

Random Context Grammars

Jiří Techet Tomáš Masopust Alexander Meduna

Department of Information Systems
Faculty of Information Technology
Brno University of Technology
Božetěchova 2, Brno 61266, Czech Republic

Modern Formal Language Theory, 2007

Random Context Grammar

Random Context Grammar

A (permitting) **random context grammar** is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R is a finite relation from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, we write

$$(p : A \rightarrow x, Q)$$

where $Q \subseteq N$ is called a **permitting context**

Derivation Step of Random Context Grammar

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y [p] \text{ in } H$$

if

1 $x \Rightarrow y [p] \text{ in } G$ and

2 $R(p) \subseteq \text{alph}(x)$

Random Context Grammar with Appearance Checking

Random Context Grammar with Appearance Checking

A **random context grammar with appearance checking** is a triple

$$H = (G, R, F)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R, F are two finite relations from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, and $F(p) = K$, we write

$$(p : A \rightarrow x, Q, K)$$

where Q and K are **permitting** and **forbidding contexts**, respectively

Derivation Step of Random Context Grammar with Appearance Checking

Forbidding Grammar

If every $(p : A \rightarrow x, Q, K)$ satisfies $Q = \emptyset$, then H is called a **forbidding grammar**.

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y [p] \text{ in } H$$

if

- 1 $x \Rightarrow y [p] \text{ in } G$
- 2 $R(p) \subseteq \text{alph}(x)$
- 3 $F(p) \cap \text{alph}(x) = \emptyset$

Example – Permitting Grammar I

Example

1 $(S \rightarrow ABC, \emptyset)$

2 $(A \rightarrow aA', \{B\})$

$(B \rightarrow bB', \{C\})$

$(C \rightarrow cC', \{A'\})$

3 $(A' \rightarrow A, \{B'\})$

$(B' \rightarrow B, \{C'\})$

$(C' \rightarrow C, \{A\})$

4 $(A \rightarrow a, \{B\})$

$(B \rightarrow b, \{C\})$

$(C \rightarrow c, \emptyset)$



Example – Permitting Grammar II

Example

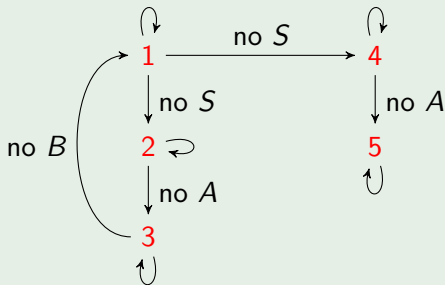
$$\begin{aligned} S &\Rightarrow ABC && 1 \\ &\Rightarrow^3 aA'bB'cC' && 2 \\ &\Rightarrow^3 aAbBcC && 3 \\ &\Rightarrow^3 aaA'bbB'ccC' && 2 \\ &\Rightarrow^3 aaAbbBccC && 3 \\ &\vdots \\ &\Rightarrow^3 a^n Ab^n Bc^n C && 3 \\ &\Rightarrow^3 a^{n+1} b^{n+1} c^{n+1} && 4 \end{aligned}$$

$$L(H) = \{a^k b^k c^k : k \geq 1\}$$

Example – Forbidding Grammar I

Example

- (1 : $S \rightarrow AA, \emptyset, \{B, D\}$)
- (2 : $A \rightarrow B, \emptyset, \{S, D\}$)
- (3 : $B \rightarrow S, \emptyset, \{A, D\}$)
- (4 : $A \rightarrow D, \emptyset, \{S, B\}$)
- (5 : $D \rightarrow a, \emptyset, \{S, A, B\}$)



Example – Forbidding Grammar II

Example

		rules
$S \Rightarrow$	AA	1
\Rightarrow^2	BB	2
\Rightarrow^2	SS	3
\Rightarrow^2	$AAAA$	1
\Rightarrow^4	$BBBB$	2
\Rightarrow^4	$SSSS$	3
	\vdots	
\Rightarrow^{2^i}	$A^{2^{i+1}}$	1
$\Rightarrow^{2^{i+1}}$	$D^{2^{i+1}}$	4
$\Rightarrow^{2^{i+1}}$	$a^{2^{i+1}}$	5

$$L(H) = \{a^{2^n} : n \geq 1\}$$



J. Dassow and Gh. Păun.

Regulated Rewriting in Formal Language Theory.

Akademie-Verlag, Berlin, 1989.



A. P. J. van der Walt.

Random context grammars.

In *Proceedings of Symposium on Formal Languages*, 1970.