New Book:

Grammars with Context Conditions
and Their Applications

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1. Introduction

The classical language theory is based on the Chomsky hierarchy of *regular*, *context-free*, *context-sensitive*, and *phrase-structure grammars*.
These grammars have several disadvantages concerning the contextual sensitivity:

Disadvantages of Regular and Context-Free Grammars

- no context sensitivity
- significantly less powerful than context-sensitive and phrase-structure grammars

Disadvantages of Context-Sensitive and Phrase-Structure Grammars

- strict conditions placed on the context surrounding the rewritten symbol
- complex form of rewriting productions
- difficult to use in practice

Advantages of Grammars with Context Conditions

- they are *based on context-independent productions*
- their context conditions are *simple and flexible*
- they are *as powerful as classical context-dependent grammars*

Classification of Their Context Conditions

- conditions placed on *derivation domains*
- conditions placed on *the use of grammatical productions*
- conditions placed on *the neighborhood of the rewritten symbols*

Sequential and Parallel Conditional Grammars

Both sequential and parallel versions of the grammars are studied. *Sequential grammars* rewrite only one symbol during a derivation step while *parallel grammars* rewrite all symbols in one derivation step.

A Context-Free Grammar is a quadruple $G = (V, T, P, S)$, where

- V is the total alpabet
- T is a finite set of terminal symbols, $T \subset V$
- P is a finite set of productions $A \rightarrow x$, where $A \in V - T$, $x \in V^*$
- S is the axiom, $S \in V - T$

Context-Free Derivation

Given uAv and uxv, where $A \in V - T, u, v, x \in V^*$,

$$uAv \Rightarrow_G uxv \text{ if and only if } A \rightarrow x \in P$$

Notation

- **CF** – the family of context-free langages
- **CS** – the family of context-sensitive languages
- **RE** – the family of recursively enumerable languages

An E0L Grammar is a quadruple $G = (V, T, P, S)$, where V, T, and S have the same meaning as in context-free grammars.

Productions of E0L grammars have the form $a \rightarrow x$, where $a \in V$, $x \in V^*$.

Derivation in E0L Grammars

Given $u = a_1 \ldots a_n$ and $v = x_1 \ldots x_n$,

$$\boxed{u \Rightarrow_G v \text{ if and only if } a_i \rightarrow x_i \in P \text{ for all } i = 1, \ldots, n}$$

Illustration

$$\begin{array}{ccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & \ldots & a_n \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow \\ x_1 & x_2 & x_3 & x_4 & x_5 & \ldots & x_n \end{array}$$

Families of languages

- **E0L** – the family of languages generated by E0L grammars
- **EP0L** – E0L grammars without erasing productions (propagating E0L grammars)

Generative power of E0L grammars

$$\mathbf{CF} \subset \mathbf{E0L} = \mathbf{EP0L} \subset \mathbf{CS}$$

2. Conditions Placed on Derivation Domains

- Classical grammars define the derivation relation over V^*, where V is an *alphabet*.
- *Grammars over word monoids* define the derivation relation over W^*, where W is a *finite language*.

Basic idea

$V = \{a, b, c, d\}$
$W = \{a, bc, d\}$

$aababd \in V^*$ – can be obtained by a concatenation of members of V;
$aababd \notin W^*$ – cannot be obtained by a concatenation of members of W;
$aabcad \in W^*$ – can be obtained by a concatenation of a, a, bc, a, d from W;

Intuitively, the string bc in W represents a *context condition* “b’s right neighbor has to be c”.

Context-free grammar over word monoid (*wm*-grammar) is a pair (G, W)

- $G = (V, T, P, S)$ is a context-free grammar
- W is a *finite language over* V
- *degree of* (G, W) is the maximal length of strings in W

The relation of a direct derivation from u to v is defined as

$u \Rightarrow_{(G,W)} v$ if and only if $u \Rightarrow_G v$ and $u, v \in W^*$**(!)**

Families of languages

- **WM**(i), **WM** – families of languages generated by *wm*-grammars of degree i and of any degree, respectively
- **prop-WM**(i), **prop-WM** – no erasing productions are allowed

Generative power

$$\textbf{prop-WM}(1) = \textbf{WM}(1) = \textbf{CF}$$
$$\subset$$
$$\textbf{prop-WM}(2) = \textbf{prop-WM} = \textbf{CS}$$
$$\subset$$
$$\textbf{WM}(2) = \textbf{WM} = \textbf{RE}$$

Reduction of *wm*-grammars

Theorem: Every $L \in$ **RE** can be defined by a ten-nonterminal context-free grammar over a word monoid generated by an alphabet and six words of length two.

E0L grammar over word monoid (WME0L grammar) is a pair (G, W)

- $G = (V, T, P, S)$ is an E0L grammar
- W is a *finite language over* V
- *degree of* (G, W) is the maximal length of strings in W

The relation of a direct derivation from u to v is defined as

$$u \Rightarrow_{(G,W)} v \text{ if and only if } u \Rightarrow_G v \text{ and } u, v \in W^* \textbf{(!)}$$

Families of languages

- **WME0L**(i), **WME0L** – families of languages generated by WME0L grammars of degree i and of any degree, respectively
- **WMEP0L**(i), **WMEP0L** – no erasing productions are allowed

Generative power

$$\mathbf{CF}$$
$$\subset$$
$$\mathbf{WMEP0L}(1) = \mathbf{WME0L}(1) = \mathbf{EP0L} = \mathbf{E0L}$$
$$\subset$$
$$\mathbf{WMEP0L}(2) = \mathbf{CS}$$
$$\subset$$
$$\mathbf{WME0L}(2) = \mathbf{RE}$$

A Context-Conditional Grammar is a grammar with context conditions represented by strings *associated with productions*.

Types of conditions

forbidding – the string *must not* occur as a substring of the sentential form

permitting – the string *must* occur in the sentential form

A production is applicable to a sentential form if each of its permitting conditions occurs in the sentential form and any of its forbidding conditions does not.

Example

$S \Rightarrow^* ABCD \Rightarrow ?$

$(A \rightarrow x, \emptyset, \{C\})$ – cannot be applied: C occurs in $ABCD$

$(A \rightarrow y, \{AB\}, \{F\})$ – can be applied: AB is in $ABCD$, F is not in $ABCD$

A Sequential Context-Conditional Grammar is a *context-free grammar* with sets of *permitting and forbidding conditions* attached to productions.

Productions have the form

$(A \rightarrow x, Per, For)$

Per – finite set of permitting conditions, $Per \subseteq V^+$.

For – finite set of forbidding conditions, $For \subseteq V^+$.

Such a production can rewrite A to x provided that *all strings from* Per occur in the sentential form and *no string from* For occurs in the sentential form

Degree

A context-conditional grammar has *degree* (r, s) if the maximal length of permitting conditions is less or equal r and if the maximal length of forbidding conditions is less or equal s.

Families of Languages

- Context-conditional grammars with erasing productions generate **RE**.
- Context-conditional grammars without erasing productions generate exactly **CS**.

Random-Context Grammars

- permitting conditions are *nonterminal symbols*
- no forbidding conditions
- $(A \rightarrow x, Per, For)$, $Per \subseteq N$, $For = \emptyset$
- introduced by A. P. J. van der Walt, 1970

Random-Context Grammars with Appearance Checking

- permitting and forbidding conditions are *nonterminal symbols*
- $(A \rightarrow x, Per, For)$, $Per, For \subseteq N$

Forbidding Grammars

- no perimitting conditions
- forbidding conditions are *nonterminal symbols*
- $(A \rightarrow x, Per, For)$, $Per = \emptyset$, $For \subseteq N$
- M. Penttonen, 1975

A Generalized Forbidding Grammar (*gf*-grammar) is a context-conditional grammar in which every production contains *no permitting conditions*.

$$\boxed{(A \rightarrow x, Per, For) \text{ satisfies } Per = \emptyset}$$

Families of Languages

- **GF**(i) and **GF** – families of languages generated by *gf*-grammars of degree i and of any degree, respectively
- **prop-GF**(i) and **prop-GF** – no erasing productions are allowed

Generative Power

$$\textbf{prop-GF}(0) = \textbf{GF}(0) = \textbf{CF}$$
$$\subset$$
$$\textbf{GF}(1) = \textbf{F}$$
$$\subset$$
$$\textbf{GF}(2) = \textbf{GF} = \textbf{RE}$$

Reduction of *gf*-grammars

Theorem: Every $L \in$ **RE** can be defined by a generalized forbidding grammar of *degree 2* with no more than *13 conditional productions* and *15 nonterminals*.

A Simple Semi-Conditional Grammar (*ssc*-grammar) is a context-conditional grammar in which every production contains *no more than one condition*.

$$(A \rightarrow x, Per, For) \text{ satisfies } |Per| + |For| \leq 1$$

Families of Languages

- **SSC**(r, s), **SSC** – families of languages generated by *ssc*-grammars of degree (r, s) and of any degree, respectively.
- **prop-SSC**(r, s), **prop-SSC** – no erasing productions are allowed

Generative Power

$$\textbf{CF}$$
$$\subset$$
$$\textbf{prop-SSC} = \textbf{prop-SSC}(2, 1) = \textbf{prop-SSC}(1, 2) = \textbf{CS}$$
$$\subset$$
$$\textbf{SSC} = \textbf{SSC}(2, 1) = \textbf{SSC}(1, 2) = \textbf{RE}$$

Reduction of *ssc*-grammars

Theorem: Every $L \in$ **RE** can be defined by a simple semi-conditional grammar of *degree (2,1)* with no more than *12 conditional productions* and *13 nonterminals*.

E(T)0L Grammars

- an important type of L-systems (A. Lindenmayer)
- based on context-independent productions
- T = several sets of productions
- all symbols are simultaneously rewritten during a derivation step:

$$\begin{array}{cccccccc} a & a & B & a & A & c & A & d \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{array}$$

Generative Power of E(T)0L Grammars

CF $\subset$ **E0L** $\subset$ **ET0L** $\subset$ **CS**

A Context-Conditional ET0L Grammar is an *ET0L grammar* with sets of *permitting and forbidding conditions* attached to productions.

Productions

$(a \to x, Per, For)$

Per – finite set of permitting conditions, $Per \subseteq V^+$;

For – finite set of forbidding conditions, $For \subseteq V^+$.

Such a production can rewrite a to x provided that *all strings from* Per occur in the sentential form and *no string from* For occurs in the sentential form

Degree (r, s)

Defined by analogy with the degree of sequential context-conditional grammars; that is, r is the maximal length of a permitting condition in Per and s is the maximal length of a forbidding condition in For.

Forbidding ET0L Grammars

Forbidding ET0L Grammars (FET0L grammars) are context-conditional ET0L grammars with productions having only forbidding context conditions.

$$(a \rightarrow x, Per, For) \text{ satisfies } Per = \emptyset$$

Degree

The maximal length of forbidding context conditions.

Families of Languages - notation

- **F** = forbidding conditions
- **T** = several sets of productions
- **P** = without erasing productions (propagating)

Generative Power

$$\mathbf{CF}$$
$$\subset$$
$$\mathbf{FEP0L}(0) = \mathbf{FE0L}(0) = \mathbf{EP0L} = \mathbf{E0L}$$
$$\subset$$
$$\mathbf{FEP0L}(1) = \mathbf{FEPT0L}(1) = \mathbf{FE0L}(1) = \mathbf{FET0L}(1) =$$
$$\mathbf{FEPT0L}(0) = \mathbf{FET0L}(0) = \mathbf{EPT0L} = \mathbf{ET0L}$$
$$\subset$$
$$\mathbf{FEP0L}(2) = \mathbf{FEPT0L}(2) = \mathbf{FEP0L} = \mathbf{FEPT0L} = \mathbf{CS}$$
$$\subset$$
$$\mathbf{FE0L}(2) = \mathbf{FET0L}(2) = \mathbf{FE0L} = \mathbf{FET0L} = \mathbf{RE}$$

A Simple Semi-Conditional ET0L Grammar (SSC-ET0L grammar) is a context-conditional ET0L grammar in which every production contains *no more than one context condition*.

$$\boxed{(a \rightarrow x, Per, For) \text{ satisfies } |Per| + |For| \leq 1}$$

Families of Languages

Denoted by analogy with forbidding ET0L grammars; however, instead of prefix **F**, we use **SSC-** in terms of SSC-ET0L grammars.

Generative Power

$$\textbf{CF}$$
$$\subset$$
$$\textbf{SSC-EP0L}(0,0) = \textbf{SSC-E0L}(0,0) = \textbf{EP0L} = \textbf{E0L}$$
$$\subset$$
$$\textbf{SSC-EPT0L}(0,0) = \textbf{SSC-ET0L}(0,0) = \textbf{EPT0L} = \textbf{ET0L}$$
$$\subset$$
$$\textbf{SSC-EP0L}(1,2) = \textbf{SSC-EPT0L}(1,2) = \textbf{SSC-EP0L} = \textbf{SSC-EPT0L} = \textbf{CS}$$
$$\subset$$
$$\textbf{SSC-E0L}(1,2) = \textbf{SSC-ET0L}(1,2) = \textbf{SSC-E0L} = \textbf{SSC-ET0L} = \textbf{RE}$$

Continuous Context Grammars

Represented by classical *context-sensitive* and *phrase-structure* grammars. They are based on productions of the form

$$uAv \rightarrow uxv$$

The strings u and v can be interpreted as *continuous context conditions*.

Scattered Context Grammars rewrite several symbols in one step.

The rewritten symbols can be *scattered across the sentential form*; however, all of them *must occur* in the sentential form and they must occur in the *prescribed order*. Thus, they can be interpreted as *scattered context conditions*.
Example: consider a production $(B, C, D) \rightarrow (x, y, z)$. Then,

$$\begin{array}{ccccccccc} C & B & A & B & C & c & D & A & d \\ & \downarrow & & & \downarrow & & \downarrow & & \\ C & x & A & B & y & c & z & A & d \end{array}$$

Sequential and Parallel Uniform Rewriting

Aim

To make the generation of languages by phrase-structure grammars more uniform.

Results

Statement: For every phrase-structure grammar, there exists an equivalent phrase-structure grammar having the sentential forms based on a sequence of substrings, each of which represents a *permutation of symbols over a very small alphabet*.

Analogical results were achieved for

- phrase-structure grammars
- EIL grammars
- scattered context grammars

Reduction of Scattered Context Grammars

Scattered context grammars can be reduced with respect to the

- *number of nonterminals*
- *degree of context sensitivity*: number of context-sensitive productions
- *maximal context sensitivity*: maximal length of context-sensitive productions
- *total context sensitivity*: overall length of context-sensitive productions

Results

- *three-nonterminal* scattered context grammars generate **RE**
- several *simultaneous reductions* of complexity measures

5. Derivation Simulation

Some equivalent formal language models generate their language in a more similar way than others. Is it possible to formalize this similarity? Which transformations of language models satisfy this intuitive understanding of simulation?

Goals

- formalization of the similarity of rewriting processes
- more detailed approach to the equivalency of formal models
- new transformations leading to models closely simulating each other

Illustration of the Simulation Concept

Context-free grammars G_1 and G_2:

G_1: $S \rightarrow aSb$, $S \rightarrow ab$

G_2: $S \rightarrow aB$, $B \rightarrow Ab$,
$A \rightarrow aB$, $B \rightarrow b$

$$L(G_1) = L(G_2) = \{a^n b^n : n \geq 1\}$$

σ is a suitable substitution from G_2's alphabet to G_1's alphabet.

For every derivation step in G_1, there are no more than *two* corresponding derivation steps in G_2. We say that G_2 *2-closely simulates* G_1 *with respect to* σ.

G_1	$\overset{\sigma}{\longleftarrow}$	G_2
S	$\overset{S \in \sigma(S)}{\longleftarrow}$	S
		$\Downarrow$
$\Downarrow$		aB
		$\Downarrow$
aSb	$\overset{aSb \in \sigma(aAb)}{\longleftarrow}$	aAb
		$\Downarrow$
$\Downarrow$		$aaBb$
		$\Downarrow$
$aaSbb$	$\overset{aaSbb \in \sigma(aaAbb)}{\longleftarrow}$	$aaAbb$
$\Downarrow$		$\Downarrow$
$\vdots$		$\vdots$
$\Downarrow$		$\Downarrow$
$a^n b^n$	$\overset{a^n b^n \in \sigma(a^n b^n)}{\longleftarrow}$	$a^n b^n$

Results

- general *formal model of derivation simulation*
- several special cases: n*-close simulation*, *homomorphic simulation*, $\ldots$
- formalization of *grammatical simulations*

Practical Example

Theorem: For every E(0,1)L grammar G, there exists an equivalent WME0L(2) grammar G' and a homomorphism ω such that G' 1-closely homomorphically simulates G with respect to ω.

6. Applications

Three case studies in terms of a biological simulation.

Case Studies

- stagnation of a cellular organism infected by a virus
- degeneration and death of a red alga
- plant simulation controlled by a resource flow

This case study demonstrates development of a simple cellular organism infected by a virus. During a healthy development, every cell of the organism divides itself into two cells. However, when a virus infects some cells, all the organism stagnates until it is cured again.

Model

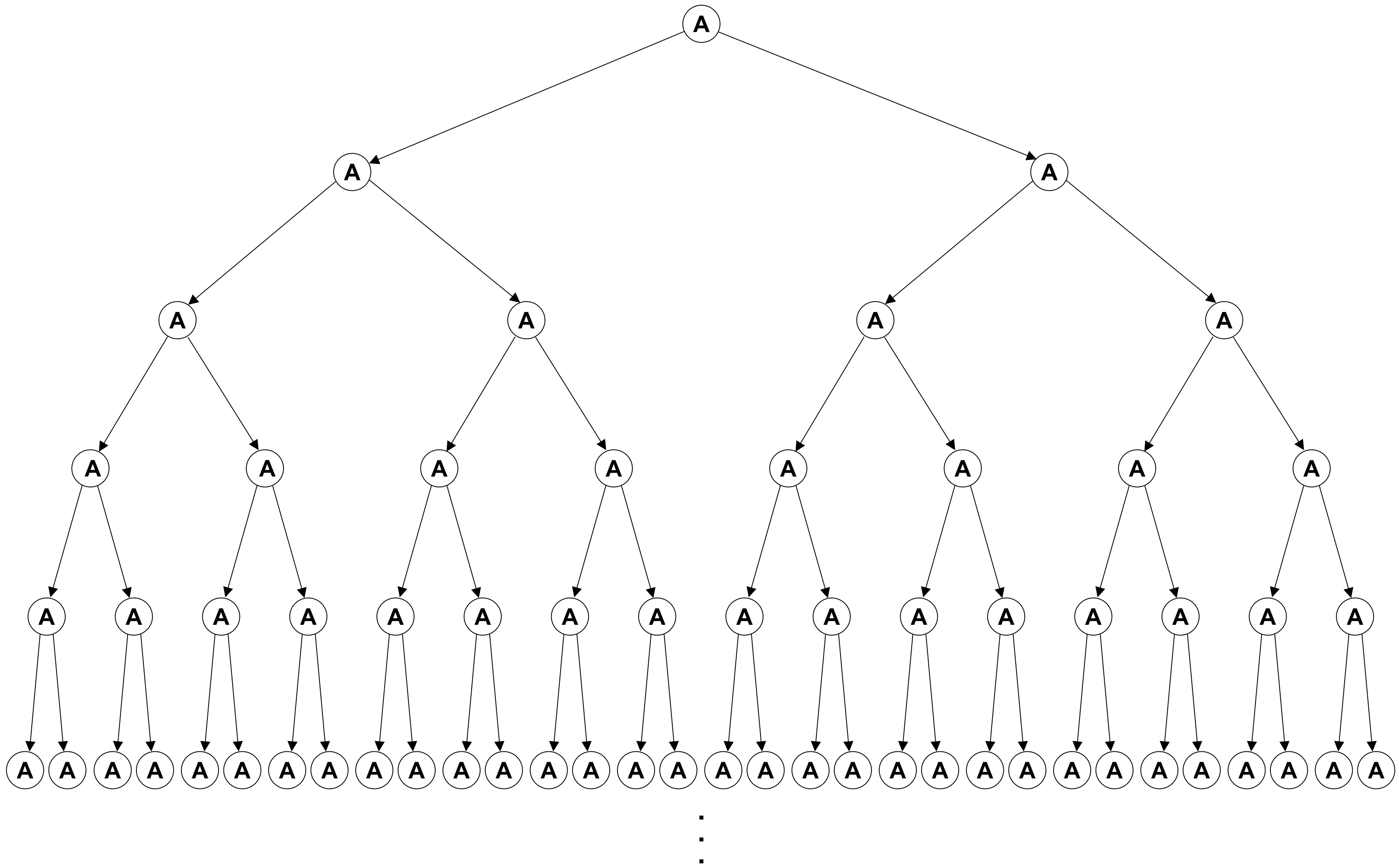
Consider a *simple semi-conditional 0L grammar*, G, with productions:

$$\begin{array}{ll} (A \to AA, \emptyset, \{B\}), & (A \to AB, \emptyset, \{B\}) \\ (A \to BA, \emptyset, \{B\}), & (B \to B, \emptyset, \emptyset), \\ (A \to A, \{B\}, \emptyset), & (B \to A, \emptyset, \emptyset), \\ (A \to B, \emptyset, \emptyset). & \end{array}$$

Symbols

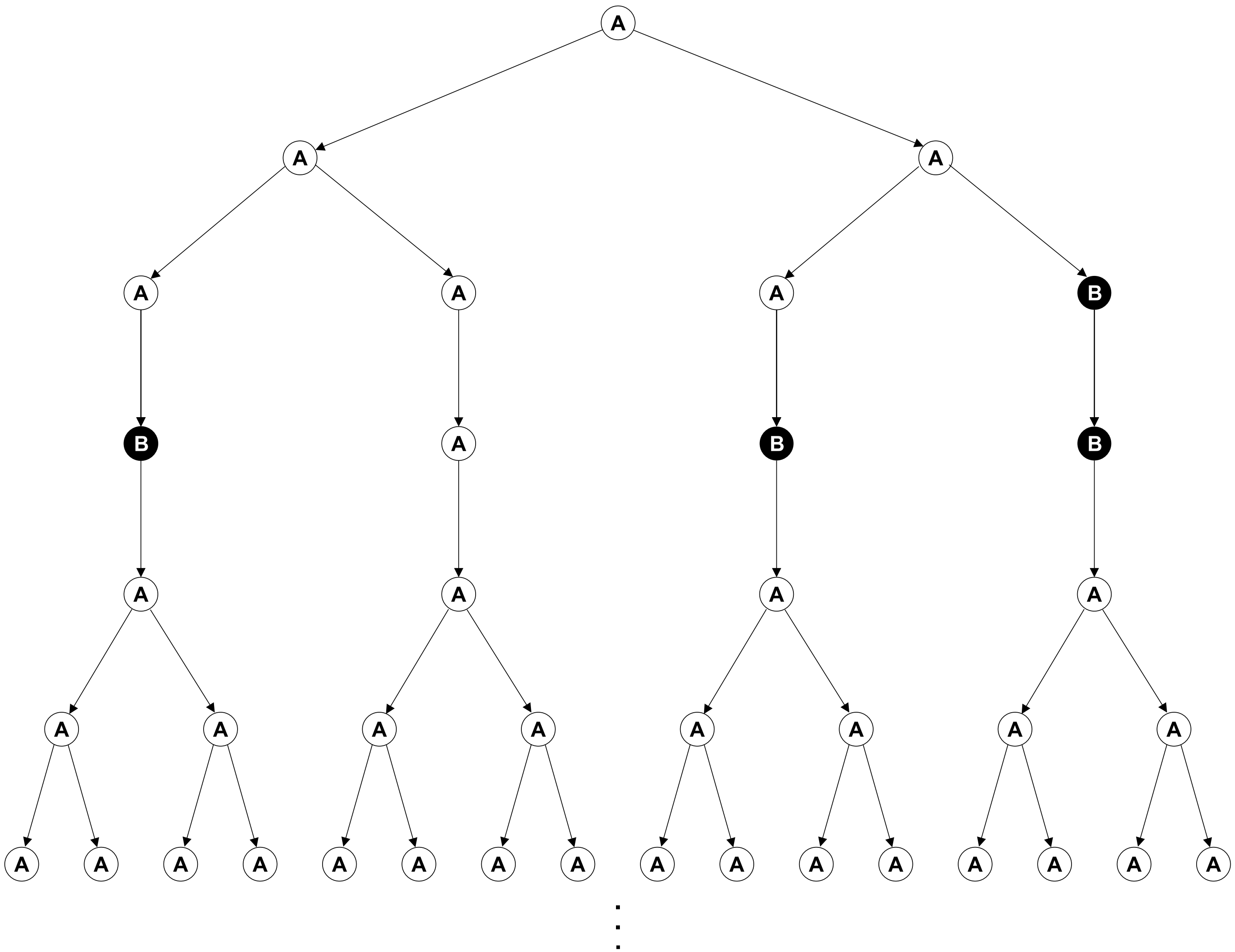
- A – healthy cell
- B – virus-infected cell

Healthy Development



Virus-Infected Cellular Organism (#3)

Development With a Stagnating Period



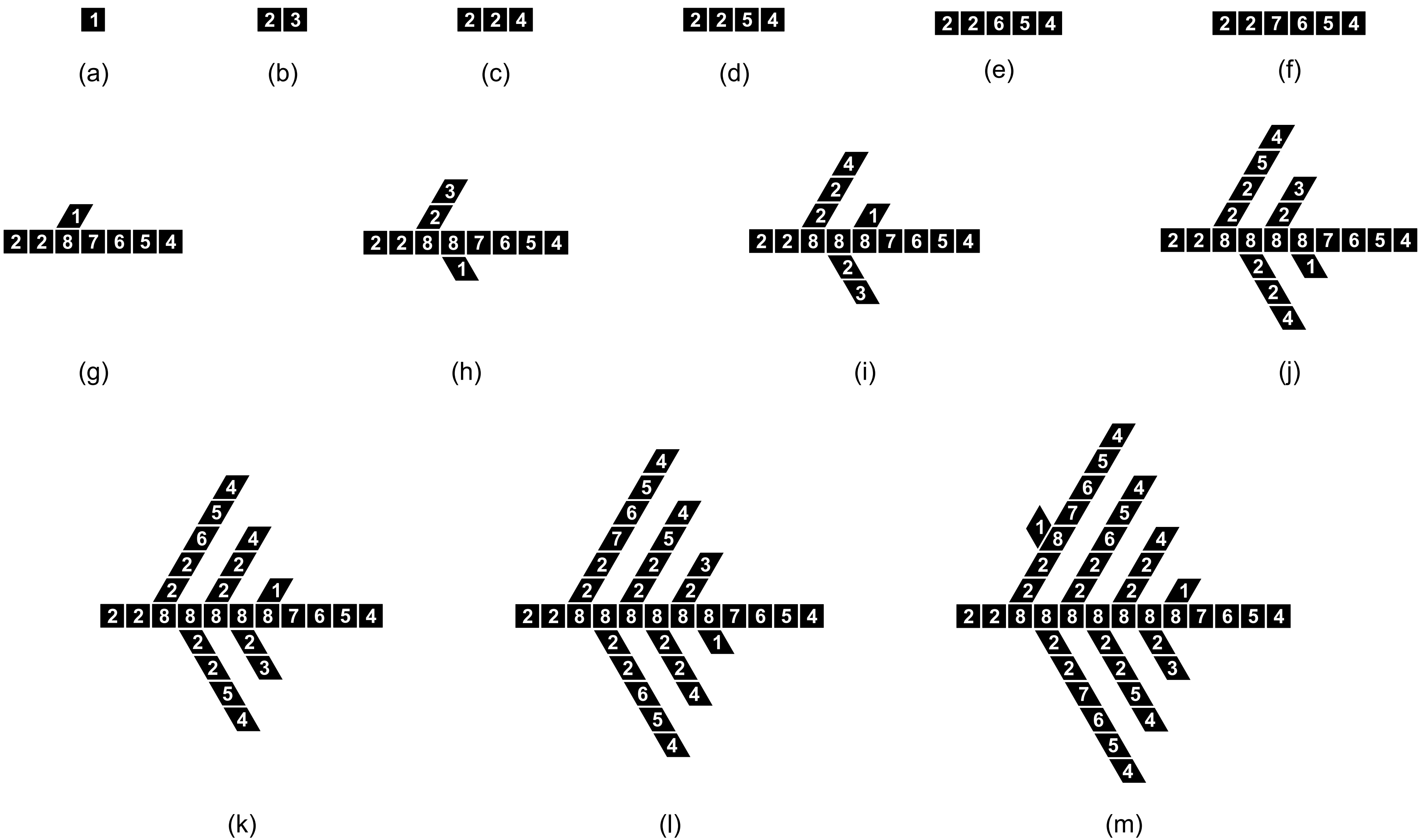
The following model describes degeneration of a red alga, where only the main stem is able to create new branches while all the other branches lengthen themselves without producing new branches.

Model

Consider a *forbidding 0L grammar*, G, with productions:

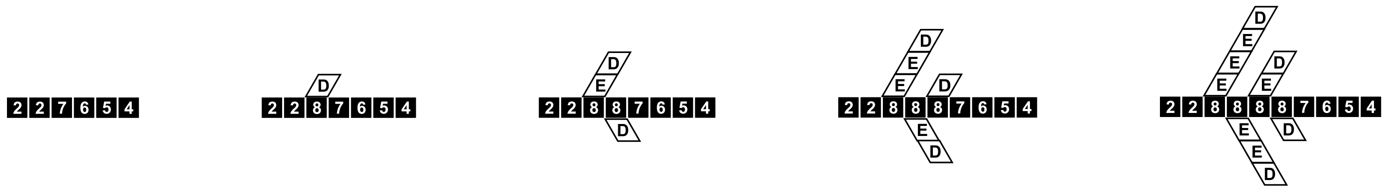
$$
\begin{array}{llll}
(1 \rightarrow 23, \emptyset), & (2 \rightarrow 2, \emptyset), & (3 \rightarrow 24, \emptyset), & (4 \rightarrow 54, \emptyset), \\
(5 \rightarrow 6, \emptyset), & (6 \rightarrow 7, \emptyset), & (7 \rightarrow 8[1], \{D\}), & (8 \rightarrow 8, \emptyset), \\
([\rightarrow [, \emptyset), & (] \rightarrow], \emptyset), & (7 \rightarrow 8[D], \emptyset), & \\
(D \rightarrow ED, \emptyset), & (E \rightarrow E, \emptyset). & &
\end{array}
$$

Healthy development

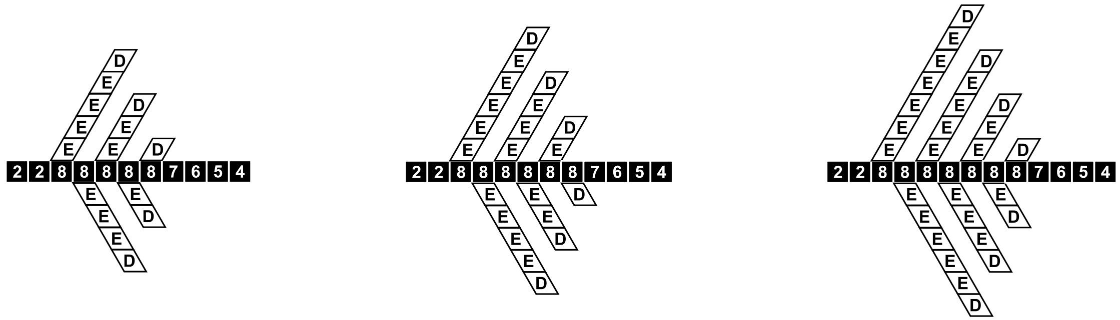


Degeneration of a Red Alga (#3)

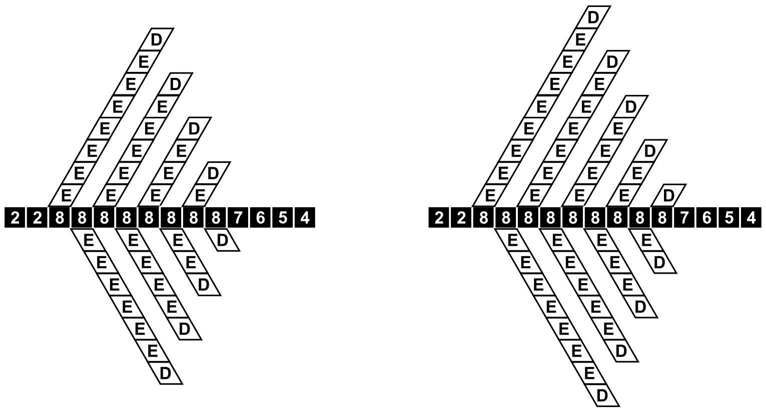
Degeneration after application of $(7 \rightarrow 8[D], \emptyset)$.



(a) (b) (c) (d) (e)



(f) (g) (h)



(i) (j)

In this case study, we extend *parametric 0L grammars* by permitting context conditions.

Parametric 0L Grammars operate on symbols with attached vectors of *parameters*.
Example of a production:

$$A(x) \ : \ x < 7 \quad \rightarrow \quad A(x+1)D(1)B(3-x)$$

This production rewrites $A(x)$ to $A(x+1)D(1)B(3-x)$ provided that $x < 7$.

Parametric 0L Grammars with Permitting Conditions introduce the concept of permitting context conditions into these grammars.
Example of a production:

$$A(x)\,?\,B(y), C(r,z) \ : \ x < r+z \quad \rightarrow \quad D(x)E(y+r)$$

This production rewrites $A(x)$ to $D(x)E(y+r)$ provided that there is an occurrence of $B(y)$ and an occurrence of $C(r,z)$ in the rewritten sentential form and the value of the logical expression $x < r+z$ is true.

Model

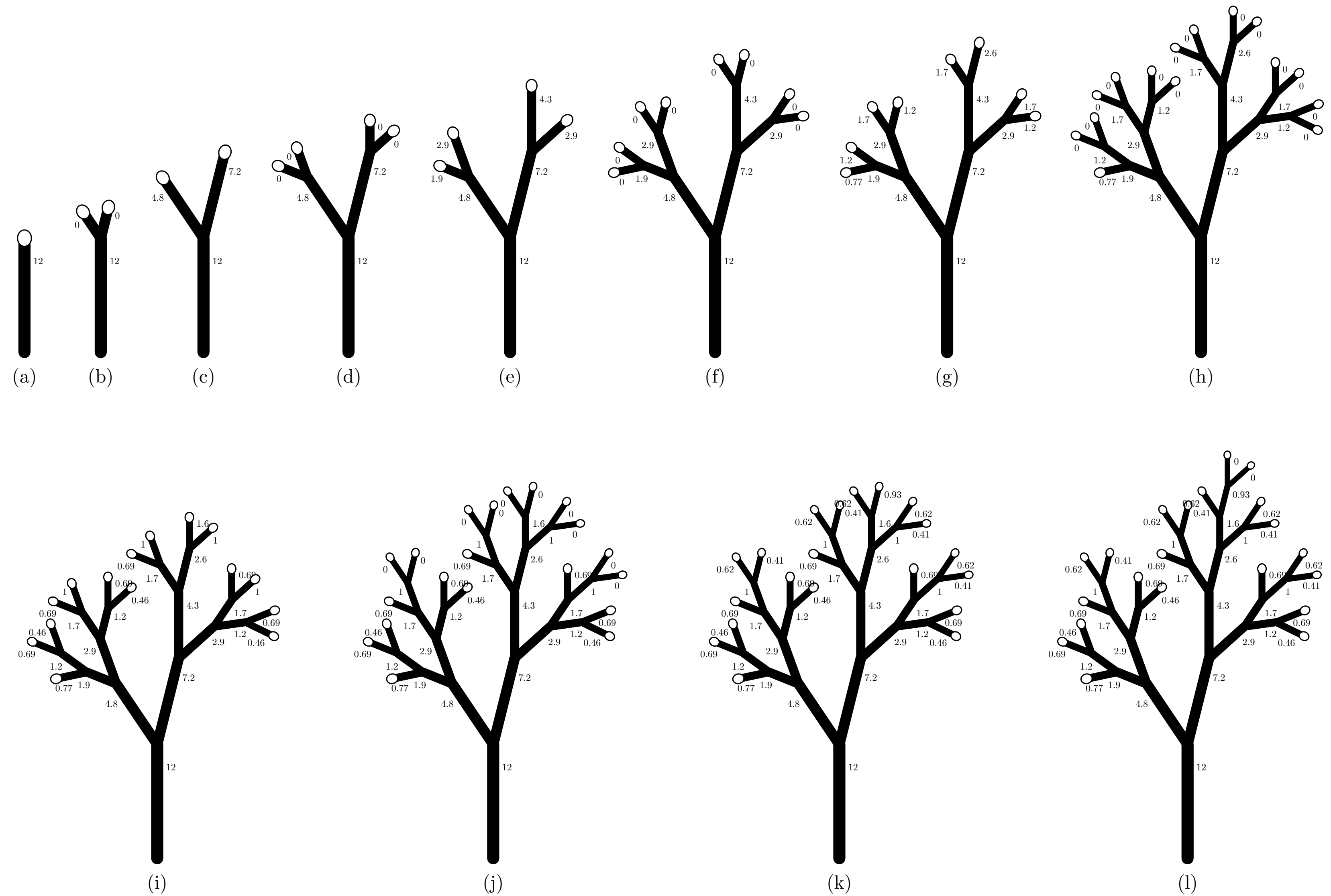
This model simulates simple resource flow distribution in a growing plant:

$$
\begin{array}{lll}
axiom & : & I(1, 1, e_{root})\, A(1) \\
p_1 & : & I(id, c, e)\ \text{?}\ I(id_p, c_p, e_p)\ :\ id_p == \lfloor id/2 \rfloor \\
 & & \rightarrow\ I(id, c, c * e_p) \\
p_2 & : & A(id)\ \text{?}\ I(id_p, c, e)\ :\ id == id_p \text{ and } e \geq e_{th} \\
 & & \rightarrow\ [+(\alpha)\, I(2*id+1, \gamma, 0)\, A(2*id+1)] \\
 & & /(\gamma)\, I(2*id, 1-\gamma, 0)\, A(2*id)
\end{array}
$$

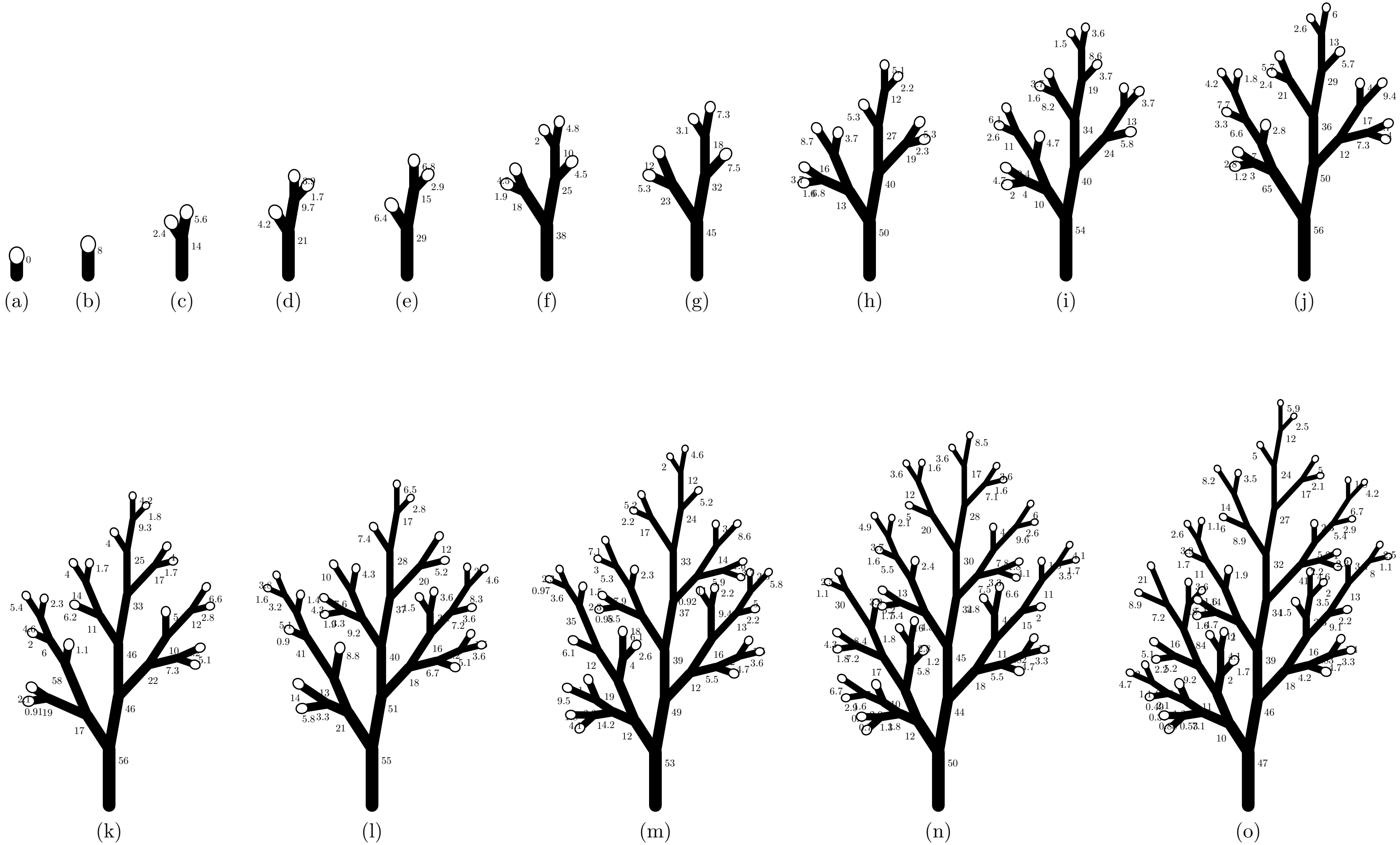
Symbols

- $I(id, c, e)$ – an *internode* with a unique identifier id, a flux coef. c, and a flux value e
- $A(id)$ – an *apex* adjacent to the internode with given identifier id
- [,] – branch delimiters
- $+(\alpha), /(\alpha)$ – rotation of branches

Developmental Stages of the Plant



A More Realistic Model Based on Context Conditions



Conclusion

Main Results

- new grammars with simple productions and easy-to-use context conditions
- new characterizations of **CS** and **RE** language families by these grammars
- reduced versions of grammars with context conditions
- sequential and parallel versions of these grammars
- formalization of the derivation similarity
- real-world applications in biology

Future Research

- investigation of another types of conditional grammars
- reduction of parallel conditional grammars
- new grammatical transformations with better derivation-simulation properties
- new applications in computer science areas, such as compilers
- new applications in other science areas, such as microbiology and genetics