Random Context Grammars

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Random Context Grammar

Random Context Grammar

A (permitting) random context grammar is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R is a finite relation from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, we write

$$(p : A \rightarrow x, Q)$$

where $Q \subseteq N$ is called a permitting context

Derivation Step of Random Context Grammar

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y\ [p] \text{ in } H$$

if

1. $x \Rightarrow y\ [p]$ in G and
2. $R(p) \subseteq \text{alph}(x)$

Random Context Grammar with Appearance Checking

Random Context Grammar with Appearance Checking

A random context grammar with appearance checking is a triple

$$H = (G, R, F)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R, F are two finite relations from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, and $F(p) = K$, we write

$$(p : A \rightarrow x, Q, K)$$

where Q and K are permitting and forbidding contexts, respectively

Derivation Step of Random Context Grammar with Appearance Checking

Forbidding Grammar

If every $(p : A \rightarrow x, Q, K)$ satisfies $Q = \emptyset$, then H is called a forbidding grammar.

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y\ [p] \text{ in } H$$

if

1. $x \Rightarrow y\ [p]$ in G
2. $R(p) \subseteq \text{alph}(x)$
3. $F(p) \cap \text{alph}(x) = \emptyset$

Example – Permitting Grammar I

Example

1. $(S \to ABC, \emptyset)$
2. $(A \to aA', \{B\})$
   $(B \to bB', \{C\})$
   $(C \to cC', \{A'\})$
3. $(A' \to A, \{B'\})$
   $(B' \to B, \{C'\})$
   $(C' \to C, \{A\})$
4. $(A \to a, \{B\})$
   $(B \to b, \{C\})$
   $(C \to c, \emptyset)$

$$\begin{tikzcd} 1 \arrow[r] \arrow[rrr, bend left] & 2 \arrow[r] & 3 \arrow[r] \arrow[l, bend left] & 4 \end{tikzcd}$$

Example – Permitting Grammar II

Example

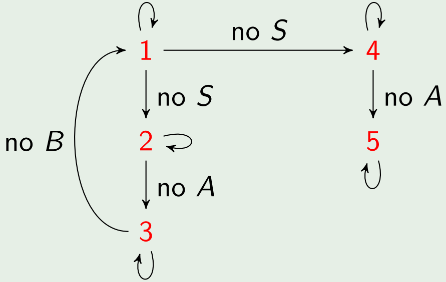
$$
\begin{array}{rll}
S \Rightarrow & ABC & 1 \\
\Rightarrow^3 & aA'bB'cC' & 2 \\
\Rightarrow^3 & aAbBcC & 3 \\
\Rightarrow^3 & aaA'bbB'ccC' & 2 \\
\Rightarrow^3 & aaAbbBccC & 3 \\
& \vdots & \\
\Rightarrow^3 & a^nAb^nBc^nC & 3 \\
\Rightarrow^3 & a^{n+1}b^{n+1}c^{n+1} & 4
\end{array}
$$

$$L(H) = \{a^k b^k c^k \ : \ k \geq 1\}$$

Example – Forbidding Grammar I

Example

$(1 : S \rightarrow AA, \emptyset, \{B, D\})$
$(2 : A \rightarrow B, \emptyset, \{S, D\})$
$(3 : B \rightarrow S, \emptyset, \{A, D\})$
$(4 : A \rightarrow D, \emptyset, \{S, B\})$
$(5 : D \rightarrow a, \emptyset, \{S, A, B\})$



Example – Forbidding Grammar II

Example

$$
\begin{array}{lll}
 & & \text{rules} \\
S \Rightarrow & AA & 1 \\
\Rightarrow^2 & BB & 2 \\
\Rightarrow^2 & SS & 3 \\
\Rightarrow^2 & AAAA & 1 \\
\Rightarrow^4 & BBBB & 2 \\
\Rightarrow^4 & SSSS & 3 \\
 & \vdots & \\
\Rightarrow^{2^i} & A^{2^{i+1}} & 1 \\
\Rightarrow^{2^{i+1}} & D^{2^{i+1}} & 4 \\
\Rightarrow^{2^{i+1}} & a^{2^{i+1}} & 5
\end{array}
$$

$$L(H) = \{a^{2^n} : n \geq 1\}$$

Bibliography

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