Scattered Context Grammars

Alexander Meduna   Jiří Techet

Department of Information Systems
Faculty of Information Technology
Brno University of Technology
Božetěchova 2, Brno 61266, Czech Republic

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Scattered Context Grammars and their Applications

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Organization of the Book

- three parts:
    I. Introduction
    II. Theory
    III. Application and Conclusion
- nine chapters
- exhaustive bibliography

Contents of the Book

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Introduction to Scattered Context Grammars

Scattered Information and Its Grammatical Formalization

- while context-sensitive grammars are suitable for modelling immediate context...

$$AAAABCAAAA \qquad BC \rightarrow AA$$

...they fail to describe scattered context dependencies efficiently

$$ABAAAAAACA \qquad BA \rightarrow AA',\ A'A \rightarrow AA',\ A'C \rightarrow AA$$

- scattered context dependencies are common in real world:

He is interested in football, isn't he?

```
... int i; int j = 10; for (i = 0; i < j; i++) { ...
```

- scattered context grammars (introduced by S. Greibach and J. Hopcroft in 1969) are convenient for describing this kind of dependencies

Scattered Context Grammars

Scattered Context Grammar

$$G = (V, T, P, S)$$

V is a finite alphabet

T is a set of terminals, $T \subset V$

S is the start symbol, $S \in V - T$

P is a finite set of productions of the form

$$(A_1, \dots, A_n) \to (x_1, \dots, x_n),$$

where $A_1, \dots, A_n \in V - T$, $x_1, \dots, x_n \in V^*$

Propagating Scattered Context Grammar

- each $(A_1, \dots, A_n) \to (x_1, \dots, x_n)$ satisfies $x_1, \dots, x_n \in V^+$

Derivation Step

Derivation Step

If $(A_1, \ldots, A_n) \to (x_1, \ldots, x_n) \in P$ and

$$u = u_1 A_1 \ldots u_n A_n u_{n+1}$$
$$v = u_1 x_1 \ldots u_n x_n u_{n+1},$$

then $u \Rightarrow v \; [(A_1, \ldots, A_n) \to (x_1, \ldots, x_n)]$

- $\mathrm{alph}(x)$ denotes the set of all symbols appearing in x

Leftmost Derivation Step

- each A_i satisfies $A_i \notin \mathrm{alph}(u_i)$

Generated Language

Generated Language

- $L(G) = \{x \in T^* : S \Rightarrow^* x\}$

Language Families

- $\mathscr{L}(SC)$ – scattered context languages
- $\mathscr{L}(PSC)$ – propagating scattered context languages

Basic Properties

Theorem

$\mathscr{L}(SC) = \mathscr{L}(RE)$.

Theorem

$\mathscr{L}(CF) \subset \mathscr{L}(PSC) \subseteq \mathscr{L}(CS)$.

Theorem

For every recursively enumerable language L there exists a propagating scattered context language L' and a homomorphism h such that $h(L') = L$.

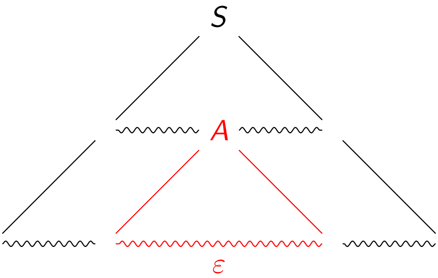
Results

Symbols Erased During Derivation

Symbols Erased During Derivation

A symbol A is erased during a derivation if the frontier of the subtree rooted at A is ε;

- if the symbol A is erased, we write $\grave{A}$,
- if the symbol A is not erased, we write $\acute{A}$

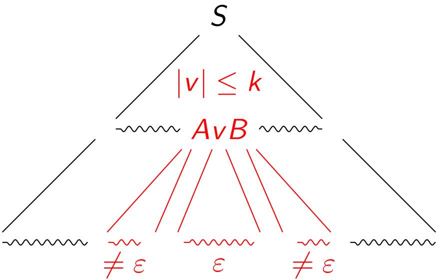


Nonterminals Erased in a Generalized k-Limited Way

Nonterminals Erased in a Generalized k-Limited Way

For $y \in L(G)$, every sentential form x in $S \Rightarrow_G^* y$ satisfies:

1. every $x = uAvBw$, where $\acute{A}$, $\acute{B}$, $\grave{v}$, satisfies $|v| \leq k$,
2. every $x = uAw$, where $\acute{A}$, satisfies: if $\grave{u}$ or $\grave{w}$, then $|u| \leq k$ or $|w| \leq k$, respectively



Results

Nonterminals Erased in Generalized k-Limited Way by SC Grammars

A scattered context grammar G erases its nonterminals in a generalized k-limited way if $L(G) = L(G, \varepsilon, k)$, where

$$L(G, \varepsilon, k) = \{x \in T^* : S \Rightarrow_G^* x, \text{ and } G \text{ erases nonterminals in a generalized } k\text{-limited way in } S \Rightarrow_G^* x\}$$

Theorem

For each $k \geq 0$ and every scattered context grammar G, there is a propagating scattered context grammar $\bar{G}$ such that $L(G, \varepsilon, k) = L(\bar{G})$.

Corollary

For every scattered context grammar G which erases its nonterminals in a generalized k-limited way, there exists a propagating scattered context grammar $\bar{G}$ such that $L(G) = L(\bar{G})$.

Linear Scattered Context Grammars

Linear Scattered Context Grammar

- is a scattered context grammar $G = (V, T, P, S)$
- P is a finite set of productions of the following two forms:
  1. $(S) \rightarrow (x_1 A_1 \dots x_k A_k x_{k+1})$, where $A_i \in (V - T) - \{S\}$, $x_j \in T^*$ for all $1 \leq i \leq k$, $1 \leq j \leq k + 1$, for some $k \geq 1$,
  2. $(A_1, \dots, A_k) \rightarrow (z_1, \dots, z_k)$, where $A_i \in (V - T) - \{S\}$, and either
     - $z_i = x_i B_i y_i$, where $x_i, y_i \in T^*$, $B_i \in (V - T) - \{S\}$, or
     - $z_i \in T^*$

     for all $1 \leq i \leq k$, for some $k \geq 1$

Linear Scattered Context Grammar of Degree n

- every $(S) \rightarrow (y_1 A_1 \dots y_m A_m y_{m+1}) \in P$ satisfies $m \leq n$

Right-Linear Scattered Context Grammars

Right-Linear Scattered Context Grammar

- is a linear scattered context grammar $G = (V, T, P, S)$
- P is a finite set of productions of the following two forms:
  1. $(S) \rightarrow (x_1 A_1 \ldots x_k A_k)$, where $A_i \in (V - T) - \{S\}$, $x_i \in T^*$ for all $1 \leq i \leq k$, for some $k \geq 1$,
  2. $(A_1, \ldots, A_k) \rightarrow (z_1, \ldots, z_k)$, where $A_i \in (V - T) - \{S\}$, and either
     - $z_i = x_i B_i$, where $x_i \in T^*$, $B_i \in (V - T) - \{S\}$, or
     - $z_i \in T^*$

     for all $1 \leq i \leq k$, for some $k \geq 1$

Language Families

- $\mathscr{L}(SC, LIN, n)$ – linear scattered context grammars of degree n
- $\mathscr{L}(SC, RLIN, n)$ – right-linear scattered context grammars of degree n

Results

Theorem

For each $n \geq 1$,

$$\begin{aligned} \mathscr{L}(SC, LIN, n) &\subset \mathscr{L}(SC, LIN, n+1), \\ \mathscr{L}(SC, RLIN, n) &\subset \mathscr{L}(SC, RLIN, n+1), \\ \mathscr{L}(SC, RLIN, n) &\subset \mathscr{L}(SC, LIN, n). \end{aligned}$$

- $\mathscr{L}(SC, LIN) = \bigcup_{n=1}^{\infty} \mathscr{L}(SC, LIN, n)$
- $\mathscr{L}(SC, RLIN) = \bigcup_{n=1}^{\infty} \mathscr{L}(SC, RLIN, n)$

Theorem

$$\mathscr{L}(CF) - \mathscr{L}(SC, LIN) \neq \emptyset,\ \mathscr{L}(CF) - \mathscr{L}(SC, RLIN) \neq \emptyset,$$

$$\mathscr{L}(SC, RLIN) \subset \mathscr{L}(SC, LIN) \subset \mathscr{L}(PSC).$$

n-Limited Derivations

- $|x|_W$ denotes the number of occurrences of symbols from set W in x

n-Limited Derivation Step

If $(A_1, \dots, A_k) \rightarrow (x_1, \dots, x_k) \in P$,

$$u = u_1 A_1 u_2 \dots u_k A_k u_{k+1},$$
$$v = u_1 x_1 u_2 \dots u_k x_k u_{k+1},$$

and u satisfies

$$|u_1 A_1 \dots u_k A_k|_{V-T} \leq n,$$

then $u \; {}_{\lim}^{n}\!\Rightarrow_G \; v$

n-Limited Derivation

- derivation $x \; {}_{\lim}^{n}\!\Rightarrow_G^* \; y$ in which every derivation step $u \; {}_{\lim}^{j}\!\Rightarrow_G \; v$ satisfies $j \leq n$

Results

Language of Order n

- $L(G, \lim, n) = \{x \in T^* : S \; {}_{\lim}^{n}\!\Rightarrow_G^* x\}$

Language Families

- $\mathscr{L}(PSC, \lim, n)$
- $\mathscr{L}(PSC, \lim, \infty) = \bigcup_{i=1}^{\infty} \mathscr{L}(PSC, \lim, i)$

Theorem

$\mathscr{L}(CF) = \mathscr{L}(PSC, \lim, 1) \subset \ldots \subset \mathscr{L}(PSC, \lim, \infty) \subset \mathscr{L}(CS)$.

Leftmost Derivations

- much simplified proof of the result proved by V. Virkkunen in 1973

Propagating Scattered Context Grammar which Uses Leftmost Derivations

- propagating scattered context grammar $G = (V, T, P, S)$ whose language is defined as

$$L(G, \mathrm{lm}) = \{x \in T^* : S \ {}_{\mathrm{lm}}\!\Rightarrow_G^* x\}$$

Language Family

- $\mathscr{L}(PSC, \mathrm{lm})$

Theorem

$\mathscr{L}(PSC, \mathrm{lm}) = \mathscr{L}(CS)$.

Maximal and Minimal Derivation

- $\text{len}((A_1, \ldots, A_n) \to (x_1, \ldots, x_n)) = |A_1 \ldots A_n| = n$

Maximal Derivation Step

Let $p \in P$. If

1. $u \Rightarrow v\ [p]$
2. for every $r \in P$ such that $u \Rightarrow w\ [r] : \text{len}(p) \geq \text{len}(r)$

then $u\ {}_{\text{max}}\!\!\Rightarrow v\ [p]$

Minimal Derivation Step

Let $p \in P$. If

1. $u \Rightarrow v\ [p]$
2. for every $r \in P$ such that $u \Rightarrow w\ [r] : \text{len}(p) \leq \text{len}(r)$

then $u\ {}_{\text{min}}\!\!\Rightarrow v\ [p]$

Results

Maximal and Minimal Languages

- $L(G, \text{max}) = \{x \in T^* : S \ {}_{\text{max}}\!\Rightarrow^* x\}$
- $L(G, \text{min}) = \{x \in T^* : S \ {}_{\text{min}}\!\Rightarrow^* x\}$

Language Families

- $\mathscr{L}(PSC, \text{max})$
- $\mathscr{L}(PSC, \text{min})$

Theorem

$\mathscr{L}(CS) = \mathscr{L}(PSC, \text{max})$.

Theorem

$\mathscr{L}(CS) = \mathscr{L}(PSC, \text{min})$.

Production Labels

Production Label

- for every grammar G, $lab(G)$ denotes the set of its production labels
- each $p \in lab(G)$ uniquely identifies one production:

$$p : (A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$$

Derivation Made by Productions

- if $x \Rightarrow y$ by $p : (A_1, \ldots, A_n) \rightarrow (x_1, \ldots, x_n)$, we write

$$x \Rightarrow y \; [p]$$

- if $x \Rightarrow^* y$ by productions labeled with $p_1, \ldots, p_n$, we write

$$x \Rightarrow^* y \; [p_1 \ldots p_n]$$

Proper Generator of Its Sentences With Their Parses

Parse (Szilard Word, Control Word)

If

$$S \Rightarrow^* x\ [\rho],$$

where $x \in T^*, \rho \in lab(G)^*$, then x is a sentence generated according to parse ρ

Proper Generator of Its Sentences With Their Parses

- $G = (V, T, P, S)$, where lab$(G) \subset T$, which satisfies

$$L(G) = \{x : x = y\rho, y \in (T - lab(G))^*, \rho \in lab(G)^*, S \Rightarrow {}^*x\ [\rho]\}$$

- leftmost generator makes every successful derivation in a leftmost way

Results

- $G = (V, P, S, T)$ is a proper generator of its sentences with their parses
- weak identity π from V^* to $(V - lab(G))^*$:
  - $\pi(a) = a$ for each $a \in (V - lab(G))$
  - $\pi(p) = \epsilon$ for each $p \in lab(G)$

Theorem

For every recursively enumerable language L, there exists a propagating scattered context grammar G such that G is a proper generator of its sentences with their parses and $L = \pi(L(G))$.

Theorem

For every recursively enumerable language L, there exists a propagating scattered context grammar $G = (V, T, P, S)$ such that G is a proper leftmost generator of its sentences with their parses, $|V - T| \leq 6$, and $L = \pi(L(G))$.

Applications in Linguistics

Applications in Linguistics

- there are scattered dependencies in natural languages

He usually, but not always, goes to work early.

- there is a scattered dependency between the subject (he) and the predicator (goes):

I usually, but not always, goes to work early.

- the dependency can be easily captured by productions of scattered context grammars:

$$(\text{He}, \text{goes}) \rightarrow (\text{I}, \text{go})$$

transforms the original sentence to

I usually, but not always, go to work early.

Example

Consider the language L consisting of these grammatical English sentences:

Your grandparents are all your grandfathers and all your grandmothers.

Your great-grandparents are all your great-grandfathers and all your great-grandmothers.

Your great-great-grandparents are all your great-great-grandfathers and all your great-great-grandmothers.

$\vdots$

In brief,

$$L = \left\{ \text{your } \{\text{great-}\}^i \text{grandparents are all your } \{\text{great-}\}^i \text{grandfathers and all your } \{\text{great-}\}^i \text{grandmothers} \; : \; i \geq 0 \right\}.$$

Example

Introduce the scattered context grammar $G = (V, T, P, S)$, where

$T = \{\text{all}, \text{and}, \text{are}, \text{grandfathers}, \text{grandmothers}, \text{grandparents}, \text{great-}, \text{your}\}$,

$V = T \cup \{S, \#\}$, and P consists of these three productions:

$$\begin{aligned}(S) &\rightarrow (\text{your } \#\text{grandparents are all your } \#\text{grandfathers} \\ &\qquad \text{and all your } \#\text{grandmothers}), \\ (\#, \#, \#) &\rightarrow (\#\text{great-}, \#\text{great-}, \#\text{great-}), \\ (\#, \#, \#) &\rightarrow (\varepsilon, \varepsilon, \varepsilon).\end{aligned}$$

Obviously, this scattered context grammar generates L; formally, $L = L(G)$.

Conclusion

Further Investigation

Possibilities of Practical Applications

- applications in compilers
- applications in natural language processing

Main Open Problem

- $\mathscr{L}(PSC) = \mathscr{L}(CS)$?

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