Multi-Grammars

Jiří Techet Tomáš Masopust Alexander Meduna

Department of Information Systems
Faculty of Information Technology
Brno University of Technology
Božetěchova 2, Brno 61266, Czech Republic

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Multisequential Grammar

Multisequential Grammar

$$G = (V, T, P, S, K)$$

where

V, T, S are defined as usual

K is a finite set of selectors of the form

$$X_1 \operatorname{act}(Y_1) \dots X_n \operatorname{act}(Y_n) X_{n+1}$$

where $n \geq 1$,

$$X_i \in \{Z^* : Z \subseteq V\}, Y_j \in \{Z : Z \subseteq V, Z \neq \emptyset\},$$

$i = 1, \dots, n+1$, and $j = 1, \dots, n$

P is a finite set of productions of the form $a \to x$, where $a \in V$, $x \in V^*$

Multisequential Grammar – Derivation Step

Direct Derivation

If there is

$$X_1 \operatorname{act}(Y_1) \dots X_n \operatorname{act}(Y_n) X_{n+1} \in K$$

satisfying

1. $u_i \in X_i$, for all $i = 1, \dots, n+1$,
2. $a_j \in Y_j$ and
3. $a_j \to x_j \in P$,

for all $j = 1, \dots, n$, then

$$u_1 a_1 \dots u_n a_n u_{n+1} \Rightarrow u_1 x_1 \dots u_n x_n u_{n+1}$$

Note

The generated language and $\Rightarrow^*$ are defined as usual

Multisequential Grammar – Example

Example

$$G = (\{S, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$\begin{aligned} P = \{ & S \to abc, \\ & a \to aa, \\ & b \to bb, \\ & c \to cc \} \end{aligned}$$

and

$$\begin{aligned} K = \{ & \mathrm{act}(\{S\}), \\ & \{a\}^* \, \mathrm{act}(\{a\}) \{b\}^* \, \mathrm{act}(\{b\}) \{c\}^* \, \mathrm{act}(\{c\}) \} \end{aligned}$$

$$L(G) = \{a^n b^n c^n \ : \ n \geq 1\}$$

Multisequential Grammar – Generative Power

Generative Power

$\mathscr{L}(MS) = \mathscr{L}(RE)$

Descriptional Complexity

Every recursively enumerable language is generated by a multisequential grammar containing 2 nonterminals and 2 selectors.

Multicontinuous Grammar

Multicontinuous Grammar

$$G = (V, T, P, S, K)$$

where

V, T, S are defined as usual

K is a finite set of selectors of the form

$$X_1 \operatorname{act}(Y_1) X_2 \dots X_n \operatorname{act}(Y_n) X_{n+1}$$

where $n \geq 1$,

$$X_i \in \{Z^* : Z \subseteq V\}, Y_j \in \{Z^+ : Z \subseteq V, Z \neq \emptyset\},$$

$i = 1, \dots, n+1$, and $j = 1, \dots, n$

P is a finite set of productions of the form $a \to x$, where $a \in V$, $x \in V^*$

Multicontinuous Grammar – Derivation Step

Direct Derivation

If there is

$$X_1 \operatorname{act}(Y_1) \dots X_n \operatorname{act}(Y_n) X_{n+1} \in K$$

satisfying

1. $u_i \in X_i$, for all $i = 1, \dots, n+1$,
2. $y_j \in Y_j$ and
3. $y_j = y_{j_1} \dots y_{j_{m_j}}$, $z_j = z_{j_1} \dots z_{j_{m_j}}$, where $y_{j_1} \to z_{j_1}, \dots, y_{j_{m_j}} \to z_{j_{m_j}} \in P$ for some $m_j \geq 1$,

for all $j = 1, \dots, n$, then

$$u_1 y_1 \dots u_n y_n u_{n+1} \Rightarrow u_1 z_1 \dots u_n z_n u_{n+1}$$

Note

The generated language and $\Rightarrow^*$ are defined as usual

Multicontinuous Grammar – Example

Example

$$G = (\{S, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$\begin{aligned} P = \{ & S \to abc, \\ & a \to aa, \\ & b \to bb, \\ & c \to cc \} \end{aligned}$$

and

$$\begin{aligned} K = \{ & \mathrm{act}(\{S\}^+), \\ & \mathrm{act}(\{a\}^+)\,\mathrm{act}(\{b\}^+)\,\mathrm{act}(\{c\}^+) \} \end{aligned}$$

$$L(G) = \{a^{2^n} b^{2^n} c^{2^n} \ : \ n \geq 0\}$$

Multicontinuous Grammar – Generative Power

Generative Power

$\mathscr{L}(MC) = \mathscr{L}(RE)$

Descriptional Complexity

Every recursively enumerable language is generated by a multicontinuous grammar containing 3 nonterminals and 2 selectors.

Multiparallel Grammar

Multiparallel Grammar

$$G = (V, T, P, S, K)$$

where

V, T, S are defined as usual

K is a finite set of selectors of the form

$$F_1 F_2 \dots F_m$$

where

$$F_j \in \{W^+ : W \subseteq V, W \neq \emptyset\},$$

$j = 1, \dots, m$, for some $m \geq 1$

P is a finite set of productions of the form $a \to x$, where $a \in V$, $x \in V^*$

Multiparallel Grammar – Derivation Step

Direct Derivation

If there is

$$\pi \in K$$

satisfying

1. either $u = S$ and $S \to v \in P$,
2. or there is $k \geq 1$ so that
   - $u = a_1 \dots a_k$, where $a_i \in V$,
   - $u \in \pi$,
   - $v = x_1 \dots x_k$ and $a_i \to x_i \in P$

   for all $i = 1, \dots, k$, then

$$u \Rightarrow v$$

Note

The generated language and $\Rightarrow^*$ are defined as usual

Multiparallel Grammar – Example

Example

$$G = (\{S, A, B, C, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$\begin{aligned} P = \{ & S \to aAbBcC, \quad S \to abc, \\ & A \to aA, \quad A \to a, \quad a \to a, \\ & B \to bB, \quad B \to b, \quad b \to b, \\ & C \to cC, \quad C \to c, \quad c \to c\} \end{aligned}$$

and

$$K = \{a\}^+\{A\}^+\{b\}^+\{B\}^+\{c\}^+\{C\}^+$$

$$L(G) = \{a^n b^n c^n \,:\, n \geq 1\}$$

Multiparallel Grammar – Generative Power

Generative Power

$\mathscr{L}(MP) = \mathscr{L}(RE)$

Descriptional Complexity

Every recursively enumerable language is generated by a multiparallel grammar containing 7 nonterminals and 4 selectors of the length 5 ($m = 5$).

Bibliography


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