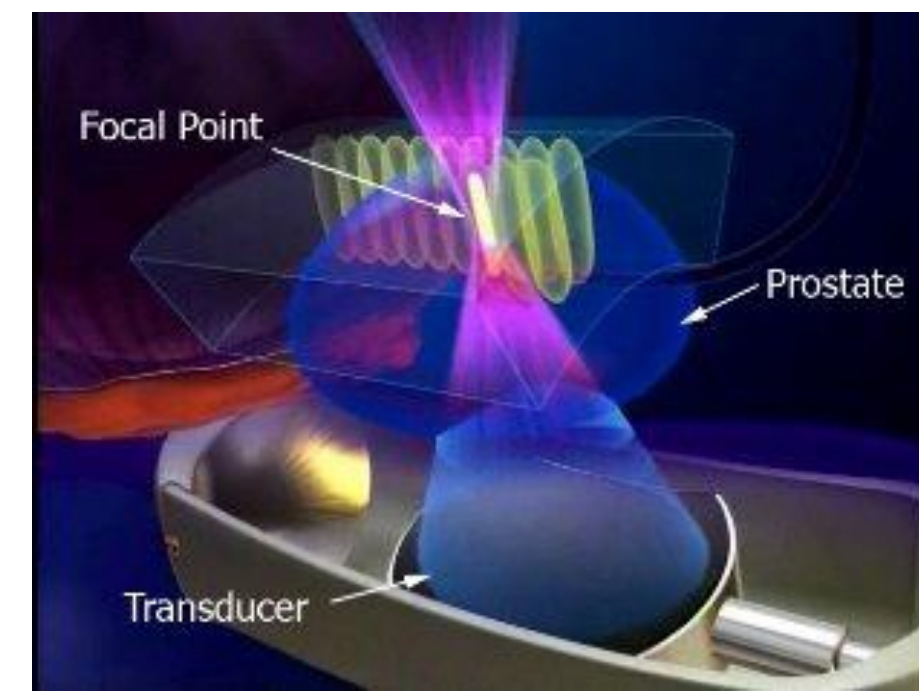
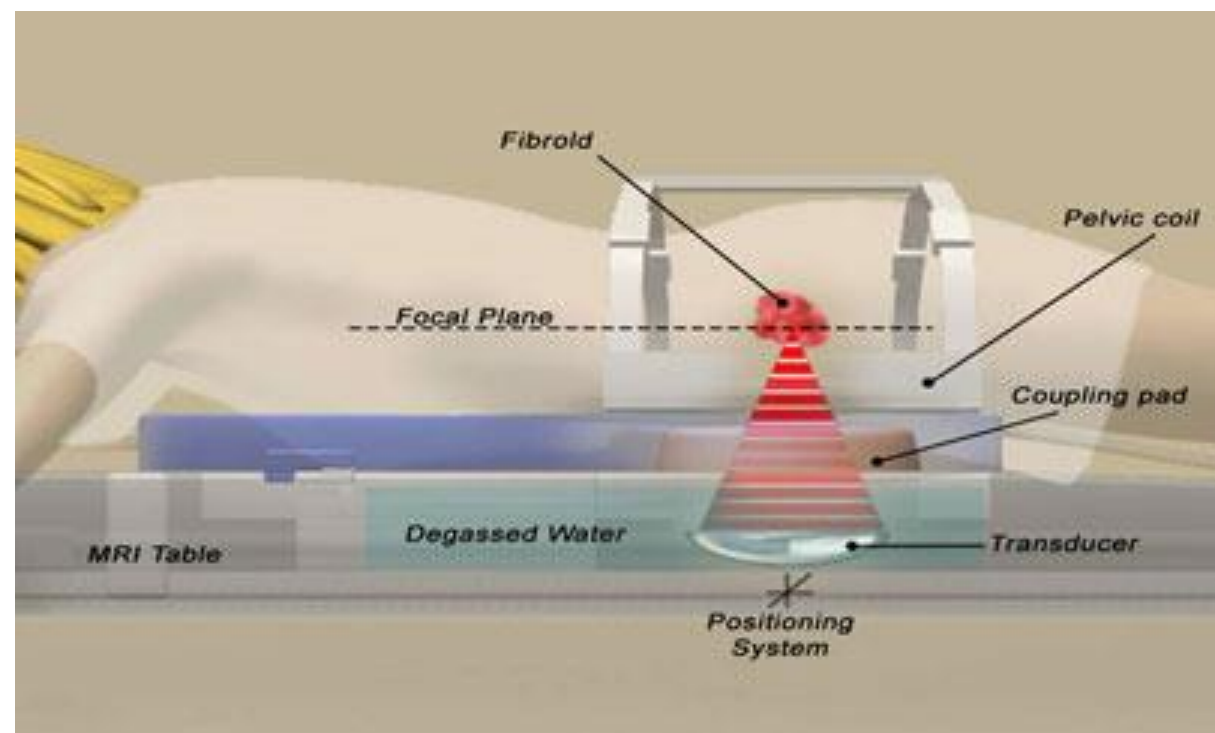


1 Overview

High-intensity focused ultrasound (HIFU) is an emerging non-invasive cancer therapy that uses tightly focused ultrasound waves to destroy tissue cells through localised heating. The treatment planning goal is to select the best transducer position and transmit parameters to accurately target the tumour. The path of the ultrasound waves can be predicted by solving acoustic equations based on mass, momentum, and energy conservation. However, this is a computationally difficult problem because the domain size is very large compared to the acoustic wavelength.



2 Nonlinear Ultrasound Wave Propagation in Tissue

The governing equations must account for the nonlinear propagation of ultrasound waves in tissue, which is a heterogeneous and absorbing medium. Accurately accounting for acoustic absorption is critical for predicting ultrasound dose under different conditions. The required acoustic equations can be written as:

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho_0} \nabla p \quad \text{momentum conservation}$$

$$\frac{\partial \rho}{\partial t} = -(\rho + \rho_0) \nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \rho_0 \quad \text{mass conservation}$$

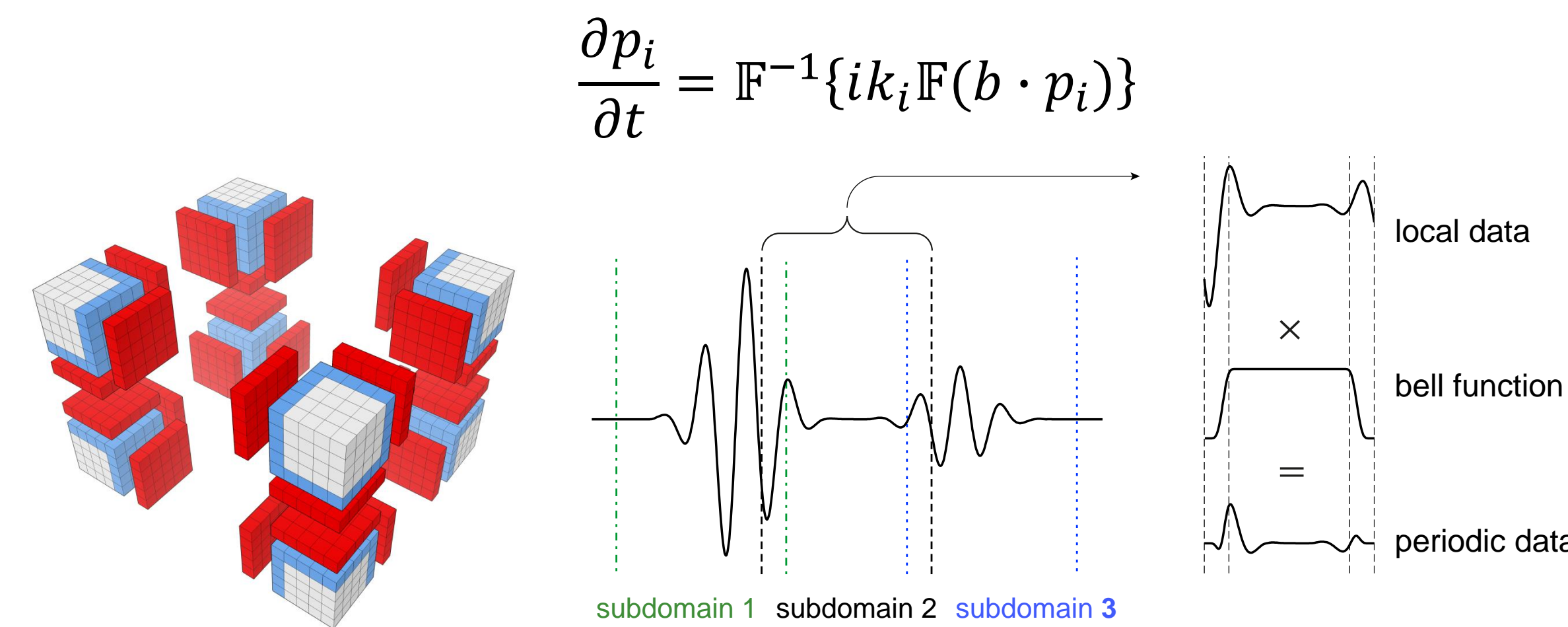
$$p = c_0^2 \left(\rho + \mathbf{d} \cdot \nabla \rho_0 + \frac{B}{2A} \frac{\rho^2}{\rho_0} - \Pi \rho \right) \quad \text{pressure-density relation}$$

These equations are discretized using the k -space pseudo-spectral method and solved iteratively. This reduces the number of required grid points per wavelength by an order of magnitude compared to finite element or finite difference methods. For uniform Cartesian grids, the gradients can be calculated using the fast Fourier transform.

3 Local Fourier Basis Decomposition

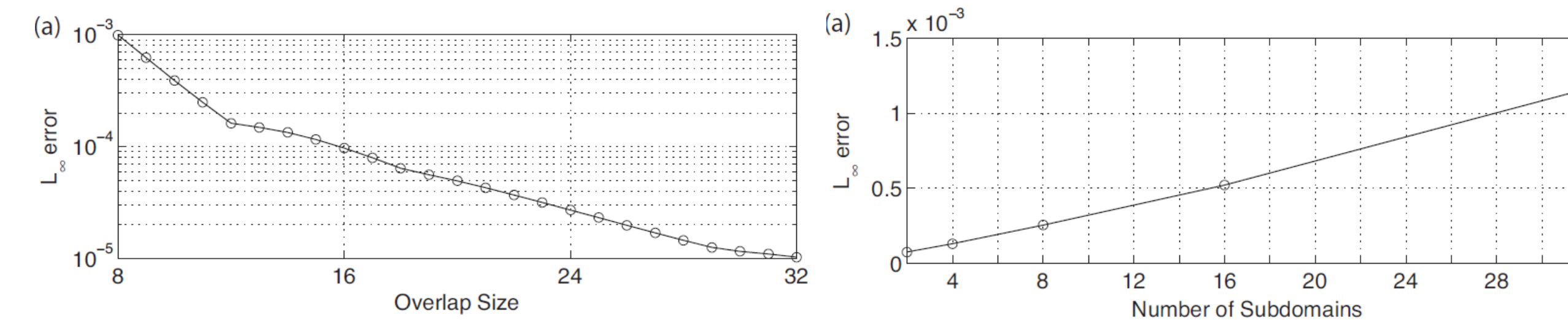
Local domain decomposition reduces the communication burden by partitioning the domain into a grid of local subdomains where gradients are calculated locally and the global communication is replaced by the nearest-neighbor halo exchange.

The gradient calculation with the halo on an i -th subdomain reads as follows (b is a bell function smoothening the subdomain interface):

$$\frac{\partial p_i}{\partial t} = \mathbb{F}^{-1} \{ i k_i \mathbb{F} (b \cdot p_i) \}$$


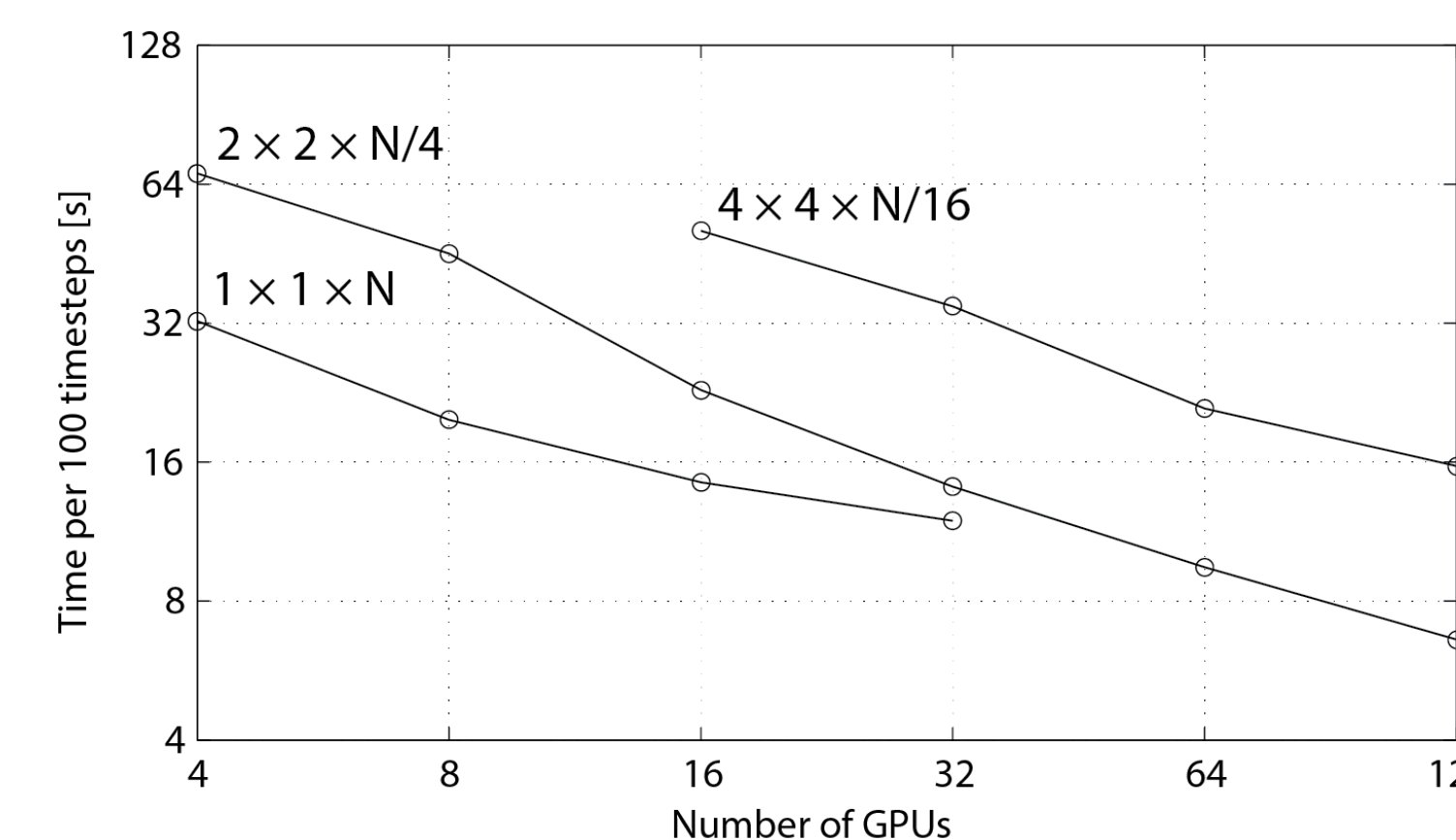
4 Local Fourier Basis Accuracy

Since the gradient is not calculated on the whole data, numeric error is introduced. Its level can be tuned by the thickness of the halo region.



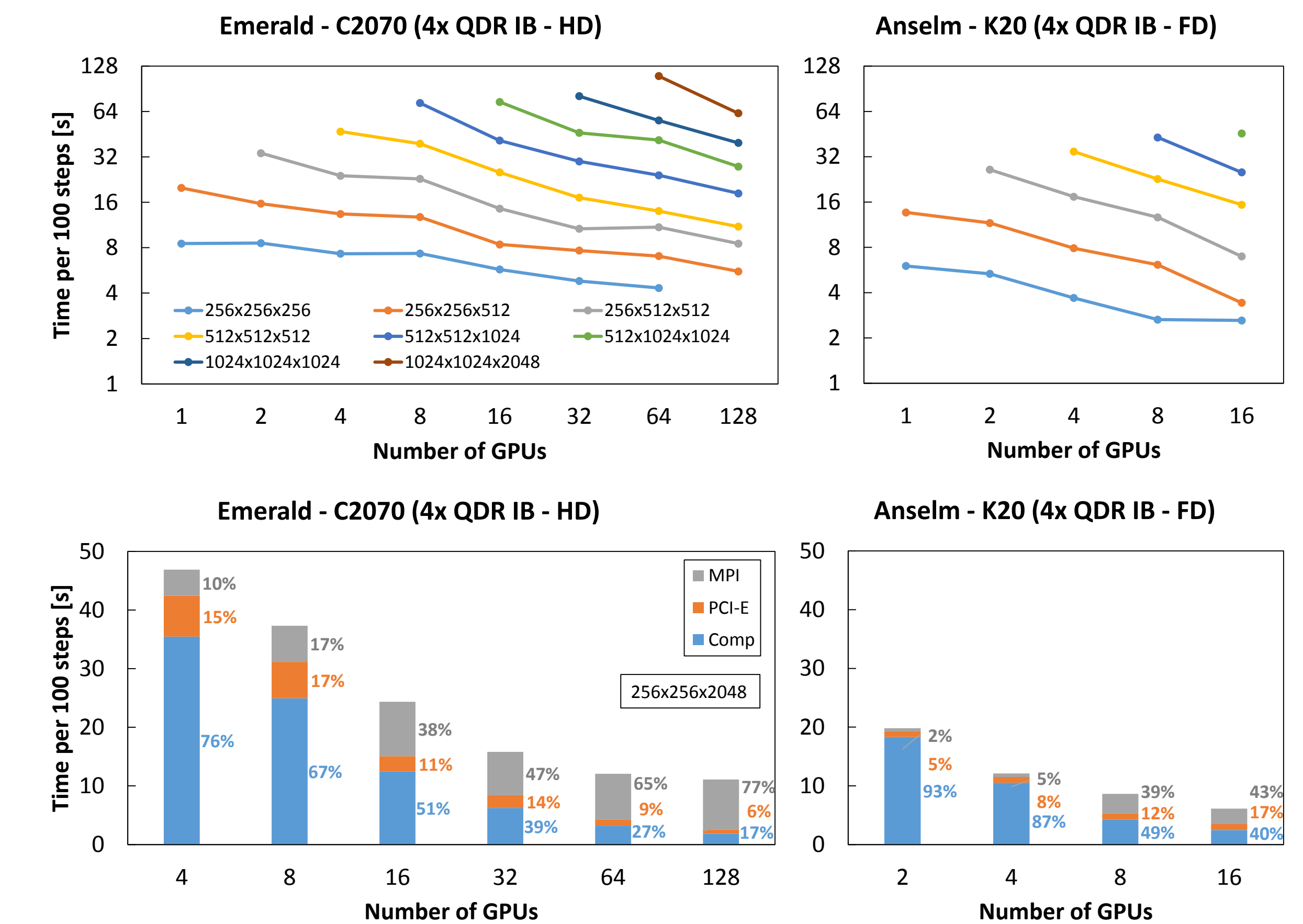
5 Performance Investigation – Decomposition Dimensionality

Because the communication complexity grows with the number of neighbors and error grows with the number of subdomains in each direction. It's best to keep dimensionality of the decomposition as low as possible.



6 Performance Investigation – Strong Scaling

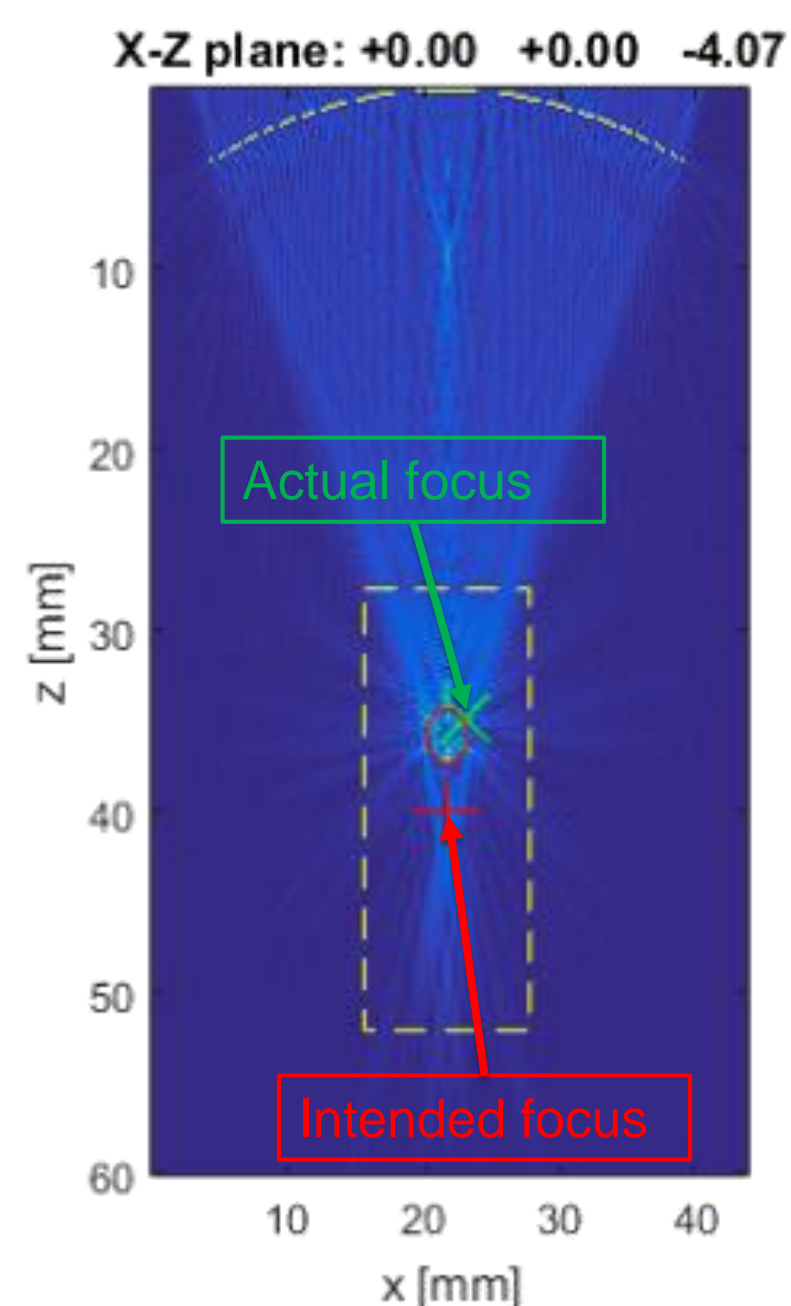
The strong scaling and run time breakdown were investigated on Anselm (1 GPU per node) and Emerald (3 GPUs per node) clusters with up to 128 GPUs.



7 Impact on Cost of Realistic Simulations

Our approach significantly reduces cost of computing a pressure field of a prostate ultrasound, which requires a domain size of 1536 x 1024 x 2048 (45mm x 30mm x 60mm) with 48,000 time steps (60μs).

Compute Resources	Simulation Time	Simulation Cost
96 GPUs	14h 09m	\$475
128 GPUs	9h 29m	\$426
128 CPU cores	6d 18h	\$1,826
256 CPU cores	3d 0h	\$1,623
512 CPU cores	2d 5h	\$2,395



SoMoPro



IT4Innovations
national supercomputing
center

