Chomsky Hierarchy

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Modern Formal Language Theory, 2007

Chomsky Hierarchy

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$$\mathscr{L}(REG) \subset \mathscr{L}(LIN) \subset \mathscr{L}(CF) \subset \mathscr{L}(CS) \subset \mathscr{L}(RE) \subset \mathscr{L}(ALL)$$

$\mathscr{L}(REG)$ family of regular languages (type 3)

$\mathscr{L}(LIN)$ family of linear languages

$\mathscr{L}(CF)$ family of context-free languages (type 2)

$\mathscr{L}(CS)$ family of context-sensitive languages (type 1)

$\mathscr{L}(RE)$ family of recursively enumerable languages (type 0)

$\mathscr{L}(ALL)$ family of all languages

Type 0 Grammars

Type 0 Grammar

$$G = (N, T, P, S)$$

N alphabet of nonterminals

T alphabet of terminals

P finite set of productions (rules) of the form

$$y \to x$$

with $y, x \in V^*$, $\text{alph}(y) \cap N \neq \emptyset$

S the start symbol, $S \in N$

- $V = N \cup T$
- $\text{alph}(y)$ is the set of all symbols occurring in $y \in V^*$

Derivation Step

Production Label

Consider a production $p : y \longrightarrow x$

p label

$y \longrightarrow x$ production

Instead of $y \longrightarrow x \in P$, we often write $p \in P$

Derivation Step

For every $u, v \in V^*$ and $p : y \longrightarrow x$,

$$uyv \Rightarrow uxv \; [p]$$

or simply

$$uyv \Rightarrow uxv$$

Notation

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For every $y_0, y_1, \ldots, y_n$, for some $n \geq 1$, such that

$$y_0 \Rightarrow y_1 \, [p_1] \Rightarrow y_2 \, [p_2] \Rightarrow \ \ldots \ \Rightarrow y_n \, [p_n],$$

where $p_i \in P$, for all $i = 1, \ldots, n$,

- $y_0 \Rightarrow^n y_n \, [p_1 \ldots p_n]$ or $y_0 \Rightarrow^n y_n$
- for every $y \in V^*$,
$$y \Rightarrow^0 y \, [\varepsilon] \text{ or } y \Rightarrow^0 y$$
- if $v \Rightarrow^m w \, [\alpha]$ for some $m \geq 1$, then
$$v \Rightarrow^+ w \, [\alpha] \text{ or } v \Rightarrow^+ w$$
- if $v \Rightarrow^m w \, [\alpha]$ for some $m \geq 0$, then
$$v \Rightarrow^* w \, [\alpha] \text{ or } v \Rightarrow^* w$$

Language of Grammar

Relation of Direct Derivation

$\Rightarrow$ direct derivation relation

$\Rightarrow^*$ transitive and reflexive closure of direct derivation relation

$\Rightarrow^+$ transitive closure of direct derivation relation

Language of Grammar

$$L(G) = \{w \in T^* : S \Rightarrow^* w\}$$

CS, *CF*, *LIN* and *REG* Grammars

CS, *CF*, *LIN* and *REG* Grammars

A type-0 grammar $G = (N, T, P, S)$ is

1. context-sensitive if for every $y \longrightarrow x \in P$,

$$|y| \leq |x|, \text{ or } y = S, x = \varepsilon$$

2. context-free if for every $y \longrightarrow x \in P$,

$$y \in N$$

3. linear if for every $y \longrightarrow x \in P$,

$$y \in N, x \in T^* \cup T^*NT^*$$

4. regular if for every $y \longrightarrow x \in P$,

$$y \in N, x \in \{\varepsilon\} \cup T \cup TN$$

Bibliography


N. Chomsky.
Three models for the description of language.
Transactions on Information Theory, 2(3):113–124, 1956.


A. Meduna.
Automata and Languages: Theory and Applications.
Springer, London, 2000.


G. Rozenberg and A. Salomaa.
Handhandbook of Formal Languages, volume 1–3.
Springer, Berlin, 1997.


A. Salomaa.
Formal Languages.
Academic Press, New York, 1973.