Other Grammars

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State Grammar

State Grammar

$$G = (V, T, K, P, S, p_0)$$

where

- V is a finite alphabet
- T is a set of terminals, $T \subset V$
- K is a finite set of states
- S is the start symbol, $S \in V - T$
- p_0 is the start state, $p_0 \in K$
- P is a finite set of productions of the form

$$(A, p) \rightarrow (x, q),$$

where $p, q \in K$, $A \in V - T$, $x \in V^*$

State Grammar – Derivation Step

Derivation Step

For $(A, p) \rightarrow (x, q) \in P$,

$$u = (rAs, p),$$
$$v = (rxs, q),$$

where $r, s \in V^*$, and for every $(B, p) \rightarrow (y, t) \in P$, $B \notin \mathrm{alph}(r)$, we write

$$u \Rightarrow v \; [(A, p) \rightarrow (x, q)]$$

Generated Language

$L(G) = \{x \in T^* \; : \; (S, p_0) \Rightarrow^* (x, q) \text{ for some } q \in K\}$

Generative Power

$\mathscr{L}(ST) = \mathscr{L}(CS)$, and $\mathscr{L}(ST, \varepsilon) = \mathscr{L}(RE)$

State Grammar – Example

Example

$$G = (\{S, A, B, C, a, b, c\}, \{a, b, c\}, \{0, 1, 2, 3, 4\}, P, S, 0)$$

$$\begin{aligned} P = \{(S, 0) &\to (ABC, 1), \\ (A, 1) &\to (aA, 2), & (A, 1) &\to (a, 4), \\ (B, 2) &\to (bB, 3), & (B, 4) &\to (b, 4), \\ (C, 3) &\to (cC, 1), & (C, 4) &\to (c, 4)\} \end{aligned}$$

$$\begin{tikzcd} 0 \arrow[r] & 1 \arrow[r] \arrow[d] & 2 \arrow[r] & 3 \arrow[ll, bend right] \\ & 4 & & \end{tikzcd}$$

$$\begin{aligned} (S, 0) &\Rightarrow (ABC, 1) \Rightarrow (aABC, 2) \Rightarrow (aAbBC, 3) \Rightarrow (aAbBcC, 1) \\ &\Rightarrow (aabBcC, 4) \Rightarrow (aabbcC, 4) \Rightarrow (aabbcc, 4) \end{aligned}$$

$$L(G) = \{a^n b^n c^n \ : \ n \geq 1\}$$

Queue Grammar

$$G = (V, T, W, F, P, s)$$

where

- V is a finite alphabet
- T is a set of terminals, $T \subset V$
- W is a finite alphabet of states
- F is a set of final states, $F \subset W$
- s is the start string, $s \in (V - T)(W - F)$
- P is a finite set of productions of the form: (a, b, x, c), where
  - $a \in V$
  - $b \in W - F$
  - $x \in V^*$
  - $c \in W$

Queue Grammar – Derivation Step

Derivation Step

If

$$u = arb,\ v = rxc,$$

where $a \in V$, $r, x \in V^*$, $b, c \in W$, and

$$(a, b, x, c) \in P,$$

then

$$u \Rightarrow v\ [(a, b, x, c)]$$

Generated Language

$L(G) = \{w \in T^* : s \Rightarrow^* wf, f \in F\}$

Generative Power

$\mathscr{L}(QG, \varepsilon) = \mathscr{L}(RE)$, and $\mathscr{L}(QG) = \mathscr{L}(CS)$

Queue Grammar – Example

Example

$$G = (\{A, a, b\}, \{a, b\}, \{\bar{e}, \bar{f}\}, \{\bar{f}\}, P, A\bar{e})$$

$$\begin{aligned} P = \{ & 1 : (A, \bar{e}, bAa, \bar{e}), \\ & 2 : (A, \bar{e}, \varepsilon, \bar{f}), \\ & 3 : (a, \bar{e}, a, \bar{e}), \\ & 4 : (b, \bar{e}, b, \bar{e})\} \end{aligned}$$

$$A\bar{e} \Rightarrow bAa\bar{e}\ [1] \Rightarrow Aab\bar{e}\ [4] \Rightarrow abbAa\bar{e}\ [1] \Rightarrow bbAaa\bar{e}\ [3] \Rightarrow bAaab\bar{e}\ [4]$$
$$\Rightarrow Aaabb\bar{e}\ [4] \Rightarrow aabb\bar{f}\ [2]$$

$$L(G) = \{a^n b^n \ : \ n \geq 0\}$$

Simple Matrix Grammar

Simple Matrix Grammar (of Degree n)

$$G = (N_1, \ldots, N_n, T, P, S)$$

where

$N_1, \ldots, N_n$ are pairwise disjoint nonterminal alphabets

P is a finite set of productions of the form

- $(S \rightarrow w)$, where $w \in T^*$
- $(S \rightarrow A_1 \ldots A_n)$, where $A_i \in N_i$, $i = 1, \ldots, n$
- $(A_1 \rightarrow w_1, \ldots, A_n \rightarrow w_n)$, where $w_i \in T^*$, $i = 1, \ldots, n$
- $(A_1 \rightarrow \alpha_1, \ldots, A_n \rightarrow \alpha_n)$, where

$$\alpha_i = x_{i1} A_{i1} \ldots x_{ik} A_{ik} y_i,$$

$$x_{ij}, y_i \in T^*,\ A_{ij} \in N_i,\ i = 1, \ldots, n,\ j = 1, \ldots, k,\ k \geq 1$$

T, S have the standard meaning, $S \notin N_1 \cup \ldots \cup N_n$

Simple Matrix Grammar – Derivation Step

Direct Derivation

If

- either $u = S$ and $(S \to v) \in P$
- or
  - $u = y_1 A_1 z_1 \dots y_n A_n z_n$
  - $v = y_1 w_1 z_1 \dots y_n w_n z_n$
  - $(A_1 \to w_1, \dots, A_n \to w_n) \in P$

  where $y_i \in T^*$, $z_i \in (N_i \cup T)^*$, for all $i = 1, \dots, n$

then

$$u \Rightarrow v$$

Note

The generated language and $\Rightarrow^*$ are defined as usual

Simple Matrix Grammar – Generative Power

(Right) Linear Simple Matrix Grammar (of Degree n)

If the productions $A_i \to \alpha_i$, $1 \leq i \leq n$, from $(A_1 \to \alpha_1, \ldots, A_n \to \alpha_n)$ are

linear then G is linear simple matrix grammar (LSM)

right linear then G is right linear simple matrix grammar ($RLSM$)

Generative Power

For all $n \geq 1$,

- $\mathscr{L}(SM, n) \subset \mathscr{L}(SM, n+1) \subset \mathscr{L}(CS)$
- $\mathscr{L}(LSM, n) \subset \mathscr{L}(LSM, n+1)$
- $\mathscr{L}(RLSM, n) \subset \mathscr{L}(RLSM, n+1)$

Right Linear Simple Matrix Grammar – Example

Example

$$G = (\{A\}, \{B\}, \{a, b\}, S, P)$$

where

$$\begin{aligned} P = \{1 &: (S \rightarrow AB), \\ 2 &: (A \rightarrow aA, B \rightarrow aB), \\ 3 &: (A \rightarrow bA, B \rightarrow bB), \\ 4 &: (A \rightarrow \varepsilon, B \rightarrow \varepsilon)\} \end{aligned}$$

$$S \Rightarrow AB\ [1] \Rightarrow aAaB\ [2] \Rightarrow abAabB\ [3] \Rightarrow abaAabaB\ [2] \Rightarrow abaaba\ [4]$$

$$L(G) = \{ww : w \in \{a, b\}^*\}$$

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