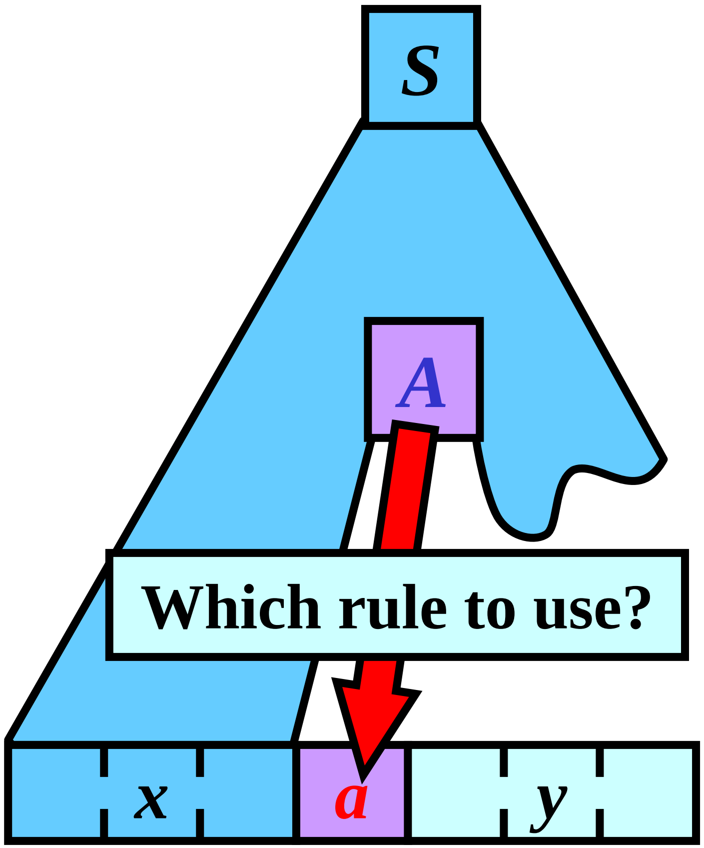
Part VIII.
Top-Down Parsing

Top-Down Parsing: Introduction

Problem:



Basic idea:

Table:

α	...	***a***	...
...			
A		$\alpha(A, a)$	
...			

Use rule r: $A \to x$

Question: Could you construct this table for **any** CFG?

Answer: **NO**

A Table-Based Selection of a Rule

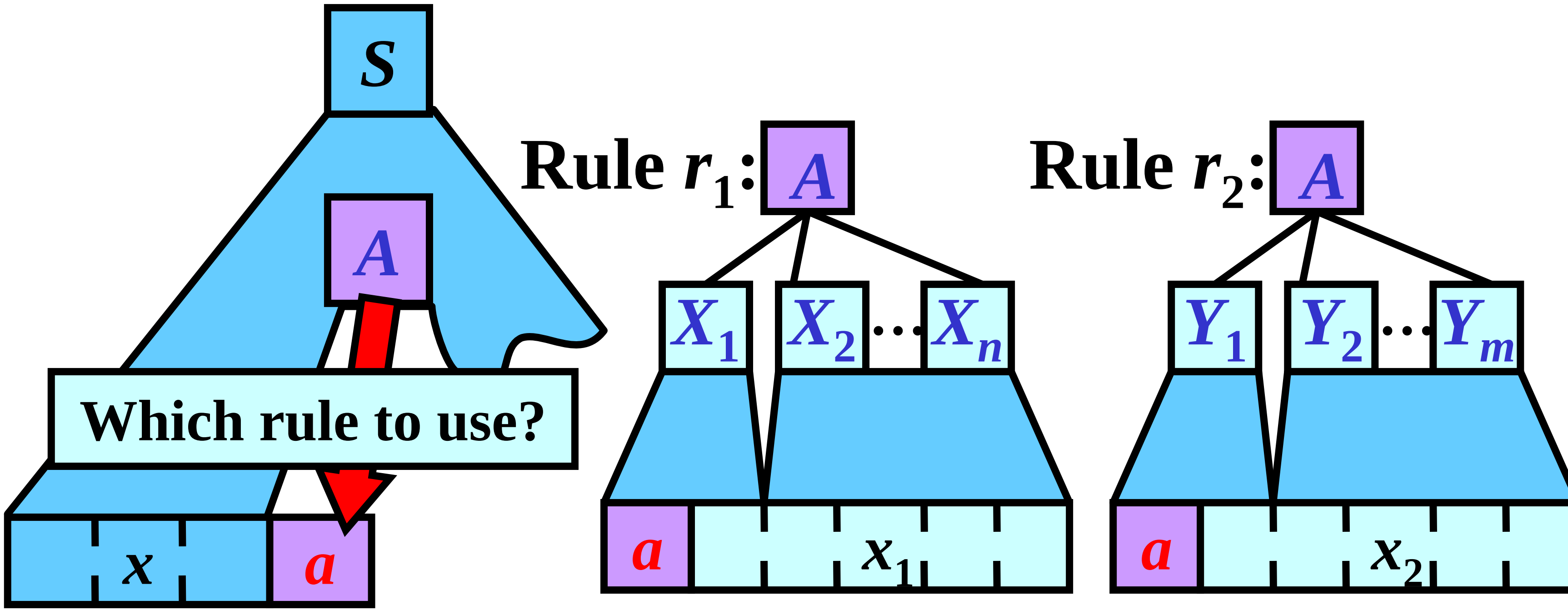


Table:

α	...	a	...
...			
A		$\alpha(A, a)$	
...			

?

Use rule r_1: $A \to X_1X_2\ldots X_n$

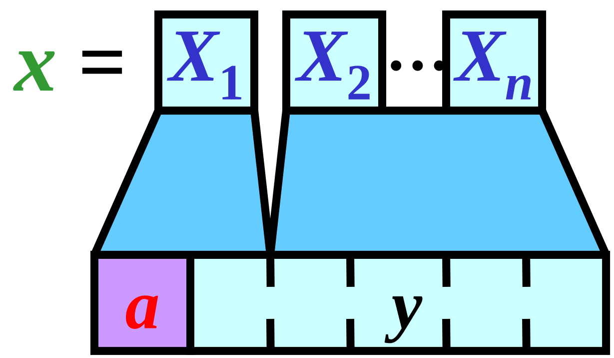
Use rule r_2: $A \to Y_1Y_2\ldots Y_m$

Set *First*

Gist: *First*(x) is the set of all terminals that can begin a sentential form derivable from x.

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $x \in (N \cup T)^*$, we define the set $First(x)$ as $First(x) = \{a: a \in T, x \Rightarrow^* ay; y \in (N \cup T)^*\}$.

Illustration:



$$x = X_1X_2 \ldots X_n \Rightarrow^* ay$$

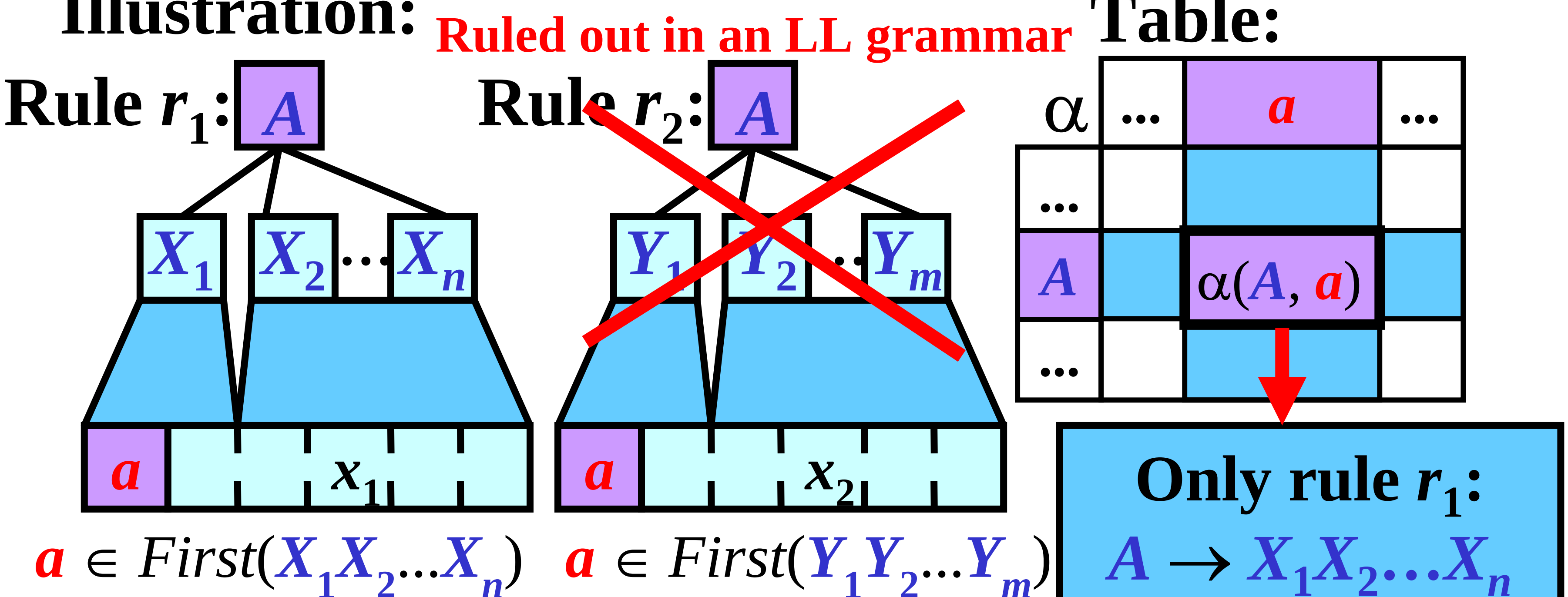
$$a \in First(x)$$

LL Grammars without ε-rules

Definition: Let $G = (N, T, P, S)$ be a CFG <u>without ε-rules</u>. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** rule $A \rightarrow X_1X_2...X_n \in P$ such that $a \in First(X_1X_2...X_n)$

Illustration:

Table:



Simple *P*rogramming *L*anguage (SPL)

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** <item> <it-list>
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

Note: G_{SPL} **is LL grammar**

Example:

```
begin
   read i;
   j := add(i, 1);
   write j;
end
```

$\in$ SPL

Algorithm: *First*(X)

- **Input:** $G = (N, T, P, S)$ **<u>without ε-rules</u>**
- **Output:** *First*(X) for every $X \in N \cup T$

- **Method:**
- for each $a \in T$: *First*(a) := $\{a\}$
- **Apply the following rule until no *First* set can be changed:**
- if $A \to X_1X_2\ldots X_n \in P$, then add *First*($X_1$) to *First*($A$)

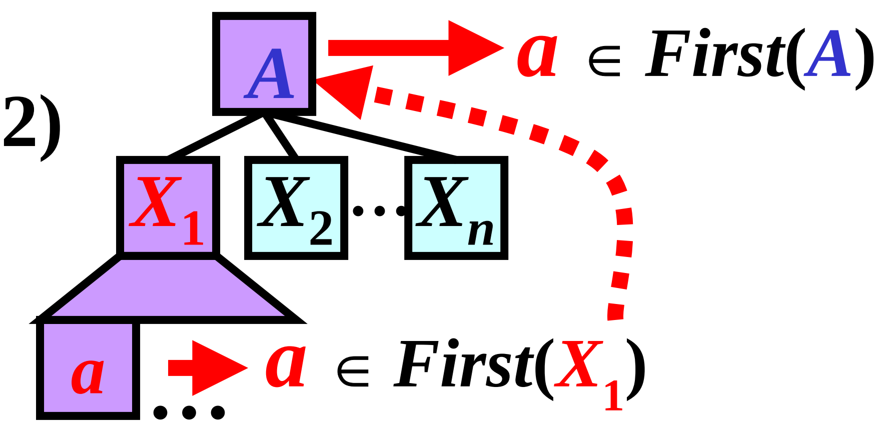
Illustration:

1) for each $a \in T$:

***First*(a) := $\{a\}$**

because $a \Rightarrow^0 a$

2)



First(*X*) for SPL: Example

First(**begin**) := {**begin**}	*First*(**id**) := {**id**}	*First*(**,**) := {**,**}
First(**end**) := {**end**}	*First*(**int**) := {**int**}	*First*(**(**) := {**(**}
First(**read**) := {**read**}	*First*(**:=**) := {**:=**}	*First*(**)**) := {**)**}
First(**write**) := {**write**}	*First*(**add**) := {**add**}	*First*(**;**) := {**;**}

<item> → **id** ∈ *P*: **add** *First*(**id**) **to** *First*(**<item>**)
<item> → **int** ∈ *P*: **add** *First*(**int**) **to** *First*(**<item>**)
Summary: *First*(**<item>**) = {**id**, **int**}

<it-list> → **)** ∈ *P*: **add** *First*(**)**) **to** *First*(**<it-list>**)
<it-list> → **,** … ∈ *P*: **add** *First*(**,**) **to** *First*(**<it-list>**)
Summary: *First*(**<it-list>**) = {**)**, **,**}

<stat> → **id** … ∈ *P*: **add** *First*(**id**) **to** *First*(**<stat>**)
<stat> → **write** … ∈ *P*: **add** *First*(**write**) **to** *First*(**<stat>**)
<stat> → **read** … ∈ *P*: **add** *First*(**read**) **to** *First*(**<stat>**)
Summary: *First*(**<stat>**) = {**id**, **write**, **read**}

<st-list> → **end** ∈ *P*: **add** *First*(**end**) **to** *First*(**<st-list>**)
<st-list> → **<stat>** … ∈ *P*: **add** *First*(**<stat>**) **to** *First*(**<st-list>**)
Summary: *First*(**<st-list>**) = {**id**, **write**, **read**, **end**}

<prog> → **begin** … ∈ *P*: **add** *First*(**begin**) **to** *First*(**<prog>**)
Summary: *First*(**<prog>**) = {**begin**}

Construction of LL Table

α	...	*a*	...
...			
A		α(*A*, *a*)	
...			

$\alpha(A, a) = A \rightarrow X_1X_2\ldots X_n \in P$ if $a \in First(X_1)$; otherwise, $\alpha(A, a)$ is blank ⇒ **ERROR**

Task: LL table for SPL

	id	**int**	**:=**	**⋯**
<prog>				
<st-list>	2			
<stat>	6			
<it-list>				
<item>	10			

id ∈ ***First***(**<stat>**)

id ∈ ***First***(**id**)

id ∈ ***First***(**id**)

Rule r**:** $A \rightarrow X_1X_2\ldots X_n$	$First(X_1)$
1: <prog> → begin …	{begin}
2: <st-list> → <stat>…	{id, write, read}
3: <st-list> → end	{end}
4: <stat> → read …	{read}
5: <stat> → write …	{write}
6: <stat> → id …	{id}
7: <it-list> → , …	{,}
8: <it-list> →)	{)}
9: <item> → int	{int}
10: <item> → id	{id}

Construct the rest analogically.

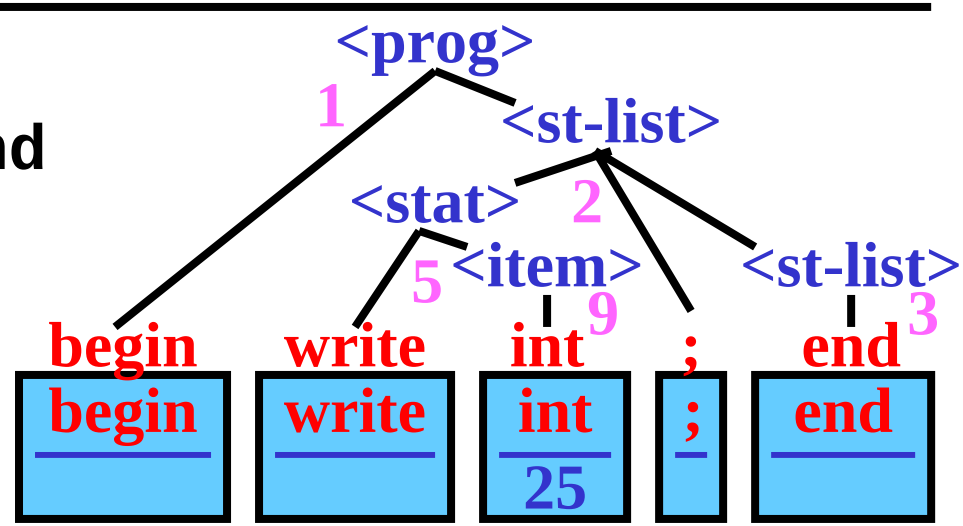
Parsing Based on LL Table: Example

1: <prog> → **begin** <st-list>
2: <st-list> → <stat> **;** <st-list>
3: <st-list> → **end**
4: <stat> → **read** **id**
5: <stat> → **write** <item>
6: <stat> → **id** **:=** **add** **(** …
7: <it-list> → **,** <item> <it-list>
8: <it-list> → **)**
9: <item> → **int**
10: <item> → **id**

	beg	end	rd	wr	id	int	,	(	)	;	:=	add
<prog>	1											
<st-list>		3	2	2	2							
<stat>			4	5	6							
<it-list>							7		8			
<item>					10	9						

Source program:

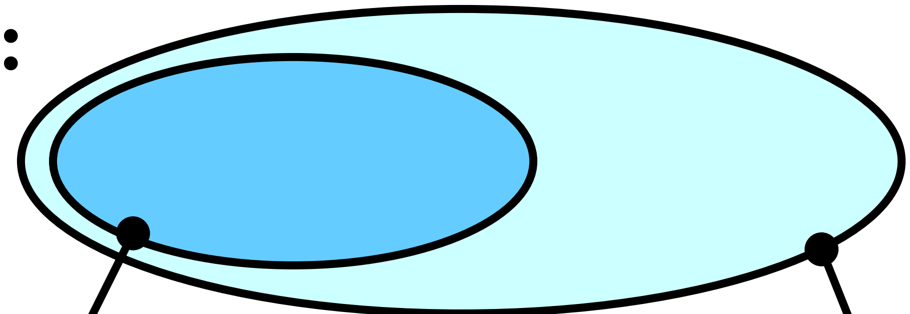
`begin write 25; end`



LL Grammars: Useful Transformations

Generaly: CFG are stronger that LL grammars

Illustration:



The family of languages generated by LL grammars $\subset$ **The family of languages generated by CFGs**

- **Some** CFGs can be converted to equivalent LL grammars

Basic conversions:

1) Factorization

2) Left recursion replacement

Note: A rule of the form $\boldsymbol{A} \to \boldsymbol{Ax}$, where $\boldsymbol{A} \in N$, $\boldsymbol{x} \in (N \cup T)^*$ is called a *left recursive rule*.

Factorization

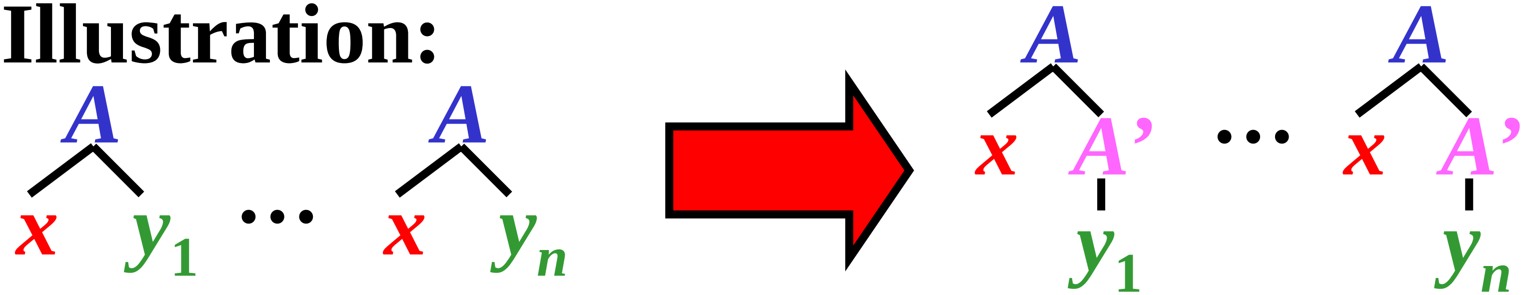
Idea: Replace rules of the form

$$A \to xy_1,\ A \to xy_2, \ldots, A \to xy_n \text{ with}$$

$$A \to xA',\ A' \to y_1,\ A' \to y_2, \ldots, A' \to y_n,$$

where A' is a new nonterminal

Illustration:



Example:

<stat> → write id
<stat> → write int

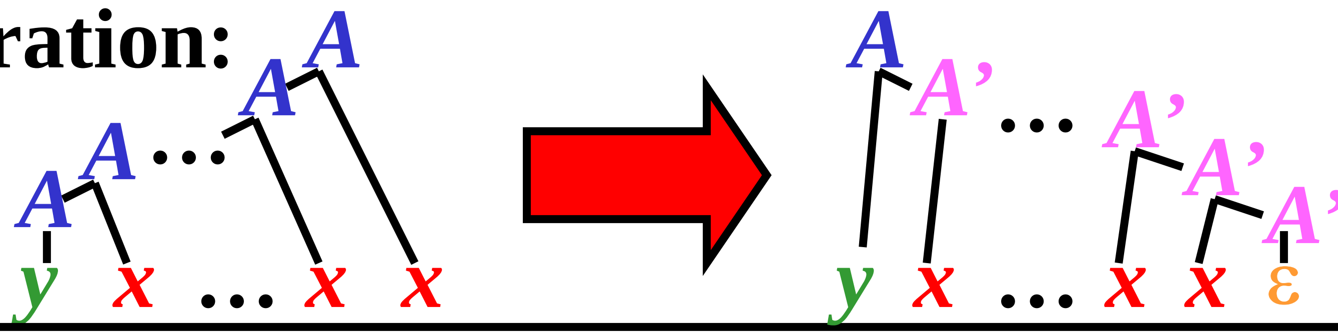
⇒

<stat> → write <item>
<item> → id
<item> → int

Left Recursion Replacement

Idea: Replace rules of the form $A \to Ax$, $A \to y$ with $A \to yA'$, $A' \to xA'$, $A' \to \varepsilon$, where A' is a new nonterminal.

Illustration:



Example:

$E \to E+T$
$E \to T$

⇒ $E \to TE'$, $E' \to +TE'$, $E' \to \varepsilon$

$T \to T*F$
$T \to F$

⇒ $T \to FT'$, $T' \to *FT'$, $T' \to \varepsilon$

$F \to (E)$
$F \to i$

$F \to (E)$
$F \to i$

LL Grammars with ε-rules: Introduction

Why ε-rules?

- elimination of the left recursion introduces ε-rule
- ε-rules often make the language specification clearer

Simplification of this part:

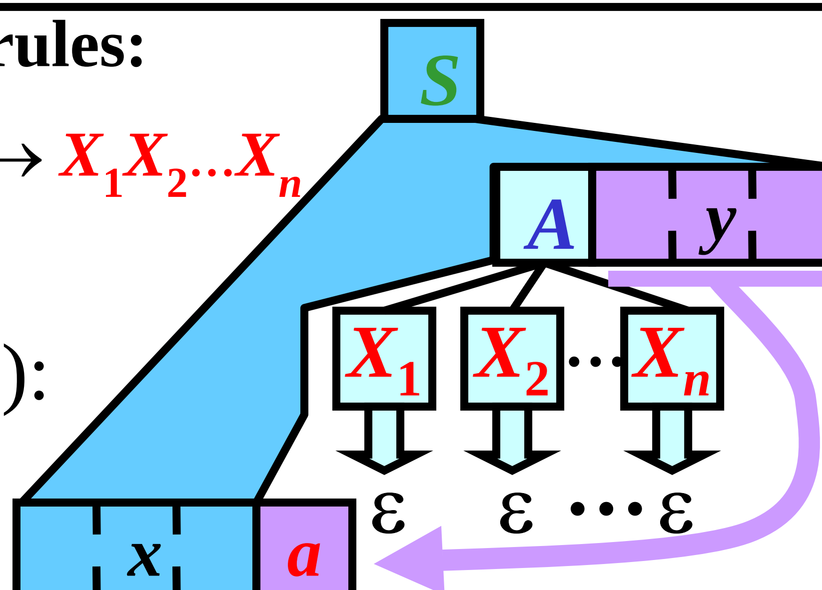
Assume that every input string of tokens ends with $.

Note: $ acts as an *end marker*.

Main problem with ε-rules:

Rule r: $A \rightarrow X_1X_2\ldots X_n$

Maybe: $a \notin First(A)$:



Note: We must define other sets: *Empty*, *Follow* and *Predict.*

Grammar for Arithmetical Expressions

- $G_{expr3} = (N, T, P, E)$, where
- $N = \{E, E', F, F', T\}$,
- $T = \{i, +, *, (,)\}$,
- $P = \{$ 1: $E \rightarrow TE'$, 2: $E' \rightarrow +TE'$,
  3: $E' \rightarrow \varepsilon$, 4: $T \rightarrow FT'$,
  5: $T' \rightarrow *FT'$, 6: $T' \rightarrow \varepsilon$,
  7: $F \rightarrow (E)$, 8: $F \rightarrow i$ $\}$

Example:

$$(i + i)*(i + i) \in L(G_{expr3})$$

Set *Empty*

Gist: *Empty*(x) is the set that include ε if x derives the empty string; otherwise, *Empty*(x) is empty

Definition: Let $G = (N, T, P, S)$ be a CFG.
$Empty(x) = \{\varepsilon\}$ if $x \Rightarrow^* \varepsilon$; otherwise,
$Empty(x) = \emptyset$, where $x \in (N \cup T)^*$.

Illustration: $x = \boxed{X_1}\,\boxed{X_2} \cdots \boxed{X_n}$

$$\underbrace{\varepsilon \quad \varepsilon \cdots \varepsilon}_{\varepsilon}$$

$$x = X_1X_2 \ldots X_n \Rightarrow^* \varepsilon$$

$$Empty(x) = \{\varepsilon\}$$

Algorithm: *Empty*(X)

- **Input:** $G = (N, T, P, S)$
- **Output:** $Empty(X)$ for every $X \in N \cup T$

- **Method:**
- **for each** $a \in T$: $Empty(a) := \emptyset$
- **for each** $A \in N$:

  if $A \to \varepsilon \in P$ **then** $Empty(A) := \{\varepsilon\}$

  else $Empty(A) := \emptyset$
- **Apply the following rule until no *Empty* set can be changed:**
  - **if** $A \to X_1X_2\ldots X_n \in P$ **and** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots, n$ **then** $Empty(A) = \{\varepsilon\}$

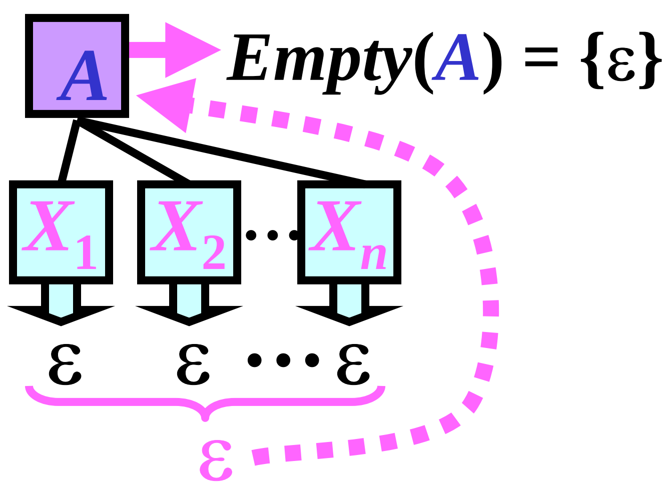
Previous Algorithm: Illustration

1) for each $\boldsymbol{a} \in T$: $Empty(\boldsymbol{a}) := \varnothing$ because $\boldsymbol{a} \not\Rightarrow^{*} \varepsilon$

2) for each $\boldsymbol{r}$**:** $\boldsymbol{A} \rightarrow \varepsilon \in \boldsymbol{P}$: $Empty(\boldsymbol{A}) := \{\varepsilon\}$ because $\boldsymbol{A} \Rightarrow^{\mathbf{1}} \varepsilon$ **[*r*]**

3) Apply the following rules until no *Empty* set can be changed:

• **if** $\boldsymbol{A} \rightarrow \boldsymbol{X_1 X_2} \ldots \boldsymbol{X_n} \in P$ **and** $Empty(\boldsymbol{X_i}) = \{\varepsilon\}$ for all $i = 1,\ldots,n$ **then** $Empty(\boldsymbol{A}) = \{\varepsilon\}$



$Empty(X)$ for G_{expr3}: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \to TE'$, 2: $E' \to +TE'$, 3: $E' \to \varepsilon$, 4: $T \to FT'$
5: $T' \to *FT'$, 6: $T' \to \varepsilon$, 7: $F \to (E)$, 8: $F \to i$ $\}$

Initialization:

$Empty(\, i \,) := \emptyset$	$Empty(E) := \emptyset$
$Empty(\, + \,) := \emptyset$	$Empty(E') := \{\varepsilon\}$
$Empty(\, * \,) := \emptyset$	$Empty(T) := \emptyset$
$Empty(\, (\,) := \emptyset$	$Empty(T') := \{\varepsilon\}$
$Empty(\,) \,) := \emptyset$	$Empty(F) := \emptyset$

- **No *Empty* set can be changed.**

Algorithm: *First*(*X*)

- **Input:** $G = (N, T, P, S)$
- **Output:** $First(X)$ for every $X \in N \cup T$

- **Method:**
- **for each** $a \in T$: $First(a) := \{a\}$
- **for each** $A \in N$: $First(A) := \emptyset$
- **Apply the following rule until no *First* set can be changed:**
- **if** $A \to X_1X_2\ldots X_{k-1}X_k\ldots X_n \in P$ **then**
  - add all symbols from $First(X_1)$ to $First(A)$
  - **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots, k$-1, where $k \leq n$ **then** add all symbols from $First(X_k)$ to $First(A)$

Previous Algorithm: Illustration

1) for each $a \in T$: *First*(a) := {a} because $a \Rightarrow^{0} a$

2) for each $A \in N$: *First*(A) := ∅ (inicialization)

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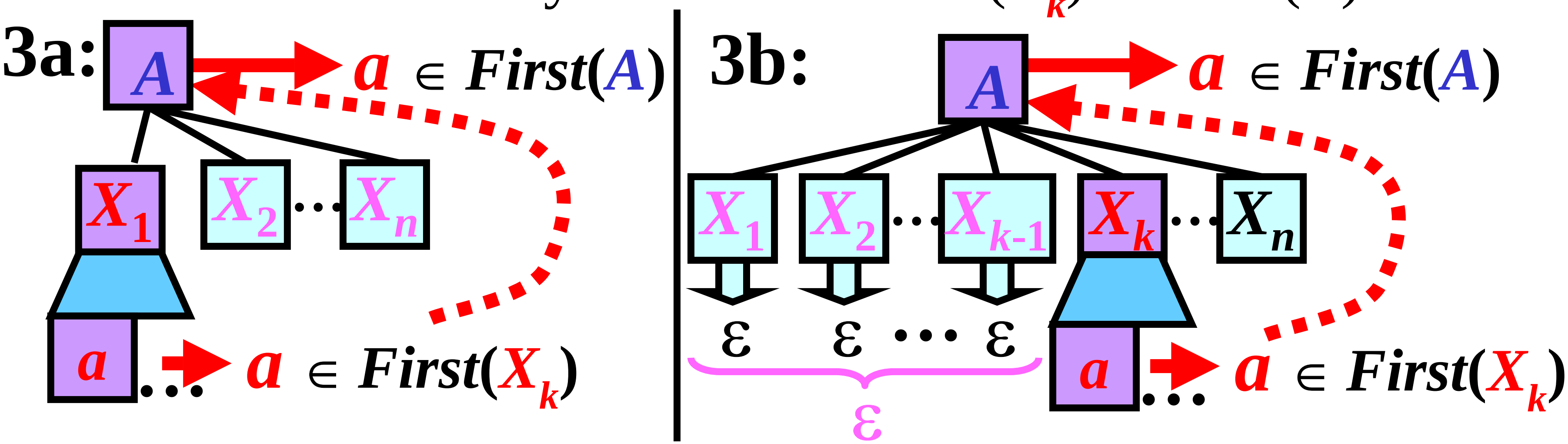
3) Apply the following rules until no *First* set or *Empty* set can be changed:

- **if** $A \rightarrow X_1X_2 \dots X_{k-1}X_k \dots X_n \in P$ **then**
  - **3a)** add all symbols from *First*(X_1) to *First*(A)
  - **3b) if** *Empty*(X_i) = {ε} for all $i = 1,\dots, k-1$, where $k < n$ **then** add all symbols from *First*(X_k) to *First*(A):



First(*X*) for G_{expr3}: Example

Initialization:

First(**i**)	:= {**i**}	*First*(***E***)	:= ∅
First(+)	:= {+}	*First*(***E'***)	:= ∅
First(*****)	:= {*****}	*First*(***T***)	:= ∅
First(**(**)	:= { **(** }	*First*(***T'***)	:= ∅
First(**)**)	:= { **)** }	*First*(***F***)	:= ∅

F → **i** ∈ *P*: **add** *First*(**i**) = {**i**} **to** *First*(***F***)
F → **(*E*)** ∈ *P*: **add** *First*(**(**) = {**(**} **to** *First*(***F***)
Summary: *First*(*F*) = {i, (}

T' → ***FT'*** ∈ *P*: **add** *First* (*****) = {*****} **to** *First*(***T'***)
Summary: *First*(*T'*) = {*}

T → ***FT'*** ∈ *P*: **add** *First* (***F***) = {**i**, **(**} **to** *First*(***T***)
Summary: *First*(*T*) = {i, (}

E' → **+*TE'*** ∈ *P*: **add** *First* (+) = {+} **to** *First*(***E'***)
Summary: *First*(*E'*) = {+}

E → ***TE'*** ∈ *P*: **add** *First* (***T***) = {**i**, **(**} **to** *First*(***E***)
Summary: *First*(*E*) = {i, (}

- **No *First* set can be changed.**

First(*X*) & *Empty*(*X*) for G_{expr3}: Summary

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 1: $E \to TE'$, 2: $E' \to +TE'$, 3: $E' \to \varepsilon$, 4: $T \to FT'$
5: $T' \to *FT'$, 6: $T' \to \varepsilon$, 7: $F \to (E)$, 8: $F \to i$ $\}$

Set *Empty* for all $X \in N \cup T$:

Empty(i)	$:= \emptyset$	*Empty*(E)	$:= \emptyset$
Empty($+$)	$:= \emptyset$	*Empty*(E')	$:= \{\varepsilon\}$
Empty($*$)	$:= \emptyset$	*Empty*(T)	$:= \emptyset$
Empty($($)	$:= \emptyset$	*Empty*(T')	$:= \{\varepsilon\}$
Empty($)$)	$:= \emptyset$	*Empty*(F)	$:= \emptyset$

Set *First* for all $X \in N \cup T$:

First(i)	$:= \{i\}$	*First*(E)	$:= \{i, (\}$
First($+$)	$:= \{+\}$	*First*(E')	$:= \{+\}$
First($*$)	$:= \{*\}$	*First*(T)	$:= \{i, (\}$
First($($)	$:= \{ (\}$	*First*(T')	$:= \{*\}$
First($)$)	$:= \{) \}$	*First*(F)	$:= \{i, (\}$

Note: for each $a \in T$: *Empty*(a) = $\emptyset$, *First*(a) = $\{a\}$

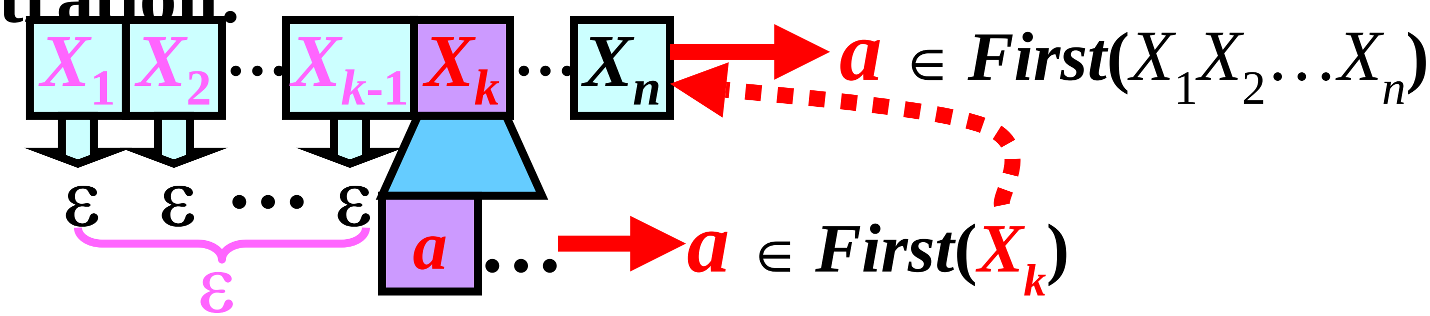
Algorithm: $First(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $First(X)$ & $Empty(X)$ for every $X \in N \cup T$; $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $First(X_1X_2\ldots X_n)$

- **Method:**
- $First(X_1X_2\ldots X_n) := First(X_1)$
- **Apply the following rule until nothing can be added to $\boldsymbol{First(X_1X_2 \ldots X_{k-1} X_k \ldots X_n)}$:**
  - **if** $Empty(\boldsymbol{X_i}) = \{\varepsilon\}$ for all $i = 1,\ldots,k\text{-}1$, where $k \leq n$ **then** add all symbols from $First(\boldsymbol{X_k})$ to $First(X_1X_2\ldots X_n)$

! Note: $First(\varepsilon) = \varnothing$

Illustration:



$First(X_1X_2\ldots X_n)$: Example

$G_{expr3} = (N, T, P, E)$, where: $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \to TE'$, 2: $E' \to +TE'$, 3: $E' \to \varepsilon$, 4: $T \to FT'$
5: $T' \to *FT'$, 6: $T' \to \varepsilon$, 7: $F \to (E)$, 8: $F \to i$ $\}$

Set *Empty* & *First* for all $X \in N$:	$Empty(E)$	$:= \emptyset$	$First(E)$	$:= \{i, (\}$
	$Empty(E')$	$:= \{\varepsilon\}$	$First(E')$	$:= \{+\}$
	$Empty(T)$	$:= \emptyset$	$First(T)$	$:= \{i, (\}$
	$Empty(T')$	$:= \{\varepsilon\}$	$First(T')$	$:= \{*\}$
	$Empty(F)$	$:= \emptyset$	$First(F)$	$:= \{i, (\}$

Task: $First(E'T'FET)$

1) $First(\underline{E'}T'FET) := First(E') = \{+\}$

2) $First(E'\underline{T'}FET)$: add $First(T') = \{*\}$ to $First(E'T'FET)$

$Empty(E') = \{\varepsilon\}$

3) $First(E'T'\underline{F}ET)$: add $First(F) = \{i, (\}$ to $First(E'T'FET)$

$Empty(E') = Empty(T') = \{\varepsilon\}$

Summary: $First(E'T'FET) = \{+, *, i, (\}$

Algorithm: $Empty(X_1X_2\ldots X_n)$

- **Input:** $G = (N, T, P, S)$; $Empty(X)$ for every $X \in N \cup T$;
  $x = X_1X_2\ldots X_n$, where $x \in (N \cup T)^+$
- **Output:** $Empty(X_1X_2\ldots X_n)$

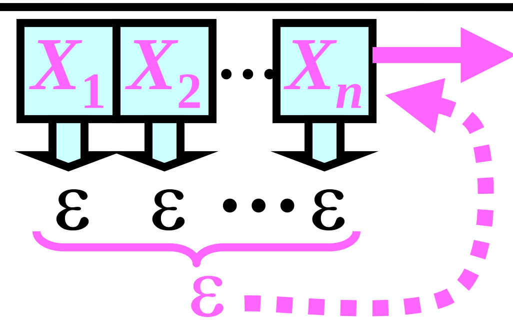
- **Method:**
- **if** $Empty(X_i) = \{\varepsilon\}$ for all $i = 1,\ldots,n$ **then**
  $Empty(X_1X_2\ldots X_n) := \{\varepsilon\}$

  else

  $Empty(X_1X_2\ldots X_n) := \emptyset$

! Note: $Empty(\varepsilon) = \{\varepsilon\}$

Illustration:



$\boldsymbol{Empty}(X_1X_2\ldots X_n) = \{\varepsilon\}$

$Empty(X_1X_2\ldots X_n)$: Example

$G_{expr3} = (N, T, P, \mathbf{E})$, where: $N = \{\mathbf{E}, \mathbf{F}, \mathbf{T}\}$, $T = \{\mathbf{i}, \mathbf{+}, \mathbf{*}, \mathbf{(}, \mathbf{)}\}$,

$P = \{$ 1: $\mathbf{E} \to \mathbf{TE'}$, 2: $\mathbf{E'} \to \mathbf{+TE'}$, 3: $\mathbf{E'} \to \varepsilon$, 4: $\mathbf{T} \to \mathbf{FT'}$

5: $\mathbf{T'} \to \mathbf{*FT'}$, 6: $\mathbf{T'} \to \varepsilon$, 7: $\mathbf{F} \to \mathbf{(E)}$, 8: $\mathbf{F} \to \mathbf{i}$ $\}$

Set *Empty* for all $X \in N$:

$Empty(\mathbf{E}) := \emptyset$
$Empty(\mathbf{E'}) := \{\varepsilon\}$
$Empty(\mathbf{T}) := \emptyset$
$Empty(\mathbf{T'}) := \{\varepsilon\}$
$Empty(\mathbf{F}) := \emptyset$

Task: $Empty(\mathbf{E'T'})$

$Empty(\mathbf{E'}) = Empty(\mathbf{T'}) = \{\varepsilon\}$, **so** $Empty(\mathbf{E'T'}) = \{\varepsilon\}$

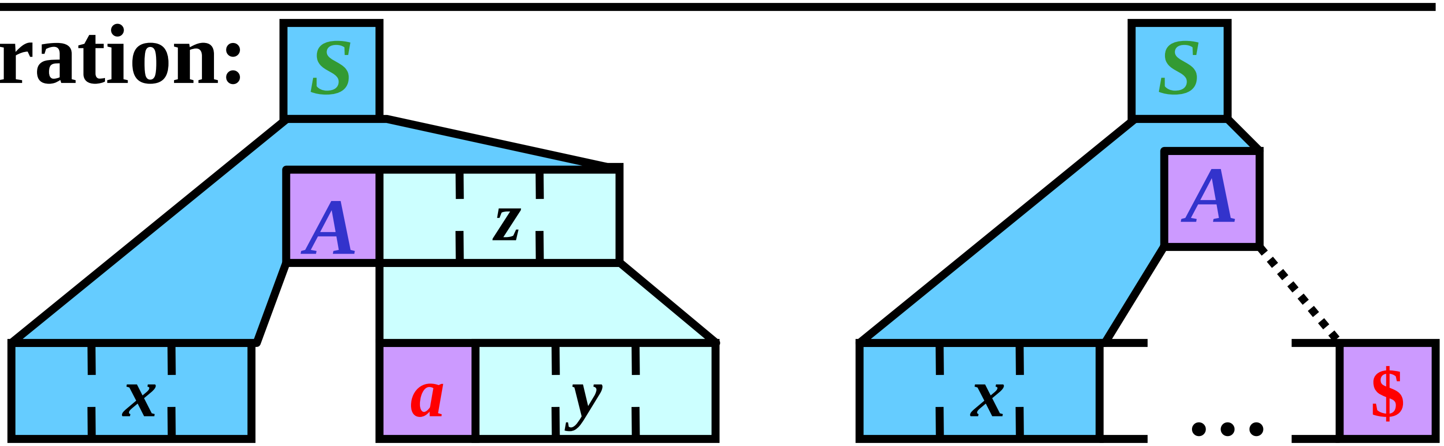
Set *Follow*

Gist: *Follow*(*A*) is the set of all terminals that can come right after *A* in a sentential form of *G*

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \in N$, we define the set $Follow(A)$ as

$$Follow(A) = \{a: a \in T, S \Rightarrow^* xAay, x, y \in (N \cup T)^*\} \cup \{\$: S \Rightarrow^* xA, x \in (N \cup T)^*\}$$

Illustration:



$S \Rightarrow^* xAz \Rightarrow^* xAay$

$a \in Follow(A)$

$S \Rightarrow^* xA$

$\$ \in Follow(A)$

Algorithm: *Follow*(*A*)

- **Input:** $G = (N, T, P, \boldsymbol{S})$;
- **Output:** *Follow*(*A*) for every $A \in N$

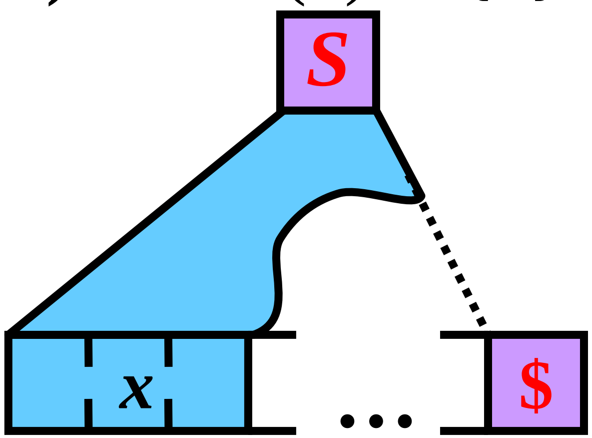
- **Method:**
- *Follow*(**S**) := {**$**};
- **Apply the following rules until no *Follow* set can be changed:**
- **if** $\boldsymbol{A} \rightarrow \boldsymbol{xBy} \in P$ **then**
  - **if** $y \neq \varepsilon$ **then**

    add all symbols from *First*(*y*) to *Follow*(**B**);
  - **if** *Empty*(*y*) **=** {ε} **then**

    add all symbols from *Follow*(**A**) to *Follow*(**B**);

Previous Algorithm: Illustration

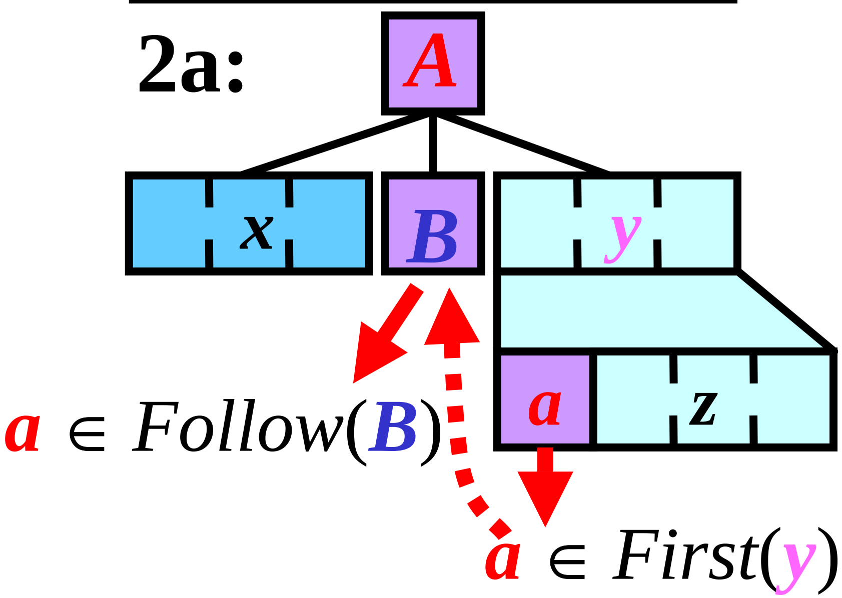
1) *Follow*(S) := {\$}



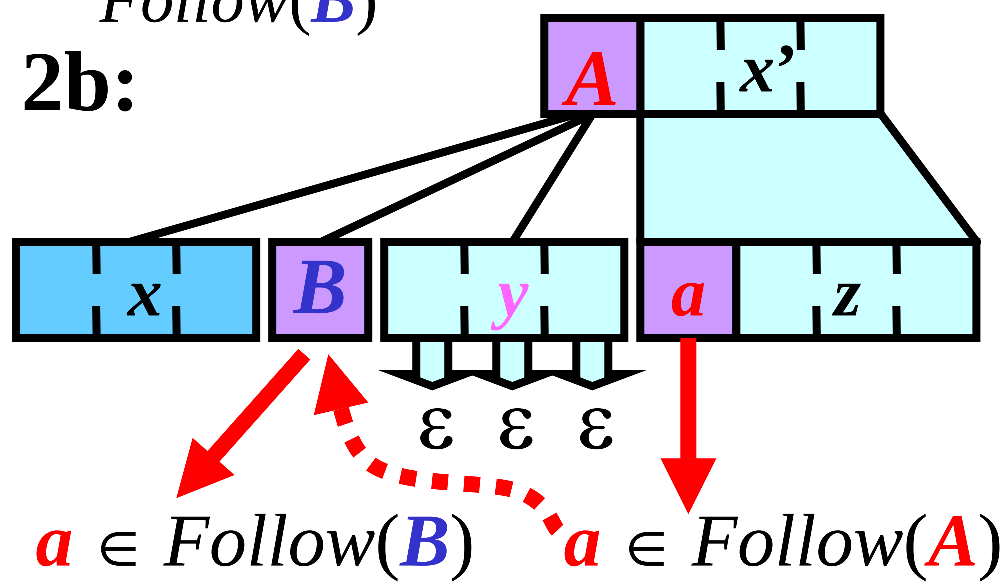
2) Apply the following rules until no *Follow* set can be changed:

- **if** $A \rightarrow xBy \in P$ **then**
  - **2a) if** $y \neq \varepsilon$ **then** add all symbols from *First*(y) to *Follow*(B)
  - **2b) if** *Empty*(y) = {ε} **then** add all symbols from *Follow*(A) to *Follow*(B)

2a:



2b:



$Follow(X)$ for G_{expr3}: Example 1/3

$First(E) := \{i, (\}$	$Empty(E) := \emptyset$	$Follow(E) := \emptyset$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \emptyset$
$First(T) := \{i, (\}$	$Empty(T) := \emptyset$	$Follow(T) := \emptyset$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \emptyset$
$First(F) := \{i, (\}$	$Empty(F) := \emptyset$	$Follow(F) := \emptyset$

0) $Follow(E) := \{\$\}$

---

1) $F \to (E\underbrace{)}_{\neq \varepsilon} \in P$: **add** $First()) = \{)\}$ **to** $Follow(E)$

Summary: $Follow(E) = \{\$,)\}$

---

2) $E \to TE'\underbrace{\ }_{\varepsilon} \in P$: **add** $Follow(E) = \{\$,)\}$ **to** $Follow(E')$

ε: $Empty(\varepsilon) = \{\varepsilon\}$

$E \to T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** $First(E') = \{+\}$ **to** $Follow(T)$

$E \to T\underbrace{E'} \in P$: **add** $Follow(E) = \{\$,)\}$ **to** $Follow(T)$

$Empty(E') = \{\varepsilon\}$

Summary: $Follow(E') = \{\$,)\}$, $Follow(T) = \{+, \$,)\}$

Follow(*X*) for G_{expr3}: Example 2/3

First(*E*)	:= {*i*, (}	*Empty*(*E*)	:= ∅	*Follow*(*E*)	:= {\$,)}
First(*E'*)	:= {+}	*Empty*(*E'*)	:= {ε}	*Follow*(*E'*)	:= {\$,)}
First(*T*)	:= {*i*, (}	*Empty*(*T*)	:= ∅	*Follow*(*T*)	:= {+, \$,)}
First(*T'*)	:= {*}	*Empty*(*T'*)	:= {ε}	*Follow*(*T'*)	:= ∅
First(*F*)	:= {*i*, (}	*Empty*(*F*)	:= ∅	*Follow*(*F*)	:= ∅

3) $E' \rightarrow +TE'\underbrace{}_{\varepsilon} \in P$: **add** *Follow*(*E'*) = {\$,)} **to** *Follow*(*E'*)

ε: *Empty*(ε) = {ε}

$E' \rightarrow +T\underbrace{E'}_{\neq \varepsilon} \in P$: **add** *First*(*E'*) = {+} **to** *Follow*(*T*)

$E' \rightarrow +T\underbrace{E'} \in P$: **add** *Follow*(*E'*) = {\$,)} **to** *Follow*(*T*)

Empty(*E'*) = {ε}

Summary: Nothing is changed

4) $T \rightarrow FT'\underbrace{}_{\varepsilon} \in P$: **add** *Follow*(*T*) = {+, \$,)} **to** *Follow*(*T'*)

ε: *Empty*(ε) = {ε}

$T \rightarrow F\underbrace{T'}_{\neq \varepsilon} \in P$: **add** *First*(*T'*) = {*} **to** *Follow*(*F*)

$T \rightarrow F\underbrace{T'} \in P$: **add** *Follow*(*T*) = {+, \$,)} **to** *Follow*(*F*)

Empty(*T'*) = {ε}

Summary: *Follow*(*T'*) = {+, \$,)}, *Follow*(*F*) = {*, +, \$,)}

$Follow(X)$ for G_{expr3}: Example 3/3

$First(E)$	:= {**i**, **(**}	$Empty(E)$	:= ∅	$Follow(E)$	:= {$, **)**}
$First(E')$	:= {+}	$Empty(E')$	:= {ε}	$Follow(E')$	:= {$, **)**}
$First(T)$	:= {**i**, **(**}	$Empty(T)$	:= ∅	$Follow(T)$	:= {+, $, **)**}
$First(T')$	:= {*}	$Empty(T')$	:= {ε}	$Follow(T')$	:= {+, $, **)**}
$First(F)$	:= {**i**, **(**}	$Empty(F)$	:= ∅	$Follow(F)$	:= {*, +, $, **)**}

5) $T' \rightarrow *FT'\underbrace{}_{\varepsilon} \in P$: **add** $Follow(T')$ = {+, $, **)**} **to** $Follow(T')$

ε: $Empty(\varepsilon) = \{\varepsilon\}$

$T' \rightarrow *F\underbrace{T'}_{\neq \varepsilon} \in P$: **add** $First(T')$ = {*} **to** $Follow(F)$

$T' \rightarrow *F\underbrace{T'} \in P$: **add** $Follow(T')$ = {+, $, **)**} **to** $Follow(F)$

$Empty(T') = \{\varepsilon\}$

End: Nothing is changed.

Summary:
$Follow(E)$:= {$, **)**}
$Follow(E')$:= {$, **)**}
$Follow(T)$:= {+, $, **)**}
$Follow(T')$:= {+, $, **)**}
$Follow(F)$:= {*, +, $, **)**}

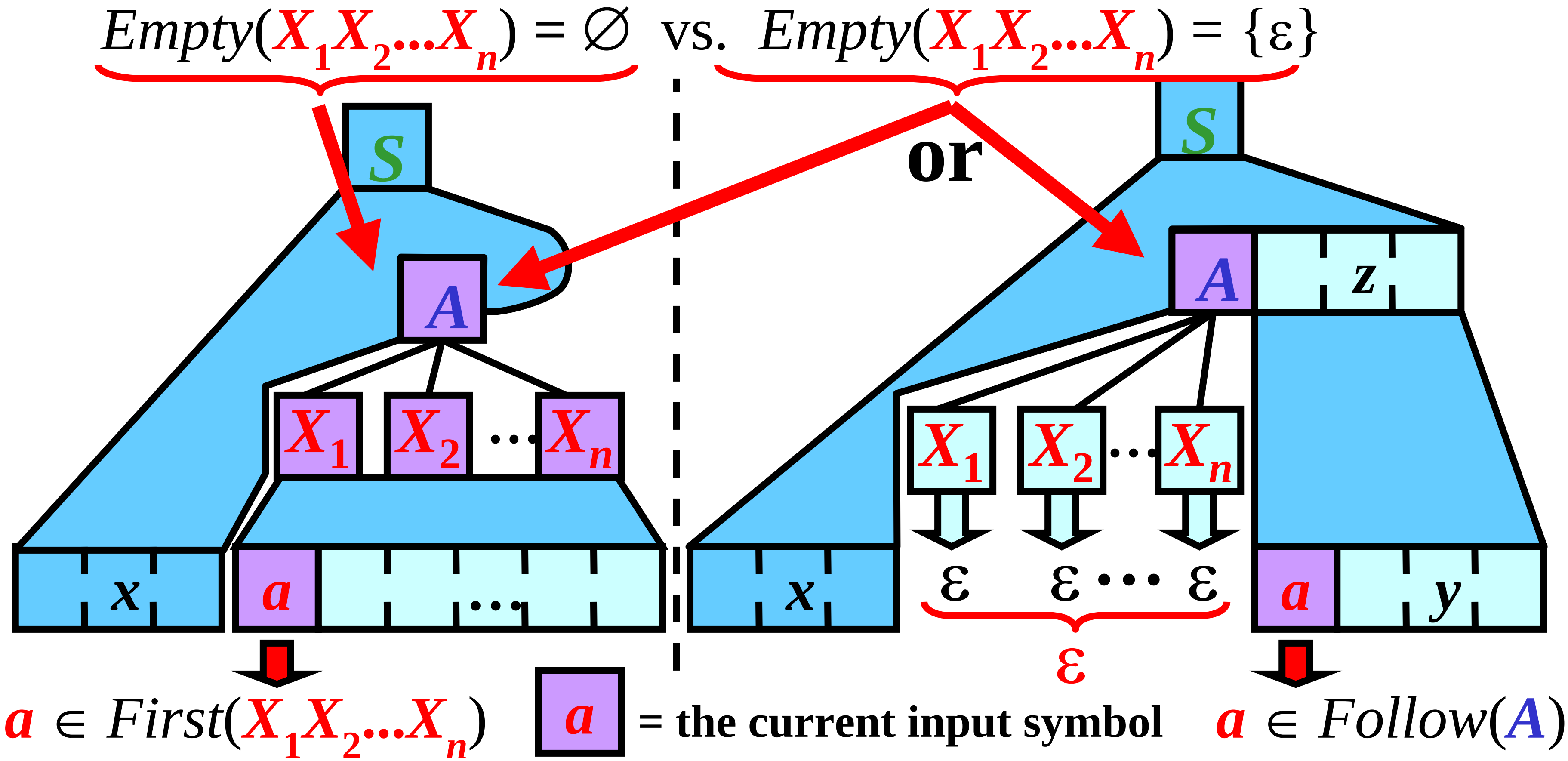
Set *Predict*

Gist: ***Predict*(*A* → *x*) is the set of all terminals that can begin a string obtained by a derivation started by using *A* → *x*.**

Definition: Let $G = (N, T, P, S)$ be a CFG. For every $A \to x \in P$, we define $Predict(A \to x)$ so that

- if $Empty(x) = \{\varepsilon\}$ then
  $Predict(A \to x) = First(x) \cup Follow(A)$
- if $Empty(x) = \emptyset$ then
  $Predict(A \to x) = First(x)$

Set $Predict(A \to X_1X_2...X_n)$: Illustration



Summary: if $Empty(X_1X_2...X_n) = \{\varepsilon\}$ then
$Predict(A \to X_1X_2...X_n) = First(X_1X_2...X_n) \cup Follow(A)$;
otherwise, $Predict(A \to X_1X_2...X_n) = First(X_1X_2...X_n)$

$Predict(A \to x)$ for G_{expr3}: Example 1/2

$First(E) := \{i, (\}$	$Empty(E) := \varnothing$	$Follow(E) := \{\$,)\}$
$First(E') := \{+\}$	$Empty(E') := \{\varepsilon\}$	$Follow(E') := \{\$,)\}$
$First(T) := \{i, (\}$	$Empty(T) := \varnothing$	$Follow(T) := \{+, \$,)\}$
$First(T') := \{*\}$	$Empty(T') := \{\varepsilon\}$	$Follow(T') := \{+, \$,)\}$
$First(F) := \{i, (\}$	$Empty(F) := \varnothing$	$Follow(F) := \{*, +, \$,)\}$

1: $E \to TE'$

$Empty(TE') = \varnothing$ because $Empty(T) = \varnothing$

$Predict(1) := First(TE') = First(T) = \{i, (\}$

2: $E' \to +TE'$

$Empty(+TE') = \varnothing$ because $Empty(T) = \varnothing$

$Predict(2) := First(+TE') = First(+) = \{+\}$

3: $E' \to \varepsilon$

$Empty(\varepsilon) = \{\varepsilon\}$

$Predict(3) := First(\varepsilon) \cup Follow(E') = \varnothing \cup \{\$,)\} = \{\$,)\}$

4: $T \to FT'$

$Empty(FT') = \varnothing$ because $Empty(F) = \varnothing$

$Predict(4) := First(FT') = First(F) = \{i, (\}$

Predict($A \to x$) for G_{expr3}: Example 2/2

$First(E)$	:= {i, (}	$Empty(E)$	:= ∅	$Follow(E)$	:= {$,)}
$First(E')$	:= {+}	$Empty(E')$	:= {ε}	$Follow(E')$	:= {$,)}
$First(T)$	:= {i, (}	$Empty(T)$	:= ∅	$Follow(T)$	:= {+, $,)}
$First(T')$	:= {*}	$Empty(T')$	:= {ε}	$Follow(T')$	:= {+, $,)}
$First(F)$	:= {i, (}	$Empty(F)$	:= ∅	$Follow(F)$	:= {*, +, $,)}

5: $T' \to *FT'$

$Empty(*FT') = \varnothing$ because $Empty(F) = \varnothing$

$Predict(5) := First(*FT') = First(*) = \{*\}$

6: $T' \to \varepsilon$

$Empty(\varepsilon) = \{\varepsilon\}$

$Predict(6) := First(\varepsilon) \cup Follow(T') = \varnothing \cup \{+, \$,)\} = \{+, \$,)\}$

7: $F \to (E)$

$Empty((E)) = \varnothing$ because $Empty(E) = \varnothing$

$Predict(7) := First((E)) = First(() = \{(\}$

8: $F \to i$

$Empty(i) = \varnothing$

$Predict(8) := First(i) = \{i\}$

Construction of LL Table

α	...	a	...
...			
A		$\alpha(A, a)$	
...			

$\alpha(A, a) = A \to X_1X_2\ldots X_n \in P$ if $a \in Predict(A \to X_1X_2\ldots X_n)$; otherwise, $\alpha(A, a)$ is blank.

Task: LL table for G_{expr1}

	i	+	*	(	)	$
E	1					
E'						
T	4					
T'						
F	8					

$i \in Predict(1)$

$i \in Predict(4)$

$i \in Predict(8)$

Rule r	$Predict(r)$
1: $E \to TE'$	{i, (}
2: $E' \to +TE'$	{+}
3: $E' \to \varepsilon$	{$,)}
4: $T \to FT'$	{i, (}
5: $T' \to *FT'$	{*}
6: $T' \to \varepsilon$	{+, $,)}
7: $F \to (E)$	{(}
8: $F \to i$	{i}

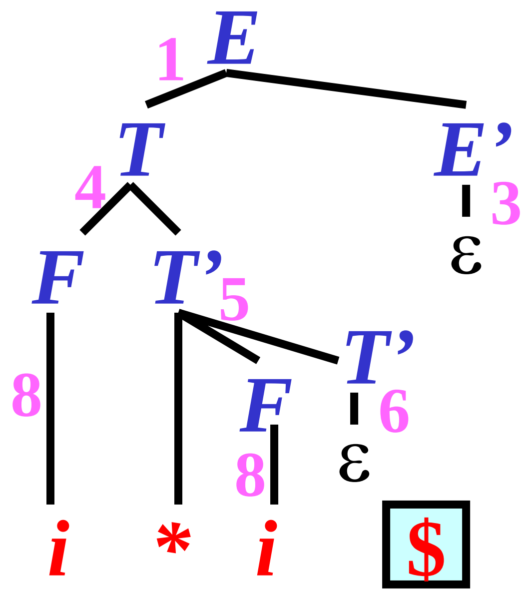
Construct the rest analogically.

Parsing Based on LL Table: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

1: $E \to TE'$
2: $E' \to +TE'$
3: $E' \to \varepsilon$
4: $T \to FT'$
5: $T' \to *FT'$
6: $T' \to \varepsilon$
7: $F \to (E)$
8: $F \to i$

Question: $i * i \in L(G_{expr3})$**?**



LL Grammars with ε-rules: Definition

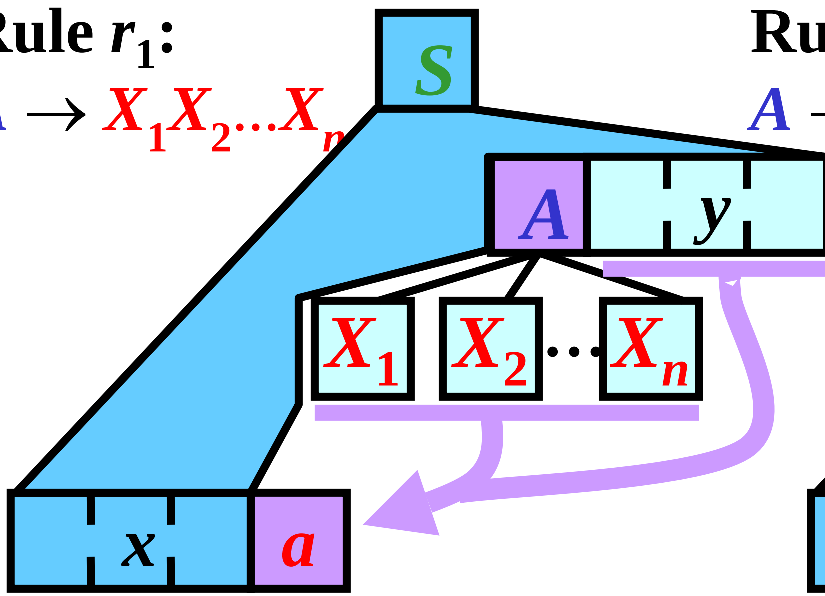
Definition: Let $G = (N, T, P, S)$ be a CFG. G is an *LL grammar* if for every $a \in T$ and every $A \in N$ there is **no more than one** A-rule $A \to X_1X_2 \ldots X_n \in P$ such that $a \in Predict(A \to X_1X_2 \ldots X_n)$

Illustration:

Ruled out in an LL grammar

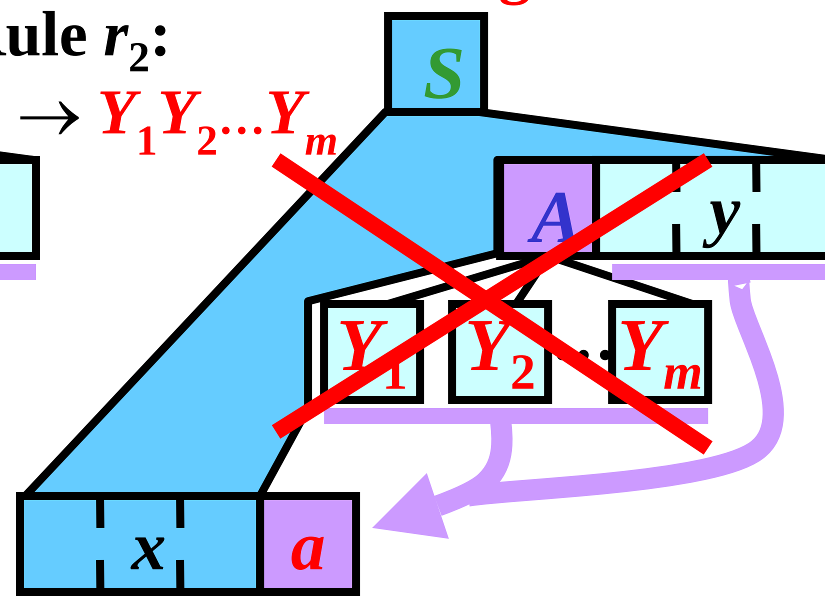
Rule r_1:

$A \to X_1X_2 \ldots X_n$



Rule r_2:

$A \to Y_1Y_2 \ldots Y_m$



$a \in Predict(A \to X_1X_2 \ldots X_n)$

$a \in Predict(A \to Y_1Y_2 \ldots Y_m)$

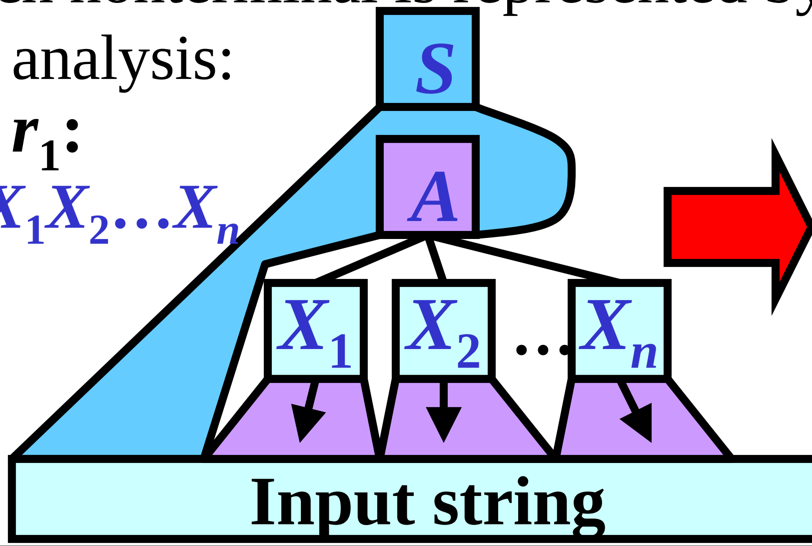
LL Analyzer Implementation

1) Recursive-Descent Parsing

- Each nonterminal is represented by a procedure, which perform its analysis:

Rule r_1:

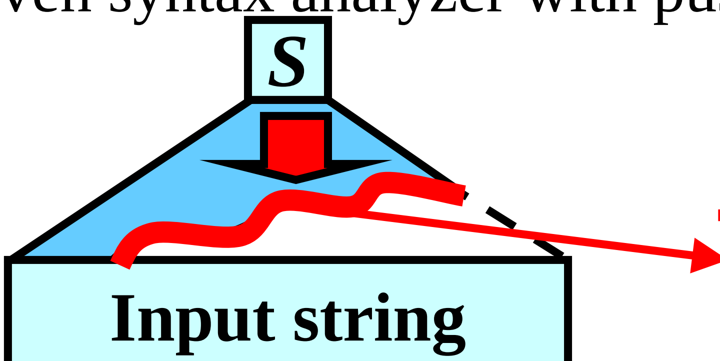
$A \to X_1X_2\ldots X_n$



<u>function</u> A: **boolean**;
<u>begin</u>
{X_1 analysis}
{X_2 analysis}
…
{X_n analysis}
<u>end</u>

2) Predictive Parsing

- Table-driven syntax analyzer with pushdown



These symbols are in the pushdown.

Recursive Descent: Example 1/4

```
Procedure GetNextToken;
begin
{ this procedure get the next token to global variable "token"}
end
```

• For $E \in N$: Rule 1: $E \rightarrow TE'$

```
function E: boolean;
begin
   E := false;
   if token in ['i', '('] then
         { simulation of rule 1: E → TE' }
          E := T and E1;
end;
```

$\boxed{1}$

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

• For $T \in N$: Rule 4: $T \rightarrow FT'$

```
function T: boolean;
begin
   T := false;
   if token in ['i', '('] then
         { simulation of rule 4: T → FT' }
         T := F and T1;
end;
```

$\boxed{4}$

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Recursive Descent: Example 2/4

- For *E’* ∈ *N*: Rules 2: *E’* → +*TE’*, 3: *E’* → ε

```
function E1: boolean;
begin
   E1 := false;
   if token = '+' then begin
        { simulation of rule 2: E’ → +TE’ }
        GetNextToken;
        E1 := T and E1;
   end
   else
   if token in [')', '$'] then
        { simulation of rule 3: E’ → ε}
        E1 := true;
end;
```

	i	+	*	(	)	$
E	1			1		
E’		2			3	3
T	4			4		
T’		6	5		6	6
F	8			7		

Recursive Descent: Example 3/4

- For *T’* ∈ *N*: Rules 5: *T’* → ***FT’**, 6: *T’* → ε

```
function T1: boolean;
begin
  T1 := false;
  if token = '*' then begin
    { simulation of rule 5: T’ → *FT’ }
    GetNextToken;
    T1 := F and T1;
  end
  else
  if token in ['+', ')', '$'] then
    { simulation of rule 6: T’ → ε}
    T1 := true;
end;
```

	i	+	*	(	)	$
E	1			1		
E’		2			3	3
T	4			4		
T’		6	5		6	6
F	8			7		

Recursive Descent: Example 4/4

- For $F \in N$: Rules 7: $F \to (E)$, 8: $F \to i$

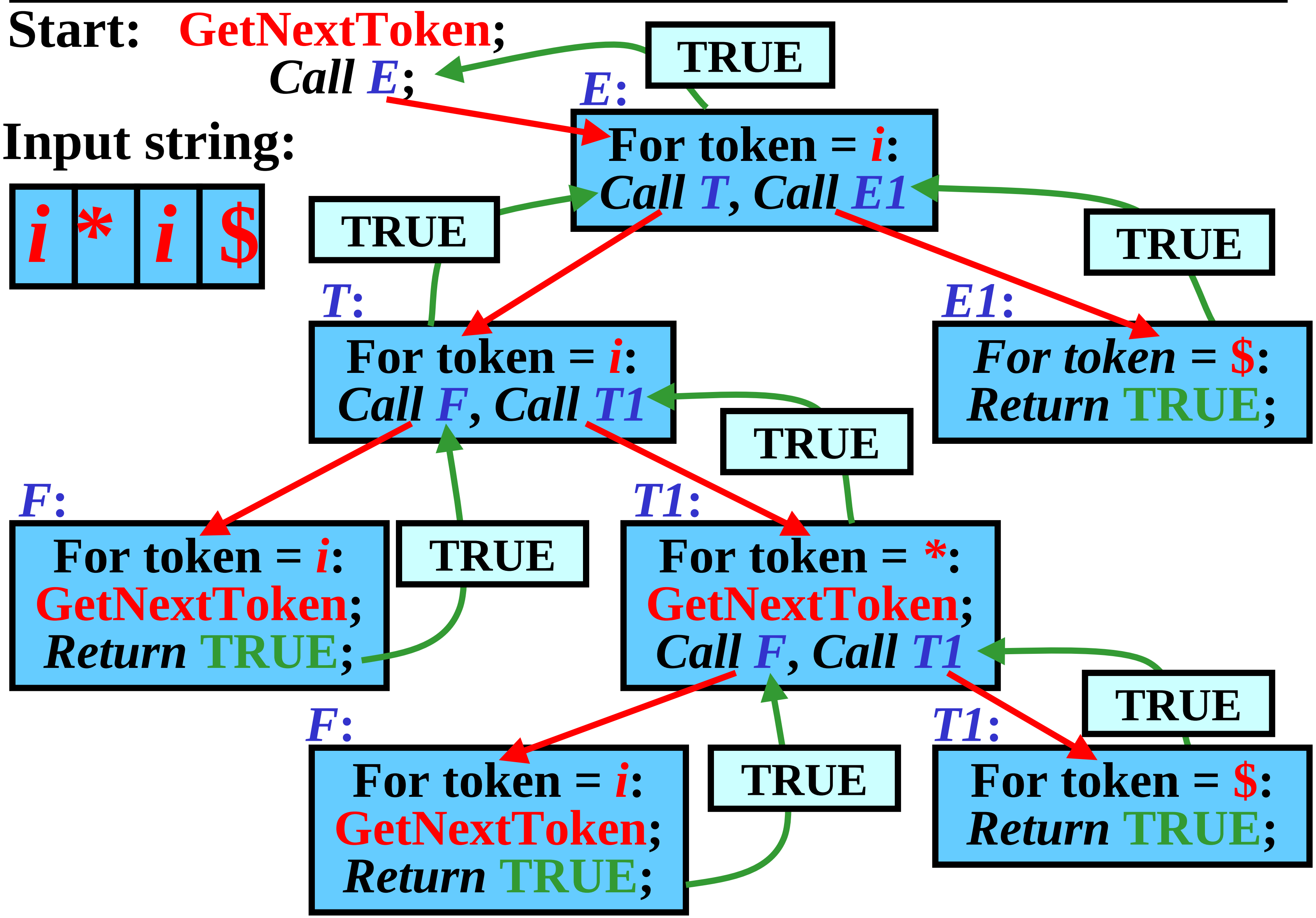
```
function F: boolean;
begin
   F := false;
   if token = '(' then begin
      { simulation of rule 7: F → (E) }
      GetNextToken;
      if E then begin
         F := (token = ')');
         GetNextToken;
      end;
   end
   else
   if token = 'i' then begin
      { simulation of rule 8: F → i }
      F := true;
      GetNextToken;
   end;
end;
```

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Main body:

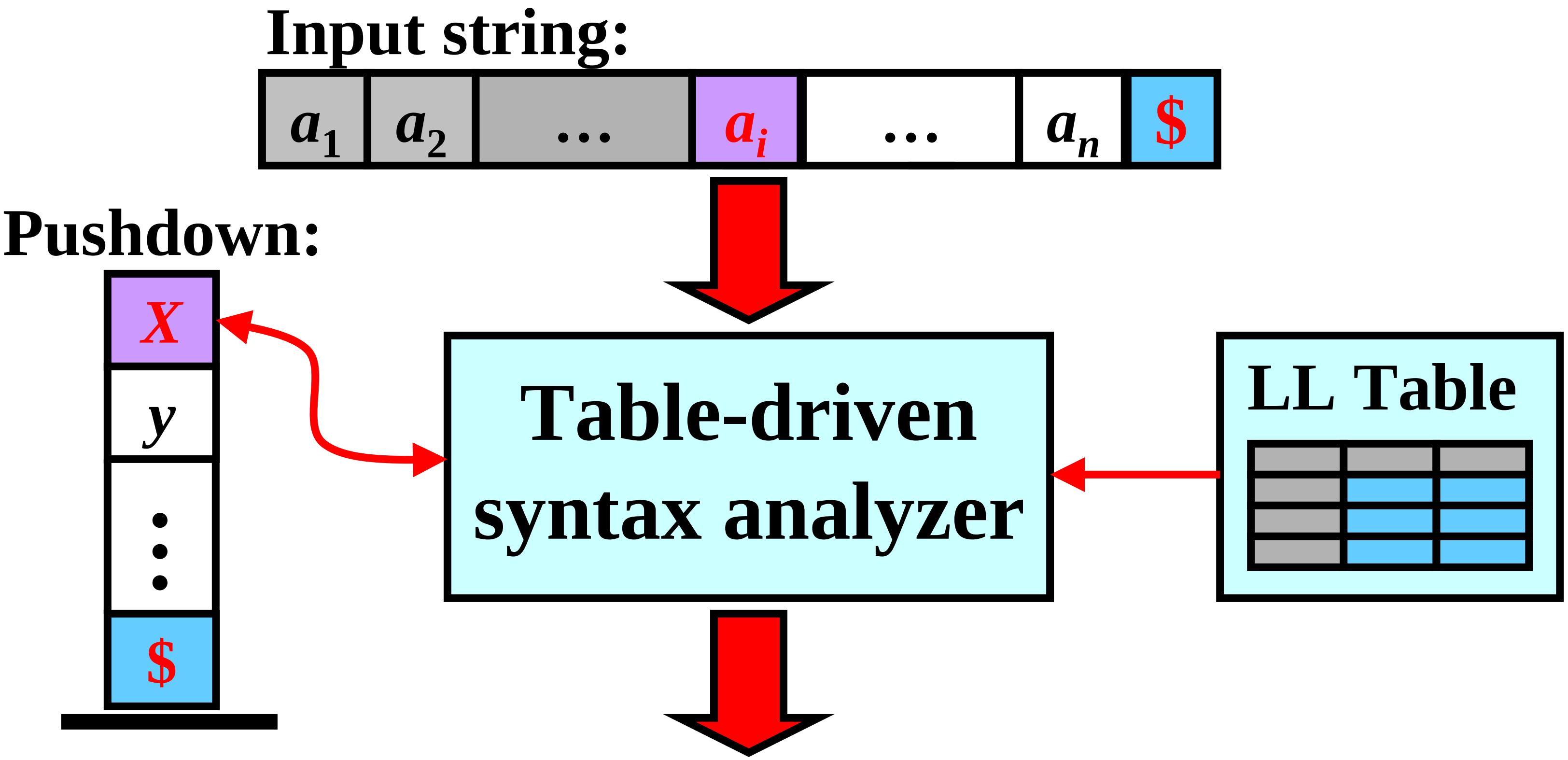
```
begin
   GetNextToken;
   if E then
      write('OK')
   else
      write('ERROR')
end.
```

Recursive Descent: Illustration for *i*i$*



Predictive Parsing

- **Model of table-driven syntax analyzer:**



Left parse = sequence of rules used in the leftmost derivation of the input string.

Table-Driven Parsing: Algorithm

- **Input:** LL-table for $G=(N, T, P, S)$; $x \in T^*$
- **Output:** Left parse of x if $x \in L(G)$; otherwise, error

---

- **Method:**
- push(**\$**) & push(S) onto the pushdown;
- **while** the pushdown is not empty **do**
  - let X = the pushdown top and a = the current token
  - **case** X **of:**
    - X **= \$:** **if** a **= \$ then success**
      else error;
    - $X \in T$**:** **if** X **=** a **then** pop(X) & read next a from input string
      else error;
    - $X \in N$**:** **if** r**:** $X \rightarrow x \in$ LL-table[X, a] **then**
      replace X with reversal(x) on the pushdown & write r to output
      else error;

end

Table-Driven Parsing: Example

	i	+	*	(	)	$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Rules:

1: $E \to TE'$
2: $E' \to +TE'$
3: $E' \to \varepsilon$
4: $T \to FT'$
5: $T' \to *FT'$
6: $T' \to \varepsilon$
7: $F \to (E)$
8: $F \to i$

Input string: *i* * *i* $

Pushdown	Input	Rule	Derivation
$E	i*i$	1: $E \to TE'$	$\underline{E} \Rightarrow \underline{T}E'$
$E'T	i*i$	4: $T \to FT'$	$\Rightarrow \underline{F}T'E'$
$E'T'F	i*i$	8: $F \to i$	$\Rightarrow i\underline{T'}E'$
$E'T'i	i*i$		
$E'T'	*i$	5: $T' \to *FT'$	$\Rightarrow i*\underline{F}T'E'$
$E'T'F*	*i$		
$E'T'F	i$	8: $F \to i$	$\Rightarrow i*i\underline{T'}E'$
$E'T'i	i$		
$E'T'	$	6: $T' \to \varepsilon$	$\Rightarrow i*i\underline{E'}$
$E'	$	3: $E' \to \varepsilon$	$\Rightarrow i*i$
$	$		

Success

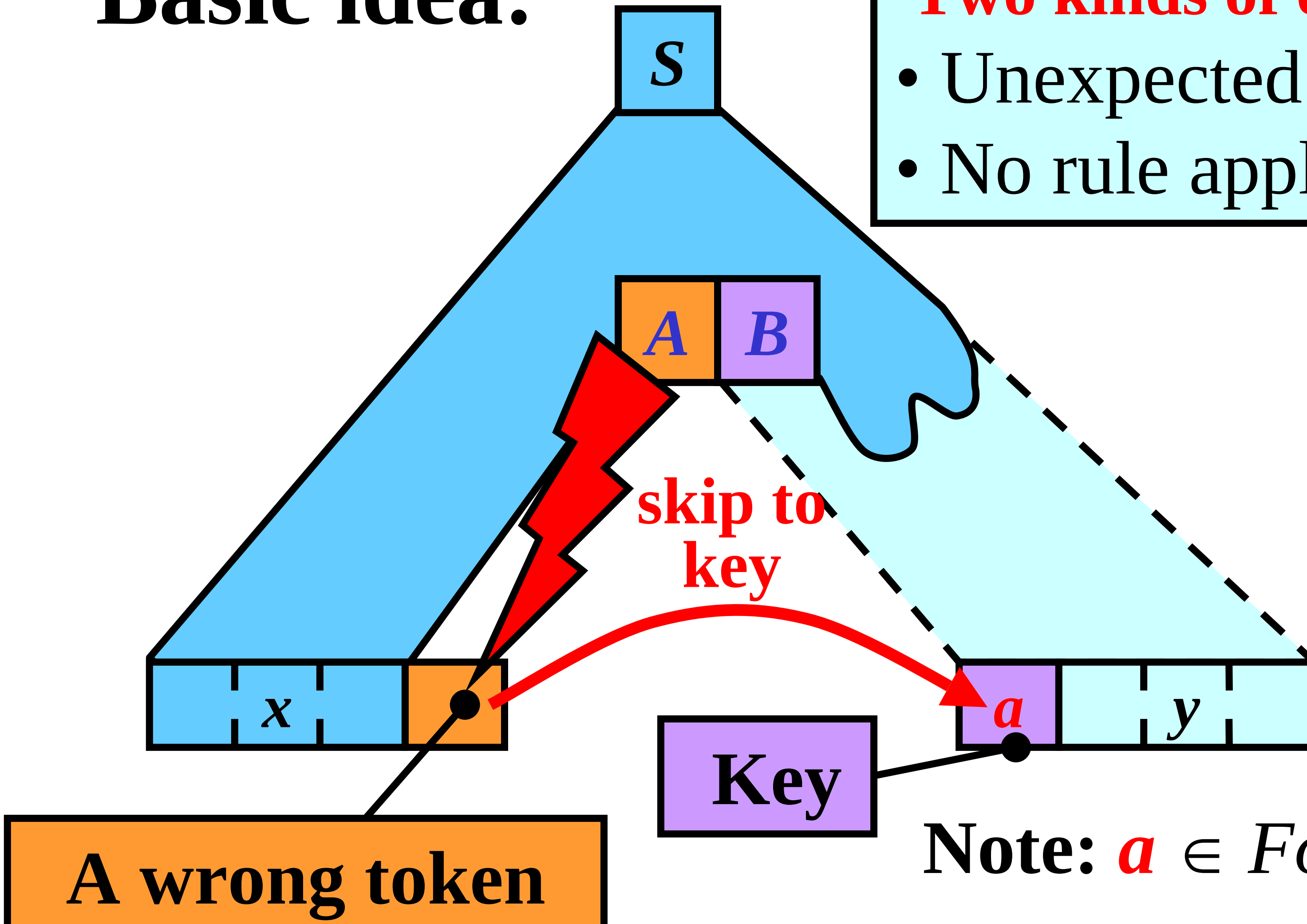
Left parse: 1485863

Handling Errors: Introduction

Basic idea:

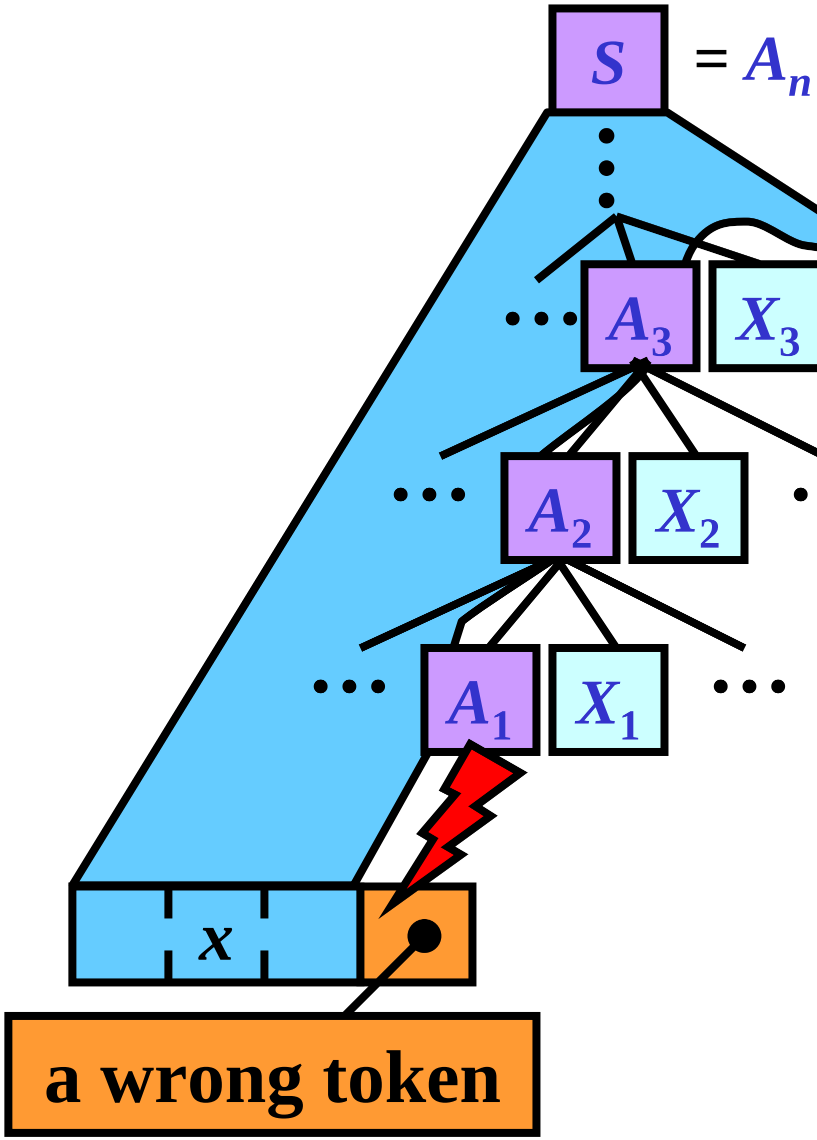
Two kinds of errors:

- Unexpected token
- No rule applicable



Note: $a \in Follow(A)$

Panic-Mode (Hartmann) Error Recovery



- Let ***Context***(A_1) =
  Follow(A_1) ∪
  Follow(A_2) ∪
  …
  Follow(A_n)

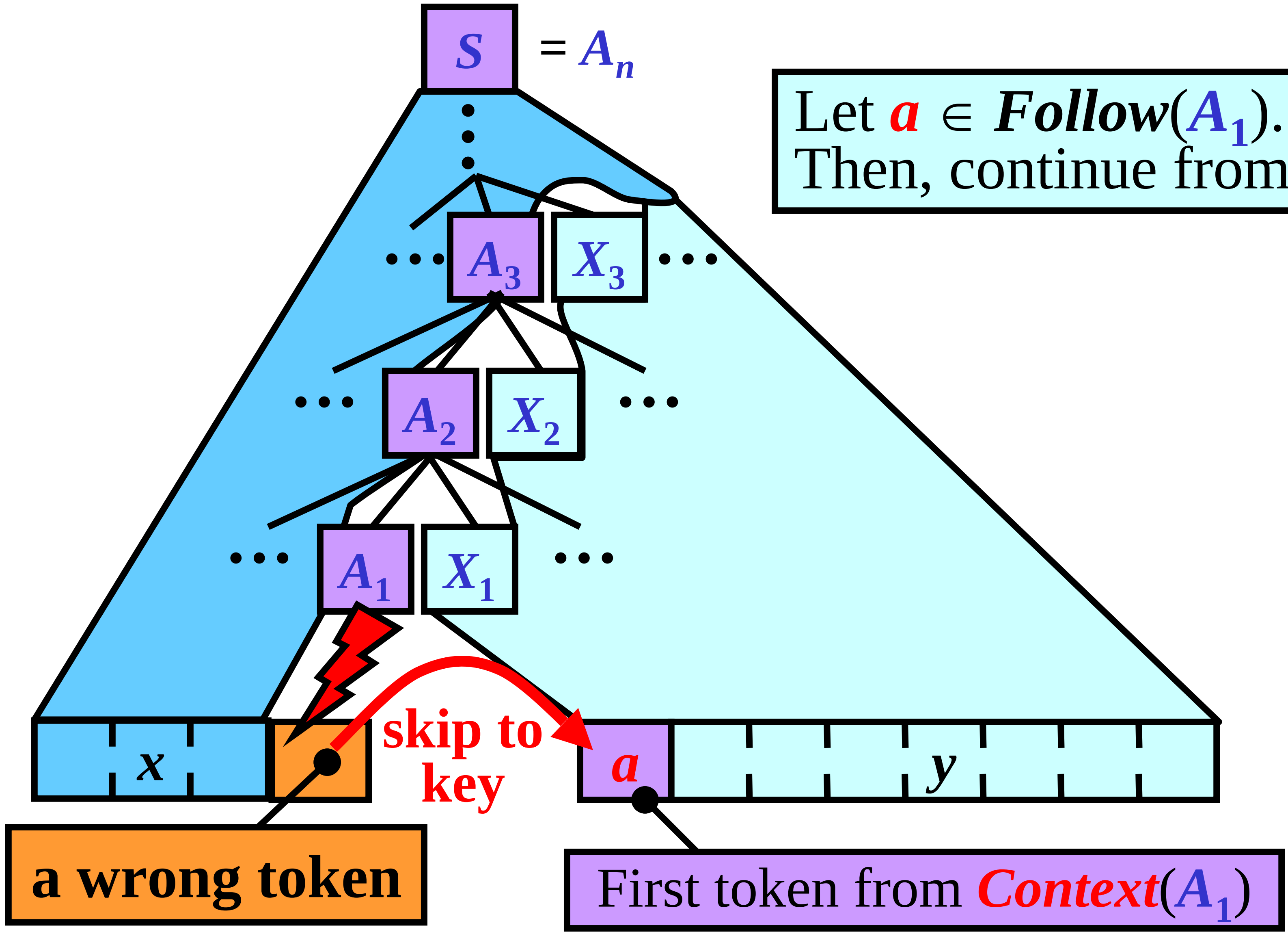
repeat
- a := GetNextToken;
  {These tokens are skipped}

until a in ***Context***(A_1)

if a in ***Follow***(A_i) **then**
continue with parsing from the symbol X_i.

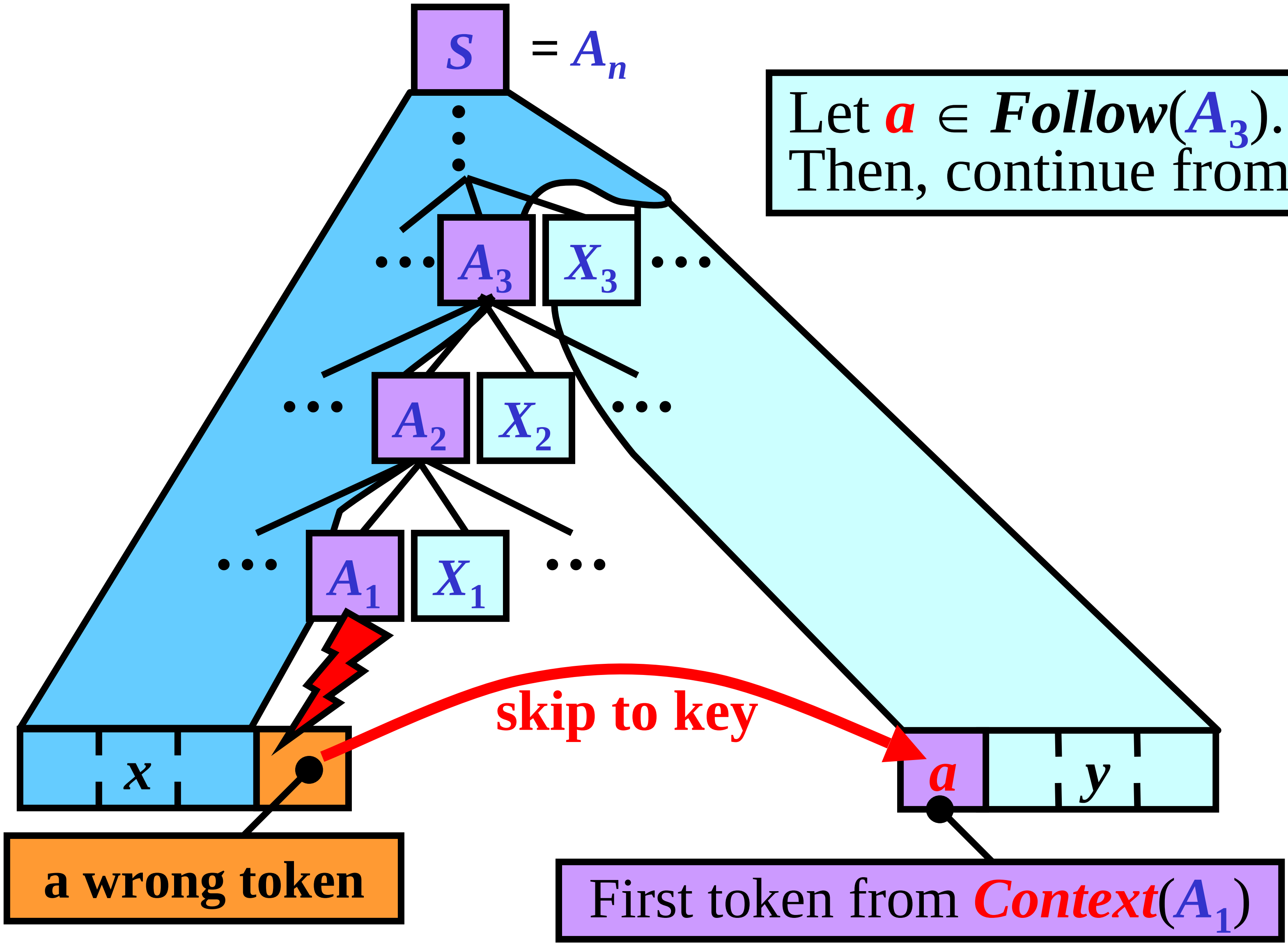
Panic-Mode Recovery: Illustration 1/2

Let $a \in Follow(A_1)$.
Then, continue from X_1



Panic-Mode Recovery: Illustration 2/2

Let $\boldsymbol{a} \in \textbf{\textit{Follow}}(A_3)$.
Then, continue from X_3



Context(*X*) for Predictive Parser: Variant I

For $G = (N, T, P, S)$,

Context*(*A*) = *Follow*(*A*) for every *A* $\in$ *N

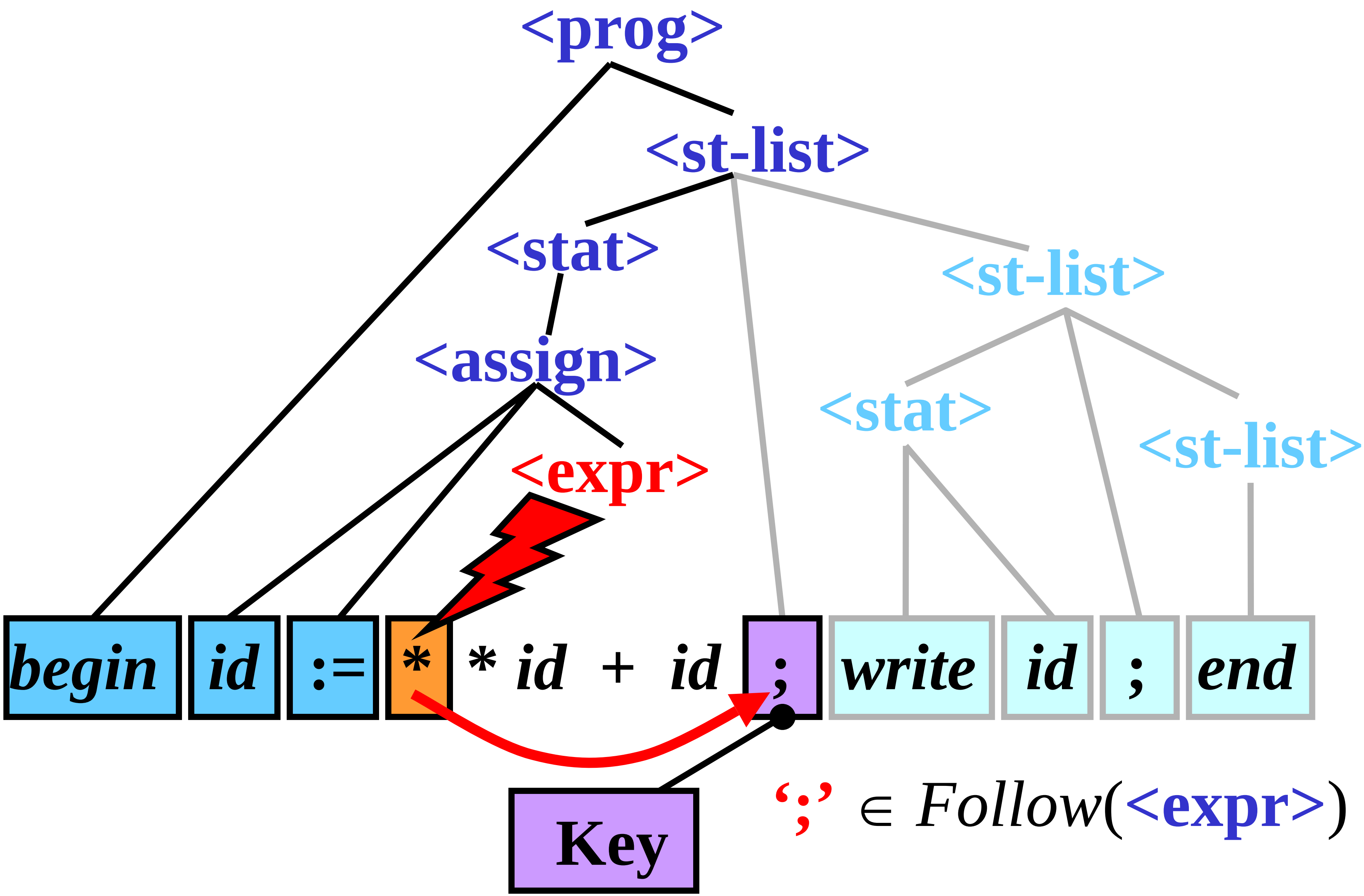
- **Method:**
- Let ***A*** be pushdown top & no rule is applicable:
- **repeat**

  a := GetNextToken;

  {These tokens are skipped}

  until ***a*** in ***Context***(***A***)
- pop ***A*** from the pushdown;

Variant I: Example



‘;’ ∈ *Follow*(**<expr>**)

Context(*X*) for Predictive Parser: Variant II

For $G = (N, T, P, S)$,

$Context(A) = First(A) \cup Follow(A)$ for every $A \in N$

- **Method:**
- Let ***A*** be pushdown top & no rule is applicable:
- **repeat**

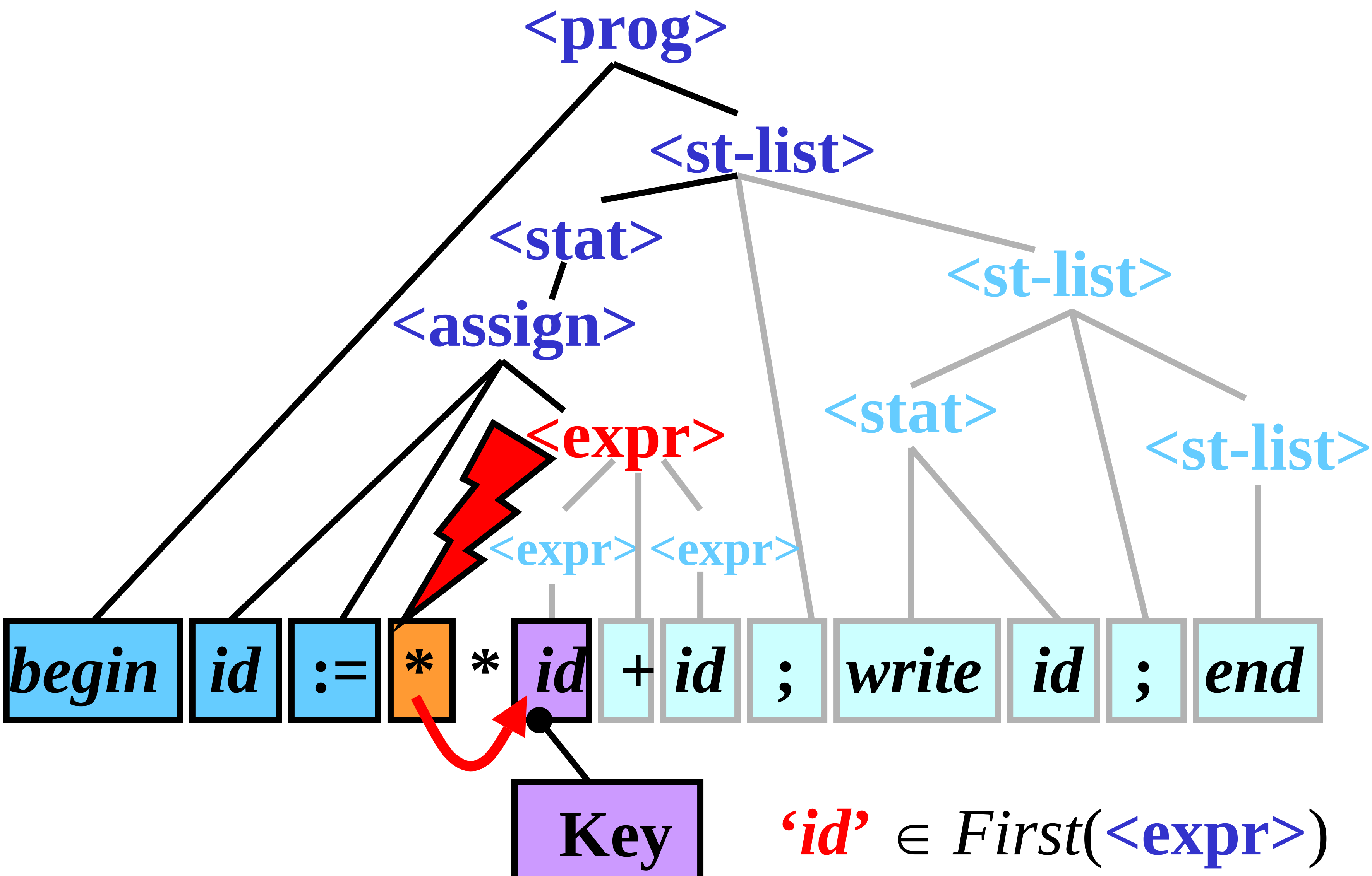
  a := GetNextToken;

  {These tokens are skipped}

  until ***a*** in ***Context***(***A***)
- **if** ***a*** $\in$ *First*(***A***) **then** resume according to ***A***

  else pop ***A*** from the pushdown // ***a*** $\in$ *Follow*(***A***)

Variant II: Example



‘*id*’ ∈ *First*(**<expr>**)