FIT VUT Brno

Digital Images and Formal Languages – 2nd part

TID – course

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Digital Images and Formal Languages – 2nd part

- Outline

I. Motivation – our aim
II. Last episode - repetition
III. Gray-scale image - continues
IV. Image transformation and WFA
V. Image compression
VI. Note to color images
VII. Other transformations and WFT
VIII. Conclusion

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- *Motivation:*
  - Human eye can recognize approximately 16.7 millions of colors.
  - Hence we do not usually require to store more information.
  - ***Digital image***
    - Information about image points represented in binary format.
    - We need to know:
      - How to represent image model in efficient way.
      - **How to do operations like zoom, compression, filtering etc.**

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- Last episode - repetition:
  - ***Digital Image representation by raster graphics.***
  - ***Different image formats based on raster***
  - ***The basics of formal image representation:***
    - **Pixel representation, string address**
    - **Set of pixels (strings) is image.**
    - **Black and white image (f: $\Sigma^m \rightarrow \{0, 1\}$)**
    - **How to generate fractals and regular images precisely**
    - **Note to gray scale**

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- *Gray-scale image*
  - *In case of gray-scale images the pixel values are real numbers, so multi-resolution image is then described by function **g**: **Σ** * → **R**, R stands for real numbers.*
  - *For the letter we usually require that the resolution levels are compatible.*
  - *The compatibility is formalized by requiring the **g** is an **average preserving function**.*
  - *Function is average preserving (**ap**) if:*

$$f(w) = \frac{1}{4}[f(w0) + f(w1) + f(w2) + f(w3)]$$

  - *for all w from **Σ** **

Show example.

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- *Grey-scale image & WFA – weighed finite automaton*
  - WFA def:

An m-state *weighted finite automaton* (WFA) A over alphabet Σ is defined by
(i) a row vector $I^A \in I\!R^{1\times m}$ (called the initial distribution),
(ii) a column vector $F^A \in I\!R^{m\times 1}$ (the final distribution), and
(iii) weight matrices $W_a^A \in I\!R^{m\times m}$ for all $a \in \Sigma$.

  - WFA ***A*** defines a multi-resolution function f_A over **Σ** by

$$f_A(a_1 a_2 \ldots a_k) = I^A \cdot W_{a_1}^A \cdot W_{a_2}^A \cdot \ldots \cdot W_{a_k}^A \cdot F^A.$$

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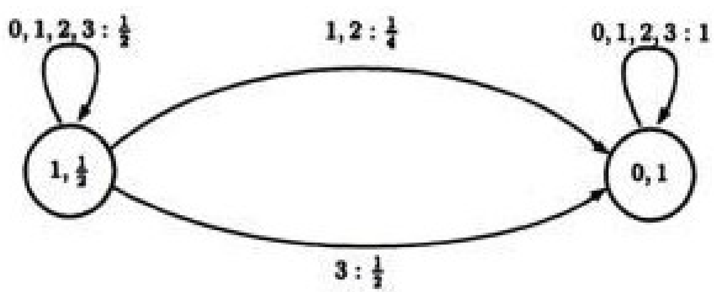
- *Grey-scale image & WFA – example*
  - Consider **WFA** *A* over Σ = {0, 1, 2, 3}, the initial distribution *I* = (0, 1), the final distribution *F* = (1/2, 1) and weight matrices:

$$W_0 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}, W_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & 1 \end{pmatrix}, W_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & 1 \end{pmatrix} \text{ and } W_3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$$

  - Then the WFA we can show as diagram:



Show example.

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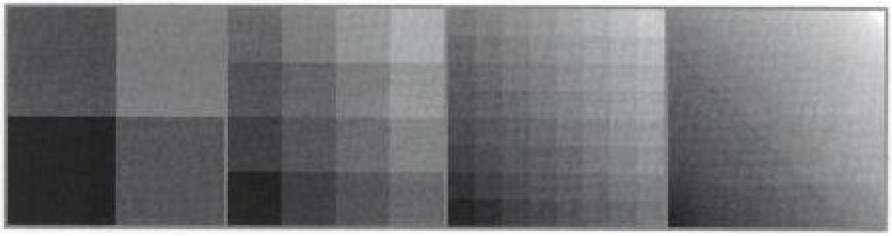
- *Grey-scale image & WFA – example continues*
  - The image corresponding to ***A*** for resolutions 2 x 2, 4 x 4, 8 x 8 and 256 x256 is here (note $\boldsymbol{f}_{\mathbf{A}}$ is linear grayness function):



  - WFA is called *average preserving* (**ap**) WFA if:

$$\sum_{a \in \Sigma} W_a^A \cdot F^A = p \cdot F^A,$$

  - where $p = |\Sigma|$ is cardinality of Σ. In other words a WFA is **ap** if *F* is eigenvector corresponding to eigenvalue *p.*

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- ***Grey-scale image & WFA – practical aspects.***
  - For given image we need to construct image description (ap-WFA), it is **encoder** (see efficient algorithm [1]).
  - From given image description (ap-WFA) we need to reconstruct original image, it is called **decoder** (see efficient algorithm [1]).
  - By this way the **compression** of image is obtained (see [2]).

  - Note that **det. ap-WFA** is weaker than **nondet. ap-WFA**. Hence det. ap-WFA generates only countable unions of fractals or contrast level greyness functions, but not smoothly growing grayness functions (used in photos). See example of linear greyness function on previous slide.

  - **Image operations** (transformations) can be obtained by suitable WFA matrices transformation.

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- ***Grey-scale image & WFA – practical aspects continue.***
  - Operation **ZOOMING**: for an arbitrary multi-resolution image ***f*** over Σ and string ***u*** (*u* is from Σ^*), let $\boldsymbol{f_u}$ denote the multi-resolution image:

$$f_u(w) = f(uw), \text{ for every } w \in \Sigma^*.$$

  - Where $\boldsymbol{f_u}$ is the image obtained from image ***f*** by zooming to sub-square with address ***u***. It means: to get $\boldsymbol{f_u}$ we need to change initial distribution ***I*** (from *f*) to $\boldsymbol{I_u} = \boldsymbol{I}\,\boldsymbol{W}_{a1} \ldots \boldsymbol{W}_{ak}$, where $\boldsymbol{u} = \boldsymbol{a}_1, \ldots \boldsymbol{a}_k$.
- ***From grey-scale to color (we like colors).***

  Similar to greyness images. If we consider YIQ color model (intensity, hue and saturation) then we need WFA with three initial distributions for each model component.

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- ***Images & WFT – weighed finite transducers***
  - WFT def:

Consider alphabet $\Sigma = \{0,1,2,3\}$. Analogously to WFA, an n-state *weighted finite transducer* (WFT) M is specified by
(i) weight matrices $W_{a,b} \in \mathbb{R}^{n\times n}$ for all $a \in \Sigma \cup \{\varepsilon\}$ and $b \in \Sigma \cup \{\varepsilon\}$,
(ii) a row vector $I \in \mathbb{R}^{1\times n}$, called the initial distribution, and
(iii) a column vector $F \in \mathbb{R}^{n\times 1}$, called the final distribution.
The WFT M is called ε-free if weight matrices $W_{\varepsilon,\varepsilon}$, $W_{a,\varepsilon}$ and $W_{\varepsilon,b}$ are zero matrices for all $a \in \Sigma$ and $b \in \Sigma$.

  - Note ***WFT ≠ WTF***

The WFT M defines function $f_M : \Sigma^* \times \Sigma^* \longrightarrow \mathbb{R}$, called weighted relation, by $f_M(u,v) = I \cdot W_{u,v} \cdot F$, for all $u \in \Sigma^*, v \in \Sigma^*$, where

$$W_{u,v} = \sum_{\substack{a_1 \dots a_k = u \\ b_1 \dots b_k = v}} W_{a_1,b_1} \cdot W_{a_2,b_2} \cdot \ldots \cdot W_{a_k,b_k}, \qquad (4)$$

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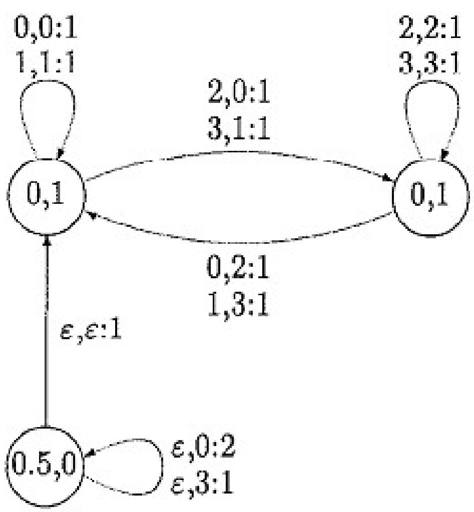
- ***Images & WFT – practical aspects.***
  - WFT are necessary to represent **any transformations** which involves **moving** (scaled) **values between pixels**.
  - Practical example: the composition of affine transformation Squeeze and fractal copying.



(a) WFT α

(b) Image α(Carol)

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Digital Images and Formal Languages – conclusion

- All images of regular character and fractals can be perfectly (with infinite precision) described by regular expression (FA) as was shown at 1st part.
- All grey-scale and color images can be represented by nondet. ap-WFA, this leads to image compression and we are also able to handle some image operations like zooming.
- All image operations (transformations) which involve moving greyness values between pixels need to be handled by WFT representation.
- ***That is all guys.***

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References

- **[1]** Culik, Kari: Image compression using WFA, Computer and Graphics 17.
- **[2]** Culik, Kari: Finite state transformations of images, Proceedings of ICALP 95.
- **[3]** Culik, Kari: Efficient inference algorithm for WFA, in Fractal image compression, ed. Y. Fisher, Springer-Verlag.

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1. End of presentation

- **Thank you for your attention.**
- **Any questions?**