Cooperating Distributed Grammar Systems

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CD Grammar Systems

CD Grammar System

A cooperating distributed (CD) grammar system of degree n, $n \geq 1$, is a construct

$$\Gamma = (N, T, S, P_1, \ldots, P_n)$$

where

N, T, S are defined as usual

P_i is a finite set of context-free productions, called component of Γ, for each $i \in \{1, \ldots, n\}$

Notation

- the ith grammar $G_i = (N, T, P_i, S)$
- $y \not\Rightarrow z$ – y does not directly derive z

CD Grammar Systems – Derivation Modes I

Terminating Mode

For each $i = 1, \ldots, n$, terminating derivation by the ith component

$$x \;{}_{i}{\Rightarrow}^{t}\; y$$

iff

1. $x \Rightarrow^* y$ in $G_i = (N, T, P_i, S)$ and
2. $y \not\Rightarrow z$ for all $z \in (N \cup T)^*$

Note

$y \in T^*$ is not required

CD Grammar Systems – Derivation Modes II

k-Step Derivation in G_i

$$x \;{}_i{\Rightarrow}^{=k}\; y$$

iff $x \Rightarrow^{k} y$ in G_i

At Most k-Step Derivation in G_i

$$x \;{}_i{\Rightarrow}^{\leq k}\; y$$

iff $x \Rightarrow^{j} y$ in G_i for some $j \leq k$

At Least k-Step Derivation in G_i

$$x \;{}_i{\Rightarrow}^{\geq k}\; y$$

iff $x \Rightarrow^{j} y$ in G_i for some $j \geq k$

CD Grammar Systems – Generated Language

Set of Derivation Modes

$$D = \{*, t\} \cup \{\leq k, = k, \geq k \; : \; k = 1, 2, \ldots\}$$

Set of Possible Derivations

$$F(G_j, u, f) = \{v \; : \; u \;{}_j\!\Rightarrow^f \; v\}, \text{ where } j \in \{1, \ldots, n\}, f \in D, u \in V^*$$

Generated Language

$$L_f(\Gamma) = \{ \; w \in T^* \; : \; \text{there are } v_0, v_1, \ldots, v_m \text{ such that } v_i \in F(G_{j_i}, v_{i-1}, f), i = 1, \ldots, m, j_i \in \{1, \ldots, n\}, v_0 = S, v_m = w, \text{ for some } m \geq 1\}$$

CD Grammar Systems – Example I

Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$
$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k \,:\, k \geq 1\}} =$$
$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) =$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) =$$

CD Grammar Systems – Example I

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$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k \,:\, k \geq 1\}} = \{a^n b^n c^m \;:\; m, n \geq 1\}$$
$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) =$$

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CD Grammar Systems – Example I

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CD Grammar Systems – Example I

Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \to S, S \to AB, A' \to A, B' \to B\}$$
$$P_2 = \{A \to aA'b, B \to cB', A \to ab, B \to c\}$$

$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k \,:\, k \geq 1\}} = \{a^n b^n c^m \;:\; m, n \geq 1\}$$
$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) = \{a^n b^n c^n \;:\; n \geq 1\}$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) = \emptyset$$

Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$
$$P_2 = \{A \rightarrow S\}$$
$$P_3 = \{A \rightarrow a\}$$

$$L_t(\Gamma) =$$

CD Grammar Systems – Example II

Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$
$$P_2 = \{A \rightarrow S\}$$
$$P_3 = \{A \rightarrow a\}$$

$$L_t(\Gamma) = \{a^{2^n} : n \geq 1\}$$

Denotation of CD Language Families

Denotation of CD Language Families

$$CD_x^y(f)$$

where

f derivation mode, $f \in D$

y
- nothing – no ε-productions
- ε – ε-productions allowed

x
- n – degree at most n, $n \geq 1$
- ∞ – the number of components is not limited

Note

$CD_\infty(=)$ the union of all families $CD_\infty(=k)$ for $k = 1, 2, \ldots$

$CD_\infty(\geq)$ the union of all families $CD_\infty(\geq k)$ for $k = 1, 2, \ldots$

CD Grammar Systems – Generative Power

Theorem

- $CD_{\infty}^{y}(f) = \mathscr{L}(CF)$, *for all* $f \in \{= 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathscr{L}(CF) = CD_{1}^{y}(f) \subset CD_{2}^{y}(f) \subseteq CD_{r}^{y}(f) \subseteq CD_{\infty}^{y}(f) \subseteq \mathscr{L}(M)$, *for all* $f \in \{= k, \geq k : k \geq 2\}$, $r \geq 3$
- $CD_{r}^{y}(\geq k) \subseteq CD_{r}^{y}(\geq k + 1)$
- $CD_{\infty}^{y}(\geq) \subseteq CD_{\infty}^{y}(=)$
- $\mathscr{L}(CF) = CD_{1}^{y}(t) = CD_{2}^{y}(t) \subset CD_{3}^{y}(t) = CD_{\infty}^{y}(t) = \mathscr{L}(ET0L)$

Hybrid CD Grammar Systems

- combination of different modes is possible

Hybrid CD Grammar System

$$\Gamma = (N, T, S, (P_1, f_1), \ldots, (P_n, f_n))$$

where

N, T, S, P_i are defined as in the case of CD grammar systems

f_i is the mode of the ith component, $f_i \in D$ for all $i \in \{1, \ldots, n\}$

Generated Language

$$L(\Gamma) = \{w \in T^* : \text{ there are } v_0, v_1, \ldots, v_m \text{ such that } v_i \in F(G_{j_i}, v_{i-1}, f_{j_i}), i = 1, \ldots, m, j_i \in \{1, \ldots, n\}, v_0 = S, v_m = w, \text{ for some } m \geq 1\}$$

Denotation of Hybrid CD Language Families

Denotation of Hybrid CD Language Families

$$XCD^{y}_{x,v}(f)$$

where

x, y, f are defined as in the case of CD grammar systems

v
- m – each P_i contains at most m productions, $m \geq 1$
- ∞, nothing – any number of productions

X
- nothing – nondeterministic
- D – deterministic (for each P_i, $A \rightarrow u, A \rightarrow w \in P_i$ satisfy $u = w$)
- H – hybrid (then, (f) is not written)

Example

HCD^{ε}_{n} is a hybrid CD grammar system with at most n components

Hybrid CD Grammar Systems – Generative Power

Theorem

- $CD_{\infty,\infty}(f) = CD_{\infty,1}(f) = \mathscr{L}(CF)$, *for all* $f \in \{=1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathscr{L}(CF) \subset CD^{\varepsilon}_{\infty,1}(t) \subset CD^{\varepsilon}_{\infty,2}(t) \subseteq CD^{\varepsilon}_{\infty,3}(t) \subseteq CD^{\varepsilon}_{\infty,4}(t) \subseteq CD^{\varepsilon}_{\infty,5}(t) = CD^{\varepsilon}_{\infty,\infty}(t) = \mathscr{L}(ET0L)$
- $CD_{n,m}(f) \subset CD_{n+1,m}(f)$, $f \in \{*, t\}$
- $CD_{n,m}(f) \subset CD_{n,m+1}(f)$, $f \in \{*, t\}$

Theorem

- $\mathscr{L}(CF) = HCD_1 \subset HCD_2 \subseteq HCD_3 \subseteq HCD_4 = HCD_{\infty} = \mathscr{L}(M, ac)$
- $\mathscr{L}(ET0L) \subset HCD_4$
- $CD_{\infty}(=) \subset HCD_3$

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