Additional types of L-systems

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Repetition 1/3

- 0L is triple $H = (V, P, w)$, where:
  - V is a finite alphabet of symbols,
  - P is a finite set of rules of the form $a \rightarrow x$, where $a \in V$, $x \in V^*$,
  - $w \in V^+$ is an axiom.

Repetition 2/3

- Direct derivation (⇒):
  - Let be $u=a_1...a_n$, $v=x_1...x_n \in V^*$.
  - We say that v is directly derived from u if and only if $a_i \rightarrow x_i \in P$, $i \in \{1,...,n\}$ and we write $u \Rightarrow v$.
  - $\Rightarrow^*$ is transitive and reflexive closure of $\Rightarrow$

Repetition 3/3

- D0L – **D**eterministic, only one rule $a \rightarrow x$ for each $a \in V$.
- P0L – **P**roduction, for every rule $a \rightarrow x$, $x \neq \varepsilon$.
- E0L – **E**xtended, is $H = (V, T, P, w)$, where:
  - V, P, w is same as in an 0L,
  - T is finite alphabet, $T \subseteq V$.
- T0L – **T**ables, is $H = (V, P_i, w)$ and for every $i \in \{1,\ldots, n\}$, $H_i = (V_i, P_i, w)$ is an 0L system.

Stochastic 0L-systems 1/3

- Stochastic 0L is a quadruplet $H = (V, P, w, \pi)$, where:
  - V, P, w is same as in an 0L,
  - π is called *probability distribution*, and it is a function $\pi: P \rightarrow (0, 1]$,
  - it is assumed that for every $a \in V$, the sum of probabilities of all productions with the predecessor a is equal to 1.

Stochastic 0L-systems 2/3

- Direct *stochastic* derivation ($\Rightarrow$):
  - Let be $u, v \in V^*$.
  - We say that *v* is directly derived from *u* if for each occurrence of the symbol $a \in V$ in the word *u* is randomly chosen rule $p \in P$ with predecessor *a*, the probability of applying production *p* with the predecessor *a* is equal to $\pi(p)$. Thus, in one stochastic derivation step, different rules can be applied for different occurrence of symbol *a*.
  - $\Rightarrow^*$ is transitive and reflexive closure of $\Rightarrow$

Stochastic 0L-systems 3/3

- Example:
  - Let *H* be the stochastic 0L:
    - $V = \{a\}$,
    - $P = \{$ 1: $a \rightarrow a$,
      2: $a \rightarrow aa$,
      3: $a \rightarrow aaa\}$,
    - $w = a$,
    - $\pi(1) = 0.33$,
      $\pi(2) = 0.33$,
      $\pi(3) = 0.34$.
  - $L(H) = \{a^n, n \geq 1\}$

Illustration

a
⇓ [2]
aa
⇓ [12]
aaa
⇓ [313]
aaaaaaa

Parametric 0L-system 1/3

- Parametric 0L is a quadruplet
  $H = (V, \Sigma, P, w)$, where:
  - V is a finite alphabet of symbols
  - Σ is a finite set of formal parameters
  - P is a finite set of rules of the form $a(p){:}c \rightarrow x$, where $a(p) \in (V \times \Sigma^*)$, $c \in C(\Sigma)$, $x \in (V \times E(\Sigma))^*$
  - $w \in (V \times R^*)^+$ is an axiom

- C means all correct conditions over Σ.
- E means all correct expressions over Σ.
- Member of $(V \times \Sigma^*)$ is called *module*.

Parametric 0L-system 2/3

- Direct derivation ($\Rightarrow$):
  - Let u be a word of modules $u = a_1 ... a_n$, we say that $v = x_1 ... x_n$ is direct derived from u if and only if there exists a sequence of productions $p_i: a_i \to x_i \in P$, $i \in \{1,...,n\}$ and we write $u \Rightarrow v$.
  - $\Rightarrow^*$ is transitive and reflexive closure of $\Rightarrow$

Parametric 0L-system 3/3

- Example:
  - Let *H* be the parametric 0L-system:
    - $V = \{a, b\}$,
    - $\Sigma = \{i, j, k\}$,
    - $P = \{$ 1: $a(k): (k \geq 10) \rightarrow a(10)$,
      2: $a(k): (k < 10) \rightarrow b(k,0)$,
      3: $b(i,j): (i = j) \rightarrow a(i+j)$,
      4: $b(i,j): (i > j) \rightarrow b(i,j+1)$,
      5: $b(i,j): (i < j) \rightarrow b(i+1,j)\}$,
    - $w = a(2)b(5,4)$.

Illustration

a(2)b(5,4)

⇓ [24]

b(2,0)b(5,5)

⇓ [43]

b(2,1)a(10)

⇓ [41]

b(2,2)a(10)

Context-sensitive L-systems 1/3

- (l,k)-system is triple $H = (V, P, w)$, where:
  - V, w is same as in 0L,
  - P is an ordered finite set of rules of the form $a < b > c \rightarrow x$, where $b \in V$ and $a, c, x \in V^*$
  - l or k denotes length of a left or right context

Note: 1L or 2L systems are specific types of (l,k)-systems

Context-sensitive L-systems 2/3

- Direct derivation ($\Rightarrow$):
  - Let be $u=u_1...u_n$, $v=x_1...x_n \in V^*$.
  - We say that v is directly derived from u if and only if there is a sequence of productions p_i: $a_i < b_i > c_i \rightarrow x_i \in P$, such that p_i is applicable on u_i, $i \in \{1,...,n\}$ and we write $u \Rightarrow v$.
  - $\Rightarrow^*$ is transitive and reflexive closure of $\Rightarrow$

Context-sensitive L-systems 3/3

- Example:
  - Let H be the (k,l)-system:
    - $V = \{a, b\}$,
    - $P = \{$ 1: $b < a \rightarrow b$,
      2: $b \rightarrow a$,
      3: $a \rightarrow a$,
      4: $b \rightarrow b\}$,
    - w = baa.
  - $L(H) = \{baa, aba, aab\}$

<u>Illustration</u>

baa

⇓ [213]

aba

⇓ [321]

aab

⇓ [334]

aab

Conclusion

- Stochastic 0L-systems can be used for simulation real organisms
- Parametric 0L-systems can compute some important/characteristic values during derivation
- Context-sensitive L-systems are more powerful