

Workspace Theorems for Regular-Controlled Grammars

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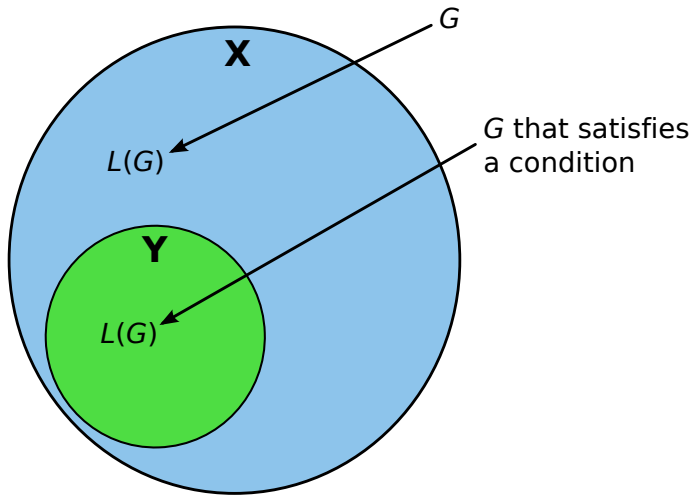
Based on

 [Alexander Meduna and Petr Zemek.](#)

Workspace Theorems for Regular-Controlled Grammars.

Theoretical Computer Science, 2011 (to appear)

- **Introduction**
- **Classical Workspace Theorem**
- **New Workspace Theorem**
- **Idea Behind the Proof**
- **Concluding Remarks and Discussion**



Phrase-Structure Grammars

Definition

A *phrase-structure grammar* is a quadruple

$$G = (N, T, P, S)$$

where

- N is an alphabet of *nonterminals*,
- T is an alphabet of *terminals* ($N \cap T = \emptyset$),
- P is a finite set of *rules* of the form

$$x \rightarrow y$$

where $x \in (N \cup T)^* N (N \cup T)^*$ and $y \in (N \cup T)^*$,

- $S \in N$ is the *starting nonterminal*.

Phrase-Structure Grammars

Definition

The relation of a *direct derivation*, denoted by \Rightarrow , is defined as follows:

$$uxv \Rightarrow uyv \text{ in } G$$

if and only if

- $u, v \in (N \cup T)^*$,
- $x \rightarrow y \in P$.

Definition

The *language of G* , denoted by $L(G)$, is defined as

$$L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$$

where \Rightarrow^* is the reflexive-transitive closure of \Rightarrow .

RE: the family of recursively enumerable languages

Context-Sensitive Grammars

Definition

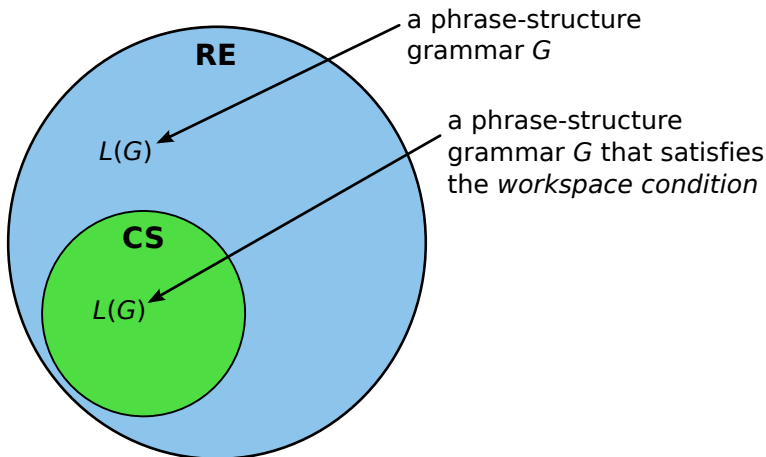
A phrase-structure grammar $G = (N, T, P, S)$ is a *context-sensitive grammar* if every $x \rightarrow y \in P$ is of the form

$$uAv \rightarrow uzv$$

where $u, v \in (N \cup T)^*$, $A \in N$, and $z \in (N \cup T)^+$.

CS: the family of context-sensitive languages

Basic Idea



Theorem

Let $G = (N, T, P, S)$ be a phrase-structure grammar. If there exists some $k \geq 1$ such that for every (nonempty) $w \in L(G)$, there exists a derivation

$$S \Rightarrow x_1 \Rightarrow x_2 \Rightarrow \cdots \Rightarrow x_n = w$$

where

$$|x_i| \leq k|w|$$

for all $1 \leq i \leq n$, then

$$L(G) \in \mathbf{CS}$$

$$|x_i| \leq \overbrace{|w w \cdots w|}^{k \text{ times}}$$

Context-Free Grammars

Definition

A phrase-structure grammar $G = (N, T, P, S)$ is a *context-free grammar* if every $x \rightarrow y \in P$ satisfies

$$x \in N$$

If $uAv \Rightarrow uyv$ by $r = (A \rightarrow y) \in P$, then we write

$$uAv \Rightarrow uyv [r]$$

If $S \Rightarrow x_1 [r_1] \Rightarrow \dots \Rightarrow x_n [r_n]$, then we write

$$S \Rightarrow^* x_n [r_1 \dots r_n]$$

CF: the family of context-free languages

Regular-Controlled Grammars

Definition

A *regular-controlled (context-free) grammar* is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$ is a context-free grammar,
- $R \subseteq P^*$ is a *regular control language*.

Definition

The *language generated by H* , denoted by $L(H)$, is defined as

$$L(H) = \{w \in T^* \mid S \Rightarrow^* w [q] \text{ with } q \in R\}$$

Example of a Regular-Controlled Grammar

Example

Consider the following regular-controlled grammar H :

$$1: S \rightarrow ABC$$

$$2: A \rightarrow aA$$

$$3: B \rightarrow bB$$

$$4: C \rightarrow cC$$

$$5: A \rightarrow \varepsilon$$

$$6: B \rightarrow \varepsilon$$

$$7: C \rightarrow \varepsilon$$

$$R = \{1\}\{234\}^*\{567\}$$

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$$S \Rightarrow ABC \Rightarrow aABC \Rightarrow aAbBC \Rightarrow aAbBcC \Rightarrow abBcC \Rightarrow abcC \\ \Rightarrow abc$$

$$L(H) = \{a^n b^n c^n \mid n \geq 0\}$$

Denotation of Language Families

RC: the family of languages generated by regular-controlled grammars

RC^{-ε}: the family of languages generated by regular-controlled grammars without *erasing rules*

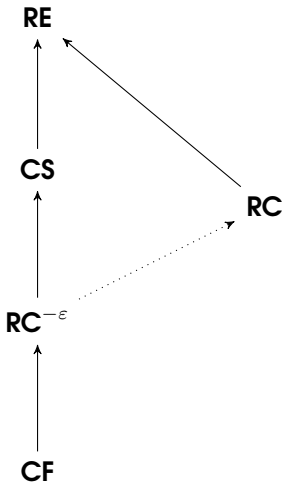
$$A \rightarrow \varepsilon$$

RE: the family of recursively enumerable languages

CS: the family of context-sensitive languages

CF: the family of context-free languages

Relationships Between Language Families

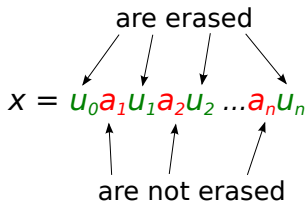


Basic Idea

Definition

A regular-controlled grammar H works within a k -limited workspace provided that for all (nonempty) $w \in L(H)$, there exists a derivation $S \Rightarrow^* w$ in which every sentential form x satisfies the following condition:

for every occurrence of a symbol in x which is not erased, x contains at most k occurrences of symbols which are erased.



$$k|a_1 a_2 \dots a_n| \geq |u_0 u_1 \dots u_n|$$

Example

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For every $a^n b^n c^n \in L(H)$, where $n \geq 1$, there exists

$$S \Rightarrow^* a^n A b^n B c^n C \Rightarrow^* a^n b^n c^n [\varrho] \text{ with } \varrho \in R$$

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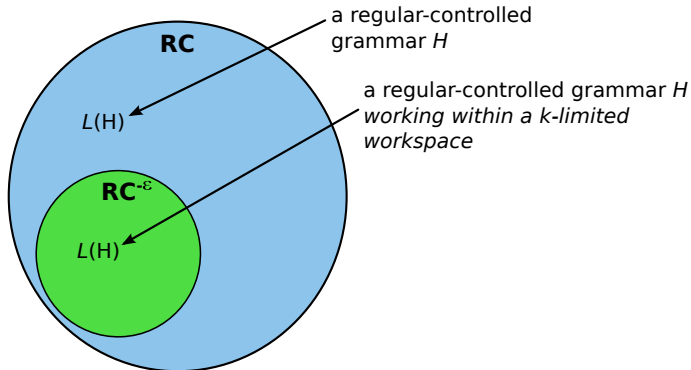
$$S \Rightarrow^* a^n A b^n B c^n C \Rightarrow^* a^n b^n c^n [\varrho] \text{ with } \varrho \in R$$

Hence, H works within a 1-limited workspace.

Theorem

Let H be a regular-controlled grammar working within a k -limited workspace, for some $k \geq 0$. Then,

$$L(H) \in \mathbf{RC}^{-\varepsilon}$$



We use compound nonterminals of the form

$$\langle X, y \rangle$$

where

- X is a symbol that is not erased throughout the rest of the derivation,
- y is a k -limited string of symbols that are erased throughout the rest of the derivation.

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Illustration

$$\langle A, DE \rangle \langle b, \varepsilon \rangle \langle b, A \rangle$$



1 For each $A \rightarrow x_0 X_1 x_1 X_2 x_2 \cdots X_m x_m$ and $\langle A, y \rangle$, we introduce

$$\langle A, y \rangle \rightarrow \langle X_1, y x_0 x_1 \cdots x_m \rangle \langle X_2, \varepsilon \rangle \cdots \langle X_m, \varepsilon \rangle$$



- 1 For each $A \rightarrow x_0x_1x_1x_2x_2 \cdots x_mx_m$ and $\langle A, y \rangle$, we introduce

$$\langle A, y \rangle \rightarrow \langle X_1, yx_0x_1 \cdots x_m \rangle \langle X_2, \varepsilon \rangle \cdots \langle X_m, \varepsilon \rangle$$

Illustration

For $A \rightarrow aA$ and $\langle A, \varepsilon \rangle$, we introduce

$$\langle A, \varepsilon \rangle \rightarrow \langle a, \varepsilon \rangle \langle A, \varepsilon \rangle$$

$$\langle A, \varepsilon \rangle \rightarrow \langle a, A \rangle$$

- ② For each $A \rightarrow y$ and $\langle X, uAv \rangle$, we introduce

$$\langle X, uAv \rangle \rightarrow \langle X, uyv \rangle$$

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Illustration

For $A \rightarrow BC$ and $\langle a, AD \rangle$, we introduce

$$\langle a, AD \rangle \rightarrow \langle a, BCD \rangle$$

- 2 For each $A \rightarrow y$ and $\langle X, uAv \rangle$, we introduce

$$\langle X, uAv \rangle \rightarrow \langle X, uyv \rangle$$

Illustration

For $A \rightarrow BC$ and $\langle a, AD \rangle$, we introduce

$$\langle a, AD \rangle \rightarrow \langle a, BCD \rangle$$

Illustration

For $A \rightarrow \varepsilon$ and $\langle a, AD \rangle$, we introduce

$$\langle a, AD \rangle \rightarrow \langle a, D \rangle$$

- 3 For each $\langle X, uAv \rangle$ and $\langle Y, y \rangle$, we introduce

$$\begin{aligned}\langle X, uAv \rangle &\rightarrow \langle X, uv \rangle \\ \langle Y, y \rangle &\rightarrow \langle Y, yA \rangle\end{aligned}$$

+ regulation!

- 3 For each $\langle X, uAv \rangle$ and $\langle Y, y \rangle$, we introduce

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+ regulation!

Illustration

For $\langle C, AD \rangle$ and $\langle a, B \rangle$, we introduce

$$\begin{aligned}\langle C, AD \rangle &\rightarrow \langle C, D \rangle \\ \langle a, B \rangle &\rightarrow \langle a, BA \rangle\end{aligned}$$

- 4 For each terminal a , we introduce

$$\langle a, \varepsilon \rangle \rightarrow a$$

- 4 For each terminal a , we introduce

$$\langle a, \varepsilon \rangle \rightarrow a$$

Illustration

For a, b, c , we introduce

$$\langle a, \varepsilon \rangle \rightarrow a$$

$$\langle b, \varepsilon \rangle \rightarrow b$$

$$\langle c, \varepsilon \rangle \rightarrow c$$

Example

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$\langle S, \varepsilon \rangle$

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$$\langle S, \varepsilon \rangle \Rightarrow \langle A, \varepsilon \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle \quad [\langle S, \varepsilon \rangle \rightarrow \langle A, \varepsilon \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle]$$

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 & \Rightarrow \langle a, \varepsilon \rangle \langle b, \varepsilon \rangle \langle c, \varepsilon \rangle & [\langle c, C \rangle \rightarrow \langle c, \varepsilon \rangle] \\
 & \Rightarrow a \langle b, \varepsilon \rangle \langle c, \varepsilon \rangle & [\langle a, \varepsilon \rangle \rightarrow a]
 \end{array}$$

Example

Consider the regular-controlled grammar H :

$$1: S \rightarrow ABC$$

$$2: A \rightarrow aA$$

$$3: B \rightarrow bB$$

$$4: C \rightarrow cC$$

$$5: A \rightarrow \varepsilon$$

$$6: B \rightarrow \varepsilon$$

$$7: C \rightarrow \varepsilon$$

$$\begin{array}{ll}
 \langle S, \varepsilon \rangle & \Rightarrow \langle A, \varepsilon \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle & [\langle S, \varepsilon \rangle \rightarrow \langle A, \varepsilon \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle] \\
 & \Rightarrow \langle a, A \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle & [\langle A, \varepsilon \rangle \rightarrow \langle a, A \rangle] \\
 & \Rightarrow \langle a, A \rangle \langle b, B \rangle \langle C, \varepsilon \rangle & [\langle B, \varepsilon \rangle \rightarrow \langle b, B \rangle] \\
 & \Rightarrow \langle a, A \rangle \langle b, B \rangle \langle c, C \rangle & [\langle C, \varepsilon \rangle \rightarrow \langle c, C \rangle] \\
 & \Rightarrow \langle a, \varepsilon \rangle \langle b, B \rangle \langle c, C \rangle & [\langle a, A \rangle \rightarrow \langle a, \varepsilon \rangle] \\
 & \Rightarrow \langle a, \varepsilon \rangle \langle b, \varepsilon \rangle \langle c, C \rangle & [\langle b, B \rangle \rightarrow \langle b, \varepsilon \rangle] \\
 & \Rightarrow \langle a, \varepsilon \rangle \langle b, \varepsilon \rangle \langle c, \varepsilon \rangle & [\langle c, C \rangle \rightarrow \langle c, \varepsilon \rangle] \\
 & \Rightarrow a \langle b, \varepsilon \rangle \langle c, \varepsilon \rangle & [\langle a, \varepsilon \rangle \rightarrow a] \\
 & \Rightarrow ab \langle c, \varepsilon \rangle & [\langle b, \varepsilon \rangle \rightarrow b]
 \end{array}$$

Example

Consider the regular-controlled grammar H :

$$1: S \rightarrow ABC$$

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$$\begin{array}{ll}
 \langle S, \varepsilon \rangle & \Rightarrow \langle A, \varepsilon \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle & [\langle S, \varepsilon \rangle \rightarrow \langle A, \varepsilon \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle] \\
 & \Rightarrow \langle a, A \rangle \langle B, \varepsilon \rangle \langle C, \varepsilon \rangle & [\langle A, \varepsilon \rangle \rightarrow \langle a, A \rangle] \\
 & \Rightarrow \langle a, A \rangle \langle b, B \rangle \langle C, \varepsilon \rangle & [\langle B, \varepsilon \rangle \rightarrow \langle b, B \rangle] \\
 & \Rightarrow \langle a, A \rangle \langle b, B \rangle \langle c, C \rangle & [\langle C, \varepsilon \rangle \rightarrow \langle c, C \rangle] \\
 & \Rightarrow \langle a, \varepsilon \rangle \langle b, B \rangle \langle c, C \rangle & [\langle a, A \rangle \rightarrow \langle a, \varepsilon \rangle] \\
 & \Rightarrow \langle a, \varepsilon \rangle \langle b, \varepsilon \rangle \langle c, C \rangle & [\langle b, B \rangle \rightarrow \langle b, \varepsilon \rangle] \\
 & \Rightarrow \langle a, \varepsilon \rangle \langle b, \varepsilon \rangle \langle c, \varepsilon \rangle & [\langle c, C \rangle \rightarrow \langle c, \varepsilon \rangle] \\
 & \Rightarrow a \langle b, \varepsilon \rangle \langle c, \varepsilon \rangle & [\langle a, \varepsilon \rangle \rightarrow a] \\
 & \Rightarrow ab \langle c, \varepsilon \rangle & [\langle b, \varepsilon \rangle \rightarrow b] \\
 & \Rightarrow abc & [\langle c, \varepsilon \rangle \rightarrow c]
 \end{array}$$



- Regular-controlled grammars with *appearance checking*.
- Workspace theorems for other regulated grammars.
- Does the new workspace theorem hold for the classical condition as well?



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Discussion