# Programmed Grammars

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# Programmed Grammar

## Programmed Grammar

A programmed grammar is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- $R$ is a finite relation on $P$

## Notation

If $p : A \to x \in P$, $R(p) = Q$, we write

$$(p : A \to x, Q)$$

# Generated Language

## Derivation Step

For $(x, p), (y, q) \in V^* \times P$,

$$(x, p) \Rightarrow (y, q) \text{ in } H$$

if

1. $x \Rightarrow y\ [p]$ in $G$
2. $q \in R(p)$

## Generated Language

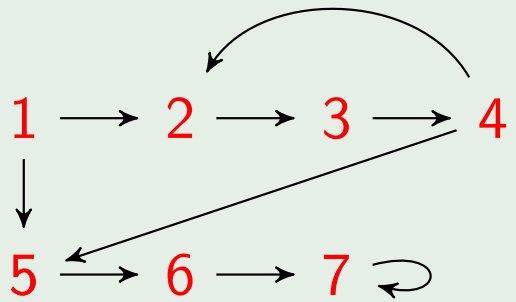
$$L(H) = \{x \in T^* : (S, p) \Rightarrow^* (x, p') \text{ for some } p, p' \in P\}$$

# Programmed Grammar – Example

## Example

$(1 : S \to ABC, \{2, 5\})$
$(2 : A \to aA, \{3\})$
$(3 : B \to bB, \{4\})$
$(4 : C \to cC, \{2, 5\})$
$(5 : A \to a, \{6\})$
$(6 : B \to b, \{7\})$
$(7 : C \to c, \{7\})$

$$
\begin{aligned}
(S, 1) &\Rightarrow (ABC, 2) \\
&\Rightarrow (aABC, 3) \\
&\Rightarrow (aAbBC, 4) \\
&\Rightarrow (aAbBcC, 5) \\
&\Rightarrow (aabBcC, 6) \\
&\Rightarrow (aabbcC, 7) \\
&\Rightarrow (aabbcc, 7)
\end{aligned}
$$




$$L(H) = \{a^n b^n c^n \ : \ n \geq 1\}$$

# Programmed Grammar with Appearance Checking

## Programmed Grammar with Appearance Checking

A programmed grammar with appearance checking is a triple

$$H = (G, R, F)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- $R, F$ are finite relations on $P$

## Notation

If $p : A \to x \in P$, $R(p) = Q$, and $F(p) = H$, we write

$$(p : A \to x, Q, H)$$

where $Q$ and $H$ are success and failure fields, respectively

# Derivation Step

## Derivation Step

For $(x, p), (y, q) \in V^* \times P$,

$$(x, p) \Rightarrow (y, q) \text{ in } H$$

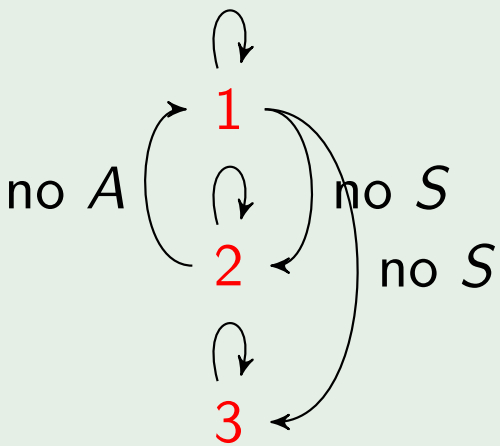
if

- either $x \Rightarrow y\ [p]$ in $G$ and $q \in R(p)$
- or $x = y$, $q \in F(p)$, $p$ is not applicable to $x$

# Example I

## Example

$(1 : S \to AA, \{1\}, \{2, 3\})$
$(2 : A \to S, \{2\}, \{1\})$
$(3 : A \to a, \{3\}, \emptyset)$




$L(H) = \{a^{2^n} : n \geq 1\}$

$(S,1) \Rightarrow (AA,1) \Rightarrow (AA,3) \Rightarrow (Aa,3) \Rightarrow (aa,3)$
$\Rightarrow (AA,2)$
$\Rightarrow (AS,2)$
$\Rightarrow (SS,2)$
$\Rightarrow (SS,1)$
$\Rightarrow (SAA,1)$
$\Rightarrow (AAAA,1) \Rightarrow (AAAA,3) \Rightarrow (AaAA,3) \Rightarrow (AaaA,3) \Rightarrow (aaaA,3) \Rightarrow (aaaa,3)$
$\Rightarrow (AAAA,2)$
$\Rightarrow (ASAA,2) \ldots$

# Example II

## Example

$$
\begin{aligned}
&(1 : S \to SC, \{1, 2\}, \emptyset)\\
&(2 : S \to AA, \{3\}, \emptyset)\\
&(3 : A \to B, \{4\}, \{5\})\\
&(4 : C \to D, \{3\}, \{7\})\\
&(5 : C \to C, \{6\}, \emptyset)\\
&(6 : B \to A, \{6\}, \{3\})\\
&(7 : B \to A, \{7\}, \{8\})\\
&(8 : D \to A, \{9\}, \{10\})\\
&(9 : D \to C, \{9\}, \{3\})\\
&(10 : A \to a, \{10\}, \emptyset)
\end{aligned}
$$

Which language generates this grammar?

# Bibliography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