On Nondeterminism in Programmed Grammars

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Outline

- **Preliminaries and Introduction**
- **Part I: Degree of Nondeterminism**
- **Part II: Number of Nondeterministic Rules**
- **Part III: Overall Nondeterminism**
- **Concluding Remarks and Open Problems**

Acknowledgment

This presentation is partially based on: A. Meduna, L. Vrábel, P. Zemek: On Nondeterminism in Programmed Grammars, In: *AFL'11: Automata and Formal Languages 2011* (submitted).

Definition

A *programmed grammar* is a quintuple

$$G = (N, T, S, \Psi, P),$$

where

- N is an alphabet of *nonterminals*;
- T is an alphabet of *terminals* ($N \cap T = \emptyset$);
- $S \in N$ is the *starting nonterminal*;
- Ψ is an alphabet of *rule labels*;
- P is a finite set of *rules* of the form

  $$(r\colon A \rightarrow x, \sigma_r),$$

  where $r \in \Psi$, $A \in N$, $x \in (N \cup T)^*$, and $\sigma_r \subseteq \Psi$.

Definition

The relation of a *direct derivation*, symbolically denoted by $\Rightarrow$, is defined over $(N \cup T)^* \times \Psi$ as follows:

$$(u, r) \Rightarrow (v, s)$$

if and only if

$$u = u_1 A u_2,\ v = u_1 x u_2,\ (r\colon A \to x, \sigma_r) \in P, \text{ and } s \in \sigma_r.$$

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The *language generated by G*, $L(G)$, is defined as

$$L(G) = \{w \in T^* \mid (S, r) \Rightarrow^* (w, s), \text{ for some } r, s \in \Psi\}.$$

Programmed Grammars – Definition

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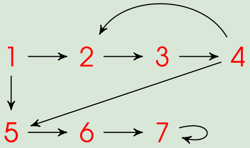
P . . . the family of languages generated by programmed grammars

Programmed Grammars – Example

Example

$$
\begin{aligned}
&(1\colon S \to ABC, \{2,5\})\\
&(2\colon A \to aA, \{3\})\\
&(3\colon B \to bB, \{4\})\\
&(4\colon C \to cC, \{2,5\})\\
&(5\colon A \to a, \{6\})\\
&(6\colon B \to b, \{7\})\\
&(7\colon C \to c, \{7\})
\end{aligned}
$$

$$
\begin{aligned}
(S,1) &\Rightarrow (ABC,2)\\
&\Rightarrow (aABC,3)\\
&\Rightarrow (aAbBC,4)\\
&\Rightarrow (aAbBcC,5)\\
&\Rightarrow (aabBcC,6)\\
&\Rightarrow (aabbcC,7)\\
&\Rightarrow (aabbcc,7)
\end{aligned}
$$



$$L(G) = \{a^n b^n c^n \mid n \geq 1\}$$

Definition

Let $G = (N, T, S, \Psi, P)$ be a programmed grammar. G is of *degree of nondeterminism n*, where $n \geq 1$, if every $(r\colon A \to x, \sigma_r) \in P$ satisfies

$$\text{card}(\sigma_r) \leq n.$$

By $\text{dnd}(G)$, we denote the degree of nondeterminism of G.

$\mathbf{DND}(\mathbf{P}, n)$ . . . the family of languages generated by programmed grammars of degree of nondeterminism n

Question

What happens if we limit the degree of nondeterminism?

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Theorem

$\mathbf{DND}(\mathbf{P}, 1) = \mathbf{FIN}$

FIN . . . the family of finite languages

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Theorem

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Theorem

$\mathbf{DND}(\mathbf{P}, 2) = \mathbf{P}$

Part II: Number of Nondeterministic Rules

Question

What happens if we limit the number of nondeterministic rules?

Question

What happens if we limit the number of nondeterministic rules?

${}_n\mathbf{P}$ . . . the family of languages generated by programmed grammars with n nondeterministic rules

Question

What happens if we limit the number of nondeterministic rules?

${}_{n}\mathbf{P}$ . . . the family of languages generated by programmed grammars with n nondeterministic rules

Theorem

${}_{1}\mathbf{P} = \mathbf{P}$

Definition

Let $G = (N, T, S, \Psi, P)$ be a programmed grammar. For each $(r\colon A \to x, \sigma_r) \in P$, let $\zeta(r)$ be defined as

$$\zeta(r) = \begin{cases} \mathrm{card}(\sigma_r) & \text{if } \mathrm{card}(\sigma_r) \geq 2 \\ 0 & \text{otherwise.} \end{cases}$$

The *overall nondeterminism of G* is denoted by $\mathrm{ond}(G)$ and defined as

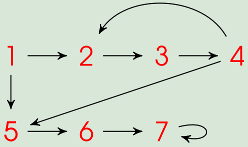
$$\mathrm{ond}(G) = \sum_{r \in \Psi} \zeta(r).$$

$\mathbf{OND}(\mathbf{P}, n)$ . . . the family of languages generated by programmed grammars with overall nondeterminism n

Part III: Overall Nondeterminism – Example

Example

$(1\colon S \to ABC, \{2,5\})$
$(2\colon A \to aA, \{3\})$
$(3\colon B \to bB, \{4\})$
$(4\colon C \to cC, \{2,5\})$
$(5\colon A \to a, \{6\})$
$(6\colon B \to b, \{7\})$
$(7\colon C \to c, \{7\})$



ond(G)

$\mathrm{ond}(G) = 4$

Part III: Overall Nondeterminism

Question

What happens if we limit the overall nondeterminism?

Question

What happens if we limit the overall nondeterminism?

Theorem

$\mathbf{OND}(\mathbf{P}, n) \subset \mathbf{OND}(\mathbf{P}, n + 1)$

Open Problems

- Appearance checking?
- Propagating programmed grammars?

References

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The thank you slide.