Parallel Communicating Grammar Systems

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PC Grammar Systems

PC Grammar System

A parallel communicating (PC) grammar system of degree n, $n \geq 1$, is a construct

$$\Gamma = (N, K, T, (S_1, P_1), \ldots, (S_n, P_n))$$

where

K is a finite set of query symbols, $K = \{Q_1, \ldots, Q_n\}$

P_i is a finite set of productions of the form

$$A \to x$$

with $A \in N$ and $x \in (N \cup T \cup K)^*$, for all $i = 1, \ldots, n$

S_i is the start symbol of the ith component, $S_i \in N$ for all $i = 1, \ldots, n$

N, T are defined as usual, N, K, T are pairwise disjoint

PC Grammar Systems – Derivation I

Two Kinds of Derivation Steps

- generating
- communicating

g-Step

If

- either $x_i \Rightarrow y_i$ in $G_i = (N \cup K, T, P_i, S_i)$,
- or $x_i = y_i \in T^*$

for all $1 \leq i \leq n$, then

$$(x_1, \ldots, x_n) \ {}_g\!\Rightarrow (y_1, \ldots, y_n)$$

PC Grammar Systems – Derivation II

c-Step

- set $z_i = x_i$ for all $i = 1, \dots, n$

For each $i = 1, \dots, n$, if

$$\mathrm{alph}(x_i) \cap K \neq \emptyset$$

and for each Q_j in x_i,

$$\mathrm{alph}(x_j) \cap K = \emptyset,$$

then for each Q_j in x_i

1. set $z_j = S_j$,
2. replace Q_j with x_j in x_i,
3. set z_i to the string resulting from (2)

Perform

$$(x_1, \dots, x_n) \;{}_c\!\Rightarrow (y_1, \dots, y_n)$$

with $y_i = z_i$, for all $i = 1, \dots, n$

PC Grammar Systems – Generated Language

Direct Derivation

If either

$$(x_1, \ldots, x_n) \ {}_g\!\Rightarrow (y_1, \ldots, y_n)$$

or

$$(x_1, \ldots, x_n) \ {}_c\!\Rightarrow (y_1, \ldots, y_n)$$

then

$$(x_1, \ldots, x_n) \Rightarrow (y_1, \ldots, y_n)$$

Generated Language

$$L(\Gamma) = \{x \in T^* \ : \ (S_1, S_2, \ldots, S_n) \Rightarrow^* (x, \alpha_2, \ldots, \alpha_n), \\ \alpha_i \in (N \cup T \cup K)^*, \ \text{for all } i = 2, \ldots, n\}$$

Centralized PC Grammar Systems

- only P_1 can produce query symbols

Centralized PC Grammar System

Let

$$\Gamma = (N, K, T, (S_1, P_1), \ldots, (S_n, P_n))$$

be a PC grammar system. Γ is centralized if for all $A \rightarrow x \in P_i$, where $i = 2, \ldots, n$,

$$\mathrm{alph}(x) \cap K = \emptyset$$

Returning and Non-Returning PC Grammar Systems

Returning PC Grammar System

After communicating, each component that has sent its string to another component returns to its axiom.

- generated language denoted by $L_r(\Gamma)$

Non-Returning PC Grammar System

After communicating, each component that has sent its string to another component continues to process the current string. That is, remove (1) in the basic definition.

- generated language denoted by $L_{nr}(\Gamma)$

PC Grammar Systems – Example

Example

$$\Gamma = (\{S_1, S_1', S_2, S_3\}, K, \{a, b\}, (S_1, P_1), (S_2, P_2), (S_3, P_3))$$

where

$$\begin{aligned}
P_1 &= \{S_1 \rightarrow abc, S_1 \rightarrow a^2b^2c^2, S_1 \rightarrow aS_1', S_1 \rightarrow a^3Q_2, \\
&\qquad S_1' \rightarrow aS_1', S_1' \rightarrow a^3Q_2, S_2 \rightarrow b^2Q_3, S_3 \rightarrow c\} \\
P_2 &= \{S_2 \rightarrow bS_2\} \\
P_3 &= \{S_3 \rightarrow cS_3\}
\end{aligned}$$

(S_1, S_2, S_3)

PC Grammar Systems – Example

Example

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$$(S_1, S_2, S_3) \Rightarrow (aS_1', bS_2, cS_3)$$

PC Grammar Systems – Example

Example

$$\Gamma = (\{S_1, S_1', S_2, S_3\}, K, \{a, b\}, (S_1, P_1), (S_2, P_2), (S_3, P_3))$$

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$$(S_1, S_2, S_3) \Rightarrow (aS_1', bS_2, cS_3) \Rightarrow^* (a^nS_1', b^nS_2, c^nS_3)$$

PC Grammar Systems – Example

Example

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$$\begin{aligned} (S_1, S_2, S_3) &\Rightarrow (aS_1', bS_2, cS_3) \Rightarrow^* (a^nS_1', b^nS_2, c^nS_3) \\ &\Rightarrow (a^{n+3}Q_2, b^{n+1}S_2, c^{n+1}S_3) \end{aligned}$$

PC Grammar Systems – Example

Example

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PC Grammar Systems – Example

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P_2 = \{&S_2 \rightarrow bS_2\}\\
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\end{aligned}$$

$$\begin{aligned}
(S_1, S_2, S_3) &\Rightarrow (aS_1', bS_2, cS_3) \Rightarrow^* (a^nS_1', b^nS_2, c^nS_3)\\
&\Rightarrow (a^{n+3}Q_2, b^{n+1}S_2, c^{n+1}S_3) \Rightarrow (a^{n+3}b^{n+1}S_2, S_2, c^{n+1}S_3)\\
&\Rightarrow (a^{n+3}b^{n+3}{\color{red}Q_3}, bS_2, c^{n+2}S_3)
\end{aligned}$$

PC Grammar Systems – Example

Example

$$\Gamma = (\{S_1, S_1', S_2, S_3\}, K, \{a, b\}, (S_1, P_1), (S_2, P_2), (S_3, P_3))$$

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$$\begin{aligned} (S_1, S_2, S_3) & \Rightarrow (aS_1', bS_2, cS_3) \Rightarrow^* (a^nS_1', b^nS_2, c^nS_3) \\ & \Rightarrow (a^{n+3}Q_2, b^{n+1}S_2, c^{n+1}S_3) \Rightarrow (a^{n+3}b^{n+1}S_2, S_2, c^{n+1}S_3) \\ & \Rightarrow (a^{n+3}b^{n+3}Q_3, bS_2, c^{n+2}S_3) \Rightarrow (a^{n+3}b^{n+3}c^{n+2}S_3, bS_2, S_3) \end{aligned}$$

PC Grammar Systems – Example

Example

$$\Gamma = (\{S_1, S_1', S_2, S_3\}, K, \{a, b\}, (S_1, P_1), (S_2, P_2), (S_3, P_3))$$

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P_2 &= \{S_2 \rightarrow bS_2\} \\
P_3 &= \{S_3 \rightarrow cS_3\}
\end{aligned}$$

$$\begin{aligned}
(S_1, S_2, S_3) &\Rightarrow (aS_1', bS_2, cS_3) \Rightarrow^* (a^nS_1', b^nS_2, c^nS_3) \\
&\Rightarrow (a^{n+3}Q_2, b^{n+1}S_2, c^{n+1}S_3) \Rightarrow (a^{n+3}b^{n+1}S_2, S_2, c^{n+1}S_3) \\
&\Rightarrow (a^{n+3}b^{n+3}Q_3, bS_2, c^{n+2}S_3) \Rightarrow (a^{n+3}b^{n+3}c^{n+2}S_3, bS_2, S_3) \\
&\Rightarrow (a^{n+3}b^{n+3}c^{n+3}, bbS_2, cS_3)
\end{aligned}$$

PC Grammar Systems – Example

Example

$$\Gamma = (\{S_1, S'_1, S_2, S_3\}, K, \{a, b\}, (S_1, P_1), (S_2, P_2), (S_3, P_3))$$

where

$$\begin{aligned} P_1 = \{ & S_1 \rightarrow abc, S_1 \rightarrow a^2b^2c^2, S_1 \rightarrow aS'_1, S_1 \rightarrow a^3Q_2, \\ & S'_1 \rightarrow aS'_1, S'_1 \rightarrow a^3Q_2, S_2 \rightarrow b^2Q_3, S_3 \rightarrow c\} \\ P_2 = \{ & S_2 \rightarrow bS_2\} \\ P_3 = \{ & S_3 \rightarrow cS_3\} \end{aligned}$$

$$\begin{aligned} (S_1, S_2, S_3) & \Rightarrow (aS'_1, bS_2, cS_3) \Rightarrow^* (a^nS'_1, b^nS_2, c^nS_3) \\ & \Rightarrow (a^{n+3}Q_2, b^{n+1}S_2, c^{n+1}S_3) \Rightarrow (a^{n+3}b^{n+1}S_2, S_2, c^{n+1}S_3) \\ & \Rightarrow (a^{n+3}b^{n+3}Q_3, bS_2, c^{n+2}S_3) \Rightarrow (a^{n+3}b^{n+3}c^{n+2}S_3, bS_2, S_3) \\ & \Rightarrow (a^{n+3}b^{n+3}c^{n+3}, bbS_2, cS_3) \end{aligned}$$

$$L_r(\Gamma) = L_{nr}(\Gamma) = \{a^nb^nc^n : n \geq 1\}$$

Denotation of PC Language Families

Denotation of PC Language Families

$$XPC_nY$$

X

- N – non-returning mode
- C – centralized PC grammar systems

n number of components (by analogy with CD grammar systems)

Y specification of the type of productions (REG, LIN, CF)

Example

CPC_2REG, NPC_2LIN, $NCPC_\infty$

PC Grammar Systems – Generative Power I

Theorem

- $PC_nREG - \mathscr{L}(LIN) \neq \emptyset$, *for* $n \geq 2$
- $PC_nREG - \mathscr{L}(CF) \neq \emptyset$, *for* $n \geq 3$
- $PC_nLIN - \mathscr{L}(CF) \neq \emptyset$, *for* $n \geq 2$

Theorem

$\mathscr{L}(LIN) - CPC_\infty REG \neq \emptyset$

PC Grammar Systems – Generative Power II

Theorem

$PC_n REG \subset PC_{n+1} REG$, *for* $n \geq 1$

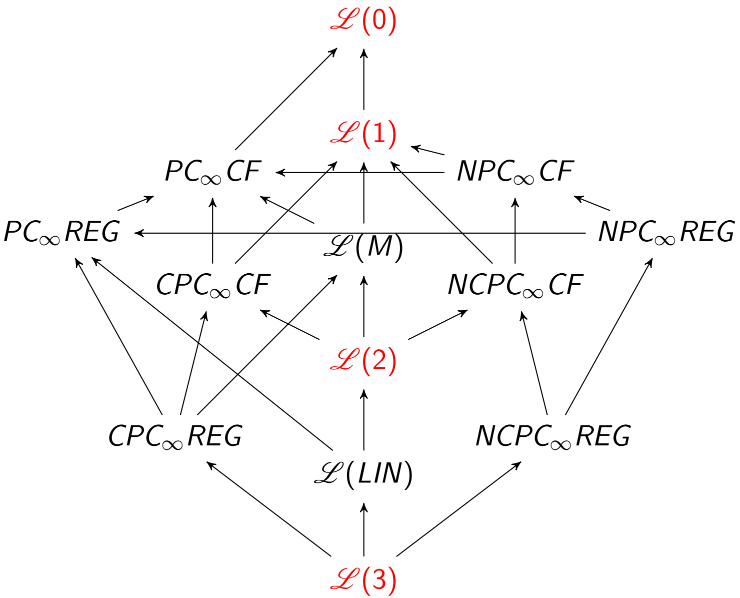
Theorem

$CPC_n REG \subset CPC_n LIN \subset CPC_n CF$, *for* $n \geq 1$

Theorem

- $NPC_\infty CF \subseteq PC_\infty CF$
- $\mathscr{L}(M) \subset PC_\infty CF$
- $\mathscr{L}(LIN) \subset PC_\infty REG$

PC Grammar Systems – The Hierarchy



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