Multi-Generative Grammar Systems

From Grammar To MGS

Grammar: $G = (N, T, P, S)$

$$S \Rightarrow \ldots \Rightarrow \ldots \Rightarrow w, \text{ where } w \in T^*$$

Grammar system: $\Gamma = (G_1, G_2, \ldots, G_n, Q)$, where

- $G_i = (N_i, T_i, P_i, S_i)$ is a CFG for all $i = 1 \ldots n$
- Q = a set of rules, which „check" generation.

$$S_1 \Rightarrow x_1 \Rightarrow \ldots \Rightarrow w_1, \text{ where } w_1 \in T_1^*$$
$$S_2 \Rightarrow x_2 \Rightarrow \ldots \Rightarrow w_2, \text{ where } w_2 \in T_2^*$$
$$\vdots$$
$$S_n \Rightarrow x_n \Rightarrow \ldots \Rightarrow w_n, \text{ where } w_n \in T_n^*$$

Checking! ($x_1, x_2, \ldots, x_n$)

Paralell generation

$(w_1, w_2, \ldots, w_n)$ = Generated multistring in Γ.

Checking: Two Approaches

1) Symbols Checking:

$$(S_1, S_2, \dots, S_n)$$
$$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$$
$$(x_1, x_2, \dots, x_n)$$
$$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$$
$$(y_1, y_2, \dots, y_n)$$
$$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$$
$$\cdots$$

2) Rules Checking:

$$(S_1, S_2, \dots, S_n)$$
$$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$$
$$(x_1, x_2, \dots, x_n)$$
$$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$$
$$(y_1, y_2, \dots, y_n)$$
$$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$$
$$\cdots$$

Checking: Two Approaches

1) Symbols Checking:

$(S_1, S_2, \ldots, S_n)$

$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$

$(x_1, x_2, \ldots, x_n)$

$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$

$(y_1, y_2, \ldots, y_n)$

$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$

$\cdots$

Checking of multiforms

2) Rules Checking:

$(S_1, S_2, \ldots, S_n)$

$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$

$(x_1, x_2, \ldots, x_n)$

$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$

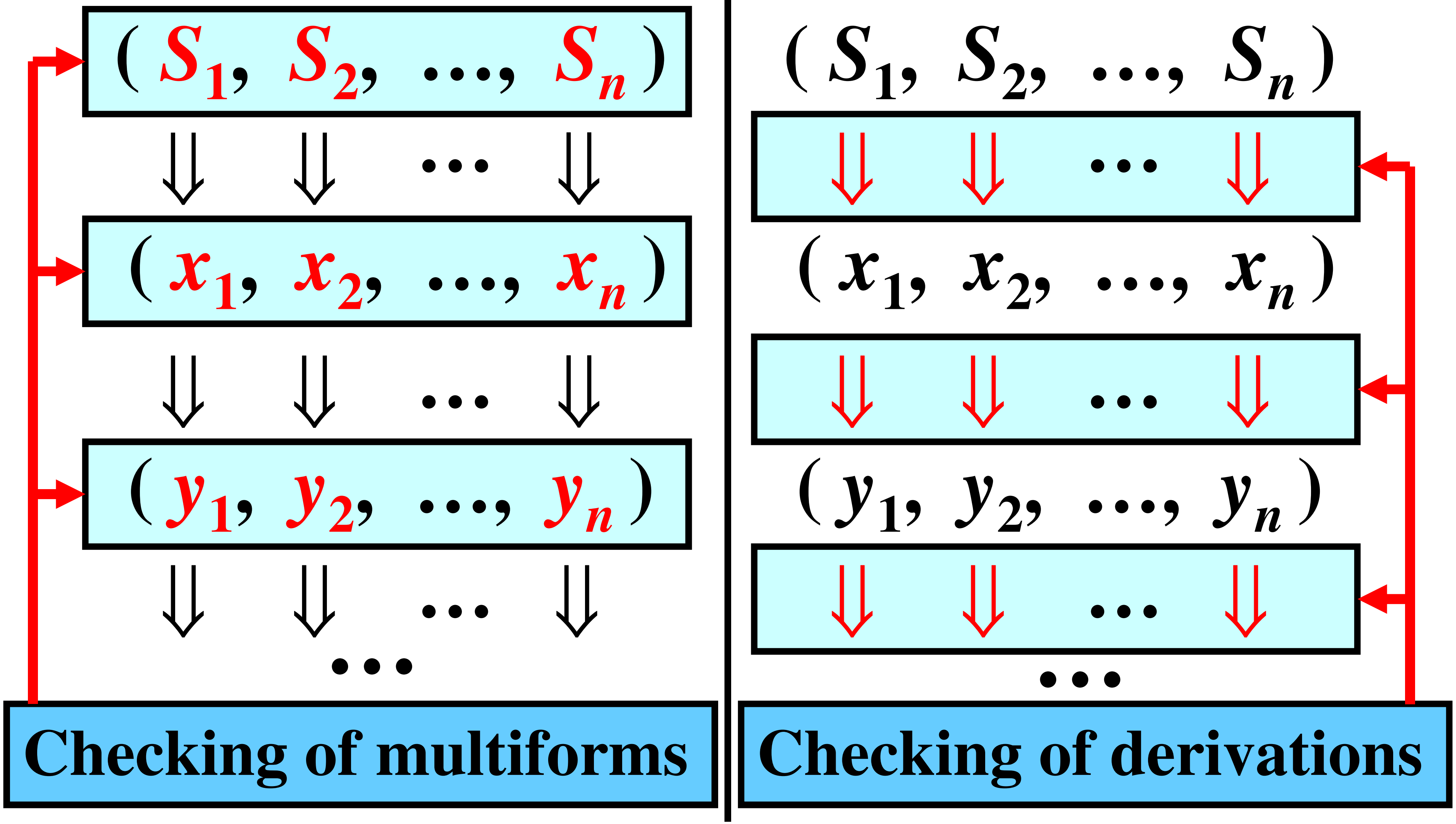
$(y_1, y_2, \ldots, y_n)$

$\Downarrow \quad \Downarrow \quad \cdots \quad \Downarrow$

$\cdots$

Checking: Two Approaches

1) Symbols Checking: **2) Rules Checking:**



Nonterminal-synchronized GS

Definition: *An n-multigenerative nonterminal-synchronized grammar system* (*n-MGN*) is n+1 tuple

$$\Gamma = (G_1, G_2, \ldots, G_n, Q), \text{ where}$$

- $G_i = (N_i, T_i, P_i, S_i)$ is a CFG for all $i = 1 \ldots n$
- Q = is a finite set of n-tuples of the form $(A_1, A_2, \ldots, A_n)$, where $A_i \in N_i$ for all $i = 1 \ldots n$

Example:

$\Gamma = (G_1, G_2, \{(S_1, S_2), (A_1, A_2)\})$, where:

$G_1 = (\{S_1, A_1\}, \{a, b, c\}, R_1, S_1)$;

$R_1 = \{S_1 \to aS_1, S_1 \to aA_1, A_1 \to bA_1c, A_1 \to bc\}$

$G_2 = (\{S_2, A_2\}, \{d\}, R_2, S_2)$;

$R_2 = \{S_2 \to S_2A_2, S_2 \to A_2, A_2 \to d\}$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \ldots , G_n, Q)$ be a n-MGN.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, $\boldsymbol{A_i} \rightarrow \boldsymbol{x_i} \in P_i$ for all $i = 1..n$ Then, $(\boldsymbol{u_1 A_1 v_1}, \boldsymbol{u_2 A_2 v_2}, \ldots, \boldsymbol{u_n A_n v_n}) \Rightarrow (\boldsymbol{u_1 x_1 v_1}, \boldsymbol{u_2 x_2 v_2}, \ldots, \boldsymbol{u_n x_n v_n})$ if $(\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$
$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(\boldsymbol{A_1}, \ldots, \boldsymbol{A_n}) \in Q$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \dots, G_n, Q)$ be a n-MGN.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, $A_i \to x_i \in P_i$
for all $i = 1..n$ Then, $(u_1A_1v_1, u_2A_2v_2, \dots, u_nA_nv_n) \Rightarrow$
$(u_1x_1v_1, u_2x_2v_2, \dots, u_nx_nv_n)$ if $(A_1, A_2, \dots, A_n) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$
$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(A_1, \dots, A_n) \in Q$

Rule: $A_1 \to x_1 \in P_1$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \dots, G_n, Q)$ be a n-MGN.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, $A_i \to x_i \in P_i$
for all $i = 1..n$ Then, $(u_1A_1v_1, u_2A_2v_2, \dots, u_nA_nv_n) \Rightarrow$
$(u_1x_1v_1, u_2x_2v_2, \dots, u_nx_nv_n)$ if $(A_1, A_2, \dots, A_n) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$
$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(A_1, \dots, A_n) \in Q$

Rule: $A_1 \to x_1 \in P_1$ **Rule:** $A_n \to x_n \in P_n$

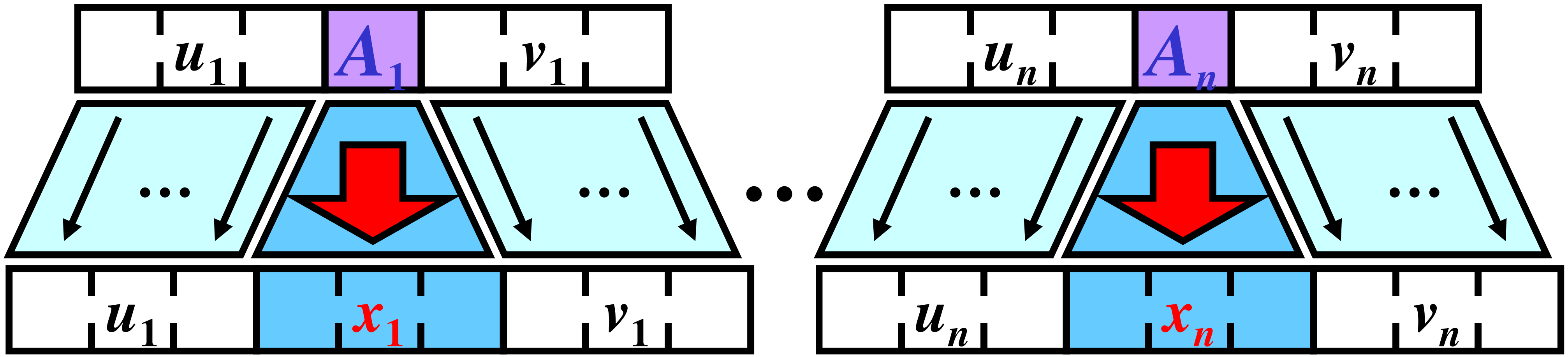
Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \ldots, G_n, Q)$ be a n-MGN.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, $A_i \to x_i \in P_i$
for all $i = 1..n$ Then, $(u_1A_1v_1, u_2A_2v_2, \ldots, u_nA_nv_n) \Rightarrow$
$(u_1x_1v_1, u_2x_2v_2, \ldots, u_nx_nv_n)$ if $(A_1, A_2, \ldots, A_n) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$
$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(A_1, \ldots, A_n) \in Q$

Rule: $A_1 \to x_1 \in P_1$ **Rule:** $A_n \to x_n \in P_n$

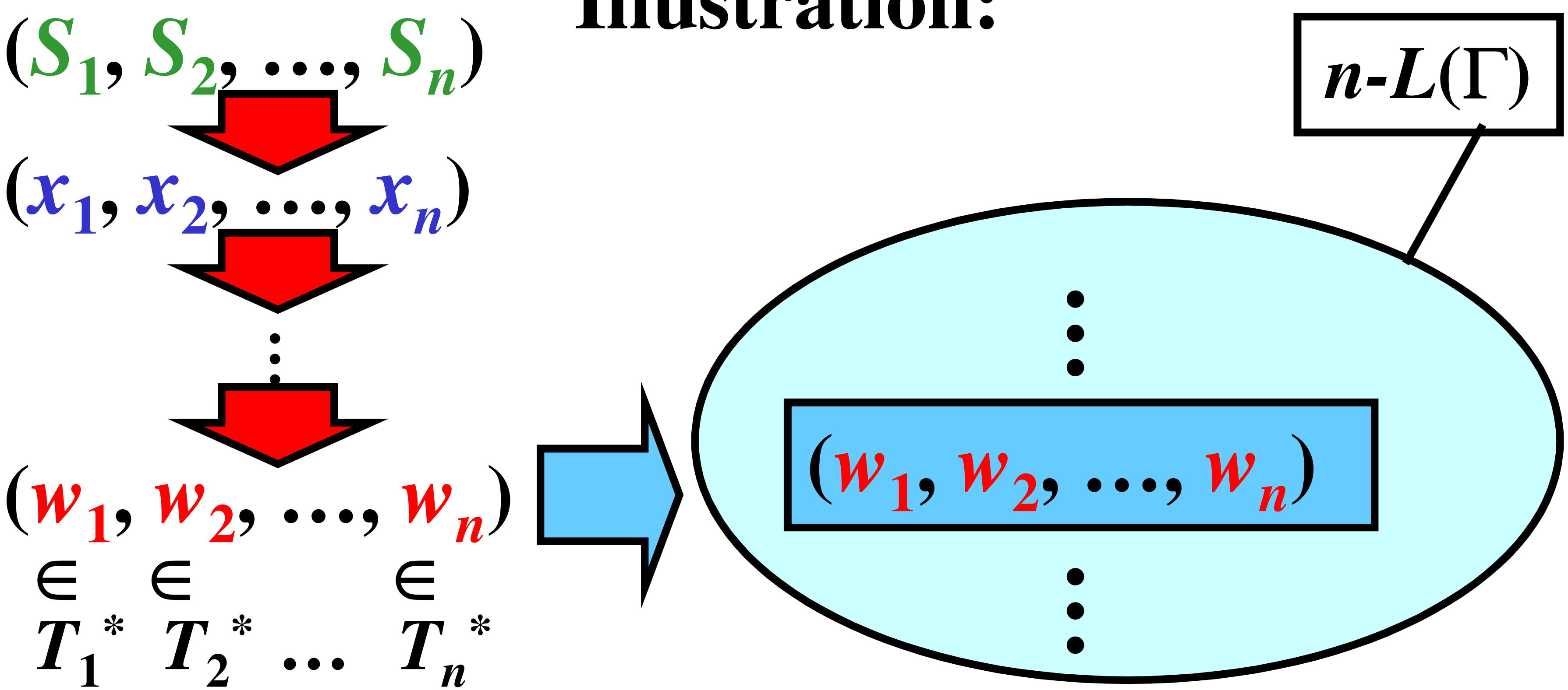


n-Language

Definition: Let $\Gamma = (G_1, G_2, \ldots, G_n, Q)$ be a n-MGN. The *n-Language of* Γ, $n\text{-}L(\Gamma)$, is defined as:

$$n\text{-}L(\Gamma) = \{(w_1, w_2, \ldots, w_n) : (S_1, S_2, \ldots, S_n) \Rightarrow^* (w_1, w_2, \ldots, w_n), \; w_i \in T_i^* \text{ for all } i = 1, \ldots, n\}$$

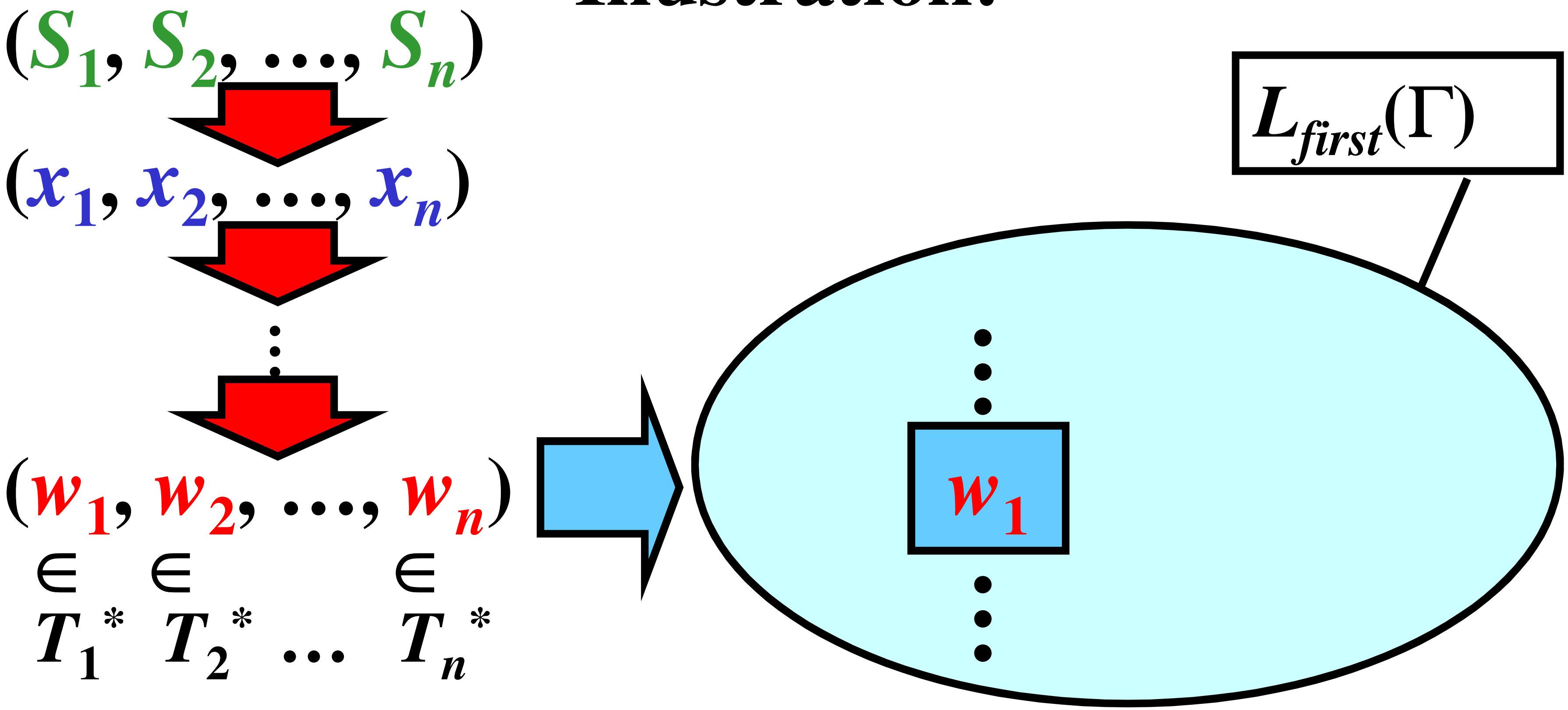
Illustration:



Generated Language in *First Mode*

Definition: The *language generated by* Γ *in the first mode*, $L_{first}(\Gamma)$, is defined as:
$L_{first}(\Gamma) = \{w_1: (w_1, w_2, \ldots, w_n) \in n\text{-}L(\Gamma)\}$

Illustration:

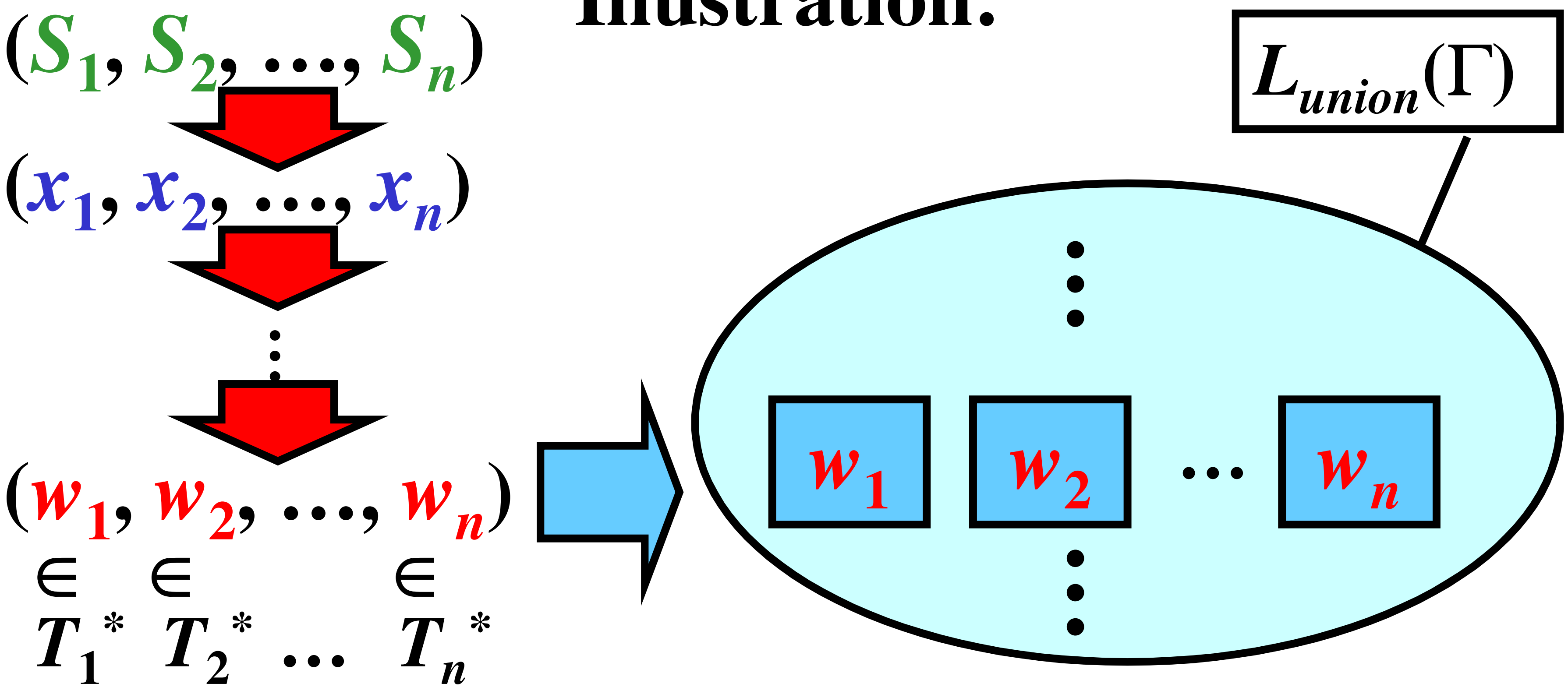


Generated Language in *Union Mode*

Definition: The *language generated by* Γ *in the union mode*, $L_{union}(\Gamma)$, is defined as:

$$L_{union}(\Gamma) = \{w: (w_1, w_2, \ldots, w_n) \in n\text{-}L(\Gamma),\ w \in \{w_i: i = 1, \ldots, n\}\}$$

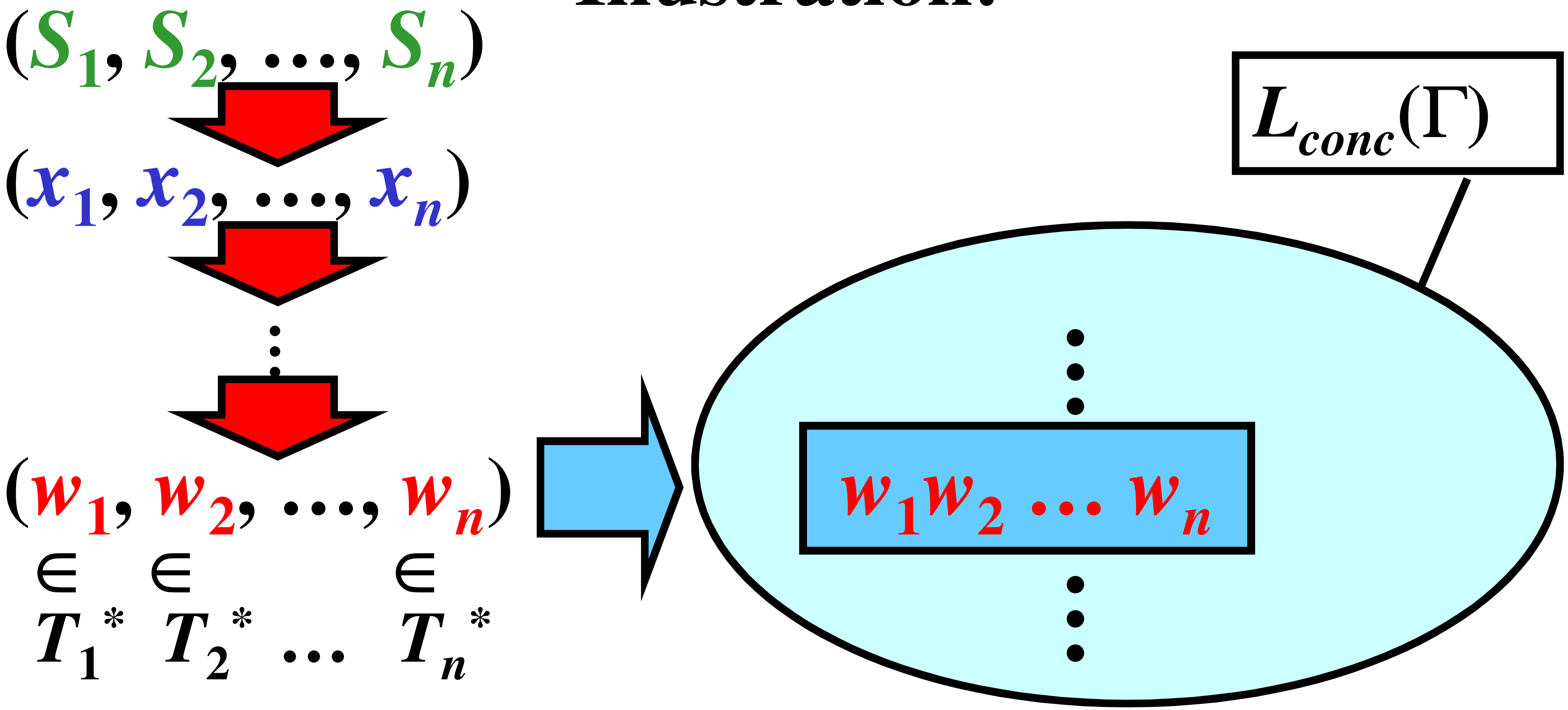
Illustration:



Generated Language in *Conc. Mode*

Definition: The *language generated by* Γ *in the concatenation mode*, $L_{conc}(\Gamma)$, is defined as:
$L_{conc}(\Gamma) = \{w_1w_2\ldots w_n: (w_1, w_2, \ldots, w_n) \in n\text{-}L(\Gamma)\}$

Illustration:



n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

S_1 S_2

- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{ S_1 \rightarrow aS_1, S_1 \rightarrow aA_1,$
$A_1 \rightarrow bA_1c, A_1 \rightarrow bc \}$

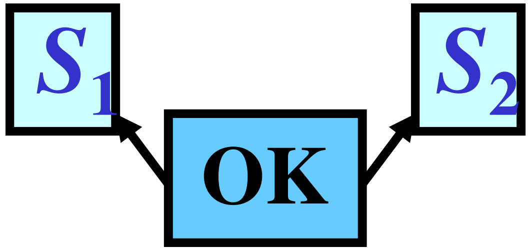
- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\}$,
$T_2 = \{d\}$,
$R_2 = \{ S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2,$
$A_2 \rightarrow d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$

n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:



- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,

$T_1 = \{a, b, c\}$,

$R_1 = \{ S_1 \rightarrow aS_1, S_1 \rightarrow aA_1,$
$A_1 \rightarrow bA_1c, A_1 \rightarrow bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\}$,

$T_2 = \{d\}$,

$R_2 = \{ S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2,$
$A_2 \rightarrow d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$

n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

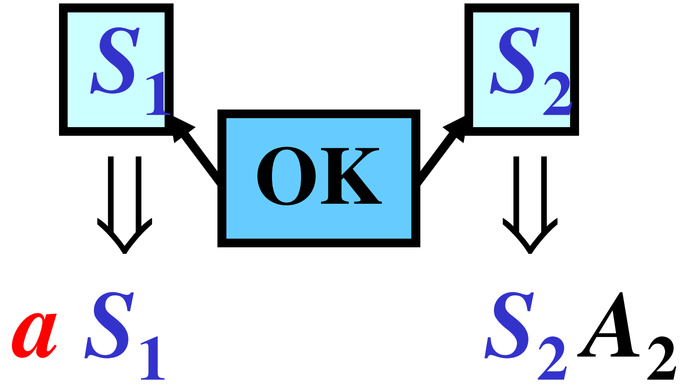
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{ S_1 \rightarrow aS_1, S_1 \rightarrow aA_1, A_1 \rightarrow bA_1c, A_1 \rightarrow bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\}$,
$T_2 = \{d\}$,
$R_2 = \{ S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2, A_2 \rightarrow d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$



n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

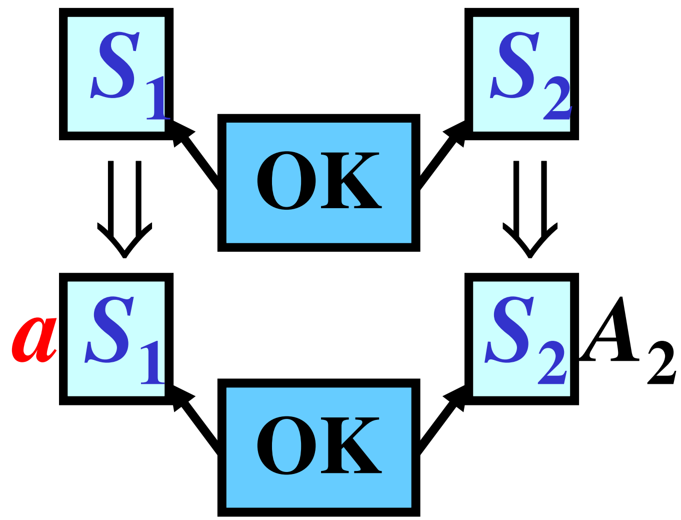
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{ S_1 \rightarrow aS_1, S_1 \rightarrow aA_1, A_1 \rightarrow bA_1c, A_1 \rightarrow bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\}$,
$T_2 = \{d\}$,
$R_2 = \{ S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2, A_2 \rightarrow d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$



n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

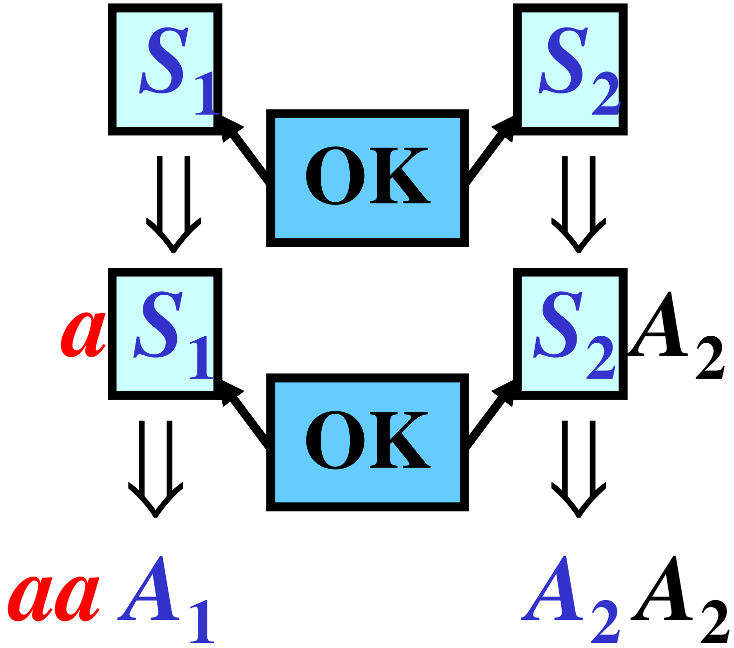
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{ S_1 \rightarrow aS_1, S_1 \rightarrow aA_1,$
$A_1 \rightarrow bA_1c, A_1 \rightarrow bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\}$,
$T_2 = \{d\}$,
$R_2 = \{ S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2,$
$A_2 \rightarrow d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$



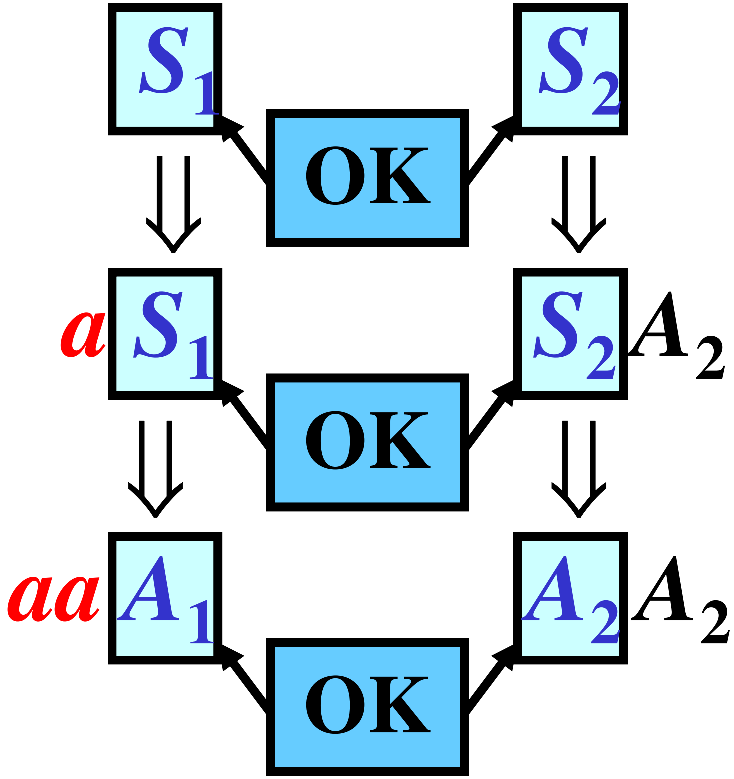
n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

- $G_1 = (N_1, T_1, R_1, S_1)$
$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{ S_1 \to aS_1, S_1 \to aA_1, A_1 \to bA_1c, A_1 \to bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$
$N_2 = \{S_2, A_2\}$,
$T_2 = \{d\}$,
$R_2 = \{ S_2 \to S_2A_2, S_2 \to A_2, A_2 \to d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$



n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

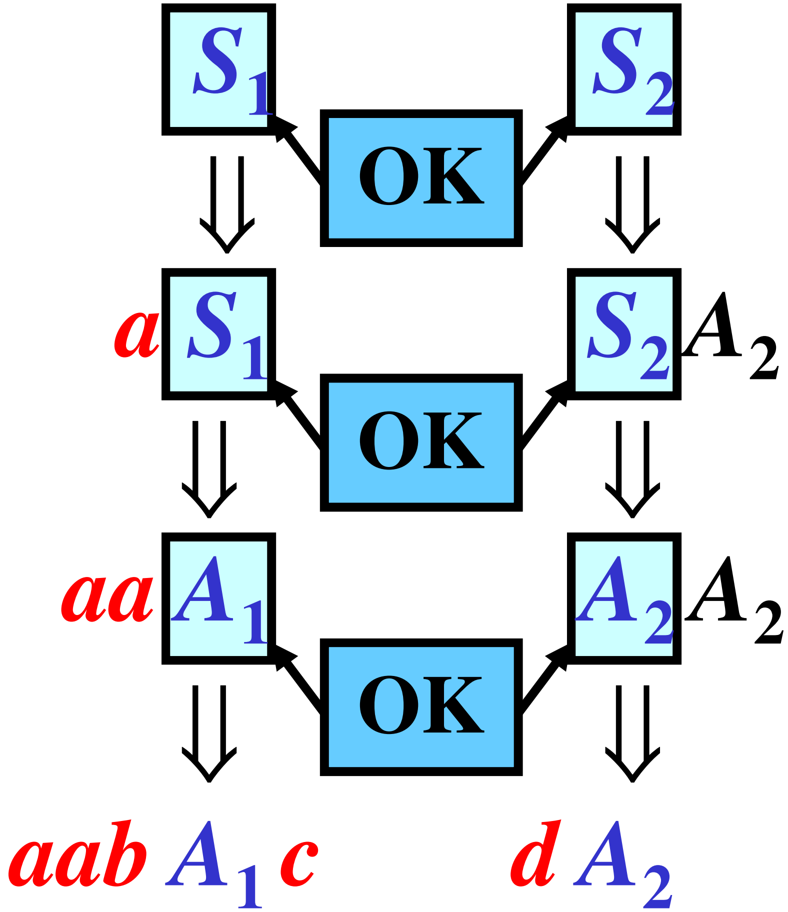
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{ S_1 \rightarrow aS_1, S_1 \rightarrow aA_1, A_1 \rightarrow bA_1c, A_1 \rightarrow bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\}$,
$T_2 = \{d\}$,
$R_2 = \{ S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2, A_2 \rightarrow d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$



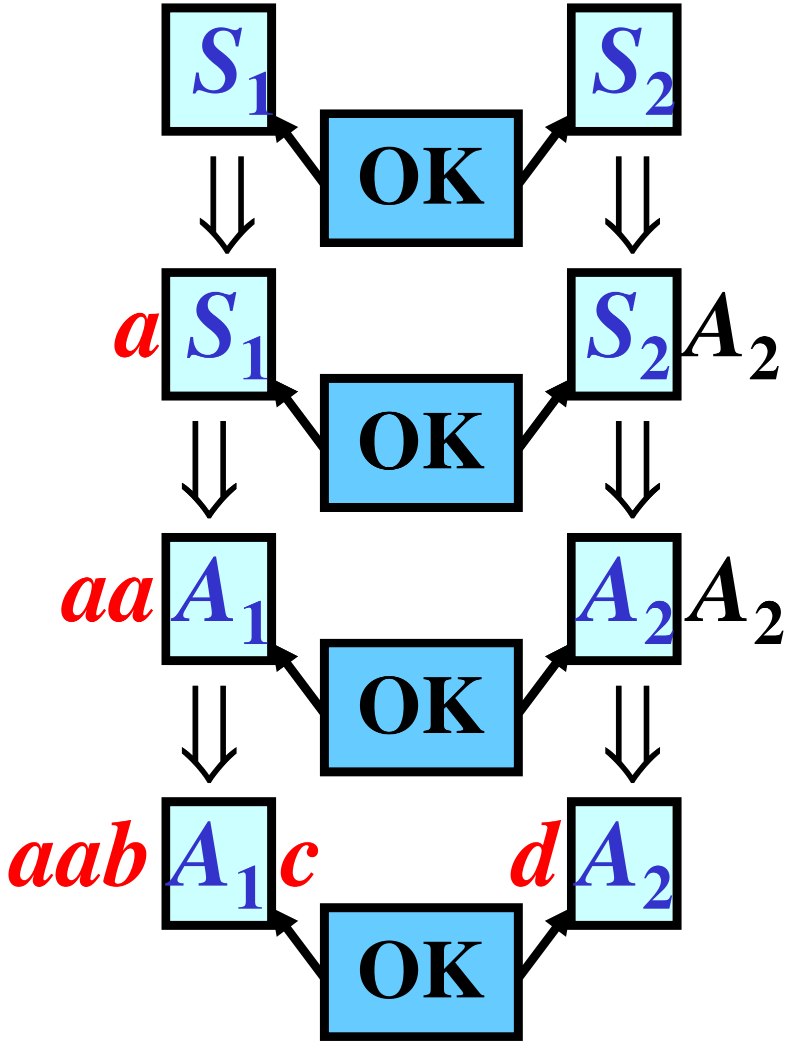
n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

- $G_1 = (N_1, T_1, R_1, S_1)$
  $N_1 = \{S_1, A_1\}$,
  $T_1 = \{a, b, c\}$,
  $R_1 = \{ S_1 \to aS_1, S_1 \to aA_1,$
  $A_1 \to bA_1c, A_1 \to bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$
  $N_2 = \{S_2, A_2\}$,
  $T_2 = \{d\}$,
  $R_2 = \{ S_2 \to S_2A_2, S_2 \to A_2,$
  $A_2 \to d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$



n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,

$T_1 = \{a, b, c\}$,

$R_1 = \{ S_1 \to aS_1, S_1 \to aA_1, A_1 \to bA_1c, A_1 \to bc \}$

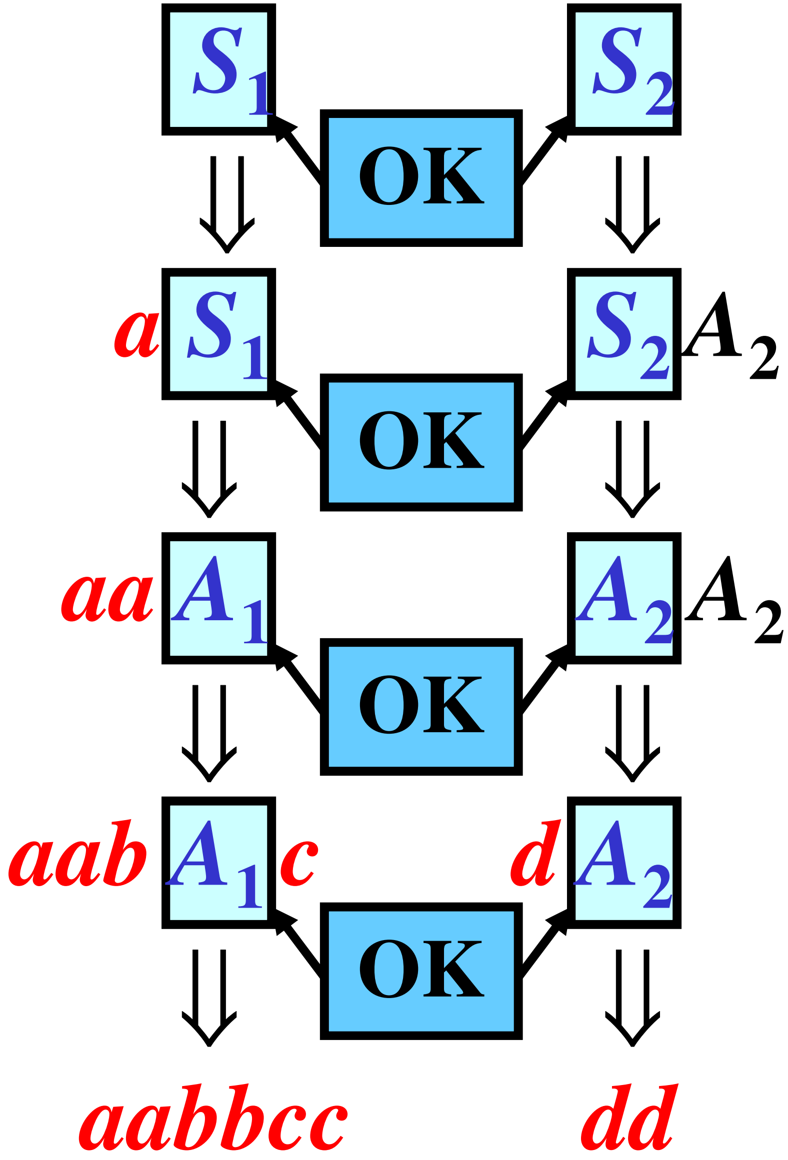
- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\}$,

$T_2 = \{d\}$,

$R_2 = \{ S_2 \to S_2A_2, S_2 \to A_2, A_2 \to d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$



n-MGN: Example 2/2

$\Gamma = (G_1, G_2, Q)$, where:

- $G_1 = (N_1, T_1, R_1, S_1)$
  $N_1 = \{S_1, A_1\}$,
  $T_1 = \{a, b, c\}$,
  $R_1 = \{ S_1 \rightarrow aS_1, S_1 \rightarrow aA_1, A_1 \rightarrow bA_1c, A_1 \rightarrow bc \}$

- $G_2 = (N_2, T_2, R_2, S_2)$
  $N_2 = \{S_2, A_2\}$,
  $T_2 = \{d\}$,
  $R_2 = \{ S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2, A_2 \rightarrow d \}$

- $Q = \{(S_1, S_2), (A_1, A_2)\}$

Note:

$$2\text{-}L(\Gamma) = \{(a^nb^nc^n, d^n) : n \geq 1\}$$

$$L(\Gamma)_{union} = \{a^nb^nc^n : n \geq 1\} \cup \{d^n : n \geq 1\}$$

$$L(\Gamma)_{conc} = \{a^nb^nc^nd^n : n \geq 1\}$$

$$L(\Gamma)_{first} = \{a^nb^nc^n : n \geq 1\}$$

Rule-synchronized GS

Definition: *An n-multigenerative rule-synchronized grammar system* (*n-MGR*) is $n+1$ tuple

$$\Gamma = (G_1, G_2, \dots, G_n, Q)$$, where

- $G_i = (N_i, T_i, P_i, S_i)$ is a CFG for all $i = 1 \dots n$
- Q = is a finite set of n-tuples of the form $(p_1, p_2, \dots, p_n)$, where $p_i \in P_i$ for all $i = 1 \dots n$

Example:

$\Gamma = (G_1, G_2, \{(1, 1), (2, 2), (3, 3), (4, 3)\})$, where:

$G_1 = (\{S_1, A_1\}, \{a, b, c\}, R_1, S_1)$;

$R_1 = \{1: S_1 \rightarrow aS_1, \quad 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c, \; 4: A_1 \rightarrow bc\}$

$G_2 = (\{S_2\}, \{d\}, R_2, S_2)$;

$R_2 = \{1: S_2 \rightarrow S_2S_2, \; 2: S_2 \rightarrow S_2, \; 3: S_2 \rightarrow d\}$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \ldots, G_n, Q)$ be a n-MGR.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, p_i: $A_i \rightarrow x_i \in P_i$
for all $i = 1..n$ Then, $(u_1A_1v_1, u_2A_2v_2, \ldots, u_nA_nv_n) \Rightarrow$
$(u_1x_1v_1, u_2x_2v_2, \ldots, u_nx_nv_n)$ if $(p_1, p_2, \ldots, p_n) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$
$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(p_1, \ldots, p_n) \in Q$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \dots , G_n, Q)$ be a n-MGR.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, p_i: $A_i \rightarrow x_i \in P_i$
for all $i = 1..n$ Then, $(u_1A_1v_1, u_2A_2v_2, \dots, u_nA_nv_n) \Rightarrow$
$(u_1x_1v_1, u_2x_2v_2, \dots, u_nx_nv_n)$ if $(p_1, p_2, \dots, p_n) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$
$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(p_1, \dots, p_n) \in Q$

Rule: p_1: $A_1 \rightarrow x_1 \in P_1$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \dots, G_n, Q)$ be a n-MGR.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, p_i: $A_i \rightarrow x_i \in P_i$
for all $i = 1..n$ Then, $(u_1A_1v_1, u_2A_2v_2, \dots, u_nA_nv_n) \Rightarrow$
$(u_1x_1v_1, u_2x_2v_2, \dots, u_nx_nv_n)$ if $(p_1, p_2, \dots, p_n) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$

$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(p_1, \dots, p_n) \in Q$

Rule: p_1: $A_1 \rightarrow x_1 \in P_1$

Rule: p_n: $A_n \rightarrow x_n \in P_n$

Direct Derivation Step

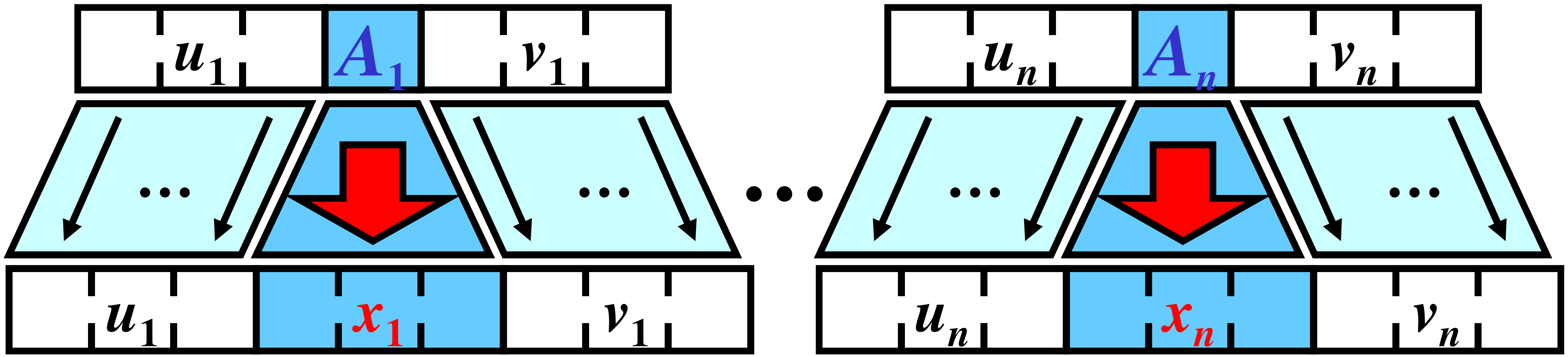
Definition: Let $\Gamma = (G_1, G_2, \ldots, G_n, Q)$ be a n-MGR.
Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, p_i: $A_i \rightarrow x_i \in P_i$ for all $i = 1..n$ Then, $(u_1A_1v_1, u_2A_2v_2, \ldots, u_nA_nv_n) \Rightarrow (u_1x_1v_1, u_2x_2v_2, \ldots, u_nx_nv_n)$ if $(p_1, p_2, \ldots, p_n) \in Q$.

Note: $\Rightarrow^+$ … transitive closure of $\Rightarrow$

$\Rightarrow^*$ … reflexive and transitive closure of $\Rightarrow$

Illustration: $(p_1, \ldots, p_n) \in Q$

Rule: p_1: $A_1 \rightarrow x_1 \in P_1$ **Rule:** p_n: $A_n \rightarrow x_n \in P_n$



Generated Multistrings & Language

Definition: The *n*-Language for n-MGR is defined analogically as the *n*-Language for n-MGN.

Definition: The language generated by n-MGR in the *X* mode, for each $X \in \{union, conc, first\}$, is defined analogically as a language generated by n-MGN in the *X* mode.

n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

S_1 S_2

- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$

n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$

S_1 $\Downarrow$ 1

S_2 1 $\Downarrow$

n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

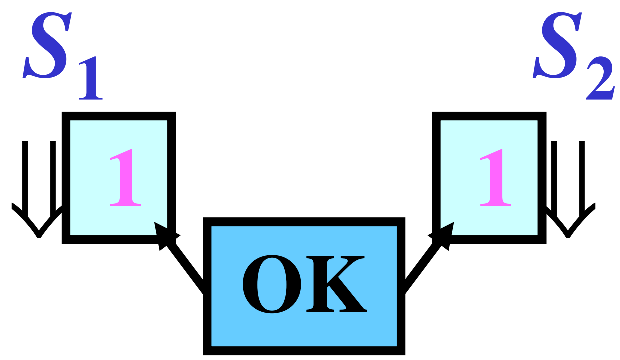
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,\ 3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,\ 3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

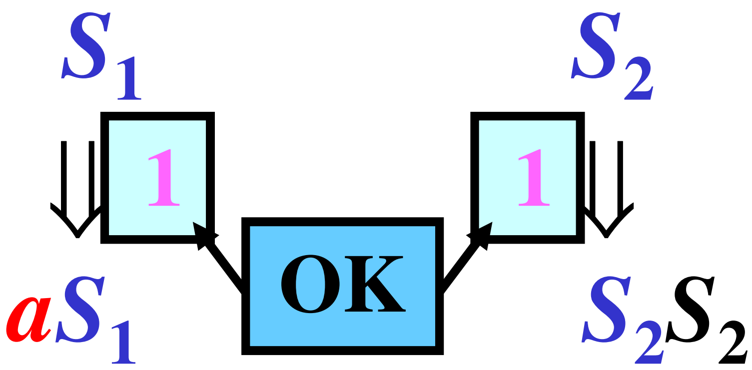
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

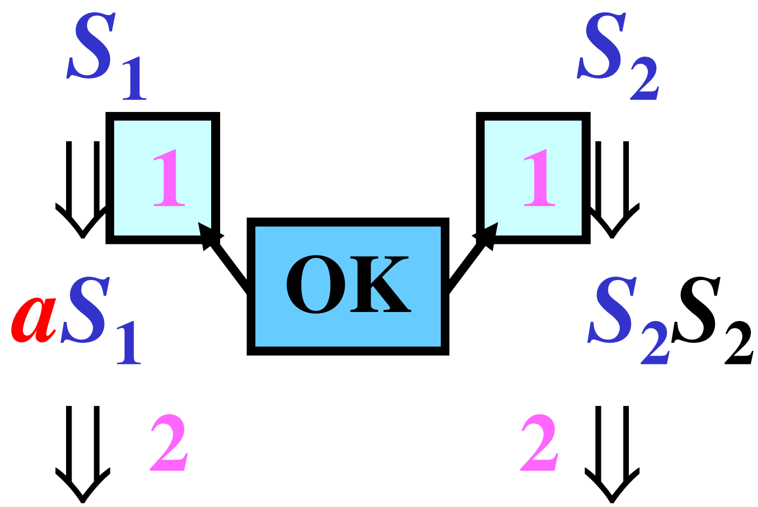
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

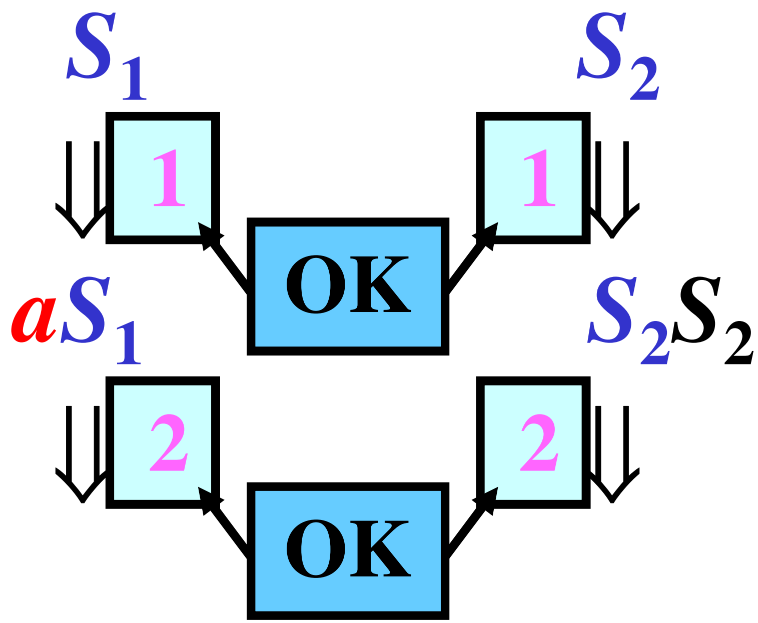
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

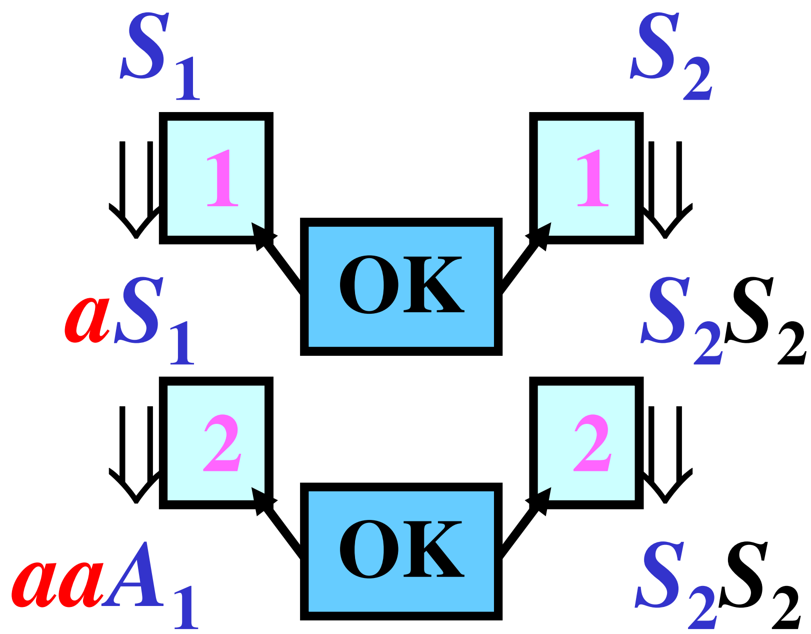
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

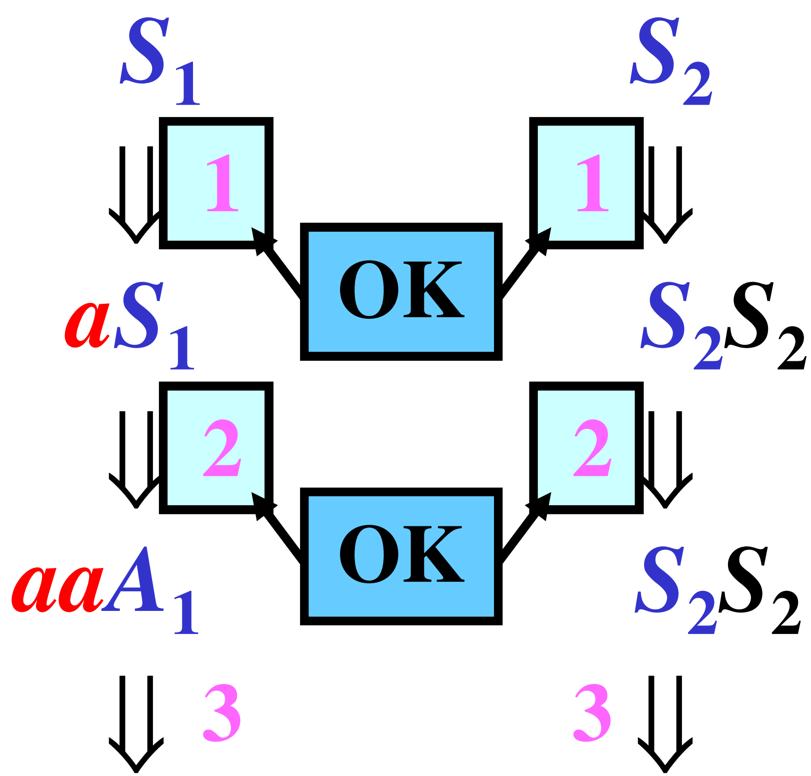
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

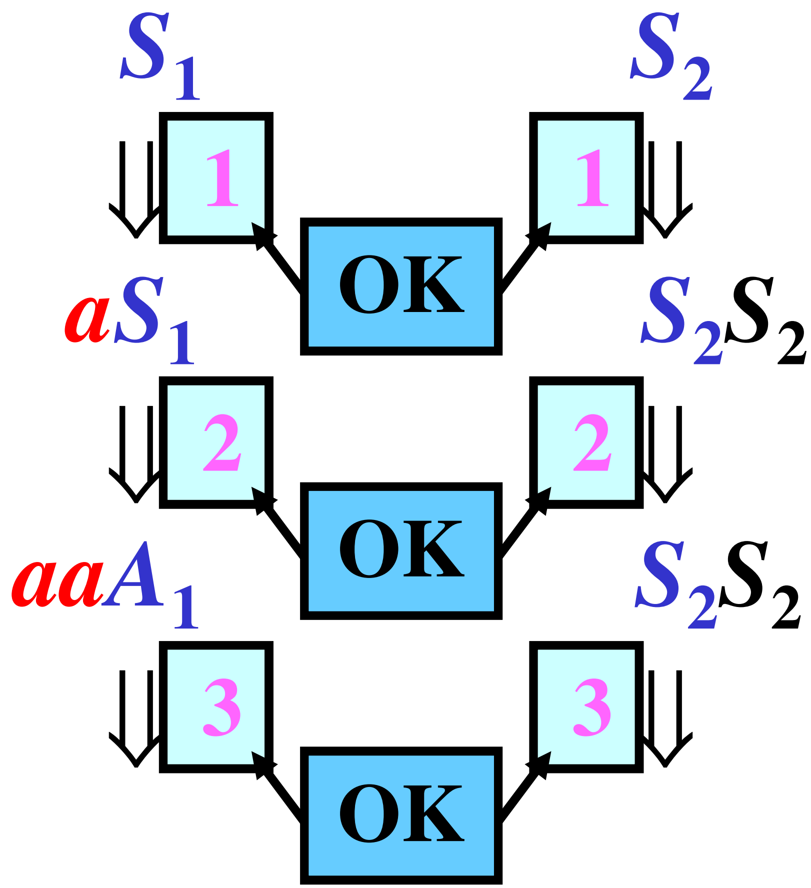
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,\ 3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,\ 3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

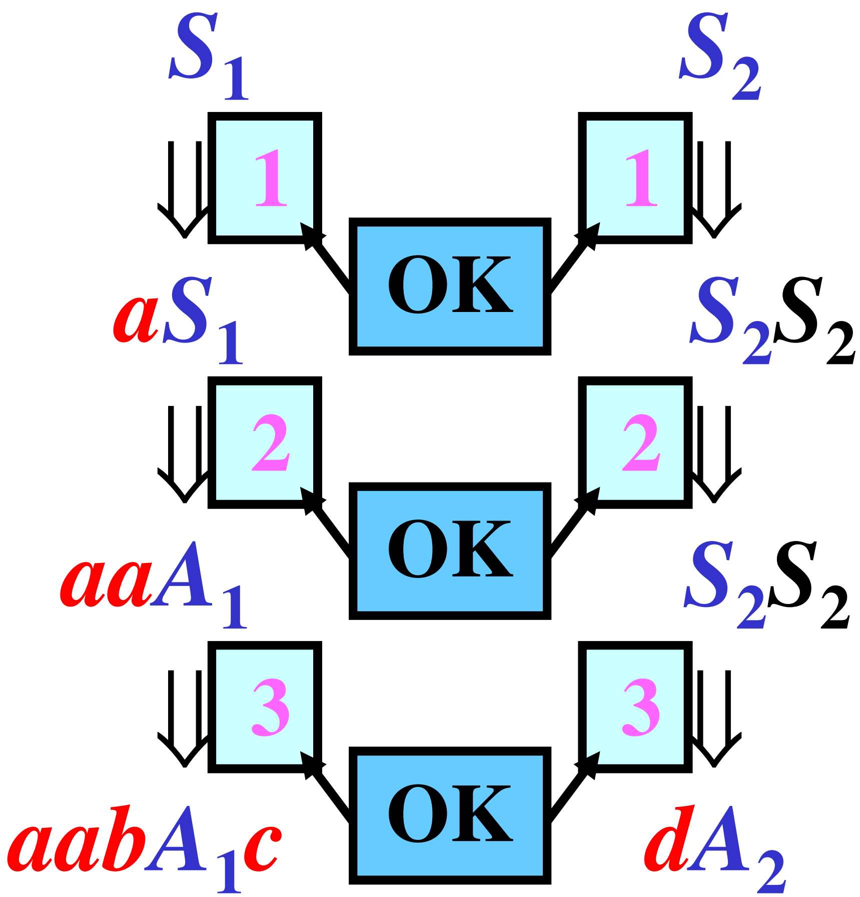
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,$
$3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,$
$3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

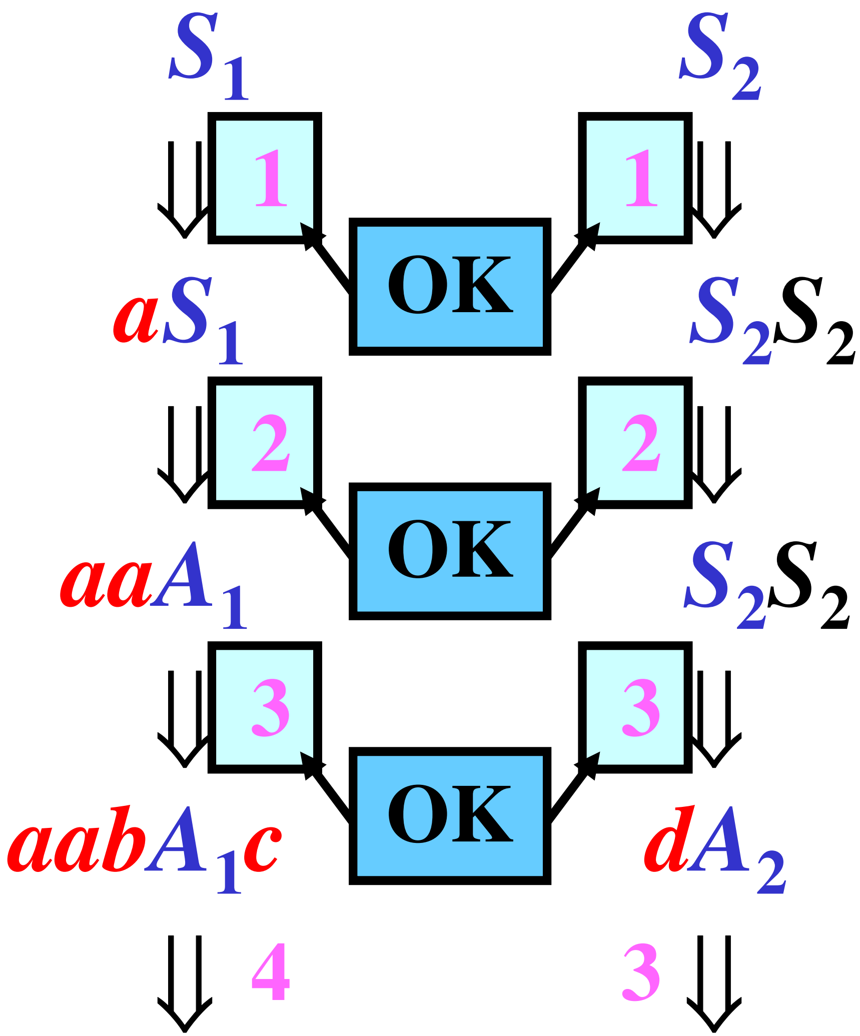
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,\ 3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,\ 3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

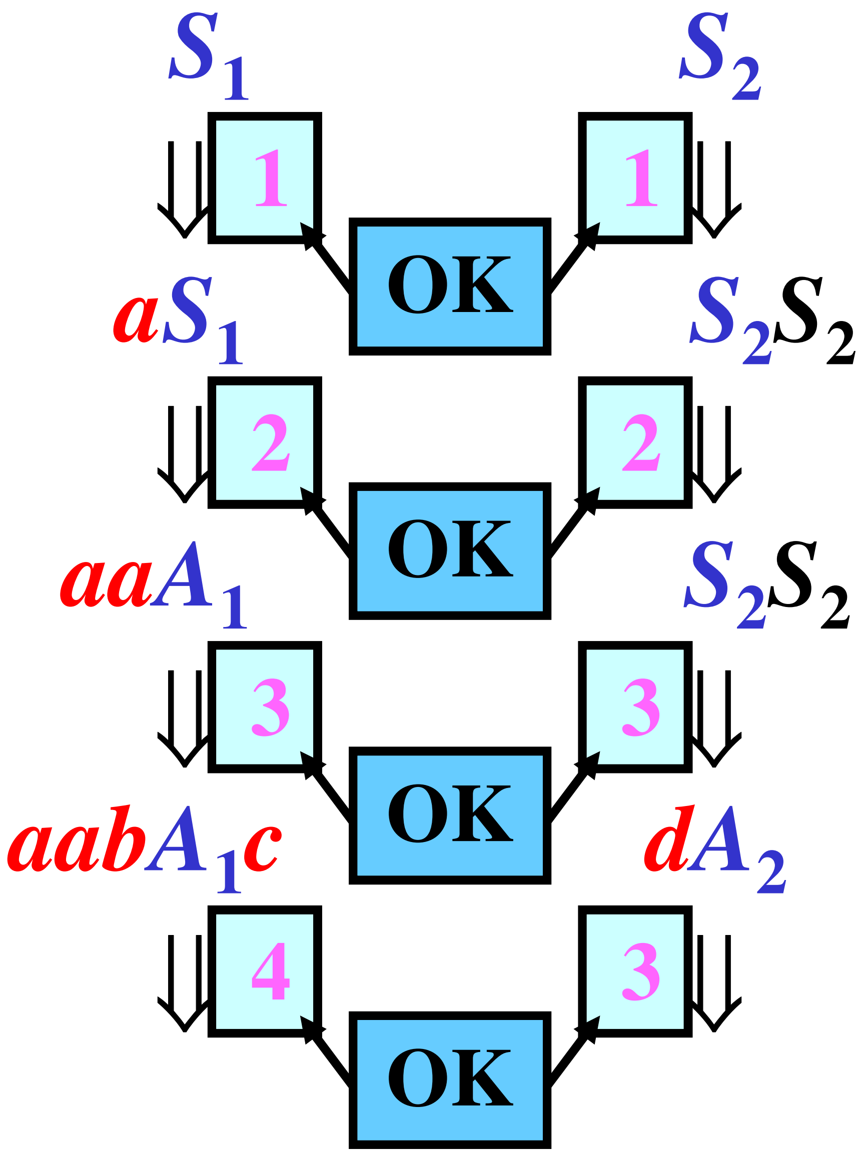
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,\ 3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,\ 3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

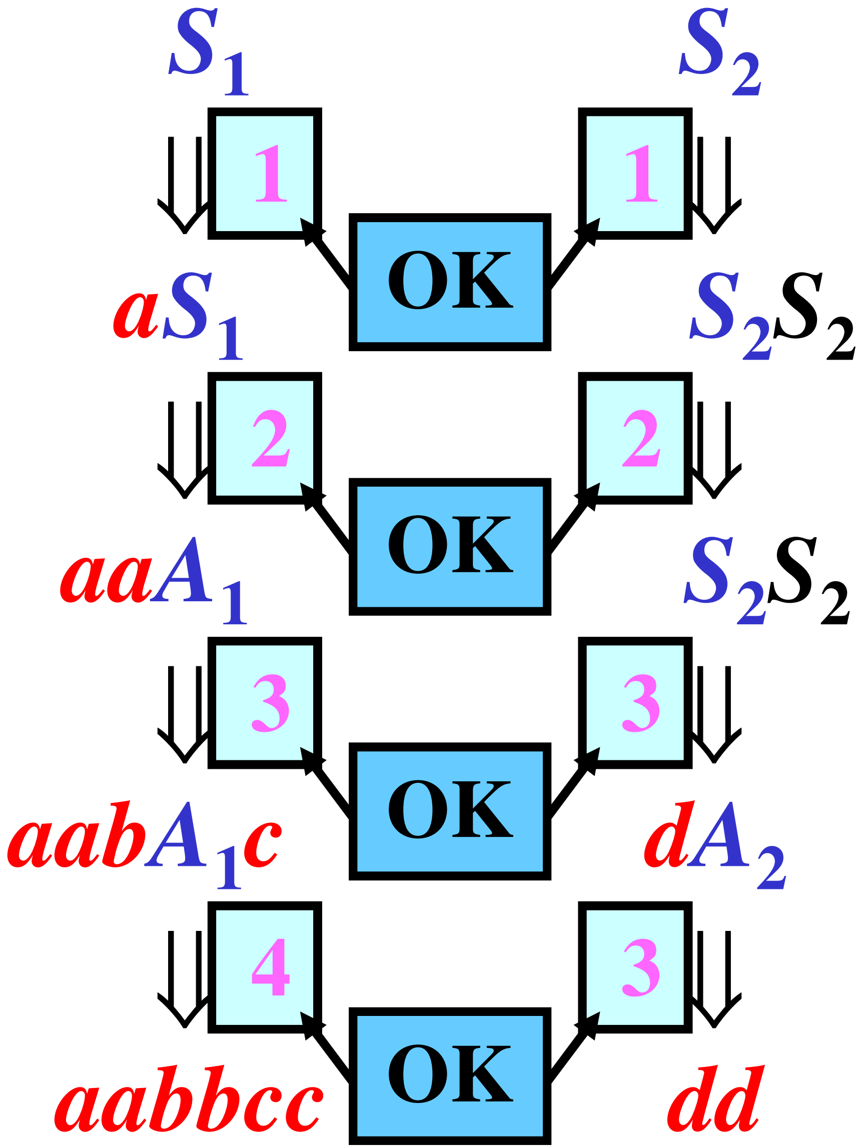
- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1: S_1 \rightarrow aS_1,\ 2: S_1 \rightarrow aA_1,\ 3: A_1 \rightarrow bA_1c,\ 4: A_1 \rightarrow bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1: S_2 \rightarrow S_2S_2,\ 2: S_2 \rightarrow S_2,\ 3: S_2 \rightarrow d\}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 2/2

$\Gamma = (G_1, G_2, Q)$, where:

- $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\}$,
$T_1 = \{a, b, c\}$,
$R_1 = \{1\text{: } S_1 \to aS_1,\ 2\text{: } S_1 \to aA_1,$
$3\text{: } A_1 \to bA_1c,\ 4\text{: } A_1 \to bc\}$

- $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\}$,
$T_2 = \{d\}$,
$R_2 = \{1\text{: } S_2 \to S_2S_2,\ 2\text{: } S_2 \to S_2,$
$3\text{: } S_2 \to d\ \}$

- $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$

Note:

$n\text{-}L(\Gamma) = \{(a^nb^nc^n, d^n) : n \geq 1\}$

$L(\Gamma)_{union} = \{a^nb^nc^n : n \geq 1\} \cup \{d^n : n \geq 1\}$

$L(\Gamma)_{conc} = \{a^nb^nc^nd^n : n \geq 1\}$

$L(\Gamma)_{first} = \{a^nb^nc^n : n \geq 1\}$

Results: Conversions Between MGSs

- **Theorem:** There exist an algorithm that converts any n-MGN to an equivalent n-MGR in the *X* mode, where *X* = {*union*, *conc*, *first*}

- **Theorem:** There exist an algorithm that converts any n-MGR to an equivalent n-MGN in the *X* mode, where *X* = {*union*, *conc*, *first*}

Illustration:

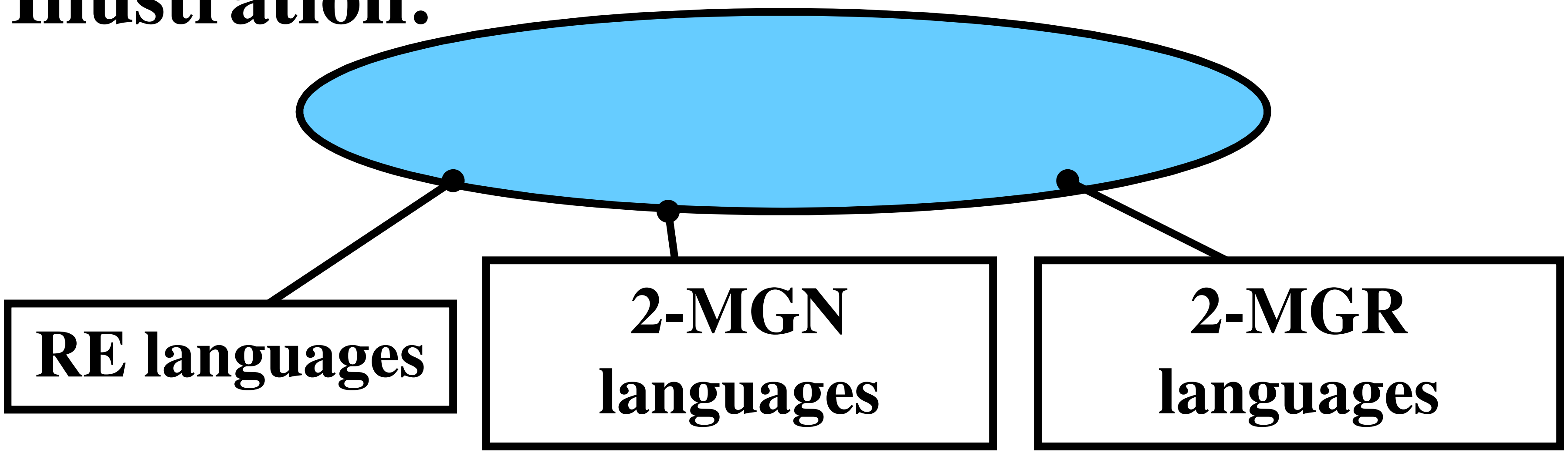
n-MGN ⇄ n-MGR

Results: Power of MGSs

- **Theorem:** Let L(n-MGN$_X$) and L(n-MGR$_X$) denote the language families defined by n-MGN in the X mode and n-MGR in the X mode resp., where $X = \{union, conc, first\}$. Then,

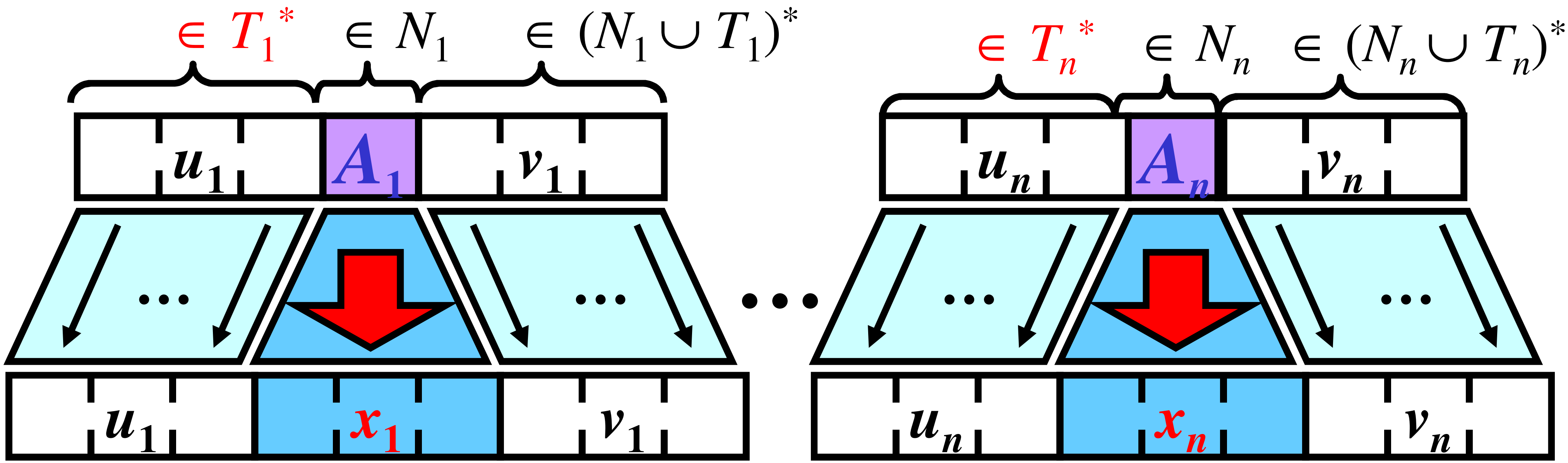
$$L(\mathbf{RE}) = L(\mathbf{2\text{-}MGR}_X) = L(\mathbf{2\text{-}MGN}_X)$$

Illustration:

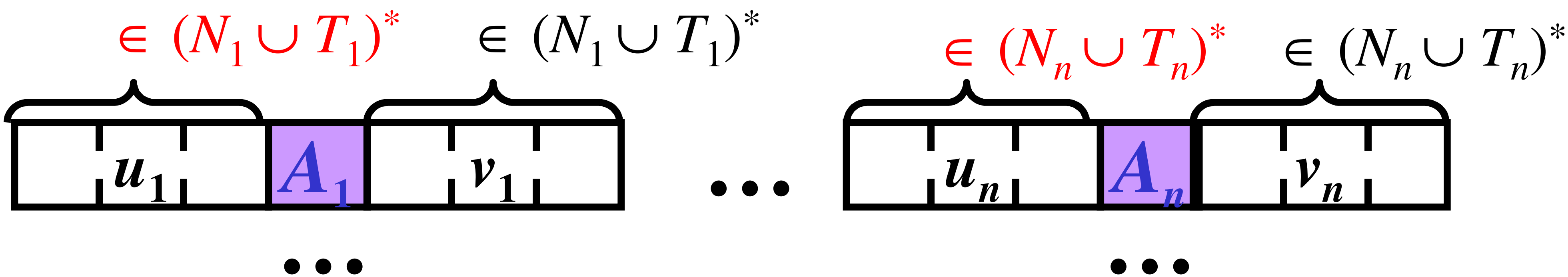


General n-MGNs and n-MGRs

- **leftmost derivations in n-MGNs/n-MGRs:**



- **general derivations in <u>General</u> n-MGNs/n-MGRs:**



Matrix Grammar

Definition: A *matrix grammar* (MG) is a pair

$$H = (G, M), \text{ where:}$$

- $G = (N, T, P, S)$ is a CFG
- M is a finite language over P ($M \subseteq P^*$)

Example:

$H = (G, M)$, **where:**

- $G = (\{S, A, B\}, \{a, b, c\}, P, S)$;

  $P = \{1: S \to AB,\ 2: A \to aA,\ 3: B \to bBc,$
  $4: A \to a,\ 5: B \to bc\}$
- $M = \{1, 23, 45\}$

Matrix Grammar: Example

$H = (G, M)$, **where:**

- $G = (\{S, A, B\}, \{a, b, c\}, P, S)$;

  $P = \{ 1: S \rightarrow AB, 2: A \rightarrow aA, 3: B \rightarrow bBc, 4: A \rightarrow a, 5: B \rightarrow bc \}$

- $M = \{1, 23, 45\}$

$\underline{S} \Rightarrow \underline{AB}\ [1] \Rightarrow abc\ [45]$

$\underline{S} \Rightarrow \underline{AB}\ [1] \Rightarrow a\underline{A}b\underline{B}c\ [23] \Rightarrow aabbcc\ [45]$

...

$$L(H) = \{a^n b^n c^n : n \geq 1\}$$

MG and General n-MGNs & n-MGRs: Conversions

• **Theorem:** For every general n-MGN & n-MGR in the *X* mode, where *X* = {*union*, *conc*, *first*}, there exists an equivalent matrix grammar.

• **Theorem:** For every matrix grammar, there exists an equivalent n-MGN & n-MGR in the *X* mode, where *X* = {*union*, *conc*, *first*}

Illustration:

General n-MGN / General n-MGR → MG → General 2-MGN / General 2-MGR

Results: Power of General MGSs

- **Theorem:** Let $L(\text{n-GMGN}_X)$ and $L(\text{n-GMGR}_X)$ denote the language families defined by general n-MGN in the X mode and general n-MGR in the X mode, respectively,where $X = \{union, conc, first\}$. Then,

$$L(\text{MG}) = L(\text{n-GMGR}_X) = L(\text{n-GMGN}_X)$$

Illustration: