Scattered Context Generators of Sentences With Their Parses

Alexander Meduna et al.

Department of Information Systems
Faculty of Information Technology
Brno University of Technology
Božetěchova 2, Brno 61266, Czech Republic

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Contents I

Outline

1 Definitions
   - Production Labels
   - Proper Generator of its Sentences with Their Parses

2 Generation of Sentences With Their Parses by Scattered Context Grammars
   - Theorem 1
   - Proof 1

3 Canonical Scattered Context Generators of Sentences with Their Parses
   - Canonical Generators
   - Theorem 2
   - Proof 2

4 Conclusion
   - Future Investigation

5 Bibliography

Production Labels I

- for every grammar, G, there is a set of production labels
- we denote them $lab(G)$
- every $p \in lab(G)$ uniquely identifies one production
- we write $p : (A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$

Example

$G_1 = (\{S, A, B, C, a, b, c\}, P_1, S, \{a, b, c\})$
$lab(G_1) = \{1, 2, 3\}$
$$\begin{aligned} P_1 = \{\ & 1 : (S) \rightarrow (ABC), \\ & 2 : (A, B, C) \rightarrow (aA, bB, cC), \\ & 3 : (A, B, C) \rightarrow (\epsilon, \epsilon, \epsilon)\} \end{aligned}$$

$L(G_1) = \{a^n b^n c^n \ : \ n \geq 0\}$
G_1 generates $L(G_1)$ in a leftmost way

Production Labels II

- to express that $x \Rightarrow y$ by $p : (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$, we write $x \Rightarrow y \ [p]$

Example

$S \Rightarrow ABC \ [1] \Rightarrow aAbBcC \ [2] \Rightarrow aaAbbBccC \ [2] \Rightarrow aabbcc \ [3]$ in G_1

- to express that $x \Rightarrow^* y$ by productions labeled with $p_1, \dots, p_n$, we write $x \Rightarrow^* y \ [p_1 \dots p_n]$
- $p_1 \dots p_n \in lab(G)^*$

Example

$S \Rightarrow^* aabbcc \ [1223]$ in G_1

$1223 \in lab(G_1)^*$

Proper Generator of its Sentences with Their Parses I

Parse

If $S \Rightarrow {}^{*}x\ [\rho], x \in T^{*}, \rho \in lab(G)^{*}$, then x is a sentence generated by G according to *parse* ρ

Example

aabbcc is a sentence generated according to parse 1223 in G_1

Proper generator of its sentences with their parses

- G is a proper generator of its sentences with their parses if $L(G) = \{x : x = y\rho, y \in (T - lab(G))^{*}, \rho \in lab(G)^{*}, S \Rightarrow {}^{*}x\ [\rho]\}$

Proper Generator of its Sentences with Their Parses II

Example

$G_2 = (\{S, A, B, C, a, b, c, 1, 2, 3, \$\}, P_2, S, \{a, b, c, 1, 2, 3\})$

$lab(G_2) = \{1, 2, 3\}$

$P_2 = \{\ 1 : (S) \rightarrow (ABC1\$)$
$\quad 2 : (A, B, C, \$) \rightarrow (aA, bB, cC, 2\$)$
$\quad 3 : (A, B, C, \$) \rightarrow (\epsilon, \epsilon, \epsilon, 3)\}$

$S \Rightarrow ABC1\$\ [1] \Rightarrow aAbBcC12\$\ [2] \Rightarrow aaAbbBccC122\$\ [2] \Rightarrow$
$aabbcc1223\ [3]$

$S \Rightarrow^* aabbcc1223\ [1223]$

$L(G_2) = \{a^n b^n c^n \rho\ :\ n \geq 0, S \Rightarrow^* a^n b^n c^n \rho\ [\rho], \rho = 12^n 3\}$

G_2 is a proper generator of its sentences with their parses

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Theorem 1

- let $G = (V, P, S, T)$ be a proper generator of its sentences with their parses
- we define the weak identity π from V^* to $(V - lab(G))^*$ as
  - $\pi(a) = a$ for every $a \in (V - lab(G))$
  - $\pi(p) = \epsilon$ for every $p \in lab(G)$

Example

$\pi(aabbcc1223) = aabbcc$ in G_2

Theorem

For every recursively enumerable language, L, there exists a **PSCG**, *G, such that G is a proper generator of its sentences with their parses and $L = \pi(L(G))$.*

Problem Sketch

- idea: simulation of **SCG** by **PSCG** and adding labels of used productions
- problem: productions of **PSCG** do not contain ϵ on their right hand sides
- instead of ϵ, we can use some fill-in symbol (y)

Example

$$G_3 = (\{S, A, B, C, a, b, c, y, 1, 2, 3, \$\}, P_3, S, \{a, b, c, y, 1, 2, 3\})$$

$$lab(G_3) = \{1, 2, 3\}$$

$$\begin{aligned} P_3 = \{\ & 1 : (S) \rightarrow (ABC1\$) \\ & 2 : (A, B, C, \$) \rightarrow (aA, bB, cC, 2\$) \\ & 3 : (A, B, C, \$) \rightarrow (y, y, y, 3)\} \end{aligned}$$

$$\begin{aligned} S \Rightarrow\ & ABC1\$\ [1] \Rightarrow aAbBcC12\$\ [2] \Rightarrow aaAbbBccC122\$\ [2] \Rightarrow \\ & aaybbyccy1223\ [3] \end{aligned}$$

Problem Solution

Basic idea

1. generate the parse in front of the sentence
2. move the symbols of generated sentence in fornt of the parse
3. replace fill-in symbols with the parse

Demonstration

1. $S \Rightarrow^* 1223aaybbyccy$
2. $1223aaybbyccy \Rightarrow^* aabbcc1223\ldots yyyyyyyyy$
3. $aabbcc1223\ldots yyyyyyyyy \Rightarrow^* aabbcc1223\ldots$

Proof

- original **SCG** $\bar{G} = (\bar{V}, \bar{P}, \bar{S}, \bar{T})$
- $\Phi = \{\langle a \rangle \ : \ a \in \bar{T}\}$
- $\Gamma = \{\$_1, \$_2, \$_3\}$

Constructed **PSCG** $G = (V, P, S, T)$

$$G = (\{S, X, Y, Z\} \cup \bar{V} \cup lab(G) \cup \Phi \cup \Gamma, P, S, \bar{T} \cup lab(G))$$

Example

$$\bar{G}_4 = (\{\bar{S}, \bar{A}, \bar{B}, \bar{C}, a, b, c\}, \bar{P}_4, \bar{S}, \{a, b, c\});\ lab(\bar{G}_4) = \{\bar{1}, \bar{2}, \bar{3}\}$$

$$\begin{aligned} \bar{P}_4 = \{\ & \bar{1} : (\bar{S}) \rightarrow (\bar{A}\bar{B}\bar{C}) \\ & \bar{2} : (\bar{A}, \bar{B}, \bar{C}) \rightarrow (a\bar{A}, b\bar{B}, c\bar{C}) \\ & \bar{3} : (\bar{A}, \bar{B}, \bar{C}) \rightarrow (\epsilon, \epsilon, \epsilon)\} \end{aligned}$$

$$\begin{aligned} {\color{red}G_4} = (\ & \{S, X, Y, Z\} \cup \{\bar{S}, \bar{A}, \bar{B}, \bar{C}, a, b, c\} \cup lab(G_4) \cup \{\langle a \rangle, \langle b \rangle, \langle c \rangle\} \cup \\ & \{\$_1, \$_2, \$_3\}, P_4, S, \{a, b, c\} \cup lab(G_4)) \end{aligned}$$

Productions I

Step 1 (preparation)

Add $\lfloor 1 \rfloor : (S) \rightarrow (X\lfloor 1 \rfloor \$_1 Z\bar{S})$ to P

Example

Add $\lfloor 1 \rfloor : (S) \rightarrow (X\lfloor 1 \rfloor \$_1 Z\bar{S})$ to P_4

Productions II

- $\gamma(a) = \langle a \rangle$, $a \in \bar{T}$
  $\gamma(A) = A$, $A \in \bar{V} - \bar{T}$
  $\gamma(\epsilon) = Y$

Step 2 (simulation of the **SCG**)

For every $p : (A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n) \in \bar{P}$ add
$\lfloor p1 \rfloor : (\$_1, A_1, \ldots, A_n) \rightarrow (\lfloor p1 \rfloor \$_1, \gamma(x_1), \ldots, \gamma(x_n))$ to P;
add $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P

Example

Add $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$ to P_4
$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$ to P_4
$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$ to P_4
Add $\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Productions III

Step 3 (moving symbols before the parse)

For every $a \in \bar{T}$, add

$\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$ to P;
$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$ to P

Example

Add $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$ to P_4
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$ to P_4
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$ to P_4

$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$ to P_4
$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$ to P_4
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$ to P_4

Productions IV

Step 4 (replacing Y with the parse)

Add $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$ to P

Step 5 (removing $\$_3$)

Add $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$ to P

Example

Add $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$ to P_4
$\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$ to P_4

Example

S

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$

$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$

$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$

$\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$

$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$

$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$

$\lfloor a3 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (a, \lfloor a3 \rfloor \$_3, Y, Y)$

$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$

$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

Example

S

Step 1 $\lfloor 1 \rfloor : (S) \rightarrow (X \lfloor 1 \rfloor \$_1 Z \bar{S})$

Step 2 $\lfloor \bar{1}1 \rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1 \rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1 \rfloor \$_1, \langle a \rangle \bar{A}, \langle b \rangle \bar{B}, \langle c \rangle \bar{C})$
$\lfloor \bar{3}1 \rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1 \rfloor \$_1, Y, Y, Y)$
$\lfloor 2 \rfloor : (\$_1) \rightarrow (\lfloor 2 \rfloor \$_2)$ to P_4

Step 3 $\lfloor a2 \rfloor : (X, \$_2, Z, \langle a \rangle) \rightarrow (aX, \lfloor a2 \rfloor \$_2, Y, Z)$
$\lfloor b2 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (bX, \lfloor b2 \rfloor \$_2, Y, Z)$
$\lfloor c2 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (cX, \lfloor c2 \rfloor \$_2, Y, Z)$
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$\lfloor b3 \rfloor : (X, \$_2, Z, \langle b \rangle) \rightarrow (b, \lfloor b3 \rfloor \$_3, Y, Y)$
$\lfloor c3 \rfloor : (X, \$_2, Z, \langle c \rangle) \rightarrow (c, \lfloor c3 \rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3 \rfloor : (\$_3, Y) \rightarrow (\lfloor 3 \rfloor, \$_3)$

Step 5 $\lfloor 4 \rfloor : (\$_3) \rightarrow (\lfloor 4 \rfloor)$

Example

$X\lfloor 1\rfloor\$_1 Z\bar{S}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor \$_1 Z\bar{S}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor \$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor \$_1, \langle a\rangle \bar{A}, \langle b\rangle \bar{B}, \langle c\rangle \bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor \$_1, Y, Y, Y)$
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$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor \$_3, Y, Y)$
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Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\$_1 Z\bar{A}\bar{B}\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
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Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\$_1 Z\bar{A}\bar{B}\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$

$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$

$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$

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Example

$X\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\$_1 Z\langle a\rangle\bar{A}\langle b\rangle\bar{B}\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$

$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$

$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$

$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

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$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$

$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$

$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\$_1 Z\langle a\rangle\bar{A}\langle b\rangle\bar{B}\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\$_1 Z\langle a\rangle\langle a\rangle\bar{A}\langle b\rangle\langle b\rangle\bar{B}\langle c\rangle\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\$_1 Z\langle a\rangle\langle a\rangle\bar{A}\langle b\rangle\langle b\rangle\bar{B}\langle c\rangle\langle c\rangle\bar{C}$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\$_1 Z\langle a\rangle\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\$_1 Z\langle a\rangle\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\$_2 Z\langle a\rangle\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$X\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\$_2 Z\langle a\rangle\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aX\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\$_2 Y Z\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\$_2 Y Z\langle a\rangle Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aaX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\$_2 Y Y Z Y\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aaX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\$_2 YYZY\langle b\rangle\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\$_2 YYYYZ\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabX\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\$_2 YYYYZ\langle b\rangle Y\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\$_2 YYYYYZY\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\$_2 YYYYYZY\langle c\rangle\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcX\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\$_2YYYYYYYZ$
$\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcX\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\$_2 YYYYYYZ\langle c\rangle Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor \$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor \$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor \$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor \$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor \$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor \$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor \$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor \$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor \$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\$_3 YYYYYYY$
YYY

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\$_3 Y\,YYYYY\,YYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\$_3YYYYY$
$YYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\$_3 Y YYY$
$YYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3YYY$
$YYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor\bar{1}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{2}1\rfloor\lfloor\bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y Y YYYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor\bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor\bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$

$\lfloor\bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$

$\lfloor\bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor\bar{3}1\rfloor\$_1, Y, Y, Y)$

$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$

$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$

$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$

$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$

$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$

$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y YYYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3$
$YYYYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\$_3 YYYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\$_3 Y YYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\$_3 YYYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor \$_3 Y YYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor \$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor \$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor \$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor \$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor \$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor \$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor \$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor \$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor \$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\$_3 YYYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y YYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 YYY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y YY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 YY$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3 Y$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\$_3$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor\$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor\$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor\$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor\$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor\$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor\$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor\$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor\$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor\$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor\$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor\$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 4\rfloor$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor \$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor \$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor \$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor \$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor \$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor \$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor \$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor \$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor \$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Example

$aabbcc\lfloor 1\rfloor\lfloor \bar{1}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{2}1\rfloor\lfloor \bar{3}1\rfloor\lfloor 2\rfloor\lfloor a2\rfloor\lfloor a2\rfloor\lfloor b2\rfloor\lfloor b2\rfloor\lfloor c2\rfloor\lfloor c3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor$
$\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 3\rfloor\lfloor 4\rfloor$

Step 1 $\lfloor 1\rfloor : (S) \rightarrow (X\lfloor 1\rfloor \$_1 Z\bar{S})$

Step 2 $\lfloor \bar{1}1\rfloor : (\$_1, \bar{S}) \rightarrow (\lfloor \bar{1}1\rfloor \$_1, \bar{A}\bar{B}\bar{C})$
$\lfloor \bar{2}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{2}1\rfloor \$_1, \langle a\rangle\bar{A}, \langle b\rangle\bar{B}, \langle c\rangle\bar{C})$
$\lfloor \bar{3}1\rfloor : (\$_1, \bar{A}, \bar{B}, \bar{C}) \rightarrow (\lfloor \bar{3}1\rfloor \$_1, Y, Y, Y)$
$\lfloor 2\rfloor : (\$_1) \rightarrow (\lfloor 2\rfloor \$_2)$ to P_4

Step 3 $\lfloor a2\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (aX, \lfloor a2\rfloor \$_2, Y, Z)$
$\lfloor b2\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (bX, \lfloor b2\rfloor \$_2, Y, Z)$
$\lfloor c2\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (cX, \lfloor c2\rfloor \$_2, Y, Z)$
$\lfloor a3\rfloor : (X, \$_2, Z, \langle a\rangle) \rightarrow (a, \lfloor a3\rfloor \$_3, Y, Y)$
$\lfloor b3\rfloor : (X, \$_2, Z, \langle b\rangle) \rightarrow (b, \lfloor b3\rfloor \$_3, Y, Y)$
$\lfloor c3\rfloor : (X, \$_2, Z, \langle c\rangle) \rightarrow (c, \lfloor c3\rfloor \$_3, Y, Y)$

Step 4 $\lfloor 3\rfloor : (\$_3, Y) \rightarrow (\lfloor 3\rfloor, \$_3)$

Step 5 $\lfloor 4\rfloor : (\$_3) \rightarrow (\lfloor 4\rfloor)$

Additional Productions

- 2 additional productions for generation of sentence which corresponds to ϵ in $\bar{G}$

Step 1 (without symbols X, Z)

Add $\lfloor 1_\epsilon \rfloor : (S) \rightarrow (\lfloor 1_\epsilon \rfloor \$_1 \bar{S})$ to P

Step 2 (skipping step 3)

Add $\lfloor 2_\epsilon \rfloor : (\$_1) \rightarrow (\lfloor 2_\epsilon \rfloor \$_3)$ to P

Example

Add $\lfloor 1_\epsilon \rfloor : (S) \rightarrow (\lfloor 1_\epsilon \rfloor \$_1 \bar{S})$ to P_4
$\lfloor 2_\epsilon \rfloor : (\$_1) \rightarrow (\lfloor 2_\epsilon \rfloor \$_3)$ to P_4

Outline

1 Definitions
- Production Labels
- Proper Generator of its Sentences with Their Parses

2 Generation of Sentences With Their Parses by Scattered Context Grammars
- Theorem 1
- Proof 1

3 Canonical Scattered Context Generators of Sentences with Their Parses
- Canonical Generators
- Theorem 2
- Proof 2

4 Conclusion
- Future Investigation

5 Bibliography

Leftmost derivation step in **SCG**

Derivation step in **SCG**

if $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$

$u = u_1 A_1 \dots u_n A_n u_{n+1}$

$v = u_1 x_1 \dots u_n x_n u_{n+1}$

then $u \Rightarrow v \; [(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)]$

- $alph(w)$ denotes the set of all symbols occuring in w

Example

$alph(bacaab) = \{a, b, c\}$

Leftmost derivation step in **SCG**

every $A_i \notin alph(u_i)$ for all $1 \leq i \leq n$

Language Generated in a Leftmost Way

Language generated in a leftmost way

$L(G) = \{x : x \in T^*, S \Rightarrow {}^* x\}$

- and every step in every generation of $x \in T^*$ is leftmost

Proper leftmost generator of its sentences with their parses

$L(G) = \{x : x = y\rho, y \in (T - lab(G))^*, \rho \in lab(G)^*, S \Rightarrow {}^* x\ [\rho]\}$

- and G generates $L(G)$ in a leftmost way

Language Generated in a Leftmost Way – Example

Example

$G_5 = (\{S, A, B, C, a, b, c, 1, 2, 3, 4, \$\}, P_5, S, \{a, b, c, 1, 2, 3, 4\})$

$lab(G_5) = \{1, 2, 3, 4\}$

$$\begin{aligned} P_5 = \{\ & 1 : (S) \rightarrow (ABC1\$), \\ & 2 : (A, B, C, \$) \rightarrow (AA, BB, CC, 2\$), \\ & 3 : (A, B, C, \$) \rightarrow (a, b, c, 3\$), \\ & 4 : (A, B, C, \$) \rightarrow (\epsilon, \epsilon, \epsilon, 4)\} \end{aligned}$$

$$\begin{aligned} S &\Rightarrow ABC1\$\ [1] \Rightarrow AABBCC12\$\ [2] \Rightarrow AabBCc123\$\ [3] \Rightarrow \\ &AAabBBCCc1232\$\ [2] \Rightarrow aAabBbcCc12323\$\ [3] \Rightarrow \\ &aabbcc123234\$\ [4] \end{aligned}$$

$S \Rightarrow^* aabbcc123234\ [123234]$

$L(G_5) = \{a^n b^n c^n \rho \ :\ n \geq 0, S \Rightarrow^* a^n b^n c^n \rho\ [\rho]\}$

G_5 is a proper generator of its sentences with their parses

G_5 is not a proper leftmost generator of its sentences with their parses

Theorem 2

- let $G = (V, P, S, T)$ be a proper generator of its sentences with their parses
- we define the weak identity π from V^* to $(V - lab(G))^*$ as
  - $\pi(a) = a$ for every $a \in (V - lab(G))$
  - $\pi(p) = \epsilon$ for every $p \in lab(G)$

Example

$\pi(aabbcc123234) = aabbcc$ in G_5

Theorem

For every recursively enumerable language, L, there exists a **PSCG**, *G, such that G contains no more than six nonterminals, G is a proper leftmost generator of its sentences with their parses and $L = \pi(L(G))$.*

Queue Grammar (**QG**)

- we represent the recursively enumerable language by a queue grammar

Queue Grammar $G = (V, T, W, F, s, P)$

V is a finite alphabet of symbols

T is a set of terminals, $T \subset V$

W is a finite alphabet of states

F is a set of final states, $F \subset W$

s is a starting string, $s \in (V - T)(W - F)$

P is a finite set of productions of the form: (a, b, x, c)

a $\in V$
b $\in (W - F)$
x $\in V^*$
c $\in W$

Queue Grammar – Derivation Step

Derivation Step

If $u = arb$, $v = rxc$, $a \in V$, $r, x \in V^*$, $b, c \in W$, and $(a, b, x, c) \in P$, then $u \Rightarrow v\ [(a, b, x, c)]$.

Generated Language

$L(G) = \{w : s \Rightarrow^* wf, w \in T^*, f \in F\}$

Generative Power (see [2])

$\mathcal{L}_{QG} = \mathcal{L}_{RE}$

Lemma

For every **QG** *there exists an equivalent* **QG** *which generates every string so that it first uses only productions rewriting symbols over* $(V - T)^*$, *and then only symbols over* T^* *(proof, see [5]).*

Queue Grammar – Example

Example

$G_4 = (\{A, a, b\}, \{a, b\}, \{\bar{e}, \bar{f}\}, \{\bar{f}\}, A\bar{e}, P_4)$

$$\begin{aligned} P_4 = \{\ & 1 : (A, \bar{e}, bAa, \bar{e}), \\ & 2 : (A, \bar{e}, \varepsilon, \bar{f}), \\ & 3 : (a, \bar{e}, a, \bar{e}), \\ & 4 : (b, \bar{e}, b, \bar{e})\} \end{aligned}$$

$A\bar{e} \Rightarrow bAa\bar{e}\ [1] \Rightarrow Aab\bar{e}\ [4] \Rightarrow abbAa\bar{e}\ [1] \Rightarrow bbAaa\bar{e}\ [3] \Rightarrow bAaab\bar{e}\ [4]$
$\Rightarrow Aaabb\bar{e}\ [4] \Rightarrow aabb\bar{f}\ [2]$

$L(G_4) = \{a^n b^n \ :\ n \geq 0\}$

Proof Sketch

Basic idea

1. represent the recursively enumerable language by a **QG**
2. initiate the derivation
3. simulate **QG** by **PSCG**
    1. simulate generation of words from $(V - T)^*$
    2. simulate generation of words from T^+
4. check if the simulation was correct
5. complete the derivation

- every production has to add its label to the sentential form to create the parse in the correct order
- generated sentence must precede this parse

Proof

- $Q = (V, T, W, F, s, R)$, $L(Q) = L$
- α: injection from $lab(Q)$ to $\{\bar{0}\}^*\{\bar{1}\}$
- β: injection from T to $\{0\}^*\{1\}$
- $f(a) = \{\alpha(r) \; : \; r : (a, b, x, c) \in R\}$ for all $a \in V$
- $g(b) = \{\alpha(r) \; : \; r : (a, b, x, c) \in R\}$ for all $b \in W$

Constructed **PSCG**

$G = (\{S, A, B, \#, \bar{0}, \bar{1}\}, P, S, \{0, 1\} \cup lab(G))$

- the construction of P and $lab(G)$ is demonstrated on the following slides

Productions

Step 1 (initialization)

For every $\bar{a}_0 \in f(a_0)$, $\bar{q}_0 \in g(q_0)$ such that $s = a_0 q_0$, add

$\lfloor 1\bar{a}_0\bar{q}_0 \rfloor : (S) \to (A\lfloor 1\bar{a}_0\bar{q}_0 \rfloor AA\bar{q}_0 A\bar{a}_0 AB)$ to P

Step 2 (simulation of Q's productions generating words over V-T)

For every $r : (a, b, c_1 \ldots c_n, d) \in R$, $c_1, \ldots, c_n \in (V - T)$ for some $n \geq 0$ and $d \in (W - F)$, $\bar{c}_1 \in f(c_1), \ldots, \bar{c}_n \in f(c_n)$, $\bar{d} \in g(d)$, add

$$\begin{aligned}\lfloor 2r\bar{c}_1 \ldots \bar{c}_n\bar{d} \rfloor &: (A, A, A, A, A, B) \to \\ &: (A, \lfloor 2r\bar{c}_1 \ldots \bar{c}_n\bar{d} \rfloor A, \alpha(r)A, \bar{d}A, \bar{c}_1 \ldots \bar{c}_n A, B) \text{ to } P\end{aligned}$$

Step 3 (separation between steps 2 and 4)

Add $\lfloor 3 \rfloor : (A, A, A, A, A, B) \to (A, \lfloor 3 \rfloor A, A, A, B, A)$ to P

Productions I

Step 4 (simulation of Q's productions generating words over T)

For every $r : (a, b, c_1 \ldots c_n, d) \in R$, $c_1, \ldots, c_n \in T$ for some $n \geq 0$ and $d \in (W - F)$, $\bar{d} \in g(d)$, add

$\lfloor 4r\bar{d} \rfloor : (A, A, A, A, B, A) \rightarrow (\beta(c_1) \ldots \beta(c_n)A, \lfloor 4r\bar{d} \rfloor A, \alpha(r)A, \bar{d}A, B, A)$ to P

Step 5 (simulation of Q's final step)

For every $r : (a, b, c_1 \ldots c_n, d) \in R$, $c_1, \ldots, c_n \in T$ for some $n \geq 0$ and $d \in F$, add

$\lfloor 5r \rfloor : (A, A, A, A, B, A) \rightarrow (\beta(c_1) \ldots \beta(c_n), \lfloor 5r \rfloor A, \alpha(r)A, A, B, AA)$ to P

Productions II

Step 6 (simulation verification)

Add

$\lfloor 6 \rfloor : (A, \bar{0}, A, \bar{0}, A, \bar{0}, B, A, A) \rightarrow (\lfloor 6 \rfloor, A, \#, A, \#, A, B, A, A)$, and
$\lfloor 7 \rfloor : (A, \bar{1}, A, \bar{1}, A, \bar{1}, B, A, A) \rightarrow (\lfloor 7 \rfloor, A, \#, A, \#, A, B, A, A)$ to P;

Step 7 (finishing the derivation)

Add

$\lfloor 8 \rfloor : (A, A, A, B, A, A) \rightarrow (\lfloor 8 \rfloor B, \#, \#, \#, \#, \#)$,
$\lfloor 9 \rfloor : (B, \#) \rightarrow (\lfloor 9 \rfloor, B)$, and
$\lfloor 10 \rfloor : (B) \rightarrow (\lfloor 10 \rfloor)$ to P.

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4 Conclusion
- Future Investigation

5 Bibliography

Conclusion

We have proved that

- for every **RE** there is a **PSCG** which generates its sentences with their parses
- there are canonical versions of generators
- the number of needed nonterminals can be reduced

Future investigation

- which other grammars can be used as proper generators of their sentences with their parses?
  - grammar systems seem to be appropriate candidates
- is it possible to generate sentences together with other useful information (e.g. derivation trees)?

Outline

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