Other Grammars

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State Grammar

State Grammar

$$G = (V, T, K, P, S, p_0)$$

where

V is a finite alphabet

T is a set of terminals, $T \subset V$

K is a finite set of states

S is the start symbol, $S \in V - T$

p_0 is the start state, $p_0 \in K$

P is a finite set of productions of the form

$$(A, p) \rightarrow (x, q),$$

where $p, q \in K$, $A \in V - T$, $x \in V^*$

State Grammar – Derivation Step

Derivation Step

For $(A, p) \rightarrow (x, q) \in P$,

$$u = (rAs, p),$$
$$v = (rxs, q),$$

where $r, s \in V^*$, and for every $(B, p) \rightarrow (y, t) \in P$, $B \notin \text{alph}(r)$, we write

$$u \Rightarrow v \, [(A, p) \rightarrow (x, q)]$$

Generated Language

$L(G) = \{x \in T^* : (S, p_0) \Rightarrow^* (x, q) \text{ for some } q \in K\}$

Generative Power

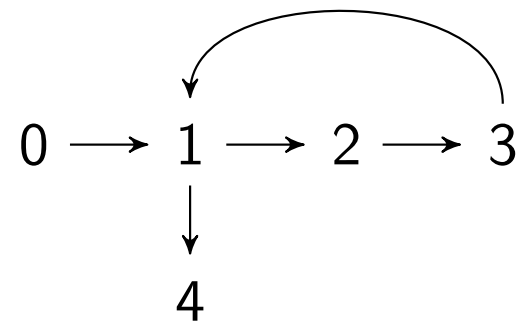
$\mathscr{L}(ST) = \mathscr{L}(CS)$, and $\mathscr{L}(ST, \varepsilon) = \mathscr{L}(RE)$

State Grammar – Example

Example

$$G = (\{S, A, B, C, a, b, c\}, \{a, b, c\}, \{0, 1, 2, 3, 4\}, P, S, 0)$$

$$\begin{aligned} P = \{&(S,0) \to (ABC,1), \\ &(A,1) \to (aA,2), \quad (A,1) \to (a,4), \\ &(B,2) \to (bB,3), \quad (B,4) \to (b,4), \\ &(C,3) \to (cC,1), \quad (C,4) \to (c,4)\} \end{aligned}$$



$$\begin{aligned} (S,0) \Rightarrow (ABC,1) \Rightarrow (aABC,2) \Rightarrow (aAbBC,3) \Rightarrow (aAbBcC,1) \\ \Rightarrow (aabBcC,4) \Rightarrow (aabbcC,4) \Rightarrow (aabbcc,4) \end{aligned}$$

$$L(G) = \{a^n b^n c^n \,:\, n \geq 1\}$$

Queue Grammar

Queue Grammar

$$G = (V, T, W, F, P, s)$$

where

- V is a finite alphabet
- T is a set of terminals, $T \subset V$
- W is a finite alphabet of states
- F is a set of final states, $F \subset W$
- s is the start string, $s \in (V - T)(W - F)$
- P is a finite set of productions of the form: (a, b, x, c), where
  - $a \in V$
  - $b \in W - F$
  - $x \in V^*$
  - $c \in W$

Queue Grammar – Derivation Step

Derivation Step

If

$$u = arb,\ v = rxc,$$

where $a \in V$, $r, x \in V^*$, $b, c \in W$, and

$$(a, b, x, c) \in P,$$

then

$$u \Rightarrow v\ [(a, b, x, c)]$$

Generated Language

$L(G) = \{w \in T^* : s \Rightarrow^* wf, f \in F\}$

Generative Power

$\mathscr{L}(QG, \varepsilon) = \mathscr{L}(RE)$, and $\mathscr{L}(QG) = \mathscr{L}(CS)$

Queue Grammar – Example

Example

$$G = (\{A, a, b\}, \{a, b\}, \{\bar{e}, \bar{f}\}, \{\bar{f}\}, P, A\bar{e})$$

$$P = \{1 : (A, \bar{e}, bAa, \bar{e}),$$
$$2 : (A, \bar{e}, \varepsilon, \bar{f}),$$
$$3 : (a, \bar{e}, a, \bar{e}),$$
$$4 : (b, \bar{e}, b, \bar{e})\}$$

$$A\bar{e} \Rightarrow bAa\bar{e}\ [1] \Rightarrow Aab\bar{e}\ [4] \Rightarrow abbAa\bar{e}\ [1] \Rightarrow bbAaa\bar{e}\ [3] \Rightarrow bAaab\bar{e}\ [4]$$
$$\Rightarrow Aaabb\bar{e}\ [4] \Rightarrow aabb\bar{f}\ [2]$$

$$L(G) = \{a^n b^n \ :\ n \geq 0\}$$

Simple Matrix Grammar

Simple Matrix Grammar (of Degree n)

$$G = (N_1, \ldots, N_n, T, P, S)$$

where

$N_1, \ldots, N_n$ are pairwise disjoint nonterminal alphabets

P is a finite set of productions of the form

- $(S \to w)$, where $w \in T^*$
- $(S \to A_1 \ldots A_n)$, where $A_i \in N_i$, $i = 1, \ldots, n$
- $(A_1 \to w_1, \ldots, A_n \to w_n)$, where $w_i \in T^*$, $i = 1, \ldots, n$
- $(A_1 \to \alpha_1, \ldots, A_n \to \alpha_n)$, where

$$\alpha_i = x_{i1} A_{i1} \ldots x_{ik} A_{ik} y_i,$$

$$x_{ij}, y_i \in T^*,\ A_{ij} \in N_i,\ i = 1, \ldots, n,\ j = 1, \ldots, k,\ k \geq 1$$

T, S have the standard meaning, $S \notin N_1 \cup \ldots \cup N_n$

Simple Matrix Grammar – Derivation Step

Direct Derivation

If

- either $u = S$ and $(S \to v) \in P$
- or
  - $u = y_1 A_1 z_1 \dots y_n A_n z_n$
  - $v = y_1 w_1 z_1 \dots y_n w_n z_n$
  - $(A_1 \to w_1, \dots, A_n \to w_n) \in P$

  where $y_i \in T^*$, $z_i \in (N_i \cup T)^*$, for all $i = 1, \dots, n$

then

$$u \Rightarrow v$$

Note

The generated language and $\Rightarrow^*$ are defined as usual

Simple Matrix Grammar – Generative Power

(Right) Linear Simple Matrix Grammar (of Degree n)

If the productions $A_i \to \alpha_i$, $1 \leq i \leq n$, from $(A_1 \to \alpha_1, \ldots, A_n \to \alpha_n)$ are

linear then G is linear simple matrix grammar (LSM)

right linear then G is right linear simple matrix grammar ($RLSM$)

Generative Power

For all $n \geq 1$,

- $\mathscr{L}(SM, n) \subset \mathscr{L}(SM, n+1) \subset \mathscr{L}(CS)$
- $\mathscr{L}(LSM, n) \subset \mathscr{L}(LSM, n+1)$
- $\mathscr{L}(RLSM, n) \subset \mathscr{L}(RLSM, n+1)$

Right Linear Simple Matrix Grammar – Example

Example

$$G = (\{A\}, \{B\}, \{a, b\}, S, P)$$

where

$$\begin{aligned} P = \{&1 : (S \to AB), \\ &2 : (A \to aA, B \to aB), \\ &3 : (A \to bA, B \to bB), \\ &4 : (A \to \varepsilon, B \to \varepsilon)\} \end{aligned}$$

$$S \Rightarrow AB\ [1] \Rightarrow aAaB\ [2] \Rightarrow abAabB\ [3] \Rightarrow abaAabaB\ [2] \Rightarrow abaaba\ [4]$$

$$L(G) = \{ww\ :\ w \in \{a, b\}^*\}$$

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