will be presented..

Proof (demonstration)

second part: $SPS_k \rightarrow PG_k$

Conversion: $SPS_k \rightarrow PG_k$

1. every $A \in N$ is of form $\langle p, i, h\rangle$, $S := \langle s, 1, 1\rangle$
2. every SPS_k's rule $p_i\# \rightarrow qy$ simulate by sequence of steps (differs for every number of #s in configuration):

$$\boxed{p\, a\underline{\#}b\#c \Rightarrow_{SPS} f\, a\underline{ab}b\#c} \qquad [p_1\# \rightarrow f\, ab]$$

$$\Updownarrow$$

a) renumbering	$a\langle p,1,2\rangle b\langle p,2,2\rangle c \Rightarrow_{PG} a\langle f'',1,1\rangle b\langle p,2,2\rangle c \Rightarrow_{PG}$
b) rewriting	$a\underline{\langle f'',1,1\rangle b}\langle f',1,1\rangle c \Rightarrow_{PG} a\underline{ab}b\langle f',1,1\rangle c \Rightarrow_{PG}$
c) finalization	$aabb\langle f,1,1\rangle c$

Modifications of SPS
another challenges...

SPS with finite index:

- deterministic variant
- accepting variant
- parallel variant

SPS without index limitation:

- generative power
- properties

References...

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