

Additional types of L-systems

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Repetition 1/3

- 0L is triple $H = (V, P, w)$, where:
 - V is a finite alphabet of symbols,
 - P is a finite set of rules of the form $a \rightarrow x$, where $a \in V, x \in V^*$,
 - $w \in V^+$ is an axiom.

Repetition 2/3

- Direct derivation (\Rightarrow):
 - Let be $u = a_1 \dots a_n$, $v = x_1 \dots x_n \in V^*$.
 - We say that v is directly derived from u if and only if $a_i \rightarrow x_i \in P$, $i \in \{1, \dots, n\}$ and we write $u \Rightarrow v$.
 - \Rightarrow^* is transitive and reflexive closure of \Rightarrow

Repetition 3/3

- D0L - **D**eterministic, only one rule $a \rightarrow x$ for each $a \in V$.
- P0L - **P**roduction, for every rule $a \rightarrow x$, $x \neq \varepsilon$.
- E0L - **E**xtended, is $H = (V, T, P, w)$, where:
 - V, P, w is same as in an 0L,
 - T is finite alphabet, $T \subseteq V$.
- T0L - **T**ables, is $H = (V, P_i, w)$ and for every $i \in \{1, \dots, n\}$, $H_i = (V_i, P_i, w)$ is an 0L system.

Stochastic 0L-systems 1/3

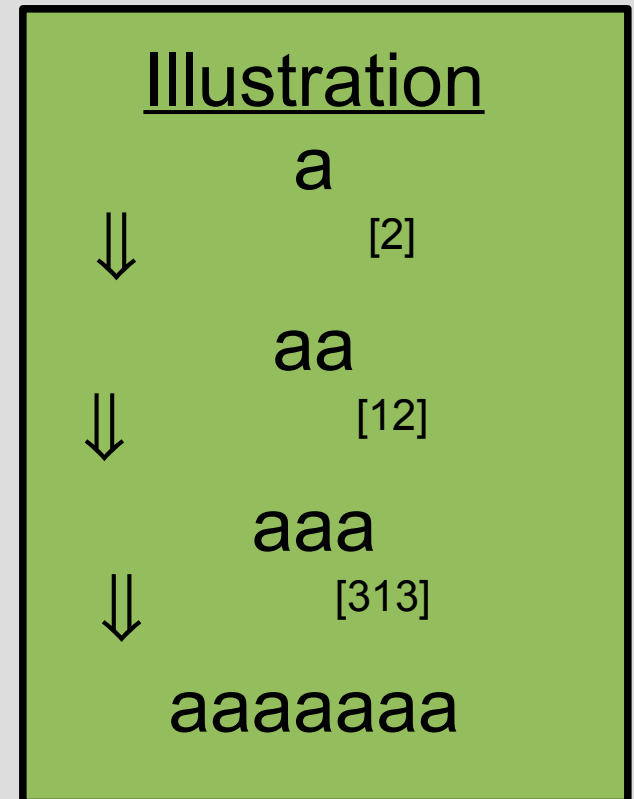
- Stochastic 0L is a quadruplet $H = (V, P, w, \pi)$, where:
 - V, P, w is same as in an 0L,
 - π is called *probability distribution*, and it is a function $\pi: P \rightarrow (0, 1]$,
 - it is assumed that for every $a \in V$, the sum of probabilities of all productions with the predecessor a is equal to 1.

Stochastic 0L-systems 2/3

- Direct *stochastic* derivation (\Rightarrow):
 - Let be $u, v \in V^*$.
 - We say that v is directly derived from u if for each occurrence of the symbol $a \in V$ in the word u is randomly chosen rule $p \in P$ with predecessor a , the probability of applying production p with the predecessor a is equal to $\pi(p)$. Thus, in one stochastic derivation step, different rules can be applied for different occurrence of symbol a .
 - \Rightarrow^* is transitive and reflexive closure of \Rightarrow

Stochastic 0L-systems 3/3

- Example:
 - Let H be the stochastic 0L:
 - $V = \{a\}$,
 - $P = \{ \begin{array}{l} 1: a \rightarrow a, \\ 2: a \rightarrow aa, \\ 3: a \rightarrow aaa \end{array} \}$,
 - $w = a$,
 - $\pi(1) = 0.33$,
 - $\pi(2) = 0.33$,
 - $\pi(3) = 0.34$.
 - $L(H) = \{a^n, n \geq 1\}$



Parametric 0L-system 1/3

- Parametric 0L is a quadruplet $H = (V, \Sigma, P, w)$, where:
 - V is a finite alphabet of symbols
 - Σ is a finite set of formal parameters
 - P is a finite set of rules of the form $a(p):c \rightarrow x$, where $a(p) \in (V \times \Sigma^*)$, $c \in C(\Sigma)$, $x \in (V \times E(\Sigma))^*$
 - $w \in (V \times R^*)^+$ is an axiom

- C means all correct conditions over Σ .
- E means all correct expressions over Σ .
- Member of $(V \times \Sigma^*)$ is called *module*.

Parametric 0L-system 2/3

- Direct derivation (\Rightarrow):
 - Let u be a word of modules $u = a_1 \dots a_n$, we say that $v = x_1 \dots x_n$ is direct derived from u if and only if there exists a sequence of productions $p_i: a_i \rightarrow x_i \in P, i \in \{1, \dots, n\}$ and we write $u \Rightarrow v$.
 - \Rightarrow^* is transitive and reflexive closure of \Rightarrow

Parametric 0L-system 3/3

- Example:
 - Let H be the parametric 0L-system:
 - $V = \{a, b\}$,
 - $\Sigma = \{i, j, k\}$,
 - $P = \{$
 - 1: $a(k): (k \geq 10) \rightarrow a(10)$,
 - 2: $a(k): (k < 10) \rightarrow b(k, 0)$,
 - 3: $b(i, j): (i = j) \rightarrow a(i + j)$,
 - 4: $b(i, j): (i > j) \rightarrow b(i, j + 1)$,
 - 5: $b(i, j): (i < j) \rightarrow b(i + 1, j)\}$,
 - $w = a(2)b(5, 4)$.

Illustration

$a(2)b(5, 4)$



[24]

$b(2, 0)b(5, 5)$



[43]

$b(2, 1)a(10)$



[41]

$b(2, 2)a(10)$

Context-sensitive L-systems 1/3

- (l,k) -system is triple $H = (V, P, w)$, where:
 - V, w is same as in 0L,
 - P is an ordered finite set of rules of the form $a < b > c \rightarrow x$, where $b \in V$ and $a, c, x \in V^*$
 - l or k denotes length of a left or right context

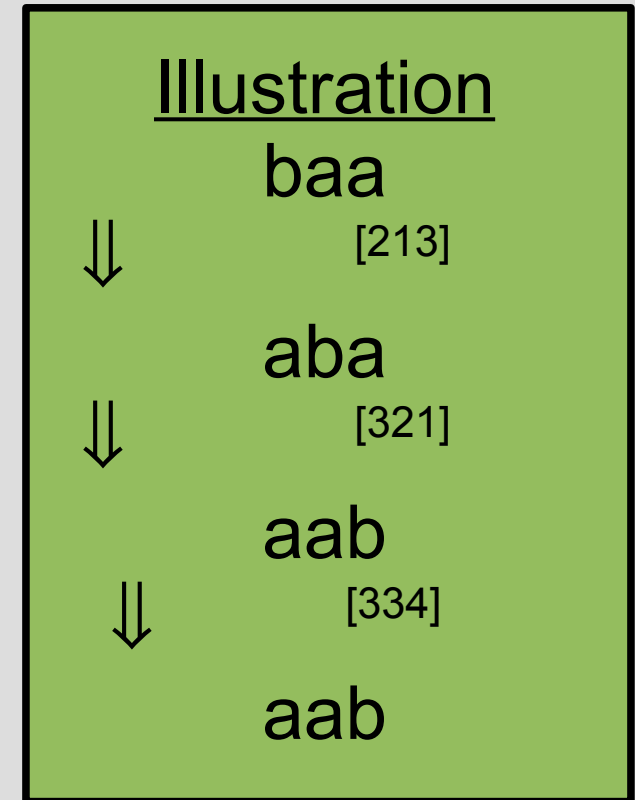
Note: 1L or 2L systems are specific types of (l,k) -systems

Context-sensitive L-systems 2/3

- Direct derivation (\Rightarrow):
 - Let be $u = u_1 \dots u_n$, $v = x_1 \dots x_n \in V^*$.
 - We say that v is directly derived from u if and only if there is a sequence of productions $p_i: a_i < b_i > c_i \rightarrow x_i \in P$, such that p_i is applicable on u_i , $i \in \{1, \dots, n\}$ and we write $u \Rightarrow v$.
 - \Rightarrow^* is transitive and reflexive closure of \Rightarrow

Context-sensitive L-systems 3/3

- Example:
 - Let H be the (k,l) -system:
 - $V = \{a, b\}$,
 - $P = \{ \begin{array}{l} 1: b < a \rightarrow b, \\ 2: b \rightarrow a, \\ 3: a \rightarrow a, \\ 4: b \rightarrow b \end{array} \}$,
 - $w = baa$.
 - $L(H) = \{baa, aba, aab\}$



Conclusion

- Stochastic 0L-systems can be used for simulation real organisms
- Parametric 0L-systems can compute some important/characteristic values during derivation
- Context-sensitive L-systems are more powerful