

Context-free languages and primitive words

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Combinatorial properties of words play an important role in mathematics and theoretical computer science. One of the well-known open problems is related to the language of primitive words. A word is called *primitive* if it is not a repetition of another word. (Thus the empty word is non-primitive.)

We conjectured that the language Q of all primitive words over a non-singleton alphabet is not context-free (Dömösi, S. Horváth, M. Ito [1991]). The problem seems to be simple but we could not solve it yet.

Apart from the conditions of Wise Lemma (D. S. Wise [1976]), Q has all well-known iteration conditions of context-free languages (P. Dömösi, S. Horváth, M. Ito, L. Kászonyi, M. Katsura [1992,1993]).¹ Another test of context-freeness is the so-called Interchanging Lemma (W. Ogden, R. J. Ross, K. Winklmann [1982]). It is also proved that Q fulfils the conditions of this test (S. Horváth [1995]). Therefore, Q resists almost all well-known tests of context-freeness.

It is also well-known that an intersection of a regular and a context-free language is again a context-free language. Therefore, if we find a regular language R such that $R \cap Q$ is not context-free then we can show that Q is not context-free. By results of L. Kászonyi and M. Katsura [1996, 1997, 1999a, 1999b], this direction also seems to be hopeless.

Maybe an appropriate homomorphic characterization of languages could help to prove our conjecture about the context-freeness of Q . (N. Chomsky and M. P. Schützenberger [1963], R. J. Stanley [1965]), S. Hirose and M. Yoneda [1985], P. Dömösi and S. Okawa [2003]).

¹Note that the applicability problem of the Wise Lemma is equivalent to the original problem.)