# Multi-Grammars

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# Multisequential Grammar

## Multisequential Grammar

$$G = (V, T, P, S, K)$$

where

$V, T, S$ are defined as usual

$K$ is a finite set of selectors of the form

$$X_1 \,\mathrm{act}(Y_1) \ldots X_n \,\mathrm{act}(Y_n) X_{n+1}$$

where $n \geq 1$,

$$X_i \in \{Z^* : Z \subseteq V\}, Y_j \in \{Z : Z \subseteq V, Z \neq \emptyset\},$$

$i = 1, \ldots, n+1$, and $j = 1, \ldots, n$

$P$ is a finite set of productions of the form $a \rightarrow x$, where $a \in V$, $x \in V^*$

# Multisequential Grammar – Derivation Step

**Direct Derivation**

If there is

$$X_1 \, \mathrm{act}(Y_1) \dots X_n \, \mathrm{act}(Y_n) X_{n+1} \in K$$

satisfying

1. $u_i \in X_i$, for all $i = 1, \dots, n+1$,
2. $a_j \in Y_j$ and
3. $a_j \to x_j \in P$,

for all $j = 1, \dots, n$, then

$$u_1 a_1 \dots u_n a_n u_{n+1} \Rightarrow u_1 x_1 \dots u_n x_n u_{n+1}$$

**Note**

The generated language and $\Rightarrow^*$ are defined as usual

# Multisequential Grammar – Example

**Example**

$$G = (\{S, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$\begin{aligned} P = \{S &\rightarrow abc, \\ a &\rightarrow aa, \\ b &\rightarrow bb, \\ c &\rightarrow cc\} \end{aligned}$$

and

$$\begin{aligned} K = \{&\mathrm{act}(\{S\}), \\ &\{a\}^* \,\mathrm{act}(\{a\})\{b\}^* \,\mathrm{act}(\{b\})\{c\}^* \,\mathrm{act}(\{c\})\} \end{aligned}$$

$$L(G) = \{a^n b^n c^n \, : \, n \geq 1\}$$

# Multisequential Grammar – Generative Power

**Generative Power**
$\mathscr{L}(MS) = \mathscr{L}(RE)$

**Descriptional Complexity**

Every recursively enumerable language is generated by a multisequential grammar containing 2 nonterminals and 2 selectors.

# Multicontinuous Grammar

## Multicontinuous Grammar

$$G = (V, T, P, S, K)$$

where

$V, T, S$ are defined as usual

$K$ is a finite set of selectors of the form

$$X_1 \, \mathrm{act}(Y_1) X_2 \dots X_n \, \mathrm{act}(Y_n) X_{n+1}$$

where $n \geq 1$,

$$X_i \in \{Z^* : Z \subseteq V\}, Y_j \in \{Z^+ : Z \subseteq V, \, Z \neq \emptyset\},$$

$i = 1, \dots, n+1$, and $j = 1, \dots, n$

$P$ is a finite set of productions of the form $a \to x$, where $a \in V$, $x \in V^*$

# Multicontinuous Grammar – Derivation Step

**Direct Derivation**

If there is

$$X_1 \operatorname{act}(Y_1) \ldots X_n \operatorname{act}(Y_n) X_{n+1} \in K$$

satisfying

1. $u_i \in X_i$, for all $i = 1, \ldots, n+1$,
2. $y_j \in Y_j$ and
3. $y_j = y_{j_1} \ldots y_{j_{m_j}}$, $z_j = z_{j_1} \ldots z_{j_{m_j}}$, where $y_{j_1} \rightarrow z_{j_1}, \ldots, y_{j_{m_j}} \rightarrow z_{j_{m_j}} \in P$ for some $m_j \geq 1$,

for all $j = 1, \ldots, n$, then

$$u_1 y_1 \ldots u_n y_n u_{n+1} \Rightarrow u_1 z_1 \ldots u_n z_n u_{n+1}$$

**Note**

The generated language and $\Rightarrow^*$ are defined as usual

# Multicontinuous Grammar – Example

**Example**

$$G = (\{S, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$\begin{aligned} P = \{S &\rightarrow abc, \\ a &\rightarrow aa, \\ b &\rightarrow bb, \\ c &\rightarrow cc\} \end{aligned}$$

and

$$\begin{aligned} K = \{&\mathrm{act}(\{S\}^+), \\ &\mathrm{act}(\{a\}^+)\,\mathrm{act}(\{b\}^+)\,\mathrm{act}(\{c\}^+)\} \end{aligned}$$

$$L(G) = \{a^{2^n} b^{2^n} c^{2^n} \;:\; n \geq 0\}$$

# Multicontinuous Grammar – Generative Power

**Generative Power**

$\mathscr{L}(MC) = \mathscr{L}(RE)$

**Descriptional Complexity**

Every recursively enumerable language is generated by a multicontinuous grammar containing 3 nonterminals and 2 selectors.

# Multiparallel Grammar

## Multiparallel Grammar

$$G = (V, T, P, S, K)$$

where

$V, T, S$ are defined as usual

$K$ is a finite set of selectors of the form

$$F_1 F_2 \dots F_m$$

where

$$F_j \in \{W^+ : W \subseteq V, W \neq \emptyset\},$$

$j = 1, \dots, m$, for some $m \geq 1$

$P$ is a finite set of productions of the form $a \rightarrow x$, where $a \in V$, $x \in V^*$

# Multiparallel Grammar – Derivation Step

**Direct Derivation**

If there is

$$\pi \in K$$

satisfying

1. either $u = S$ and $S \rightarrow v \in P$,
2. or there is $k \geq 1$ so that
   - $u = a_1 \dots a_k$, where $a_i \in V$,
   - $u \in \pi$,
   - $v = x_1 \dots x_k$ and $a_i \rightarrow x_i \in P$

   for all $i = 1, \dots, k$, then

$$u \Rightarrow v$$

**Note**

The generated language and $\Rightarrow^*$ are defined as usual

# Multiparallel Grammar – Example

**Example**

$$G = (\{S, A, B, C, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$\begin{aligned} P = \{S &\to aAbBcC, & S &\to abc, & & \\ A &\to aA, & A &\to a, & a &\to a, \\ B &\to bB, & B &\to b, & b &\to b, \\ C &\to cC, & C &\to c, & c &\to c\} \end{aligned}$$

and

$$K = \{a\}^+\{A\}^+\{b\}^+\{B\}^+\{c\}^+\{C\}^+$$

$$L(G) = \{a^n b^n c^n \,:\, n \geq 1\}$$

# Multiparallel Grammar – Generative Power

**Generative Power**
$\mathscr{L}(MP) = \mathscr{L}(RE)$

**Descriptional Complexity**

Every recursively enumerable language is generated by a multiparallel grammar containing 7 nonterminals and 4 selectors of the length 5 $(m = 5)$.

# Bibliography


H. C. M. Kleijn and G. Rozenberg.
Multi grammars.
*International Journal of Computer Mathematics*, 12:177–201, 1983.

T. Masopust.
*Regulated Formal Models and Their Reductions*.
PhD thesis, Faculty of Information Technology, Brno University of Technology, Brno, Czech Republic, 2007.