Multilanguages, multigrammar systems and multiaccepting automata systems

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n-string

is n tuple $(\omega_1, \ldots, \omega_n)$, where $\Sigma_1, \ldots, \Sigma_n$ are alphabets and for all $i \in \{1, \ldots, n\}$ $\omega_i \in \Sigma_i^*$ is string.

n-language

n-$L = \{(\omega_1, \ldots, \omega_n) : \forall i = 1, \ldots, n \ \omega_i \in \Sigma^*\}$

From n-languages to languages

Let K is a n-language:

- $L_{union}(K) = \{w_1, \ldots, w_n : (w_1, \ldots, w_n) \in K\}$
- $L_{concat}(K) = \{w_1 \ldots w_n : (w_1, \ldots, w_n) \in K\}$
- $L_{first}(K) = \{w_1 : (w_1, \ldots, w_n) \in K\}$
- $L_{intr}(K) = \{w_1 : (w_1, \ldots, w_n) \in K\colon w_1 = w_2 = \ldots = w_n\}$

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n-multigenerative nonterminal-synchronized grammar system: n-MGN

is $n+1$ tuple $\Gamma = (G_1, \dots, G_n, Q)$, where:

- $G_i = (N_i, T_i, P_i, S_i)$ for all $i \in \{1, \dots, n\}$
- Q is a set of n tuples of the form $(A_1, \dots, A_n) : A_i \in N_i$

Sential n-form

is any n tuple $\chi = (x_1, \dots, x_n)$, where $x_i \in (N_i \cup T_i)^*$ for all $i = 1, \dots, n$

Derivation step

Let $\forall i = 1, \dots, n,\ u_i \in T^*,\ A_i \in N_i,\ x_i, v_i \in (N \cup T)^*$ and:

- $\chi = (u_1 A_1 v_1, \dots, u_n A_n v_n)$ and
- $\chi = (u_1 x_1 v_1, \dots, u_n x_n v_n)$ are sential n-form,
- $(A_1, \dots, A_n) \in Q$ and $u_1 A_i v_i \Rightarrow u_1 x_i v_i$ in G_i for all $i = 1, \dots, n$

Than $\chi \Rightarrow \chi'$ in Γ.

n-language generated by Γ: n-$L(\Gamma)$

$$n\text{-}L(\Gamma) = \{(w_1, \ldots, w_n) : \ (S_1, \ldots, S_n) \Rightarrow^* (w_1, \ldots, w_n)\}$$

Example

...

n-multigenerative rule-synchronized grammar system: n-MGR

is $n+1$ tuple $\Gamma = (G_1, \ldots, G_n, Q)$, where:

- $G_i = (N_i, T_i, P_i, S_i$ for all $i \in \{1, \ldots, n\}$
- Q is a set of n tuples of the form $(p_1, \ldots, p_n) : p_i \in P_i$

Sential n-form

is any n tuple $\chi = (x_1, \ldots, x_n)$, where $x_i \in (N_i \cup T_i)^*$ for all $i = 1, \ldots, n$

Derivation step

Let $\forall i = 1, \ldots, n,\ u_i \in T^*,\ A_i \in N_i,\ x_i, v_i \in (N \cup T)^*$ and:

- $\chi = (u_1 A_1 v_1, \ldots, u_n A_n v_n)$ and
- $\chi = (u_1 x_1 v_1, \ldots, u_n x_n v_n)$ are Sential n-form,
- $(p_1, \ldots, p_n) \in Q$ and $p_i : A_i \rightarrow x_i\ \in P_i$ for all $i = 1, \ldots, n$

Than $\chi \Rightarrow \chi'$ in Γ.

n-language generated by Γ: n-$L(\Gamma)$

$$n\text{-}L(\Gamma) = \{(w_1, \ldots, w_n) : \ (S_1, \ldots, S_n) \Rightarrow^* (w_1, \ldots, w_n)\}$$

Example

...

Let X$\in$ {MGN,MGR}.

Family of *n*-languages

- n-$\mathcal{L}(n$-X) is a family of *n*-languges generated by *n*-X
- ${}_{concat}\mathcal{L}(n\text{-}X) = \{{}_{concat}L(K) : K \in n\text{-}\mathcal{L}(n\text{-}X)\}$
- ${}_{first}\mathcal{L}(n\text{-}X) = \{{}_{first}L(K) : K \in n\text{-}\mathcal{L}(n\text{-}X)\}$
- ${}_{union}\mathcal{L}(n\text{-}X) = \{{}_{union}L(K) : K \in n\text{-}\mathcal{L}(n\text{-}X)\}$
- ${}_{intr}\mathcal{L}(n\text{-}X) = \{{}_{intr}L(K) : K \in n\text{-}\mathcal{L}(n\text{-}X)\}$

Results

Let $n \geq 2$:

- ${}_{concat}\mathcal{L}(n\text{-}X) = RE$
- ${}_{first}\mathcal{L}(n\text{-}X) = RE$
- ${}_{union}\mathcal{L}(n\text{-}X) = RE$
- ${}_{intr}\mathcal{L}(n\text{-}X) = ?$

Let X$\in$ {MGN,MGR}.

Family of n-languages

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n-accepting, state-synchronized AS (n-MAS)

Let $I = \{1, \ldots, n\}$ for $n \geq 1$. Let $\forall i \in I$, $M_i = (Q_i, \Sigma, \Gamma_i, \delta_i, s_i, z_{i,0}, F_i)$ is pushdown automaton. Then, n-accepting, state-synchronized AS is defined as: $\vartheta = (M_1, \ldots, M_n, \Psi, S)$, where:

- Ψ is set of switch-rules of the form $(q_1, \ldots, q_n) \rightarrow (\hbar_1, \ldots, \hbar_n)$, where $\forall i \in I$:
  - $q_i \in Q_i$ a $\hbar_i \in \{e, d\}$
- S is n-tuple $(\hbar_1^0, \ldots, \hbar_n^0)$ and denotes start activities of the n-MAS.

n-configuration

is defined as n tuple: $\chi = (x_1^{\hbar_1}, \ldots, x_n^{\hbar_n})$, where $\forall i \in I$:

- $x_i = (z_i q_i \omega_i) \in \Gamma_i^* Q_i \Sigma^*$,
- $\hbar_i \in \{d, e\}$ denotes nonactive and active component of M_i, respectively,
- $\omega_i \in \Sigma^*$ is a input string

Move

Let $I = \{1, \ldots, n\}$, $\chi = ((\gamma_1 z_1 q_1 a_1 \omega_1)^{\hbar_1}, \ldots, (\gamma_n z_n q_n a_n \omega_n)^{\hbar_n})$ and $\chi' = ((\gamma'_1 z'_1 q'_1 \omega'_1)^{\hbar'_1}, \ldots, (\gamma'_n z'_n q'_n \omega'_n)^{\hbar'_n})$, where $\forall i \in I$:

- $q_i, q'_i \in Q_i$; $\gamma'_i, z_i, z'_i \in \Gamma^*_i$; $\gamma_i \in \Gamma \cup \{\varepsilon\}$; $\hbar_i, \hbar'_i \in \{e, d\}$
- $\omega_i, \omega'_i \in \Sigma^*$, $a_i \in \Sigma \cup \{\varepsilon\}$
- $(q'_i, \gamma'_i) \in \delta_i(q_i, \gamma_i, a_i)\ \forall i$, where $\hbar_i = e$.

ϑ moves from χ to χ', denotes by $\chi \vdash \chi'$ and

- $\forall j \in I$, where $\hbar_j = d$, $q'_j = q_j$ and $\omega'_j = a_j \omega_j$
- $\forall j \in I$, where $\hbar_j = e$, $q'_j \in Q_j$ and $\omega'_j = \omega_j$
- if $(q'_1, \ldots, q'_n) \to (g_1, \ldots, g_n) \in \Psi$, where $g_k \in \{e, d\}$ for all $k \in I$, then $\hbar'_k = g_k$,
- if $\forall (g_1, \ldots, g_n) \in \overbrace{\{e, d\} \times \ldots \times \{e, d\}}^{n\times}$: $(q'_1, \ldots, q'_n) \to (g_1, \ldots, g_n) \notin \Psi$, then for all $k \in I : \hbar'_k = \hbar_k$.

n–languages

Let $I = \{1, \dots, n\}$ and

- $\chi_0 = ((z_1 q_1 \omega_1)^{\hbar_1}, \dots, (z_n q_n \omega_n)^{\hbar_n})$ is start and
- $\chi_f = ((q'_1 \varepsilon)^{\hbar'_1}, \dots, (q'_n \varepsilon)^{\hbar'_n})$ finish

n-configuration, where $\forall i \in I$:

- $q_i, q'_i \in Q_i$, $z_i \in \Gamma^*$,
- $\hbar_i, \hbar'_1 \in \{d, e\}$,
- $\omega_i \in \Sigma^*$.

n–lanugages n-MAS are defined as:

- $n\text{-}L_\cup(\vartheta) = \{(\omega_1, \dots, \omega_n) | \chi_0 \vdash^* \chi_f; q'_j \in F_j \text{ for any } j \in I\}$
- $n\text{-}L_\cap(\vartheta) = \{(\omega_1, \dots, \omega_n) | \chi_0 \vdash^* \chi_f; q'_j \in F_j \text{ for all } j \in I\}$

Example

...

n-accepting, transition-synchronized AS (n-MAT)

Let $I = \{1, \ldots, n\}$ for $n \geq 1$. Let $\forall i \in I$, $M_i = (Q_i, \Sigma, \Gamma_i, \delta_i, s_i, z_{i,0}, F_i)$ is pushdown automaton. Then, n-accepting, transition-synchronized AS is defined as: $\vartheta = (M_1, \ldots, M_n, \Psi)$, where:

- Ψ is set of n tuples of the form $(r_1, \ldots, r_n)$ where $\forall i \in I$, $r_i \in \delta_i$

n-configuration

is defined as n tuple: $\chi = (x_1, \ldots, x_n)$, where $\forall i \in I$:

- $x_i = (z_i q_i \omega_i) \in \Gamma_i^* Q_i \Sigma^*$,
- $\omega_i \in \Sigma^*$ is a input string

Move

Let $I = \{1, \ldots, n\}$, $\chi = ((\gamma_1 z_1 q_1 a_1 \omega_1), \ldots, (\gamma_n z_n q_n a_n \omega_n))$ and $\chi' = ((\gamma'_1 z'_1 q'_1 \omega'_1), \ldots, (\gamma'_n z'_n q'_n \omega'_n))$, where $\forall i \in I$:

- $q_i, q'_i \in Q_i$; $\gamma'_i, z_i, z'_i \in \Gamma_i^*$; $\gamma_i \in \Gamma \cup \{\varepsilon\}$
- $\omega_i, \omega'_i \in \Sigma^*$, $a_i \in \Sigma \cup \{\varepsilon\}$
- $r_i : \ (q'_i, \gamma'_i) \in \delta_i(q_i, \gamma_i, a_i)$.

ϑ moves from χ to χ', denotes by $\chi \vdash \chi'$ iff $(r_1, \ldots, r_n) \in \Psi$

Example

...

Theorem

The families of *n*-languages of *n*-MGN, *n*-MGR, *n*-MAS and *n*-MAT are equivalent.

Open problems and modifications

- generative power of ${}_{intr}\mathcal{L}(n\text{-}X)$
- generative power of ${}_{x}\mathcal{L}(n\text{-}X)$ with other type of grammars
- restrictions on computation steps in multiaccepting automata systems
- hybrid automata system
- ...

Applications

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Publications

- Čermák, M.: Proceedings of the 14th conference student EEICT 2008, volume 2. System of Formal Models and Their Applications.
- Meduna, A., Lukáš, R.: Multigenerative Grammar Systems. Schedae Informaticae, Krakov, PL, 2006.
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