Scattered Context Grammars

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Based on

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Outline

Outline

Scattered Information and Its Grammatical Formalization

- while context-sensitive grammars are suitable for modeling immediate context...

$$AAAABCAAAA \qquad BC \rightarrow AA$$

...they fail to describe scattered context dependencies efficiently

$$ABAAAAAACA \qquad BA \rightarrow AA', \; A'A \rightarrow AA', \; A'C \rightarrow AA$$

- scattered context dependencies are common in real world:

He is interested in football, isn't he?

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... int i; int j = 10; for (i = 0; i < j; i++) { ...
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- scattered context grammars (introduced by S. Greibach and J. Hopcroft in 1969) are convenient for describing this kind of dependencies

Scattered Context Grammars

Scattered Context Grammar

$$G = (V, T, P, S)$$

V is a finite alphabet

T is a set of terminals, $T \subset V$

S is the start symbol, $S \in V - T$

P is a finite set of productions of the form

$$(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n),$$

where $A_1, \ldots, A_n \in V - T$, $x_1, \ldots, x_n \in V^*$

Propagating Scattered Context Grammar

- each $(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$ satisfies $x_1, \ldots, x_n \in V^+$

Derivation Step

Derivation Step

If $(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n) \in P$ and

$$u = u_1 A_1 \ldots u_n A_n u_{n+1}$$
$$v = u_1 x_1 \ldots u_n x_n u_{n+1},$$

then $u \Rightarrow v\ [(A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)]$

- $\mathrm{alph}(x)$ denotes the set of all symbols appearing in x

Leftmost Derivation Step

- each A_i satisfies $A_i \notin \mathrm{alph}(u_i)$

Generated Language

Generated Language

- $L(G) = \{x \in T^* : S \Rightarrow^* x\}$

Language Families

- $\mathscr{L}(SC)$ – scattered context languages
- $\mathscr{L}(PSC)$ – propagating scattered context languages

Theorem

$\mathscr{L}(SC) = \mathscr{L}(RE)$.

Theorem

$\mathscr{L}(CF) \subset \mathscr{L}(PSC) \subseteq \mathscr{L}(CS)$.

Example

Example

Scattered context grammar

$$G = (\{A, B, C, S, a, b, c\}, \{a, b, c\}, P, S),$$

where

$$\begin{aligned} P = \{&(S) \rightarrow (ABC), \\ &(A, B, C) \rightarrow (aA, bB, cC), \\ &(A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon)\} \end{aligned}$$

Example of derivation

$$S \Rightarrow ABC \Rightarrow aAbBcC \Rightarrow aaAbbBccC \Rightarrow aabbcc$$

Generated language

$$L(G) = \{a^n b^n c^n \ : \ n \geq 0\}$$

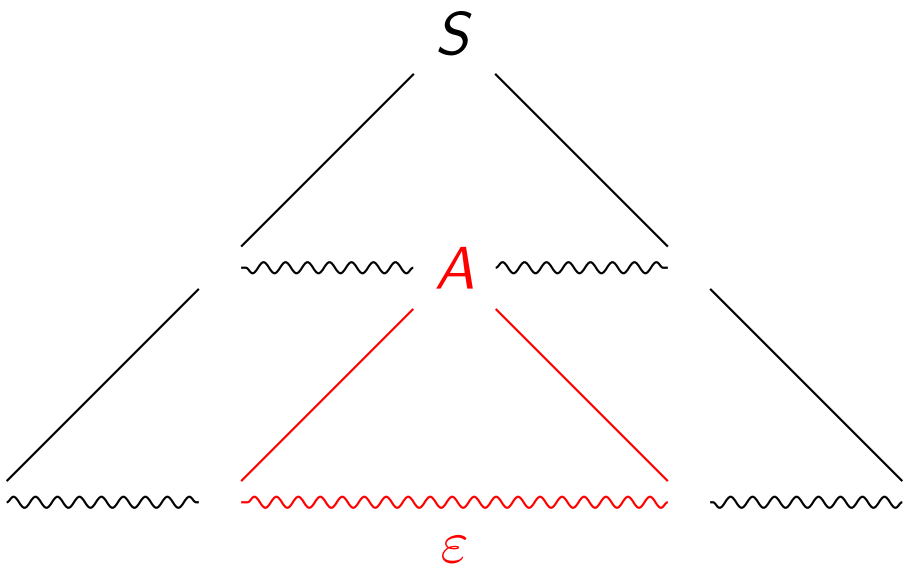
Outline

Symbols Erased During Derivation

Symbols Erased During Derivation

A symbol A is erased during a derivation if the frontier of the subtree rooted at A is ε;

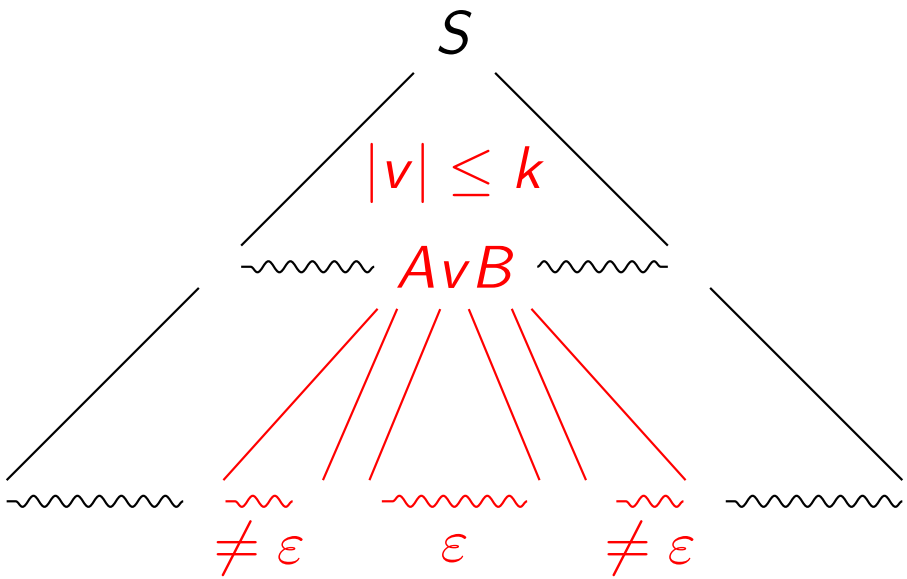
- if the symbol A is erased, we write $\grave{A}$,
- if the symbol A is not erased, we write $\acute{A}$



Nonterminals Erased in a Generalized k-Limited Way

For $y \in L(G)$, every sentential form x in $S \Rightarrow_G^* y$ satisfies:

1. every $x = uAvBw$, where $\acute{A}$, $\acute{B}$, $\grave{v}$, satisfies $|v| \leq k$,
2. every $x = uAw$, where $\acute{A}$, satisfies: if $\grave{u}$ or $\grave{w}$, then $|u| \leq k$ or $|w| \leq k$, respectively



Results

Nonterminals Erased in Generalized k-Limited Way by SC Grammars

A scattered context grammar G erases its nonterminals in a generalized k-limited way if $L(G) = L(G, \varepsilon, k)$, where

$$L(G, \varepsilon, k) = \{x \in T^* : S \Rightarrow_G^* x, \text{ and } G \text{ erases nonterminals in a generalized } k\text{-limited way in } S \Rightarrow_G^* x\}$$

Theorem

For each $k \geq 0$ and every scattered context grammar G, there is a propagating scattered context grammar $\bar{G}$ such that $L(G, \varepsilon, k) = L(\bar{G})$.

Corollary

For every scattered context grammar G which erases its nonterminals in a generalized k-limited way, there exists a propagating scattered context grammar $\bar{G}$ such that $L(G) = L(\bar{G})$.

Outline

Linear Scattered Context Grammars

Linear Scattered Context Grammar

- is a scattered context grammar $G = (V, T, P, S)$
- P is a finite set of productions of the following two forms:
  1. $(S) \rightarrow (x_1 A_1 \dots x_k A_k x_{k+1})$, where $A_i \in (V - T) - \{S\}$, $x_j \in T^*$,
  2. $(A_1, \dots, A_k) \rightarrow (z_1, \dots, z_k)$, where $A_i \in (V - T) - \{S\}$, and either
     - $z_i = x_i B_i y_i$, where $x_i, y_i \in T^*$, $B_i \in (V - T) - \{S\}$, or
     - $z_i \in T^*$

Linear Scattered Context Grammar of Degree n

- $(S) \rightarrow (x_1 A_1 \dots x_n A_n x_{n+1}) \in P$ satisfies $n \geq m$ for all $(S) \rightarrow (y_1 A_1 \dots y_m A_m y_{m+1}) \in P$

Right-Linear Scattered Context Grammars

Right-Linear Scattered Context Grammar

- is a linear scattered context grammar $G = (V, T, P, S)$
- P is a finite set of productions of the following two forms:
  1. $(S) \rightarrow (x_1 A_1 \dots x_k A_k)$, where $A_i \in (V - T) - \{S\}$, $x_i \in T^*$,
  2. $(A_1, \dots, A_k) \rightarrow (z_1, \dots, z_k)$, where $A_i \in (V - T) - \{S\}$, and either
     - $z_i = x_i B_i$, where $x_i \in T^*$, $B_i \in (V - T) - \{S\}$, or
     - $z_i \in T^*$

Language Families

- $\mathscr{L}(SC, LIN, n)$ – linear scattered context grammars of degree n
- $\mathscr{L}(SC, RLIN, n)$ – right-linear scattered context grammars of degree n

Results

Theorem

For each $n \geq 1$,

$$\begin{aligned} \mathscr{L}(SC, LIN, n) &\subset \mathscr{L}(SC, LIN, n+1), \\ \mathscr{L}(SC, RLIN, n) &\subset \mathscr{L}(SC, RLIN, n+1), \\ \mathscr{L}(SC, RLIN, n) &\subset \mathscr{L}(SC, LIN, n). \end{aligned}$$

- $\mathscr{L}(SC, LIN) = \bigcup_{n=1}^{\infty} \mathscr{L}(SC, LIN, n)$
- $\mathscr{L}(SC, RLIN) = \bigcup_{n=1}^{\infty} \mathscr{L}(SC, RLIN, n)$

Theorem

$$\mathscr{L}(CF) - \mathscr{L}(SC, LIN) \neq \emptyset, \ \mathscr{L}(CF) - \mathscr{L}(SC, RLIN) \neq \emptyset,$$

$$\mathscr{L}(SC, RLIN) \subset \mathscr{L}(SC, LIN) \subset \mathscr{L}(PSC).$$

Outline

n-Limited Derivations

- $|x|_A$ denotes the number of occurrences of symbols from A in x

n-Limited Derivation Step

If $(A_1, \ldots, A_k) \rightarrow (x_1, \ldots, x_k) \in P$,

$$u = u_1 A_1 u_2 \ldots u_k A_k u_{k+1},$$
$$v = u_1 x_1 u_2 \ldots u_k x_k u_{k+1},$$

and u satisfies

$$|u_1 A_1 \ldots u_k A_k|_{V-T} \leq n,$$

then $u \overset{n}{\underset{\text{lim}}{\Rightarrow}}_G v$

n-Limited Derivation

- derivation $x \overset{n}{\underset{\text{lim}}{\Rightarrow}}_G^* y$ in which every derivation step $u \overset{j}{\underset{\text{lim}}{\Rightarrow}}_G v$ satisfies $j \leq n$

Results

Language of Order n

- $L(G, \lim, n) = \{x \in T^* : S \overset{n}{\underset{\lim}{\Rightarrow}}_G^* x\}$

Language Families

- $\mathscr{L}(PSC, \lim, n)$
- $\mathscr{L}(PSC, \lim, \infty) = \bigcup_{i=1}^{\infty} \mathscr{L}(PSC, \lim, i)$

Theorem

$\mathscr{L}(CF) = \mathscr{L}(PSC, \lim, 1) \subset \ldots \subset \mathscr{L}(PSC, \lim, \infty) \subset \mathscr{L}(CS)$.

Outline

Leftmost Derivations

- much simplified proof of the result proved by V. Virkkunen in 1973

Propagating Scattered Context Grammar which Uses Leftmost Derivations

- propagating scattered context grammar $G = (V, T, P, S)$ whose language is defined as

$$L(G, \mathrm{lm}) = \{x \in T^* : S \ {}_{\mathrm{lm}}\Rightarrow_G^* x\}$$

Language Family

- $\mathscr{L}(PSC, \mathrm{lm})$

Theorem

$\mathscr{L}(PSC, \mathrm{lm}) = \mathscr{L}(CS)$.

Outline

Maximal and Minimal Derivation

- $\text{len}((A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)) = |A_1 \ldots A_n| = n$

Maximal Derivation Step

If

1. $u \Rightarrow v\ [p]$,
2. there is no $r \in P$, $\text{len}(r) > \text{len}(p)$, such that $u \Rightarrow w\ [r]$,

where $p \in P$, then $u \ {}_{\max}\!\Rightarrow v\ [p]$

Minimal Derivation Step

If

1. $u \Rightarrow v\ [p]$,
2. there is no $r \in P$, $\text{len}(r) < \text{len}(p)$, such that $u \Rightarrow w\ [r]$,

where $p \in P$, then $u \ {}_{\min}\!\Rightarrow v\ [p]$

Results

Maximal and Minimal Languages

- $L(G, \text{max}) = \{x \in T^* : S \ {}_{\text{max}}\Rightarrow^* x\}$
- $L(G, \text{min}) = \{x \in T^* : S \ {}_{\text{min}}\Rightarrow^* x\}$

Language Families

- $\mathscr{L}(PSC, \text{max})$
- $\mathscr{L}(PSC, \text{min})$

Theorem

$\mathscr{L}(CS) = \mathscr{L}(PSC, \text{max})$.

Theorem

$\mathscr{L}(CS) = \mathscr{L}(PSC, \text{min})$.

Outline

Production Labels

Production Label

- for every grammar G, $lab(G)$ denotes the set of its production labels
- each $p \in lab(G)$ uniquely identifies one production:

$$p : (A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$$

Derivation Made by Productions

- if $x \Rightarrow y$ by $p : (A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$, we write

$$x \Rightarrow y \ [p]$$

- if $x \Rightarrow^* y$ by productions labeled with $p_1, \ldots, p_n$, we write

$$x \Rightarrow^* y \ [p_1 \ldots p_n]$$

Proper Generator of Its Sentences With Their Parses

Parse (Szilard Word, Control Word)

If

$$S \Rightarrow^* x\,[\rho],$$

where $x \in T^*$, $\rho \in lab(G)^*$, then x is a sentence generated according to parse ρ

Proper Generator of Its Sentences With Their Parses

- $G = (V, T, P, S)$, where $\mathrm{lab}(G) \subset T$, which satisfies

$$L(G) = \{x : x = y\rho, y \in (T - lab(G))^*, \rho \in lab(G)^*, S \Rightarrow {}^*x\,[\rho]\}$$

- leftmost generator uses leftmost derivations in a successful derivation

Results

- $G = (V, P, S, T)$ is a proper generator of its sentences with their parses
- weak identity π from V^* to $(V - lab(G))^*$:
  - $\pi(a) = a$ for each $a \in V - lab(G)$
  - $\pi(p) = \epsilon$ for each $p \in lab(G)$

Theorem

For every recursively enumerable language L, there exists a propagating scattered context grammar G such that G is a proper generator of its sentences with their parses and $L = \pi(L(G))$.

Theorem

For every recursively enumerable language L, there exists a propagating scattered context grammar $G = (V, T, P, S)$ such that G is a proper leftmost generator of its sentences with their parses, $|V - T| \leq 6$, and $L = \pi(L(G))$.

Outline

Applications in Linguistics

- there are scattered dependencies in natural languages

  He usually, but not always, goes to work early.

- the subject (he) depends on the predicator (goes):

  I usually, but not always, goes to work early.

  is grammatically incorrect

- the dependency can be easily captured by productions of scattered context grammars:

  $$(\text{He}, \text{goes}) \rightarrow (\text{I}, \text{go})$$

  transforms the original sentence to

  I usually, but not always, go to work early.

- these transformations are formalized and studied on simple sentences

Transformational Scattered Context Grammar

Transformational Scattered Context Grammar

$$G = (V, T, P, I)$$

V is the total vocabulary

$T \subset V$ is a set of terminals

P is a set of scattered context productions

$I \subset V$ is the input vocabulary

Transformation T Defined by G

- $T(G) = \{(x, y) : x \Rightarrow_G^* y, x \in I^*, y \in T^*\}$
- if $(x, y) \in T(G)$, then x is transformed to y by G
- x and y are called the input and the output sentence

Clauses with *neither* and *nor*

$$G = (V, T, P, I)$$

T is the set of all words, including all their inflectional forms

$I = \{\langle x \rangle : x \in T\}$

$V = T \cup I$

$$P = \{(\langle \text{neither} \rangle, \langle \text{nor} \rangle) \rightarrow (\text{both}, \text{and})\}$$
$$\cup \{(\langle x \rangle) \rightarrow (x) : x \in T - \{\text{neither, nor}\}\}$$

Example

$\langle \text{neither} \rangle \langle \text{thomas} \rangle \langle \text{nor} \rangle \langle \text{his} \rangle \langle \text{wife} \rangle \langle \text{went} \rangle \langle \text{to} \rangle \langle \text{the} \rangle \langle \text{party} \rangle$

$\Rightarrow_G$ both $\langle \text{thomas} \rangle$and $\langle \text{his} \rangle \langle \text{wife} \rangle \langle \text{went} \rangle \langle \text{to} \rangle \langle \text{the} \rangle \langle \text{party} \rangle$

$\Rightarrow_G$ both thomas and $\langle \text{his} \rangle \langle \text{wife} \rangle \langle \text{went} \rangle \langle \text{to} \rangle \langle \text{the} \rangle \langle \text{party} \rangle$

$\Rightarrow_G$ both thomas and his $\langle \text{wife} \rangle \langle \text{went} \rangle \langle \text{to} \rangle \langle \text{the} \rangle \langle \text{party} \rangle$

$\Rightarrow_G^5$ both thomas and his wife went to the party

Existential Clauses

$$G = (V, T, P, I)$$

T is the set of all words, including all their inflectional forms

$I = \{\langle x \rangle : x \in T\}$

$V = T \cup I \cup \{X\}$

$P = \{(\langle x \rangle, \langle \text{is} \rangle) \to (\text{there is } xX, \varepsilon), (\langle x \rangle, \langle \text{are} \rangle) \to (\text{there are } xX, \varepsilon),$
$(\langle x \rangle, \langle \text{was} \rangle) \to (\text{there was } xX, \varepsilon), (\langle x \rangle, \langle \text{were} \rangle) \to (\text{there were } xX, \varepsilon)$
$\cup \{(X, \langle x \rangle) \to (X, x) : x \in T\} \cup \{(X) \to (\varepsilon)\}$

Example

$\langle \text{a} \rangle \langle \text{nurse} \rangle \langle \text{was} \rangle \langle \text{present} \rangle \Rightarrow_G \text{there was a } X \langle \text{nurse} \rangle \langle \text{present} \rangle$
$\Rightarrow_G \text{there was a } X \text{ nurse } \langle \text{present} \rangle \Rightarrow_G \text{there was a } X \text{ nurse present}$
$\Rightarrow_G \text{there was a nurse present}$

Outline

Further Investigation

Possibilities of Practical Applications

- applications in compilers
- applications in natural language processing

Theoretical Properties

- our research rises several open problems:
  - Does generalized k-limited erasing cover all possible kinds of erasing which can be performed by propagating scattered context grammars?
  - For given n, is it possible to construct a propagating scattered context grammar which rewrites the first n nonterminals without any additional restrictions?
- other papers dealing with this topic introduced several open problems as well
- the problem of whether $\mathscr{L}(PSC) = \mathscr{L}(CS)$ remains still open

Thank you