Regulated Rewriting – The Hierarchy

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Modern Formal Language Theory, 2007

Hierarchy of Language Families – Description

Notation

- *ac* appearance checking
- ε with erasing productions
- *M* matrix grammars
- *P* programmed grammars
- *RC* random context grammars
- *PER* permitting grammars
- *FOR* forbidding grammars
- **0, 1, 2** type-0, type-1, type-2 grammars, respectively

Example

$\mathscr{L}(RC, \varepsilon, ac)$ – the family of languages generated by random context grammars with erasing productions in appearance checking mode

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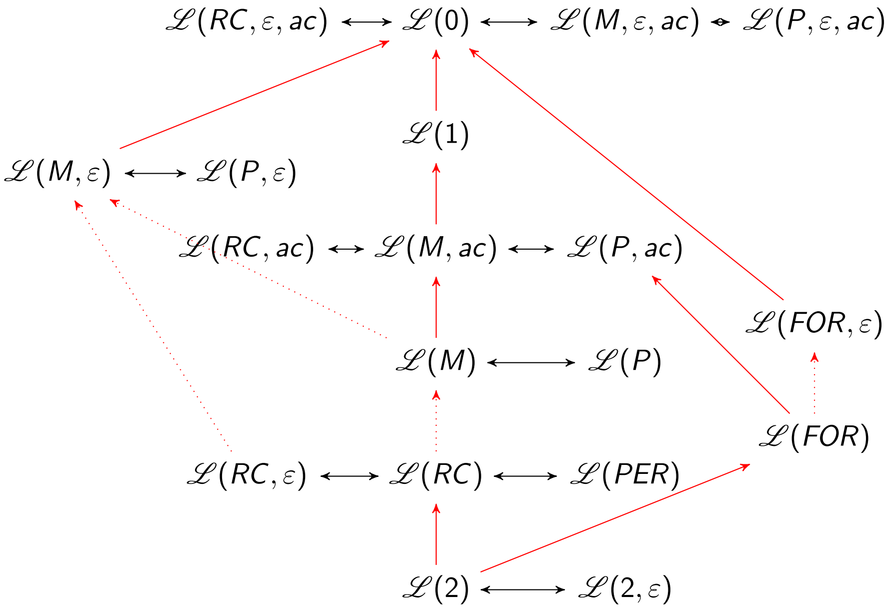

Proof of $\mathscr{L}(M) = \mathscr{L}(P)$

Theorem
$\mathscr{L}(M) = \mathscr{L}(P)$.

Proof–Basic Idea

1. $\mathscr{L}(M) \subseteq \mathscr{L}(P)$: Transform any matrix grammar to an equivalent programmed grammar
2. $\mathscr{L}(P) \subseteq \mathscr{L}(M)$: Transform any programmed grammar to an equivalent matrix grammar
3. $\mathscr{L}(M) = \mathscr{L}(P)$

Proof of $\mathscr{L}(M) \subseteq \mathscr{L}(P)$ I

Let

$$H = (G, M)$$

be a matrix grammar, where

- $G = (N, T, P, S)$ is a context-free grammar
- M is a finite language over P

Express M as

$$M = \{m_1, \ldots, m_r\}$$

for some $r \geq 1$ and

$$m_i = p_{i_1} \ldots p_{i_{k_i}}$$

with $k_i = |m_i|$. Set

$$N' = \{A' : A \in N\}$$

Define the homomorphism h from $(T \cup N)^*$ to $(T \cup N')^*$ as

- $h(a) = a$ for every $a \in T$
- $h(A) = A'$ for every $A \in N$

Proof of $\mathscr{L}(M) \subseteq \mathscr{L}(P)$ II

1. For every $p_{i_j} : A_{i_j} \to x_{i_j}$ such that $\mathrm{alph}(x_{i_j}) \cap N \neq \emptyset$ (some nonterminals occur in x_{i_j}) and $j < |m_i|$ (not the last production in m_i), introduce this programmed production:

$$(\langle i, j\rangle : A_{i_j} \to x_{i_j}, \{\langle i, j+1\rangle\})$$

2. For every $p_{i_j} : A_{i_j} \to x_{i_j}$ such that $\mathrm{alph}(x_{i_j}) \cap N = \emptyset$ (no nonterminal occurs in x_{i_j}) and $j < |m_i|$ (not the last production in m_i), introduce
   - **a** $(\langle i, j\rangle : A_{i_j} \to h(A_{i_j}), \{\langle i, j, 1\rangle, \dots, \langle i, j, m\rangle\})$
   - **b** $(\langle i, j, q\rangle : A_q \to A_q, \{\langle i, j\rangle'\})$ for $q = 1, \dots, m$ (make sure there is a nonterminal)
   - **c** $(\langle i, j\rangle' : h(A_{i_j}) \to x_{i_j}, \{\langle i, j+1\rangle\})$

   provided that $N = \{A_1, \dots, A_m\}$

3. For every $p_{i_{k_i}} : A_{i_{k_i}} \to x_{i_{k_i}}$ (simulate the last production of m_i and start simulating a new matrix), introduce

$$(\langle i, k_i\rangle : A_{i_{k_i}} \to x_{i_{k_i}}, \{\langle 1, 1\rangle, \dots, \langle r, 1\rangle\})$$

Proof of $\mathscr{L}(M) \subseteq \mathscr{L}(P)$ III

Let P' consist of all rules constructed above and

$$(\$: S' \rightarrow S, \{\langle 1, 1\rangle, \ldots, \langle r, 1\rangle\})$$

Define the programmed grammar

$$G' = (N \cup N' \cup \{S'\}, T, P', S')$$

Observe that

$$L(H) = L(G')$$

$\square$

Proof of $\mathscr{L}(P) \subseteq \mathscr{L}(M)$ I

Let

$$H = (G, R)$$

be a programmed grammar. Consider the simplified description of H which uses productions of the form

$$(p : A \to x, R(p))$$

1. If $(p : A \to x, Q)$, $(r : B \to y, R) \in P$ and $r \in Q$, then introduce the matrix

   $$(\langle A \to x \rangle \to A, A \to x, B \to \langle B \to y \rangle)$$

   (simulation will continue)
2. If $(p : A \to x, Q) \in P$, then introduce

   $$(\langle A \to x \rangle \to A, A \to x)$$

   (simulation ends)

Proof of $\mathscr{L}(P) \subseteq \mathscr{L}(M)$ II

3. If $(p : S \rightarrow x, Q) \in P$, introduce

$$(S' \rightarrow \langle S \rightarrow x \rangle)$$

(simulation starts)

Set

$$\begin{aligned} N' = N & \\ & \cup \{\langle A \rightarrow x \rangle : A \rightarrow x \in P\} \\ & \cup \{S'\} \end{aligned}$$

Define the matrix grammar

$$G' = (N', T, P', S')$$

where P' consists of all the above matrices. Observe that

$$L(H) = L(G')$$

$\square$

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