Self-Regulating Finite Automata

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Outline

Outline

n-Parallel Right-Linear Grammar (n-PRLG)
Definition

Definition

An n-PRLG, $n > 0$, is an $(n+3)$-tuple

$$G = (N_1, \ldots, N_n, T, S, P),$$

where

- N_i are mutually disjoint nonterminal alphabets, $1 \leq i \leq n$,
- T is a terminal alphabet,
- $S \notin N_1 \cup \cdots \cup N_n$,
- P contains three kinds of rules:
    1. $S \rightarrow X_1 \ldots X_n$ $X_i \in N_i$, $1 \leq i \leq n$,
    2. $X \rightarrow wY$ $X, Y \in N_i$ for some i, $1 \leq i \leq n$, $w \in T^*$, and
    3. $X \rightarrow w$ $X \in N$, $w \in T^*$.

n-Parallel Right-Linear Grammar (n-PRLG)
Derivation Step

For $x, y \in (N \cup T \cup \{S\})^*$,

$$x \Rightarrow y$$

if and only if

1. either $x = S$ and $S \rightarrow y \in P$,

2. $x = y_1 X_1 y_2 X_2 \ldots y_n X_n$
    $\qquad \downarrow \qquad \downarrow \ldots \quad \downarrow$
   $y = y_1 x_1 y_2 x_2 \ldots y_n x_n$
   $y_i \in T^*, x_i \in T^* N \cup T^*, X_i \in N_i, X_i \rightarrow x_i \in P, 1 \leq i \leq n.$

$\mathcal{L}(G) = \{w \in T^* : S \Rightarrow^+ w\}, \Rightarrow^+$ defined as usual.

$R_n = \{\mathcal{L}(G) : G$ is an n-PRLG$\}.$

n-Parallel Right-Linear Grammar (n-PRLG)
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$y = y_1 x_1 y_2 x_2 \ldots y_n x_n$

   $y_i \in T^*$, $x_i \in T^* N \cup T^*$, $X_i \in N_i$, $X_i \rightarrow x_i \in P$, $1 \leq i \leq n$.

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$y = y_1 x_1 y_2 x_2 \ldots y_n x_n$

   $y_i \in T^*, x_i \in T^* N \cup T^*, X_i \in N_i, X_i \rightarrow x_i \in P, 1 \le i \le n.$

$\mathcal{L}(G) = \{w \in T^* : S \Rightarrow^+ w\}$, $\Rightarrow^+$ defined as usual.

$R_n = \{\mathcal{L}(G) : G \text{ is an } n\text{-PRLG}\}$.

n-PRLG
Example

Example

Let $G = (\{A\}, \{B\}, \{a, b\}, S, P)$ be a 2-PRLG, where P containes rules

- $S \rightarrow AB$
- $A \rightarrow aA$
- $A \rightarrow a$
- $B \rightarrow bB$
- $B \rightarrow b$

Consider a derivation in *G*:

$$S$$

n-PRLG
Example

Example

Let $G = (\{A\}, \{B\}, \{a, b\}, S, P)$ be a 2-PRLG, where P containes rules

- $S \rightarrow AB$
- *$A \rightarrow aA$*
- $A \rightarrow a$
- *$B \rightarrow bB$*
- $B \rightarrow b$

Consider a derivation in G:

$$S \Rightarrow AB$$

n-PRLG
Example

Example

Let $G = (\{A\}, \{B\}, \{a, b\}, S, P)$ be a 2-PRLG, where P containes rules

- $S \rightarrow AB$
- $A \rightarrow aA$
- $A \rightarrow a$
- $B \rightarrow bB$
- $B \rightarrow b$

Consider a derivation in G:

$$S \Rightarrow AB \Rightarrow aAbB$$

n-PRLG
Example

Example

Let $G = (\{A\}, \{B\}, \{a, b\}, S, P)$ be a 2-PRLG, where P containes rules

- $S \rightarrow AB$
- $A \rightarrow aA$
- $A \rightarrow a$
- $B \rightarrow bB$
- $B \rightarrow b$

Consider a derivation in G:

$$S \Rightarrow AB \Rightarrow aAbB \Rightarrow aaAbbB$$

n-PRLG
Example

Example

Let $G = (\{A\}, \{B\}, \{a, b\}, S, P)$ be a 2-PRLG, where P containes rules

- $S \to AB$
- $A \to aA$
- $A \to a$
- $B \to bB$
- $B \to b$

Consider a derivation in G:

$$S \Rightarrow AB \Rightarrow aAbB \Rightarrow aaAbbB \Rightarrow a^3 b^3$$

n-PRLG
Example

Example

Let $G = (\{A\}, \{B\}, \{a, b\}, S, P)$ be a 2-PRLG, where P containes rules

- $S \rightarrow AB$
- $A \rightarrow aA$
- $A \rightarrow a$
- $B \rightarrow bB$
- $B \rightarrow b$

Consider a derivation in *G*:

$$\mathcal{L}(G) = \{a^n b^n : n \geq 1\}$$

Outline

Finite Automaton
Definition

Definition

A finite automaton is a 5-tuple, $M = (Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite set of states,
- Σ is a finite input alphabet,
- δ is a finite set of rules,
- $q_0 \in Q$ is an initial state,
- $F \subseteq Q$ is a set of final states.

Let $\psi : \delta \to \Psi$ be a bijection (Ψ is a set of rule labels).

$$r.qw \to p \text{ means } \psi(qw \to p) = r$$

$$qwy \Rightarrow py \, [r] \text{ if } qwy \in Q\Sigma^*, \, r.qw \to p \in \delta$$

$$\mathcal{L}(M) = \{w \in \Sigma^* : q_0 w \Rightarrow^* f, f \in F\}$$

n-turn First-Move Self-Regulating Finite Automaton
Definition

Definition

An n-first-SFA, $n \geq 0$, M, is a 7-tuple

$$M = (Q, \Sigma, \delta, q_0, q_t, F, R),$$

where

- $(Q, \Sigma, \delta, q_0, F)$ is a finite automaton,
- $q_t \in Q$ is a turn state,
- $R \subseteq \Psi \times \Psi$ is a finite relation on the alphabet of rule labels.

n-turn First-Move Self-Regulating Finite Automaton
Definition

Definition

M accepts *w* if there is $q_0 w \Rightarrow^* f[\mu]$, $f \in F$, such that

$$\mu = \underbrace{r_1^0 \dots r_k^0}_{k \text{ rules}} \underbrace{r_1^1 \dots r_k^1}_{k \text{ rules}} \dots \underbrace{r_1^n \dots r_k^n}_{k \text{ rules}},$$

where $k \in \mathbb{N}$, r_k^0 is the first rule of the form $qx \to q_t$, for some $q \in Q$, $x \in \Sigma^*$, and

$$(r_1^j, r_1^{j+1}) \in R$$

for all $0 \le j < n$.

$$\mathcal{L}(M) = \{w \in \Sigma^* : q_0 w \Rightarrow^* f, f \in F\}.$$

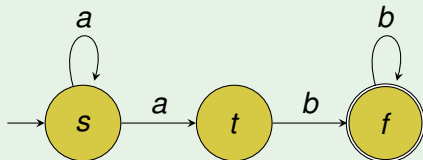The family of languages accepted by *n*-first-SFAs is denoted W_n.

Example

Consider a 1-first-SFA

$$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 3)\})$$
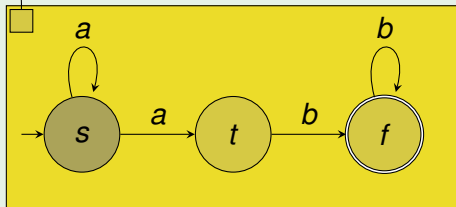
with δ containing rules

- 1. $sa \to s$
- 2. $sa \to t$
- 3. $tb \to f$
- 4. $fb \to f$

Example

$M = (\{s, t, f\}, \{a, b\}, \{1, 2, 3, 4\}, s, t, \{f\}, \{(1, 3)\})$

saabb

Example
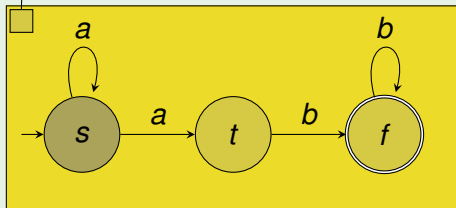
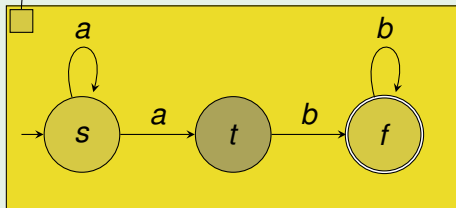$M = (\{s, t, f\}, \{a, b\}, \{1, 2, 3, 4\}, s, t, \{f\}, \{(1, 3)\})$

$saabb \Rightarrow sabb$ [1]

Example

$M = (\{s, t, f\}, \{a, b\}, \{1, 2, 3, 4\}, s, t, \{f\}, \{(1, 3)\})$
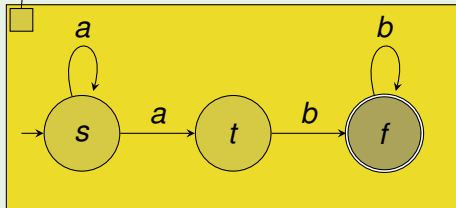

$saabb \Rightarrow sabb \,[1] \Rightarrow tbb \,[2]$

Example

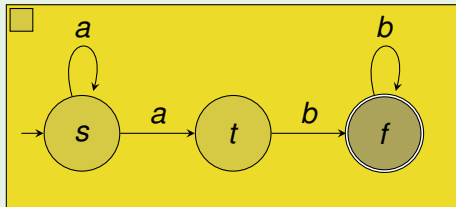$M = (\{s, t, f\}, \{a, b\}, \{1, 2, 3, 4\}, s, t, \{f\}, \{(1, 3)\})$

$saabb \Rightarrow sabb \,[1] \Rightarrow tbb \,[2] \Rightarrow fb \,[3]$

Example

$M = (\{s, t, f\}, \{a, b\}, \{1, 2, 3, 4\}, s, t, \{f\}, \{(1, 3)\})$

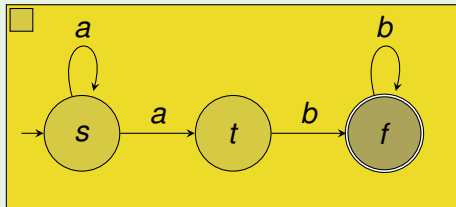| a	a	b	b

First rule after turn:

$saabb \Rightarrow sabb$ [1] $\Rightarrow tbb$ [2] $\Rightarrow fb$ [3] $\Rightarrow f$ [4]

Example

$M = (\{s, t, f\}, \{a, b\}, \{1, 2, 3, 4\}, s, t, \{f\}, \{(1, 3)\})$

| a | a | b | b | First rule after turn:

$saabb \Rightarrow sabb\ [1] \Rightarrow tbb\ [2] \Rightarrow fb\ [3] \Rightarrow f\ [4]$

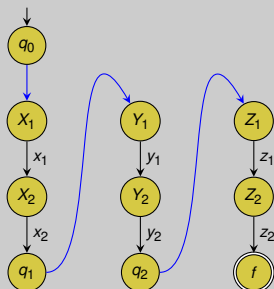$\mathcal{L}(M) = \{a^n b^n : n \geq 1\} \in CF - REG.$

Outline

Lemma

Let G be a 3-PRLG. There is a 2-first-SFA M such that $\mathcal{L}(G) = \mathcal{L}(M)$.

Proof idea.

$$S$$
$$\Downarrow$$
$$X_1 Y_1 Z_1$$
$$\Downarrow$$
$$x_1 X_2 y_1 Y_2 z_1 Z_2$$
$$\Downarrow$$
$$x_1 x_2 y_1 y_2 z_1 z_2$$

Problem

Grammar

$S \rightarrow ABC \mid DBE \in P$ and
$S \rightarrow ABE \notin P$. Then M has
transitions

- $1.q_0 \rightarrow A$
- $2.q_1 \rightarrow B$
- $3.q_2 \rightarrow C$
- $4.q_0 \rightarrow D$
- $5.q_2 \rightarrow E$

and the relation
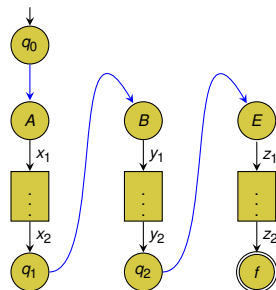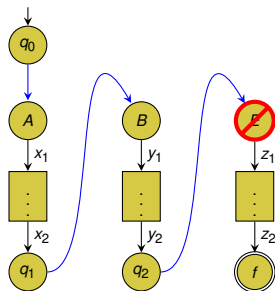$R = \{(1,2),(2,3),(4,2),(2,5),\dots\}$

Problem

Grammar

$S \rightarrow ABC \mid DBE \in P$ and
$S \rightarrow ABE \notin P$. Then M has
transitions

- 1.$q_0 \rightarrow A$
- 2.$q_1 \rightarrow B$
- 3.$q_2 \rightarrow C$
- 4.$q_0 \rightarrow D$
- 5.$q_2 \rightarrow E$

and the relation
$R = \{(1,2), (2,3), (4,2), (2,5), \dots\}$

Problem – solution

Lemma

Let G be an n-PRLG. There is an equivalent n-PRLG G′ such that

1. *if $S \to X_1 \ldots X_n$, then X_i does not occur on the right-hand side of any rule, $1 \le i \le n$;*

2. *if $S \to \alpha$, $S \to \beta$ and $\alpha \ne \beta$, then $alph(\alpha) \cap alph(\beta) = \emptyset$.*

Proof.

$S \to ABC, S \to DBE \in P$ and $alph(ABC) \cap alph(DBE) = \{B\}$.
Replace them by

$$S \to A'B'C', S \to D''B''E'',$$

$$A' \to A, B' \to B, C' \to C, D'' \to D, B'' \to B, E'' \to E.$$
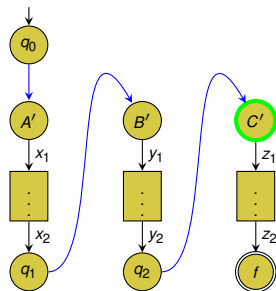
No Problem

Grammar

$S \rightarrow A'B'C' \mid D''B''E'' \in P$. Then
M has transitions

- $1. q_0 \rightarrow A'$
- $2. q_1 \rightarrow B'$
- $3. q_2 \rightarrow C'$
- $4. q_0 \rightarrow D''$
- $5. q_1 \rightarrow B''$
- $6. q_2 \rightarrow E''$

and the relation
$R = \{(1,2),(2,3),(4,5),(5,6),\dots\}$

Lemma

Let M be an 2-*first-SFA. There is an* 3-*PRLG G s. t.* $\mathcal{L}(G) = \mathcal{L}(M)$.

Proof idea.

Let $\mu = (q_0 x_0 \rightarrow q_1), (q_1 x_1 \rightarrow q_2), (q_2 x_2 \rightarrow q_t),$
$(q_t y_0 \rightarrow r_1), (r_1 y_1 \rightarrow r_2), (r_2 y_2 \rightarrow q_i),$
$(q_i z_0 \rightarrow s_1), (s_1 z_1 \rightarrow s_2), (s_2 z_2 \rightarrow q_f).$

be an acceptance of $x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2 z_2$ in *M*. Then,

$$S \Rightarrow [q_0 x_0, q_1, 0, q_t][q_t y_0, r_1, 1, q_i][q_i z_0, s_1, 2, q_f]$$
$$\Rightarrow x_0[q_1, 0, q_t] y_0[r_1, 1, q_i] z_0[s_1, 2, q_f]$$
$$\Rightarrow x_0 x_1[q_2, 0, q_t] y_0 y_1[r_2, 1, q_i] z_0 z_1[s_2, 2, q_{i_2}]$$
$$\Rightarrow x_0 x_1 x_2[q_t, 0, q_t] y_0 y_1 y_2[q_i, 1, q_i] z_0 z_1 z_2[q_f, 2, q_f]$$
$$\Rightarrow x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2$$

Lemma

Let M be an 2-first-SFA. There is an 3-PRLG G s. t. $\mathcal{L}(G) = \mathcal{L}(M)$.

Proof idea.

Let $\mu = (q_0 x_0 \rightarrow q_1), (q_1 x_1 \rightarrow q_2), (q_2 x_2 \rightarrow q_t),$
$\qquad (q_t y_0 \rightarrow r_1), (r_1 y_1 \rightarrow r_2), (r_2 y_2 \rightarrow q_i),$
$\qquad (q_i z_0 \rightarrow s_1), (s_1 z_1 \rightarrow s_2), (s_2 z_2 \rightarrow q_f).$

be an acceptance of $x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2 z_2$ in M. Then,

$$S \Rightarrow [q_0 x_0, q_1, 0, q_t][q_t y_0, r_1, 1, q_i][q_i z_0, s_1, 2, q_f]$$
$$\Rightarrow x_0[q_1, 0, q_t]y_0[r_1, 1, q_i]z_0[s_1, 2, q_f]$$
$$\Rightarrow x_0 x_1[q_2, 0, q_t]y_0 y_1[r_2, 1, q_i]z_0 z_1[s_2, 2, q_{i_2}]$$
$$\Rightarrow x_0 x_1 x_2[q_t, 0, q_t]y_0 y_1 y_2[q_i, 1, q_i]z_0 z_1 z_2[q_f, 2, q_f]$$
$$\Rightarrow x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2$$

Lemma

Let M be an 2*-first-SFA. There is an* 3*-PRLG G s. t.* $\mathcal{L}(G) = \mathcal{L}(M)$.

Proof idea.

Let $\mu = (q_0 x_0 \rightarrow q_1), (q_1 x_1 \rightarrow q_2), (q_2 x_2 \rightarrow q_t),$
$ (q_t y_0 \rightarrow r_1), (r_1 y_1 \rightarrow r_2), (r_2 y_2 \rightarrow q_i),$
$ (q_i z_0 \rightarrow s_1), (s_1 z_1 \rightarrow s_2), (s_2 z_2 \rightarrow q_f).$

be an acceptance of $x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2 z_2$ in M. Then,

$$
\begin{aligned}
S &\Rightarrow [q_0 x_0, q_1, 0, q_t][q_t y_0, r_1, 1, q_i][q_i z_0, s_1, 2, q_f] \\
&\Rightarrow x_0 [q_1, 0, q_t] y_0 [r_1, 1, q_i] z_0 [s_1, 2, q_f] \\
&\Rightarrow x_0 x_1 [q_2, 0, q_t] y_0 y_1 [r_2, 1, q_i] z_0 z_1 [s_2, 2, q_{i_2}] \\
&\Rightarrow x_0 x_1 x_2 [q_t, 0, q_t] y_0 y_1 y_2 [q_i, 1, q_i] z_0 z_1 z_2 [q_f, 2, q_f] \\
&\Rightarrow x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2
\end{aligned}
$$

Lemma

Let M be an 2*-first-SFA. There is an* 3*-PRLG G s. t.* $\mathcal{L}(G) = \mathcal{L}(M)$.

Proof idea.

Let $\mu = (q_0 x_0 \rightarrow q_1), (q_1 x_1 \rightarrow q_2), (q_2 x_2 \rightarrow q_t),$
$(q_t y_0 \rightarrow r_1), (r_1 y_1 \rightarrow r_2), (r_2 y_2 \rightarrow q_i),$
$(q_i z_0 \rightarrow s_1), (s_1 z_1 \rightarrow s_2), (s_2 z_2 \rightarrow q_f).$

be an acceptance of $x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2 z_2$ in *M*. Then,

$$S \Rightarrow [q_0 x_0, q_1, 0, q_t][q_t y_0, r_1, 1, q_i][q_i z_0, s_1, 2, q_f]$$
$$\Rightarrow x_0 [q_1, 0, q_t] y_0 [r_1, 1, q_i] z_0 [s_1, 2, q_f]$$
$$\Rightarrow x_0 x_1 [q_2, 0, q_t] y_0 y_1 [r_2, 1, q_i] z_0 z_1 [s_2, 2, q_{i_2}]$$
$$\Rightarrow x_0 x_1 x_2 [q_t, 0, q_t] y_0 y_1 y_2 [q_i, 1, q_i] z_0 z_1 z_2 [q_f, 2, q_f]$$
$$\Rightarrow x_0 x_1 x_2 y_0 y_1 y_2 z_0 z_1 z_2$$

PRLG vs. first-SFA
Equivalence

Lemma

Let G be an n-PRLG. There is an $(n-1)$-first-SFA M such that $\mathcal{L}(G) = \mathcal{L}(M)$.

Lemma

Let M be an n-first-SFA. There is an $(n+1)$-PRLG G such that $\mathcal{L}(G) = \mathcal{L}(M)$.

Theorem

For all $n \geq 0$, $W_n = R_{n+1}$.

Corollary

1. $REG = W_0 \subset W_1 \subset W_2 \subset \cdots \subset CS$.

2. $W_1 \subset CF$.

3. $W_2 \not\subseteq CF$.

4. $CF \not\subseteq W_n$ for any $n \geq 0$.

5. For all $n \geq 0$, W_n is closed under union, finite substitution, homomorphism, intersection with a regular language and right quotient with a regular language.

6. For all $n \geq 1$, W_n is not closed under intersection, complement and *inverse homomorphism*.

Outline

1. PRLG vs. first-SFA
   - Parallel Right-Linear Grammar
   - First-Move Self-Regulating Finite Automaton
   - Results

2. RLSMG vs. all-SFA
   - Right-Linear Simple Matrix Grammar
   - All-Move Self-Regulating Finite Automaton
   - Results

3. Conclusion and Discussion
   - Comparison
   - all-SPDA

n-Right-Linear Simple Matrix Grammar (n-RLSMG)
Definition

Definition

An n-RLSMG, is an $(n+3)$-tuple

$$G = (N_1, \ldots, N_n, T, S, P),$$

where

- N_i are mutually disjoint nonterminal alphabets, $1 \leq i \leq n$,
- T is a terminal alphabet,
- $S \notin N_1 \cup \cdots \cup N_n$,
- P contains three kinds of matrix rules:
  1. $[S \rightarrow X_1 \ldots X_n]$ $X_i \in N_i$, $1 \leq i \leq n$,
  2. $[X_1 \rightarrow w_1 Y_1, \ldots, X_n \rightarrow w_n Y_n]$ $w_i \in T^*$, $X_i, Y_i \in N_i$, $1 \leq i \leq n$,
  3. $[X_1 \rightarrow w_1, \ldots, X_n \rightarrow w_n]$ $X_i \in N_i$, $w_i \in T^*$, $1 \leq i \leq n$.

n-Right-Linear Simple Matrix Grammar (*n*-RLSMG)
Derivation Step

For $x, y \in (N \cup T \cup \{S\})^*$,

$$x \Rightarrow y$$

if and only if

1. either $x = S$ and $[S \rightarrow y] \in P$,

2. $x = y_1 X_1 y_2 X_2 \ldots y_n X_n$
   $\quad\quad \downarrow \quad\quad \downarrow \ldots \quad\quad \downarrow$
   $y = y_1 x_1 y_2 x_2 \ldots y_n x_n$

   $y_i \in T^*, x_i \in T^* N \cup T^*, X_i \in N_i, 1 \leq i \leq n,$
   $[X_1 \rightarrow x_1, \ldots, X_n \rightarrow x_n] \in P.$

$\mathcal{L}(G) = \{w \in T^* : S \Rightarrow^+ w\}$, $\Rightarrow^+$ is defined as usual.

$R_{[n]} = \{\mathcal{L}(G) : G \text{ is an } n\text{-RLSMG}\}.$

n-Right-Linear Simple Matrix Grammar (n-RLSMG)
Derivation Step

For $x, y \in (N \cup T \cup \{S\})^*$,

$$x \Rightarrow y$$

if and only if

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   $$\downarrow \quad \downarrow \ldots \quad \downarrow$$
   $y = y_1 x_1 y_2 x_2 \ldots y_n x_n$

   $y_i \in T^*$, $x_i \in T^* N \cup T^*$, $X_i \in N_i$, $1 \le i \le n$,
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n-Right-Linear Simple Matrix Grammar (n-RLSMG)
Derivation Step

For $x, y \in (N \cup T \cup \{S\})^*$,

$$x \Rightarrow y$$

if and only if

1. either $x = S$ and $[S \rightarrow y] \in P$,

2. $x = y_1 X_1 y_2 X_2 \ldots y_n X_n$
   $$\downarrow \quad \downarrow \ldots \quad \downarrow$$
   $y = y_1 x_1 y_2 x_2 \ldots y_n x_n$

   $y_i \in T^*$, $x_i \in T^* N \cup T^*$, $X_i \in N_i$, $1 \le i \le n$,
   $[X_1 \rightarrow x_1, \ldots, X_n \rightarrow x_n] \in P$.

$\mathcal{L}(G) = \{w \in T^* : S \Rightarrow^+ w\}$, $\Rightarrow^+$ is defined as usual.

$R_{[n]} = \{\mathcal{L}(G) : G$ is an n-RLSMG$\}$.

n-RLSMG
Example

Example

Let $G = (\{A\}, \{B\}, \{a, b\}, S, P)$ be a 2-PRLG, where P containes rules

- $[S \rightarrow AB]$
- $[A \rightarrow aA, B \rightarrow aB]$
- $[A \rightarrow bA, B \rightarrow bB]$
- $[A \rightarrow \varepsilon, B \rightarrow \varepsilon]$

Consider a derivation in G:

S

n-RLSMG
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$$S \Rightarrow AB$$

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Consider a derivation in G:

$$S \Rightarrow AB \Rightarrow aAaB$$

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Consider a derivation in G:

$$\mathcal{L}(G) = \{ww : w \in \{a, b\}^*\}$$

Outline

1. PRLG vs. first-SFA
   - Parallel Right-Linear Grammar
   - First-Move Self-Regulating Finite Automaton
   - Results

2. **RLSMG vs. all-SFA**
   - Right-Linear Simple Matrix Grammar
   - **All-Move Self-Regulating Finite Automaton**
   - Results

3. Conclusion and Discussion
   - Comparison
   - all-SPDA

n-turn All-Move Self-Regulating Finite Automaton
Definition

Definition

An n-all-SFA, $n \geq 0$, M, is a 7-tuple

$$M = (Q, \Sigma, \delta, q_0, q_t, F, R),$$

M is an n-first-SFA, and M accepts w if there is $q_0 w \Rightarrow^* f[\mu]$, $f \in F$, s.t.

$$\mu = \underbrace{r_1^0 \ldots r_k^0}_{k \text{ rules}} \underbrace{r_1^1 \ldots r_k^1}_{k \text{ rules}} \ldots \underbrace{r_1^n \ldots r_k^n}_{k \text{ rules}},$$

and

$$(r_i^j, r_i^{j+1}) \in R$$

for all $1 \leq i \leq k$, $0 \leq j < n$.
The family of languages accepted by n-all-SFAs is denoted S_n.

Example

Consider a 1-all-SFA

$$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$$

with δ containing rules

- 1.$sa \rightarrow s$
- 2.$sb \rightarrow s$
- 3.$s \rightarrow t$

- 4.$ta \rightarrow t$
- 5.$tb \rightarrow t$
- 6.$t \rightarrow f$
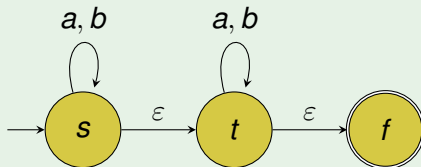
Example

$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$

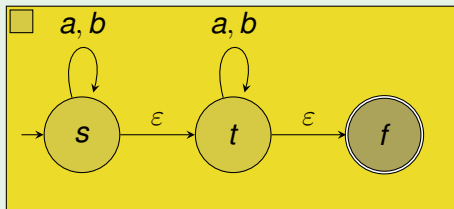| a	b	a	b
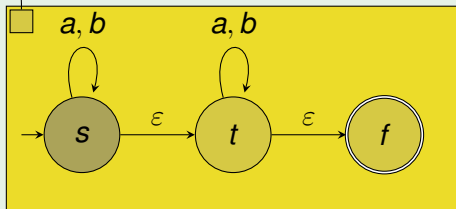
Rules after turn:

sabab

Example

$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$

a	b	a	b
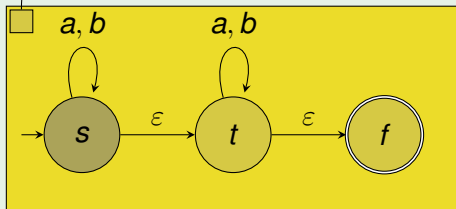
Rules after turn: 4

$sabab \Rightarrow sbab$ [1]

Example

$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$

| a | b | a | b |
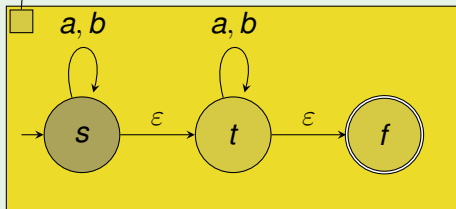
Rules after turn: 4,5

$sabab \Rightarrow sbab \; [1] \Rightarrow sab \; [2]$

Example

$$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$$

| a | b | a | b |

Rules after turn: 4,5,6
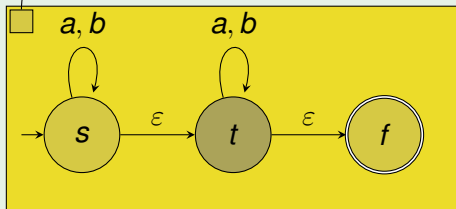
$$sabab \Rightarrow sbab\ [1] \Rightarrow sab\ [2] \Rightarrow tab\ [3]$$

Example

$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$
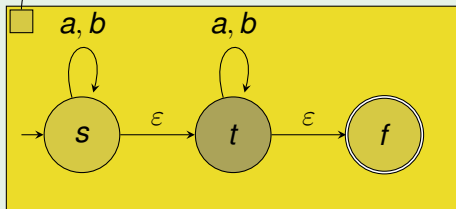
$sabab \Rightarrow sbab \; [1] \Rightarrow sab \; [2] \Rightarrow tab \; [3] \Rightarrow tb \; [4]$

Example

$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1,4), (2,5), (3,6)\})$

| a | b | a | b | Rules after turn: 4,5,6
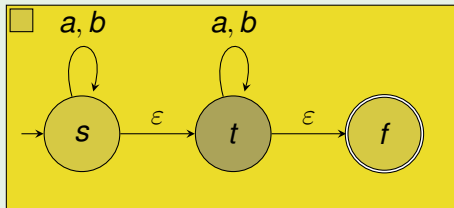
$sabab \Rightarrow sbab \ [1] \Rightarrow sab \ [2] \Rightarrow tab \ [3] \Rightarrow tb \ [4] \Rightarrow t \ [5]$

Example

$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$

| a | b | a | b | Rules after turn: 4,5,6

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Example

$M = (\{s, t, f\}, \{a, b\}, \delta, s, t, \{f\}, \{(1, 4), (2, 5), (3, 6)\})$

| a | b | a | b |

Rules after turn: 4,5,6

a, b   a, b

→ s —ε→ t —ε→ f
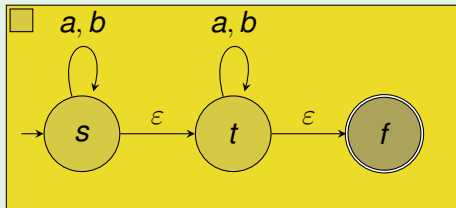
$sabab \Rightarrow sbab$ [1] $\Rightarrow sab$ [2] $\Rightarrow tab$ [3] $\Rightarrow tb$ [4] $\Rightarrow t$ [5] $\Rightarrow f$ [6]

$\mathcal{L}(M) = \{ww : w \in \{a, b\}^*\} \in CS - CF.$

Outline

RLSMG vs. all-SFA
Equivalence

Lemma

Let G be an n-RLSMG. There is an $(n-1)$-all-SFA M such that $\mathcal{L}(G) = \mathcal{L}(M)$.

Lemma

Let M be an n-all-SFA. There is an $(n+1)$-RLSMG G such that $\mathcal{L}(G) = \mathcal{L}(M)$.

Theorem

For all $n \geq 0$, $S_n = R_{[n+1]}$.

Corollary

1. $REG = S_0 \subset S_1 \subset S_2 \subset \cdots \subset CS$.

2. $S_1 \not\subseteq CF$.

3. $CF \not\subseteq S_n$, for $n \geq 0$.

4. For all $n \geq 0$, S_n is closed under union, finite substitution, homomorphism, intersection with a regular language, right quotient with a regular language and *inverse homomorphism*.

5. For all $n \geq 0$, S_n is full trio.

6. For all $n \geq 1$, S_n is not closed under intersection and complement.

Outline

Comparison

Theorem

1. $W_0 = S_0 = REG$.

2. For all $n > 0$, $W_n \subset S_n$.

3. $W_n \not\subseteq S_{n-1}$, $n \geq 1$.

4. $S_n - W \neq \emptyset$, $n \geq 1$, where $W = \bigcup_{m=1}^{\infty} W_m$.

Comparison
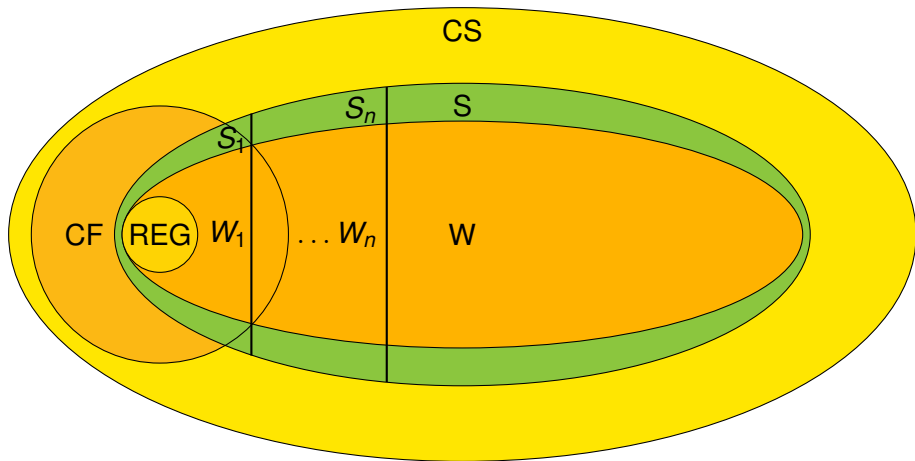
Outline

all-SPDA

Theorem

$\mathcal{L}(0\text{-}all\text{-}SPDA) = CF$.

Theorem

$\mathcal{L}(1\text{-}all\text{-}SPDA) = RE$.

For Further Reading

📚 J. Dassow and G. Paun.
Regulated Rewriting in Formal Language Theory.
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📄 R. D. Rosebrugh and D. Wood.
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Utilitas mathematica, 7:151–186, 1975.

📄 D. Wood.
m-parallel *n*-right linear simple matrix languages.
Utilitas mathematica, 8:3–28, 1975.