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A Scalable Cellular Automata Based Microscopic Traffic Simulation

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Abstract—**This paper presents a new model for simulations of very large scale traffic networks. The proposed model is based on microscopic cellular automata (CA) extended to eliminate unwanted properties of ordinary CA based models, such as stopping from maximum speed to zero in one time step. The accuracy of the model has been validated by comparisons with various fundamental diagrams. A parallel implementation developed using the proposed model allows for an almost linear speedup. This allows to run a simulation multiple in real-time, that the traffic state of very large scale networks can be precisely predicted, for example, with various scenarios.**

I. INTRODUCTION

MICROSCOPIC traffic simulation models, in contrast to macroscopic models, distinguish and trace every single vehicle and driver. The models typically employ characteristics such as vehicle lengths, speeds, accelerations, time and space headways, vehicle and engine capabi-lities, as well as some rudimentary human characteristics that describe driving behavior [1], [2].

In the past, the use of microscopic models has been limited in scope and scale of the problem. The applications were restricted to small networks with a limited number of vehicles. With the advent of fast computers a large number of microscopic simulation models have been developed. For an illustration, the SMARTEST report (1999) identified 58 microscopic simulation models of which 32 were analyzed [7]. Some of them are true microscopic simulators in the sense that they model the behavior of each individual vehicle in the traffic flow. Another and most recently short overview of microscopic traffic models (2004) can be found, for example, in [6].

The need for more precise and realistic traffic models resulted in a complicated comparison and validation schemes which are also integrating field measurements into the model [19], [20]. Field data are used for model initialization, calibration and lastly for validation. These simulation models are so called "online" traffic simulators [20]. An important condition for both online simulators and for traffic state forecasting is to simulate the traffic in networks faster than real time (multiple in real time). Hence the acceleration of microscopic models continues to grow in importance. Cellular automata (CA) based modes have been recognized as quite useful in this area, mainly because they can easily be accelerated on parallel computers or directly in hardware.

The goal of this paper is to present our extended CA based model which has been proposed in order to model road traffic more realistically and also quite efficiently. Results from this type of simulations can be used in various ITS, for example, in driver assistance or collision avoidance systems. Our approach attempts to extend the simulation beyond a few roads and intersections to large areas (e.g. we are going to model and simulate very large networks such as the whole Czech Republic with approximately 55 000 km of road segments [8]. Moreover, the given length of road segments does not distinguish the directions, the number of lanes (e.g. the real number of kilometers is much higher when including these options). For instance, the distance between Prague and Brno city is approximately 200 km, but because there are two lanes in each direction, the total length of road segments is four times greater). In contrast to specialized and expensive accelerators, such as Field Programmable Gate Array (FPGA) based machines [28], we propose to utilize a very reasonable solution based on commodity hardware.

The rest of the paper is organized as follows. CA based models for microscopic traffic simulation are introduced in Section II. In order to compare existing and proposed models, we describe a method that utilizes fundamental diagrams in Section III. In Section IV the proposed model is introduced. Results of experimental evaluation are summarized in Section V. Finally, conclusions and suggestions for future work are given in Section VI.

II. CELLULAR AUTOMATA BASED MODELS

Cellular automata based models have become popular in the area of microscopic traffic flow simulations because of their simplicity and suitability for acceleration. The first highly suitable model for acceleration of single-lane freeway traffic was introduced by Nagel and Schreckenberg in 1992 [9]. Because of the model's simplicity, it is possible to perform millions of updates in a second [13], [14]. Thus, it may be used for simulating a high volume of traffic over very large networks. Due to some critics of unrealistic behavior (such as [26]), simple CA based models are often extended. On the other hand, CA models were shown to be able to capture all basic phenomena that occur in traffic flows [10], not only in the field of vehicular traffic flow modeling, but also in other fields such as pedestrian behavior, escape and panic dynamics, etc.

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A. *Basic Model*

Nagel's and Schreckenberg's traffic model [9] was initially defined on a one-dimensional array (Fig. 1) with open or periodic boundary conditions and with every single cell representing a road segment. A local transition function (a rule) defines the new state of a cell on the basis of its current state and the state of neighboring cells. Globally viewed, it describes the movements of vehicles from one cell to another cell in a discrete way.

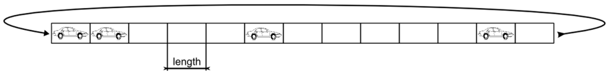

Fig. 1. CA-based traffic network with periodic boundary.

In space domain, each cell represents only a defined length of road segment, so space is coarse-grained. This coarse graininess is fundamentally different from the usual microscopic models, which adopt a semi-continuous space. Some potential lengths are shown in Table 1. The length of 7.5 m is mostly chosen because each car occupies about this amount of space in a complete jam [9]. So this is the average length of a car on the road, including a constant gap in front of and behind each car. In order to model other kinds of vehicles in CA based models, it is better to use another cell length, or even more precisely, use more smaller cells for representing a single vehicle -- especially for bigger types of vehicles [6], [12], [18].

Each CA simulation step represents an amount of time in reality. Based on some previous arguments about driver reaction time, suggestions given in [3], [4] and field data measurements [6], it is possible to represent one simulation step between 0.6 and 1.5 s. In the CA model this value is crucial because together with cell length it gives us the granularity of minimal model speed jumps. Various settings of these variables are shown in Table 1.

Normally, urban traffic road networks are very complex. Paper [15] has shown that arbitrary kinds of road and intersections can be reduced to only a few basic elements. For constructing more complicated traffic networks, we shall simply connect more various kinds of cells to the desired topology. So it is possible to build more traffic lanes [16], [17], roundabounds [18] or very complex topologies for whole cities [19], [20].

B. *Local Transition Function*

TABLE I
SPEED JUMPS [KM/H] FOR CA MODEL

Cell length [m]	Time step [s]						
	0.5	*0.7*	*0.9*	*1.0*	*1.2*	*1.4*	*1.6*
0.5	3,60	2,57	2,00	1,80	1,50	1,29	1,13
1	7,20	5,14	4,00	3,60	3,00	2,57	2,25
1.5	10,80	7,71	6,00	5,40	4,50	3,86	3,38
2	14,40	10,29	8,00	7,20	6,00	5,14	4,50
2.5	18,00	12,86	10,00	9,00	7,50	6,43	5,63
3	21,60	15,43	12,00	10,80	9,00	7,71	6,75
3.5	25,20	18,00	14,00	12,60	10,50	9,00	7,88
4	28,80	20,57	16,00	14,40	12,00	10,29	9,00
4.5	32,40	23,14	18,00	16,20	13,50	11,57	10,13
5	36,00	25,71	20,00	18,00	15,00	12,86	11,25
5.5	39,60	28,29	22,00	19,80	**16,50**	14,14	12,38
6	43,20	30,86	24,00	21,60	18,00	15,43	13,50
6.5	46,80	33,43	26,00	23,40	19,50	16,71	14,63
7	50,40	36,00	28,00	25,20	21,00	18,00	15,75
7.5	54,00	38,57	30,00	*27,00*	22,50	19,29	16,88
8	57,60	41,14	32,00	28,80	24,00	20,57	18,00

Tab. 1. Single speed jumps used in cellular automaton model in kilometers per hour (km/h) for different cell lengths and various simulation time steps. It is shown that for smaller lengths and greater time steps the model will be more accurate.

Properties of single lane traffic are modeled on the basis of integer valued probabilistic cellular automaton rules [14]. The local transition function can be formalized as follows: each vehicle in a cell has an integer velocity v_v with values between zero and v_{max}. For real vehicle speed v_{act} we use the expression:

$$v_{act} = v_v \cdot speed\_jump \qquad (1)$$

where *speed jumps* are defined in Table 1. We let $\gamma(i)$ denote the cell gap in front of cell *i*. For an arbitrary configuration, one update of the simulation system consists of the following four consecutive steps, which are performed in parallel for all vehicles [9]:

1. **Acceleration:** *if* $(v_v < v_{max})$ *and if* $(\gamma(i) > v_v)$ *then* $v_v = v_v + 1$.
2. **Slowing down:** *if a vehicle at site i sees the next vehicle at site* $i + j$ $(j \leq v_v)$, *it reduces its speed, so* $v_v = j - 1$.
3. **Randomization:** *with probability p, the velocity of each vehicle (if* $v_v > 0$*) then* $v_v = v_v - 1$.
4. **Car motion:** *each vehicle is advanced* v_v *sites.*

This simple CA based traffic model shows nontrivial and realistic behavior [9]. Step 3 is essential in simulating traffic flows since the dynamics is completely deterministic, and without this randomness, every initial configuration of vehicles and corresponding velocities reaches a stationary pattern which is shifted backwards. Fig. 5 in Section V also shows that this simple model is capable of reproducing characteristic properties of real traffic, such as certain aspects of flow-density relation, spatio-temporal evolution of jams, stop-and-go waves, etc. [9], [11], [20].

C. *Performance*

In paper [20], an online simulation model of city of Duisburg was introduced. The authors showed that for the most frequently occurring densities the road network of Duisburg can be simulated around 100 times faster than real time. It is necessary to mention that their road network is modeled using only 22 000 cells. Their simulator is not suitable for more cells. An example of explicitly implemented parallelism (using the MPI [29] message passing programming model on IBM SP2 parallel machine) in CA based model was presented in [19]. They have used the standard Nagel's and Schreckenberg's CA model to implement a traffic simulator for city of Geneva and observed a superlinear speedup for 15 000 vehicles. But, as they mentioned, their simulations are not expected to match the exact traffic situation of Geneva (!). CELLSIM is another good example of a recent CA based microscopic simulator [6]. It is tuned to be a high fidelity traffic simulation model, but on the other hand, the field data used for model validation are pretty old. Computational performance of the model, as for other CA based modes, is dependent on the number of vehicles in the system, and it was only showed, that CELSIM is able to simulate around 2 300 vehicles in shorter than real time. As computational power is increasing, we can assume that this number of vehicles may increase too. Recently, some examples of accelerating the standard CA based model were proposed using FPGAs. It was shown that road traffic simulation of entire metropolitan areas can be accelerated with reconfigurable supercomputing that combines 64-bit microprocessors and FPGAs in a high bandwidth and low latency interconnect [28]. For a realistic road description of Portland, which consists of about 6.25 million cells in a CA model, they were able to achieve a speedup of 34.4 independently of the number of vehicles. It is also necessary to mention that their streaming simulation model has used extra and expensive hardware.

III. MODEL VALIDATION

There exists some sophisticated methods for traffic simulator comparison and validation based on mathematical models (created using field measurements of some traffic parameters), or purely in field measurements and model comparison with this measured data [19], [20] and [24]. This section provides a short description of one of the methods – fundamental diagrams.

Road traffic is always in a specific state that is characterized by the flow rate – *f,* the traffic density – *d* and the vehicles mean speed – *v*. One can combine all possible homogeneous and stationary traffic states in an equilibrium function that can be described graphically by three diagrams, so-called fundamental diagrams. This diagram shows the relation between two of the three main macroscopic variables. The third variable can always be recovered by means of the relationship:

$$f = d \cdot v \qquad (2)$$

A fundamental diagram applies to a specific road and is drawn up with basic observations. The stationary and homogenous traffic is always in a state that is located on the line of Fig. 4. It should be noted that Fig. 4 is used in Section V to compare theoretical results with results obtained from our simulation model. But some special state points require extra attention:

a) Completely free flowing traffic

When vehicles are not impeded by other traffic they travel at a maximum speed of v_{max} (also called free speed). This speed is dependent, among other things, on the design speed of a road and the speed restrictions in operation at any particular time and weather. At free speed, flow rate and density will be close to zero.

b) Saturated traffic

On saturated road segments flow and speed go down to zero. The vehicles are queuing and there is a maximum density of d_{max}, which is also called jam density.

c) Capacity traffic

The capacity of a road is equal to the maximum flow rate f_{cap}. The maximum flow also has an associated capacity speed v_{cap} and capacity density d_{cap}. The diagram shows that the capacity speed lies strictly below maximum speed v_{max}.

Mathematical models for fundamental diagrams are represented with parametric mathematical expressions for the equilibrium relations given by entire fundamental diagrams. In 1934 Greenshield (for example, [21]) drew up the first formulation that was based on a small number of measurements. In his diagrams (shown in Fig. 4 with dotted lines), the capacity speed v_{cap} is half the maximum speed v_{max} and capacity density d_{cap} is half the maximum density d_{max}. This formulation is a rough simplification of observed traffic behavior. It was also shown that Greenshield's parabolic speed-flow relationship and its corresponding linear speed-density relationship are inconsistent with the field data [21]. Specifically, the data indicate that vehicles travel at a speed that is higher than half the free-speed when traveling at capacity. Even so, this model is still frequently used because of its simplicity.

Another well-known and also much used formulation of fundamental diagram is the so-called triangular diagram, or according to its author, Pipes' model [5]. His model is multi-regime in the sense that a different model is utilized for the congested versus uncongested regimes. Diagrams (in Fig. 4 sketched with a dashed line) have many advantages in dynamic traffic modeling. In his equilibrium relation the mean speed equals the maximum speed for all traffic states that have densities smaller than the capacity density. The branch of a triangle that links capacity state with saturated state has a negative constant slope.

In order to address the main laws of Greenshield's and Pipes' models into a single-regime model, Van Aerde's model can then be obtained [22], [23]. This model overcomes the shortcomings of Pipes' model in which it is assumed that vehicle speeds are insensitive to traffic density in the un-congested regime, which has been demonstrated to be inconsistent with a variety of field data from different facility types. Alternatively, the model overcomes the main shortcoming of the Greenshields model which assumes that the velocity and flow relationship is parabolic, which again is inconsistent with the field data from a variety of facility types. Corresponding diagrams are sketched with a full line in Fig. 4.

Because all the previously mentioned parameters – flow, density and velocity – are mostly identified for macroscopic models, we need to obtain these properties from the microscopic model. This is simply done for a concrete cell in topology (for flow), or for a concrete number of cells (for obtaining density and mean speed). Details can be found in [25].

IV. PROPOSED MODEL

The proposed model was designed with the aim to (i) obtain more accurate traffic behavior from traffic simulations and (ii) accelerate the simulations as much as possible using commonly available devices such as multi-core chips or graphic processing units. The proposed model extends the CA based model presented in Section II.

A. *Parameters of the Cell*

Fig. 2 shows us how the state and parameters of a cell are encoded. Instead of the traditional CA based models implementation, which utilizes several integers (to encode actual speed, speed limit, vehicle type …), we have used only one integer and encoded the parameters at the bit level. For example, for the actual speed (from 0 to 11), we need only 4 bits. This encoding allowed us to speed up the simulation significantly because the memory traffic is highly reduced.

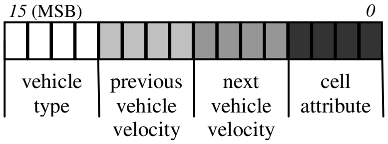

Fig. 2. Single CA cell state encoded at the bit level on 16 bits.

B. *Acceleration and Deceleration Rules*

As in the traditional CA based model, single-regime (i.e. constant acceleration) is used in our model. This gives us a reasonable approximation when the vehicle is accelerating [4].

On the other hand, deceleration has two phases. The first phase is identical with the previously described randomization step (no. 3) described in Section II. The second phase, marked in the CA based model as a slowing down step and used for collision avoidance, is modified to prevent a vehicle stopping from maximum velocity to zero in one simulation step. Elimination of this property is done in our model by the "looking ahead" technique and observing three parameters: *actual vehicle speed, speed of the leader vehicle* and the *size of cell gap* (after calculating the advance of following vehicle). If these values give us enough room for vehicle advancing in the next simulation step, nothing is done. If not, actual vehicle speed is proportionally reduced and its new value is used in the next simulation step. For example, if investigated car has a speed of 5, the cell gap gives us enough room, however, if the following vehicle has only a speed of 1, actual speed will be limited to 3. This causes us to avoid approaching a leader with extremely fast speed. This action is performed only when the follower is not decided to bypass a leading vehicle, so this updated rule (no. 2) only appears if no lane change is assumed on the road. Solely the lane changing logic of our model has been developed using the same car-following features of the model presented in this paper [27]. It was also shown that this is sufficient and fully accurate [6].

C. *Double Buffering*

Network topology is stored in two memory buffers. High-speed simulation is allowed (even for the proposed deceleration technique) using the so-called double buffering, when one simulation step is performed on one copy of buffer and consequently on another buffer copy.

D. *Other Parameters*

The cell length used in our model is 5.5 m, which is a little less than in the original CA model (7.5 m). This constant is based on our own field experiments performed in the Czech Republic [6], where a shorter gap between vehicles is observed and also vehicle sizes are becoming smaller. In order to model other kinds of vehicles, especially bigger ones, we simply use two successive cells, which means doubling the vehicle size to the length of 11 m. By defining the cell length and simulation time, we explicitly specified the minimum speed jumps to 16.5 km/h. For traditional CA based model the speed jumps are 27 km/h, as can be seen in Table 1.

We assume a neighborhood of 11. With a cell length of 5.5 m it gives us a distance of 60.5 m between the leader and the follower. As observed in [3] and [6], for separations greater than 61 m, car-following is negligible on a driver's behavior. Based on the size of the maximum neighborhood, it is also possible to calculate maximum vehicle velocity, which is exactly 181.5 km/h.

Reaction time is equivalent to one simulation time step and in our model it is 1.2 s. This value is very close to 1.21 s – the mean reaction time of unaltered drivers in car-following as suggested in [4].

V. EXPERIMENTAL RESULTS

The proposed model was implemented in the programming language C/C++ and then simulated on a dual processor *Quad Core Xeon(R) CPU E5420 @ 2.50 GHz* machine with a system memory of 16 GB running with a 64bit Linux-based operating system. In order to evaluate the model, we have used urban areas modeled using from 10 thousand to 100 million CA cells. Defining the global behavior of vehicles is necessary to determine the turns at road intersections. Since a traditional origin-destination matrix with a sufficient resolution in time and space is not available, all vehicles are guided randomly through the

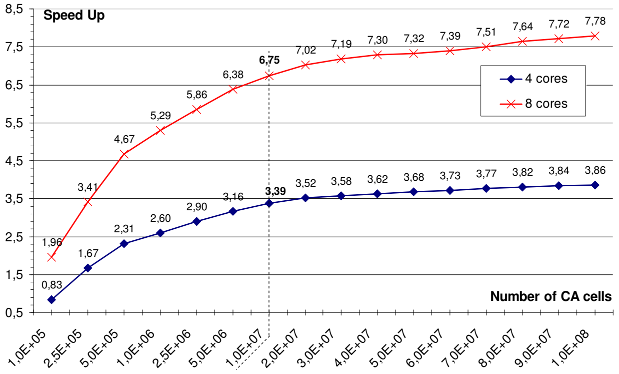

Fig. 3. Speedup obtained for 4 and 8 processor cores. The model is initialized for 50% density of vehicles.

network with respect to the given turning probabilities at road junctions over time. These probabilities were counted based on real field data, mostly from loop and camera based detectors (obtained from our industrial partner). The proposed implementation is able to simulate up $1.5 \cdot 10^6$ cells with 50% vehicle occupancy in a shorter than real time.

A. *Analysis of Scalability*

For more vehicles or simply more CA cells, the execution time has increased more quickly than the simulation time. In order to increase the level of parallelism, the model was extended using *OpenMP API* specification for parallel programming [25]. Fig. 3 shows the speedup obtained for the original model with regard to the extended model. The first run (in Fig. 3 denoted as “4 cores”) utilizes only one physical processor (i.e. 4 processor cores) where a single thread was executed per one physical processor core.

The next run (in Fig. 3 denoted as “8 cores”) is measured with both processors (i.e. with 8 processor cores). For the simplified model, which requires employing 10 million cells ($5.5 \cdot 10^4$ km of roads [24]), the obtained speedup is 3.39 using 4 cores and 6.75 using 8 cores wrt a single core processor. This is also enough to simulate the model in multiple real time, so an additional “free time” may be used in the future. The speedup is 7.78 (or 3.86) if the number of cells is 10 times higher. However, this speedup is not sufficient for real time simulation of a system consisting of 100 million cells.

TABLE II
SPEED UP FOR VARIOUS NUMBER OF PROCESSOR CORES

Number of cores	*16*	*32*	*64*	*128*	*256*	*512*
Speed up	15.50	31.00	62.00	124.00	248.01	496.03

Tab. 2. Prediction of speed up for various number of processor cores based on the assumption of constant effectiveness of 96.88 % at a maximum number of CA cells (1.0e8) and with 50 % vehicle occupancy.

If we consider that our implementation utilizes a double buffering technique, we can reach the average effectiveness of 96.88 % for both number of cores (96.5 % for 4 cores and 97.25 % for 8 cores). Hence we may predict the speedup for more cores. Table 2 shows that if we had 16 processor cores, the speedup would be 15 which is very close to the linear acceleration. Therefore, we can claim that our model is perfectly scalable.

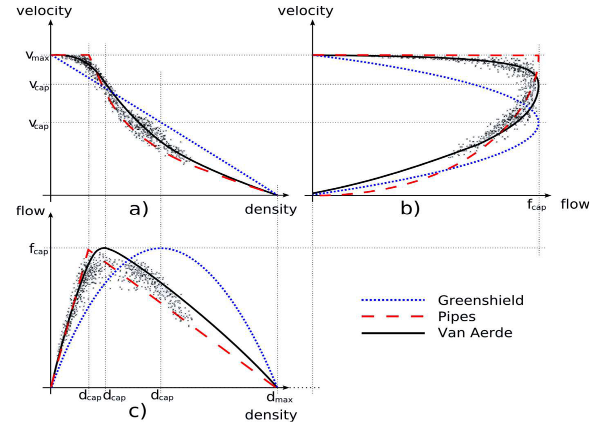

Fig. 4. Fundamental diagrams relations. They show basic relation between the vehicle velocity at one kilometer per hour, the density in the number of vehicles per one kilometer and the flow in the number of vehicles per hour. Dotted plots are measured over 2 100 time steps with the proposed CA based microsimulation model.

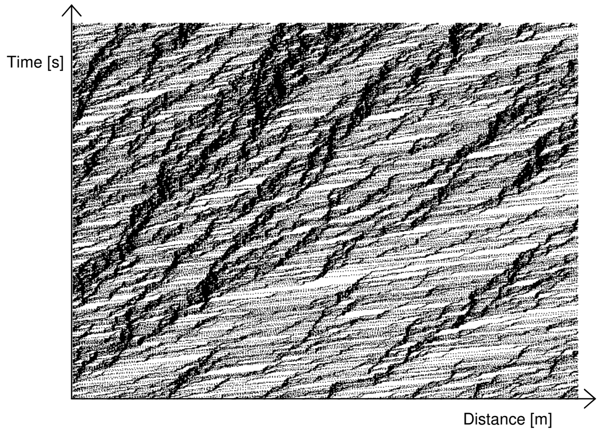

Fig. 5. Time-distance plot of density waves in high density traffic. Each trajectory represents a single vehicle. A 5.5 km long road segment is simulated before road junction for approximately 35 minutes.

B. *Analysis of Accuracy*

Fig. 4 compares all mentioned theoretical fundamental diagrams with the fundamental diagrams calculated for our model (with dots). We can observe that the data from our model best fit the Van Aerde fundamental diagram. It was shown that this model is very good in reflecting traffic stream behavior for a number of varying facility types, including a freeway or a tunnel roadway [21]. It is also important to note that our model requires a lower level of randomization probability (used for slowing down) due to our deceleration improvements, but this model is still able to reproduce basic traffic phenomena, as is shown in Fig. 5.

VI. CONCLUSION

In this paper, a model for simulations of very large scale traffic networks has been presented. The heart of the simulator is an extended microscopic CA based model, which eliminates unwanted properties of common CA based models, such as stopping from maximum speed to zero in one time step. Our model is tuned to be highly accurate, which was illustrated by comparisons with fundamental diagrams. Accuracy will also be validated on real field data in our future work. For traffic data measurement we are going to use a variety of traffic sensors, including traditional inductive loops, traffic radars and video-detection systems for single vehicle tracking in traffic network (i.e. the technology available from our industrial partners and agencies, which is already in use). Fusing data from these traffic sensors into a coherent, consistent, and reliable picture of the prevailing traffic conditions (e.g. densities, speeds, flows) will be a critical and challenging task in the online traffic management or information system, which we are going to develop.

Another very important finding of our work is that the proposed model is really suitable for acceleration. We can achieve an almost linear speedup on widely accessible multiple-core systems, which allows us to do multiple in real time simulations. In means of number, we are able to simulate 1 minute in huge traffic networks shorter than in 5 seconds! For future experiments, it is planned to couple the current data provided by the online simulation to historical data collected in a database from sensors in order to provide a very precise traffic state forecast. Hence traffic simulation of large networks must be executed multiple in real time.

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