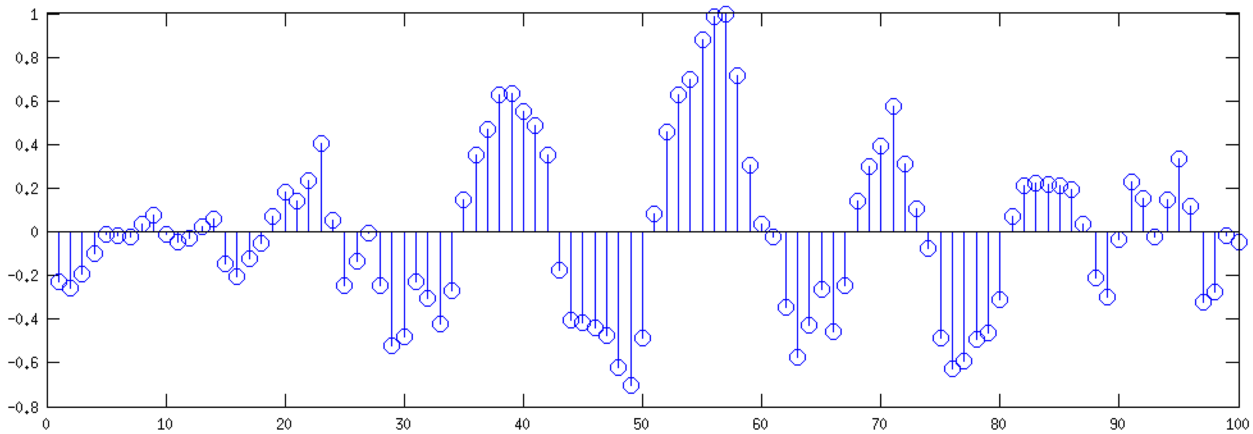
Zpracování obrazů

Honza Černocký, ÚPGM

1D signál


1
0,8
0,6
0,4
0,2
0
-0,2
-0,4
-0,6
-0,8
0
10
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30
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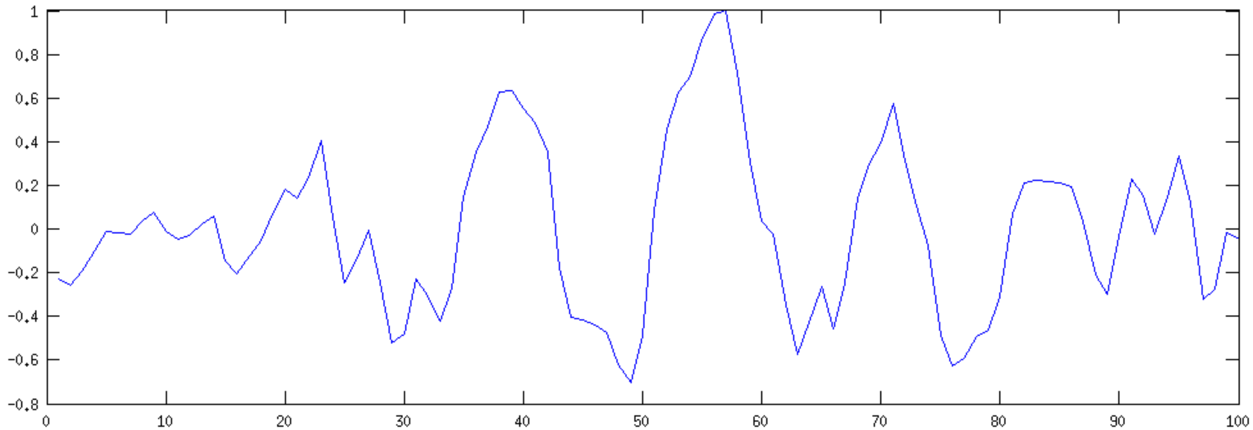
50
60
70
80
90
100


1
0,8
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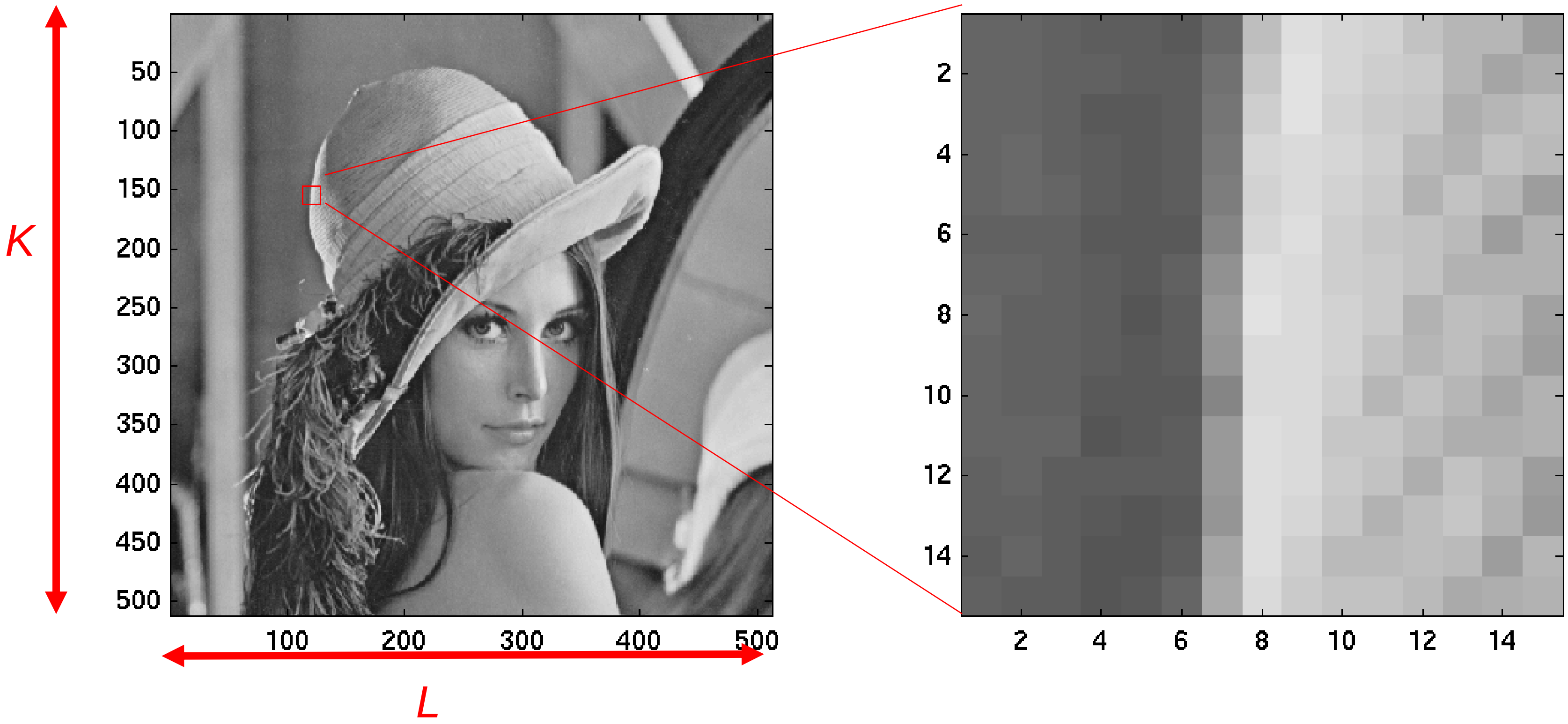
0,2
0
-0,2
-0,4
-0,6
-0,8
0
10
20
30
40
50
60
70
80
90
100

Obrázky

- 2D – šedotónový obrázek (grayscale)
- Několikrát 2D – barevné foto
- 3D – lékařské zobrazování, vektorová grafika, point-clouds (hloubková mapa, Kinect)
- Video – další dimenze je čas

Obrázek



K
L
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2
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8
10
12
14
2
4
6
8
10
12
14

Matematicky

$x[k, l]$

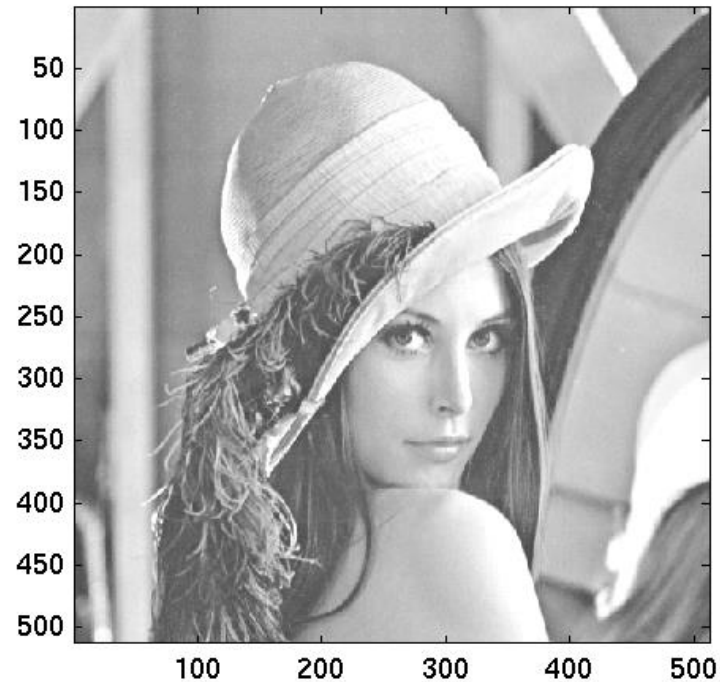
$$x[k,l] = \begin{bmatrix} x[0,0] & x[0,1] & \cdots & x[0,L-1] \\ x[1,0] & x[1,1] & \cdots & x[1,L-1] \\ \vdots & & & \vdots \\ x[K-1,0] & x[K-1,1] & \cdots & x[K-1,L-1] \end{bmatrix}$$

Representace vzorků

- Běžně kvantovány
- 8/16 bitů
- 0…255 atd
- Pro výpočty lépe [0…1]
- A jako **floats**

Operace nad pixely – jas plus

$$y[k,l] = x[k,l] + const.$$

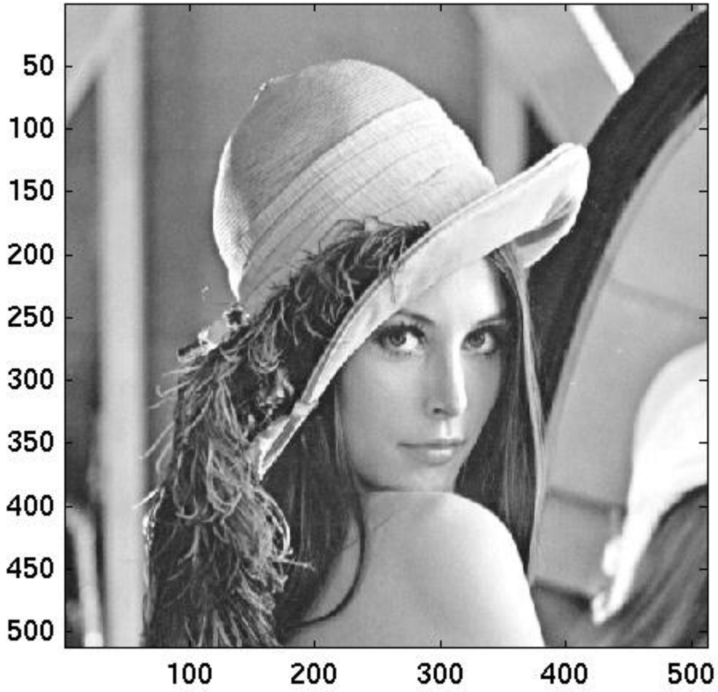


Operace nad pixely – jas minus

$$y[k,l] = x[k,l] + const.$$

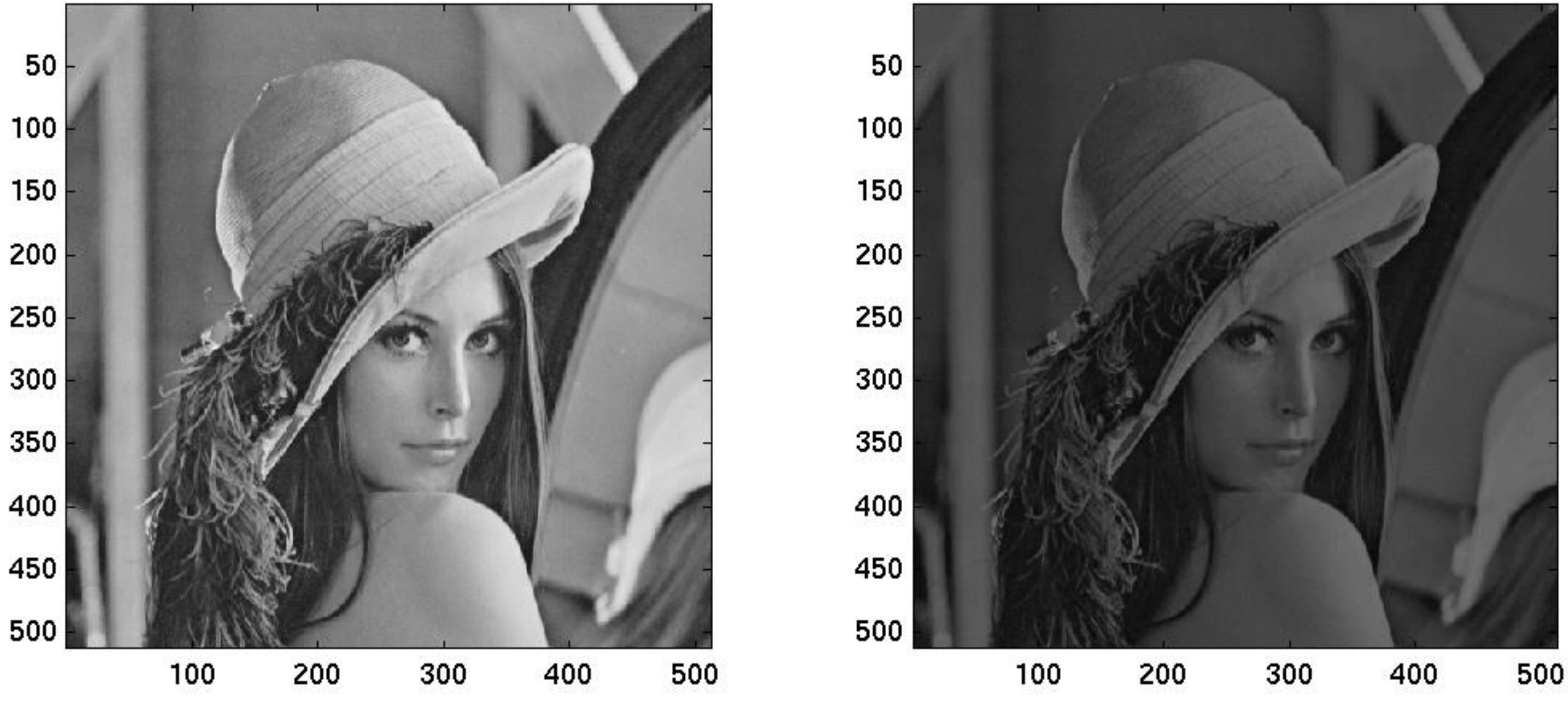
Kontrast plus

$$y[k,l] = x[k,l] \times const.$$



Kontrast minus

$$y[k,l] = x[k,l] \times const.$$



Co se špatnými hodnotami ?

x[k,l] < 0 nebo *x[k,l] > 1*

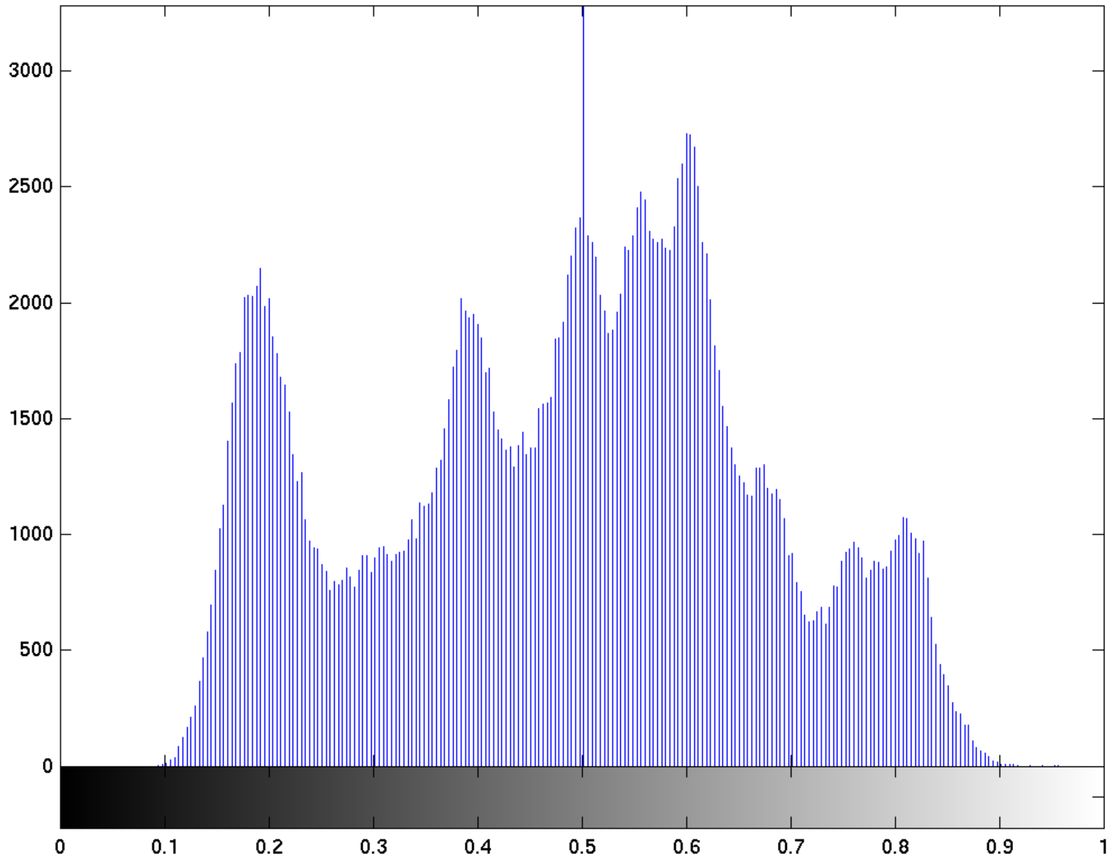
- Nechat …
- Klipovat:

$$y[k,l] = 0, \quad \text{pokud} \;\; y[k,l] < 0$$

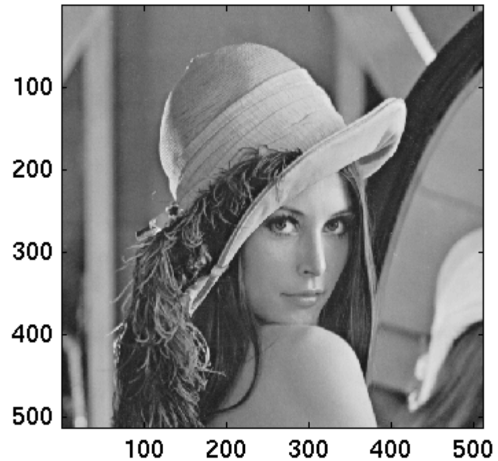
$$y[k,l] = 1, \quad \text{pokud} \;\; y[k,l] > 1$$

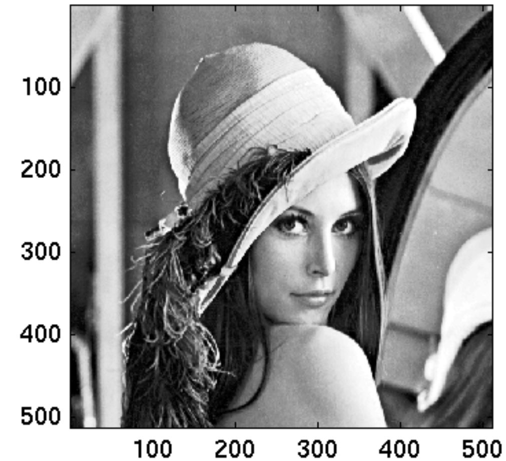
Využití statistik hodnot

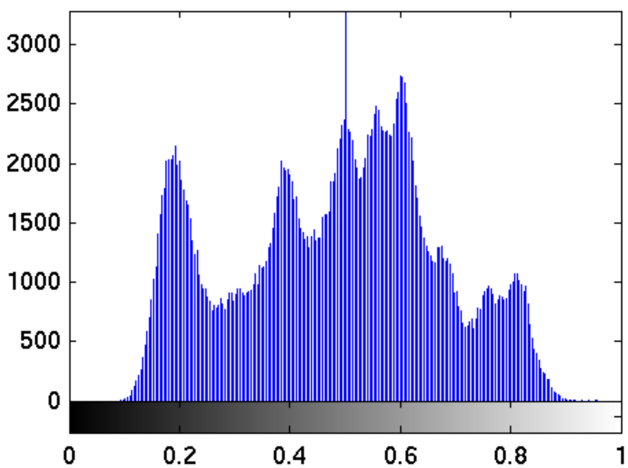
- Histogram

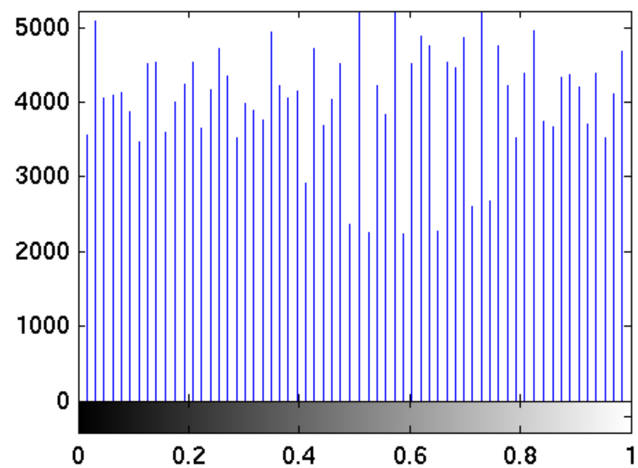


Ekvalizace histogramu

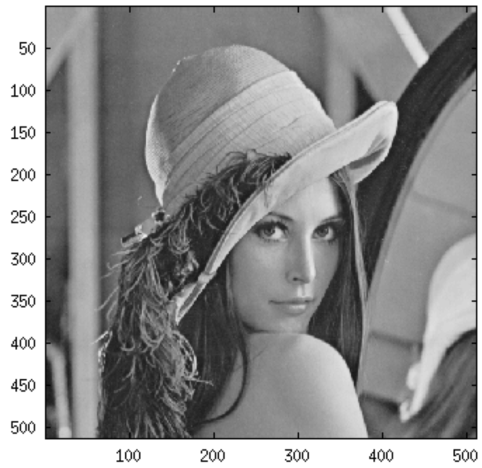








Prahování



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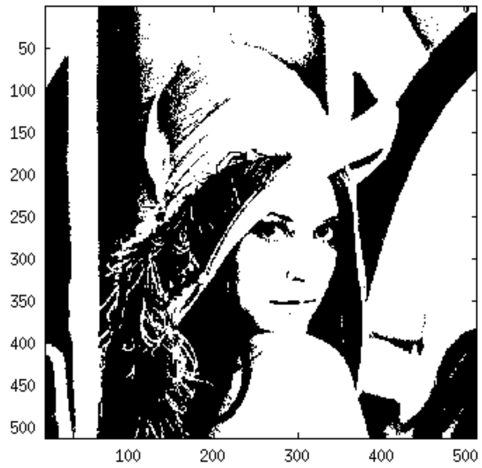

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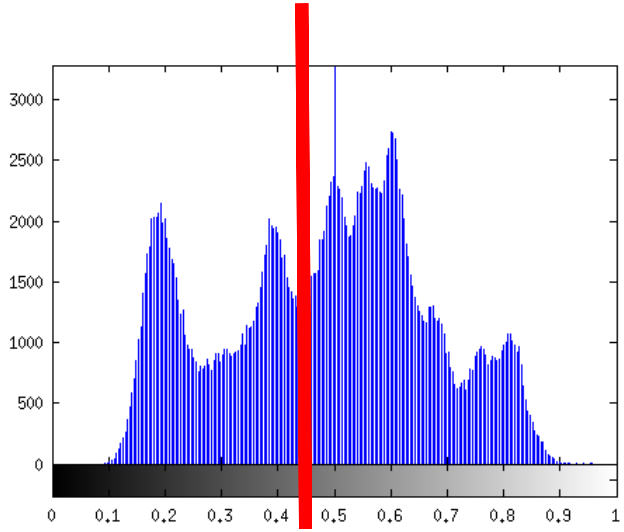

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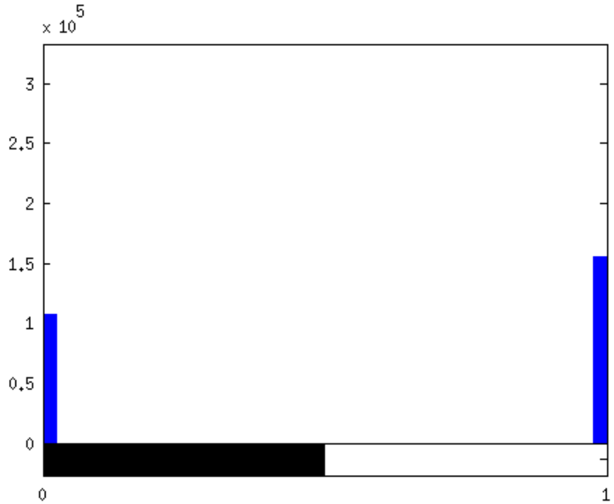
0.8
0.9
1



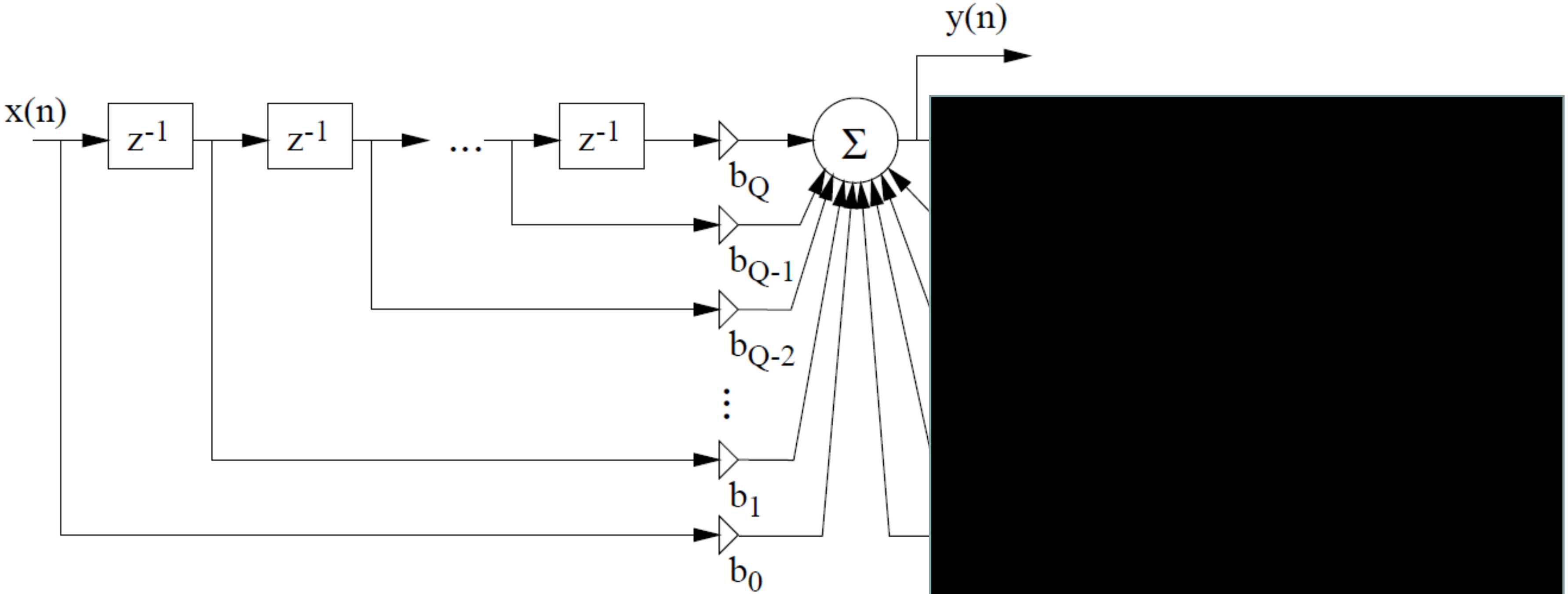
x 10^5
3
2.5
2
1.5
1
0.5
0
0
1

Filtrace

Úkol ?

Získat **nový signál** s požadovanými vlastnostmi.

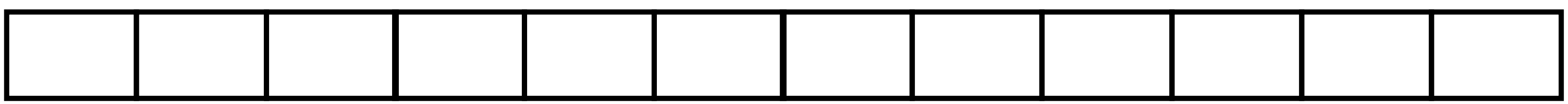
Filtrace – opakování 1D

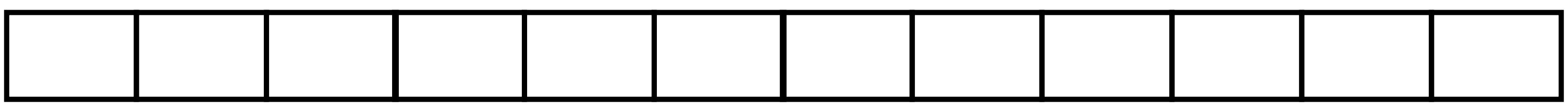


[redacted]

Filtrace – opakování 1D

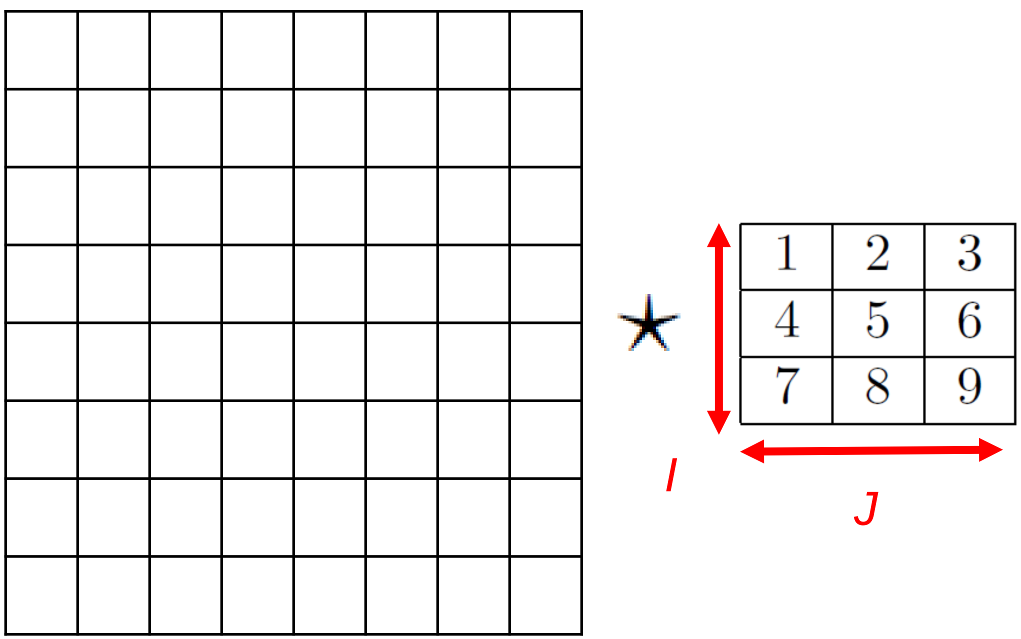
$$y[n] = x[n] \star h[n] = \sum_{k=0}^{Q} h[k]x[n-k]$$



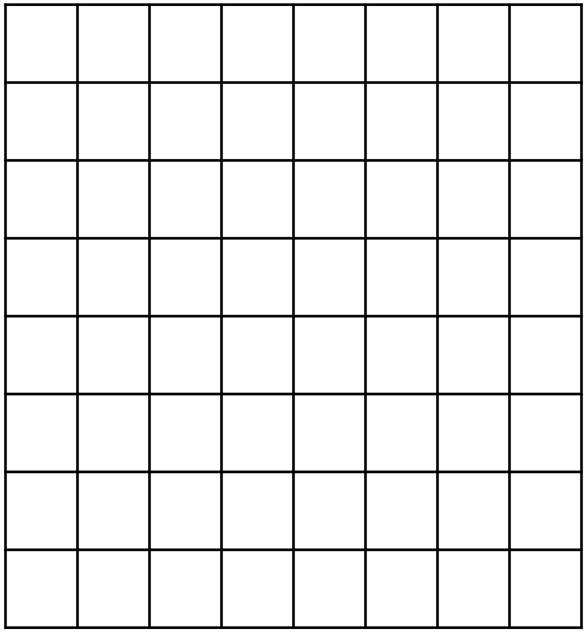


2D filtr

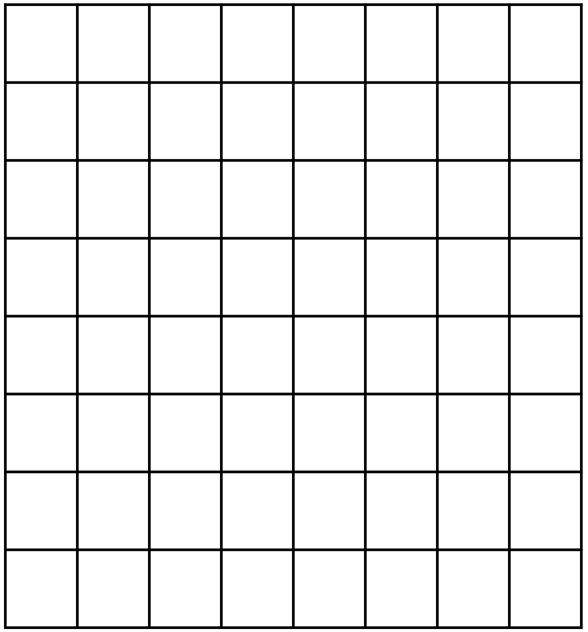
$$y[k,l] = x[k,l] \star h[k,l] = \sum_{m=-\frac{I-1}{2}}^{\frac{I-1}{2}} \sum_{n=-\frac{J-1}{2}}^{\frac{J-1}{2}} h[m,n]x[k-m,l-n]$$



Filtrace …



9	8	7
6	5	4
3	2	1



Okraj ? Třeba nuly …

Koeficienty *h[k,l]*

Říkáme jim

- Koeficienty
- Maska
- Konvoluční jádro …

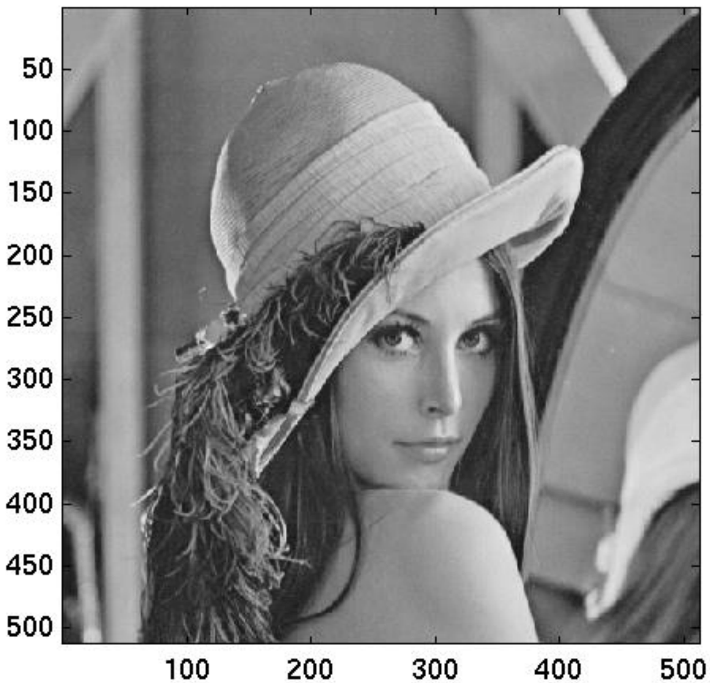
Chceme po nich

- Aby měly nějaký smysl …
- Aby neměnily dynamiku signálu

$$\sum_k \sum_l |h[k,l]| = 1$$

Drát

$$h[k,l] = [1]$$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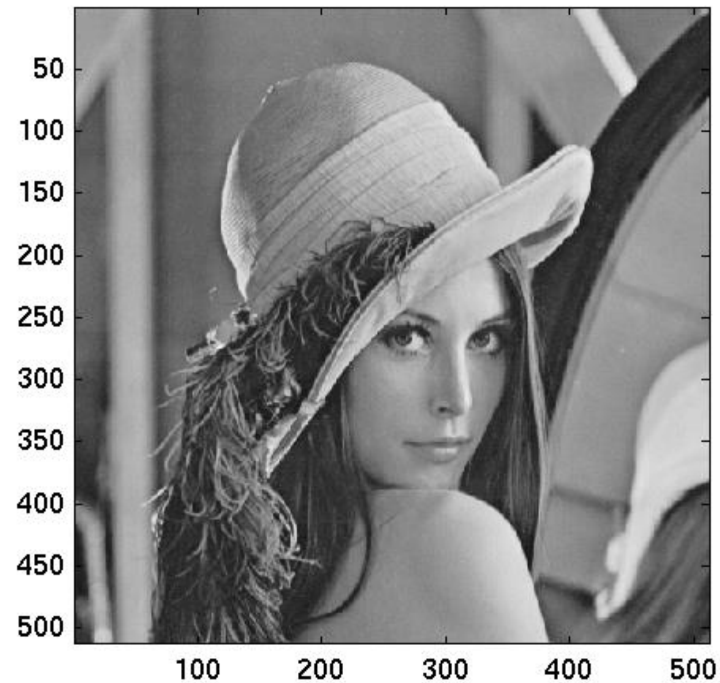
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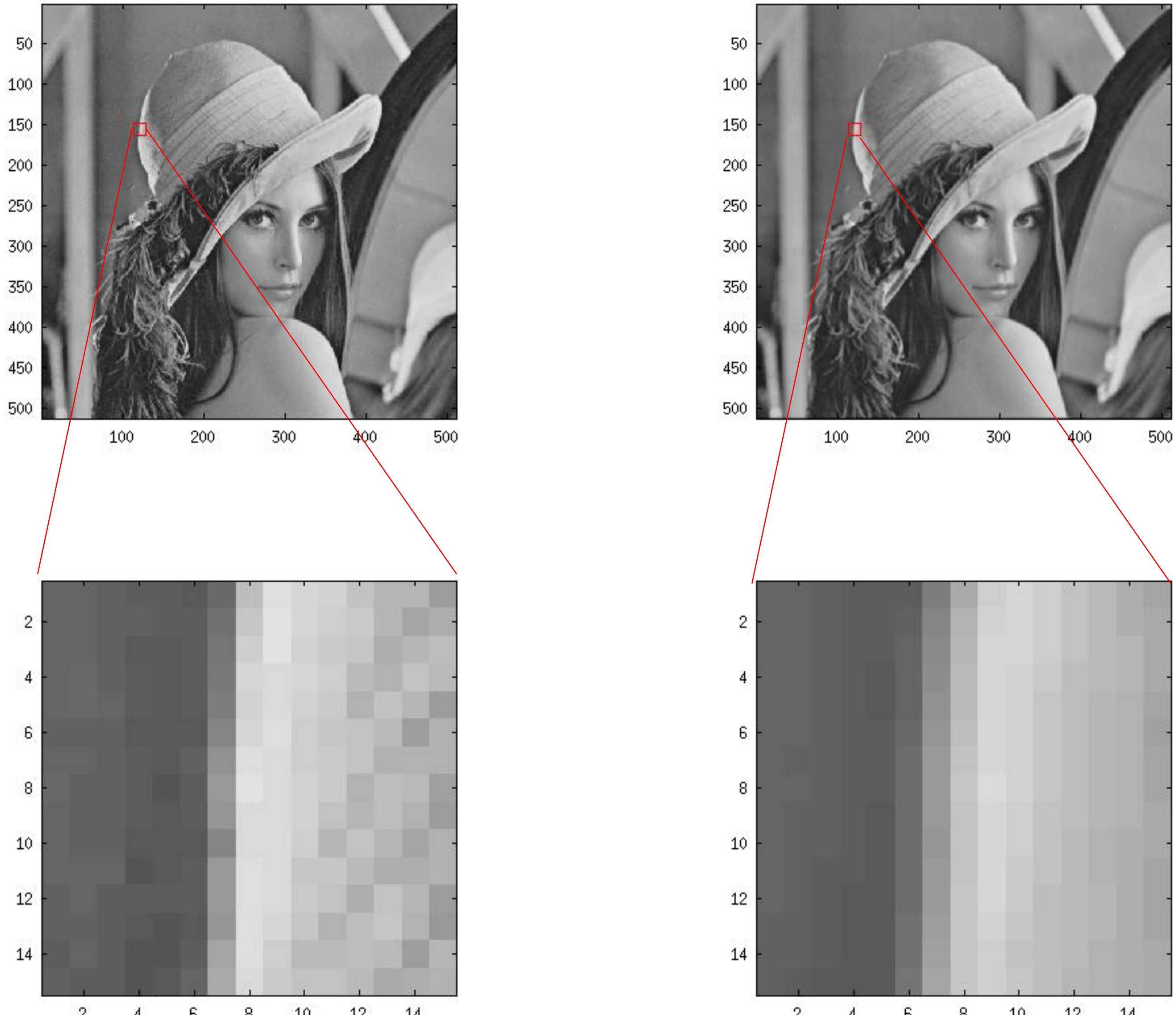
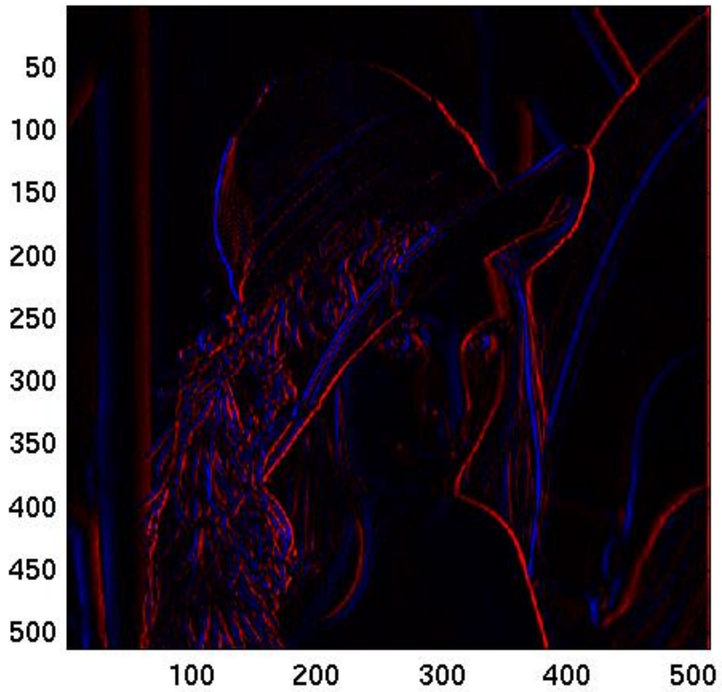
Vyhlazování

$$h[k,l] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



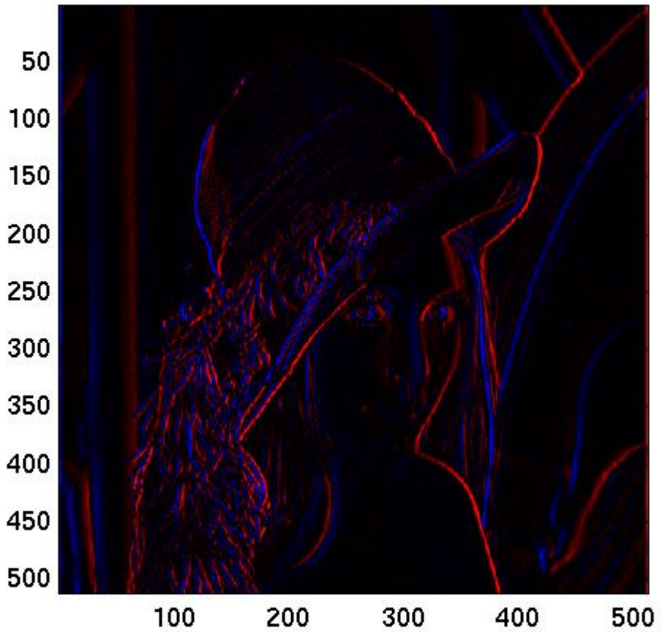
Detektor vertikálních hran

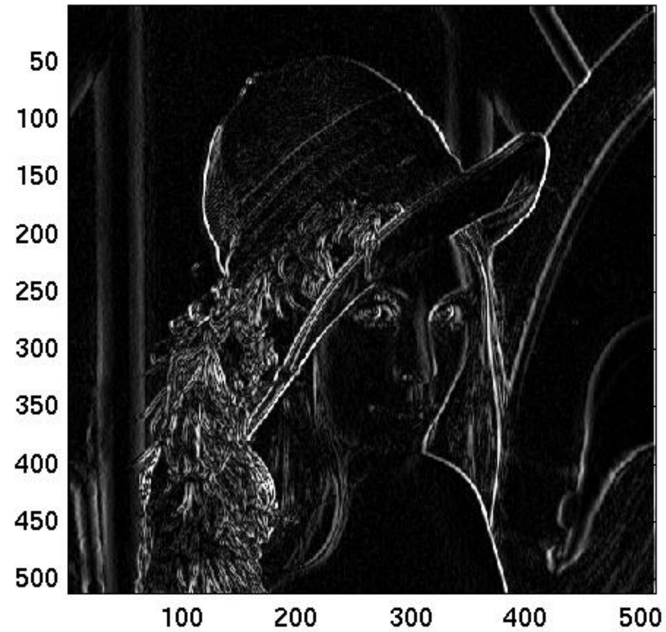
$$h_v[k,l] = \frac{1}{8}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$



Co se zápornými vzorky ?

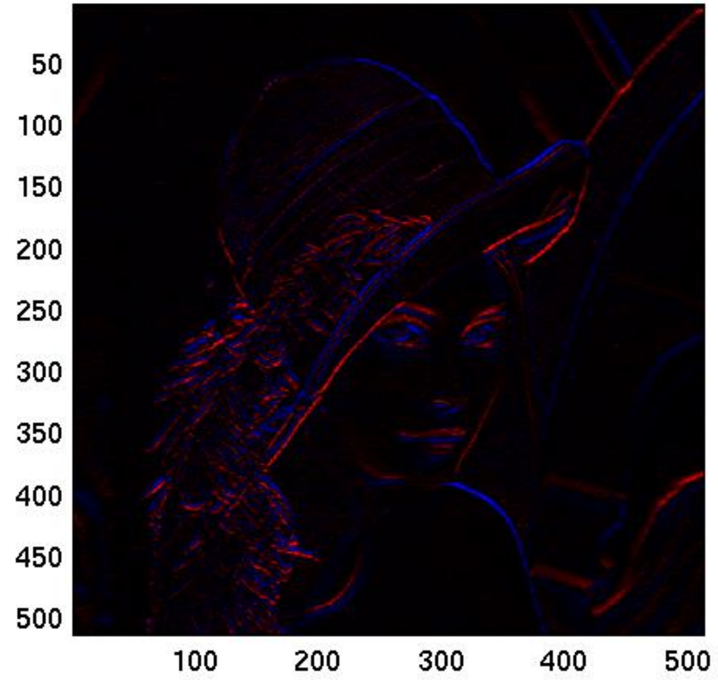
- Můžeme nechat tak pro další výpočty …
- Nebo vymyslet, jak je převést do [0…1] pro zobrazování – např. absolutní hodnota





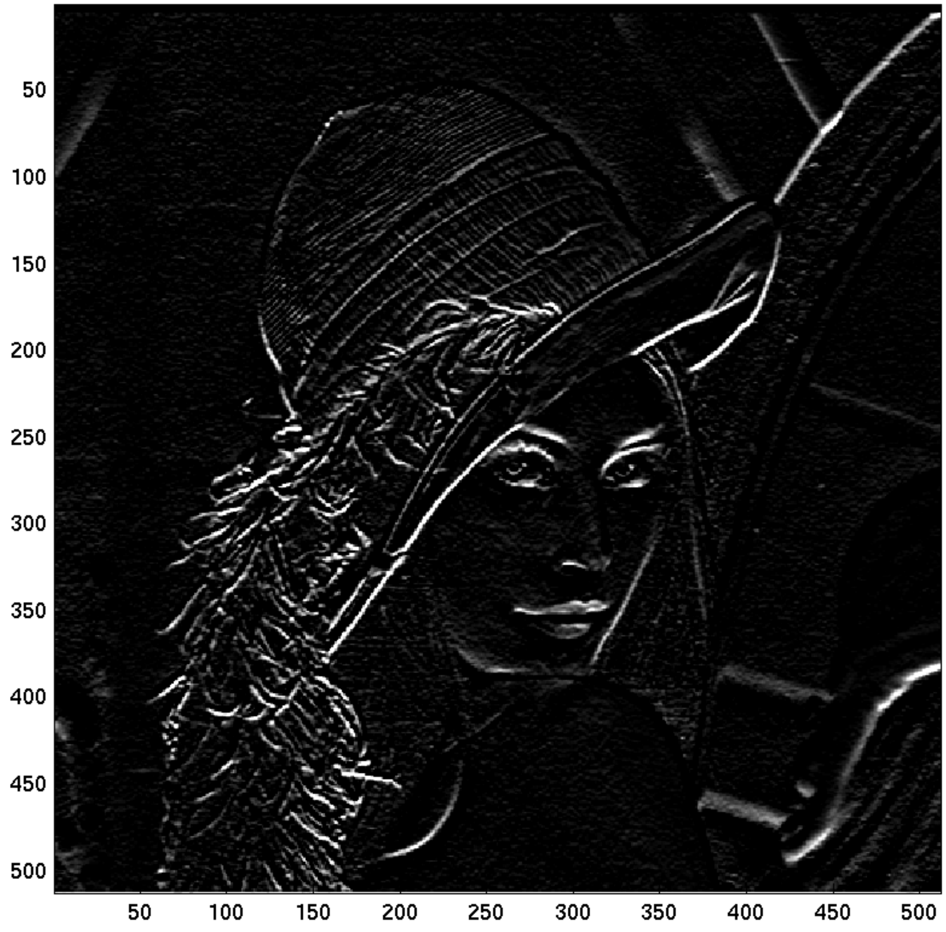
Detektor horizontálních hran

$$h_h[k,l] = \frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$



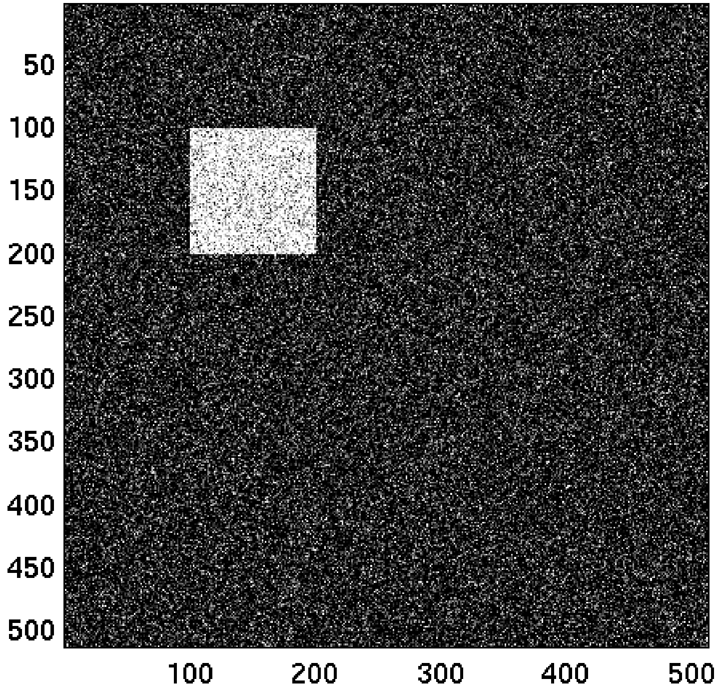
Obojí dohromady …

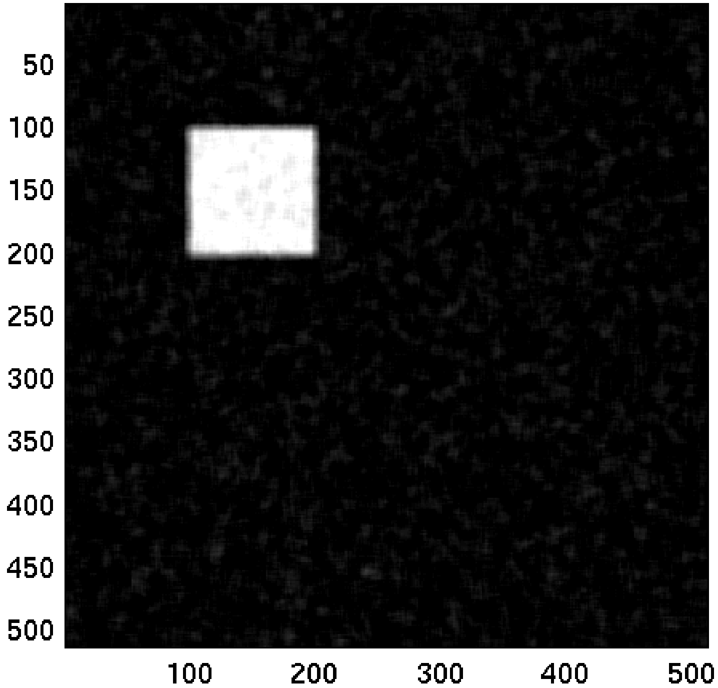
$$y[k,l] = |y_v[k,l]| + |y_h[k,l]|$$



Odšumování …

- Maska 9x9, hodnoty 1/81 … **zoom noise_low_pass.png**

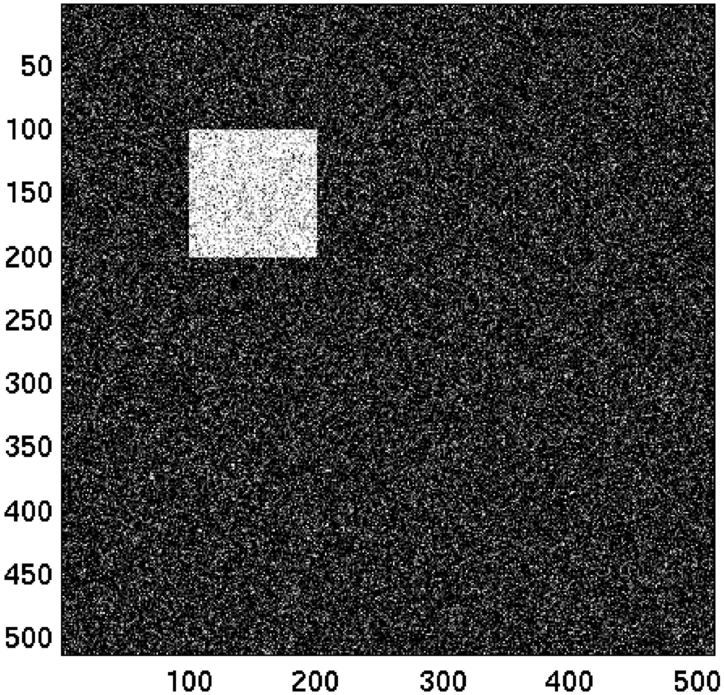


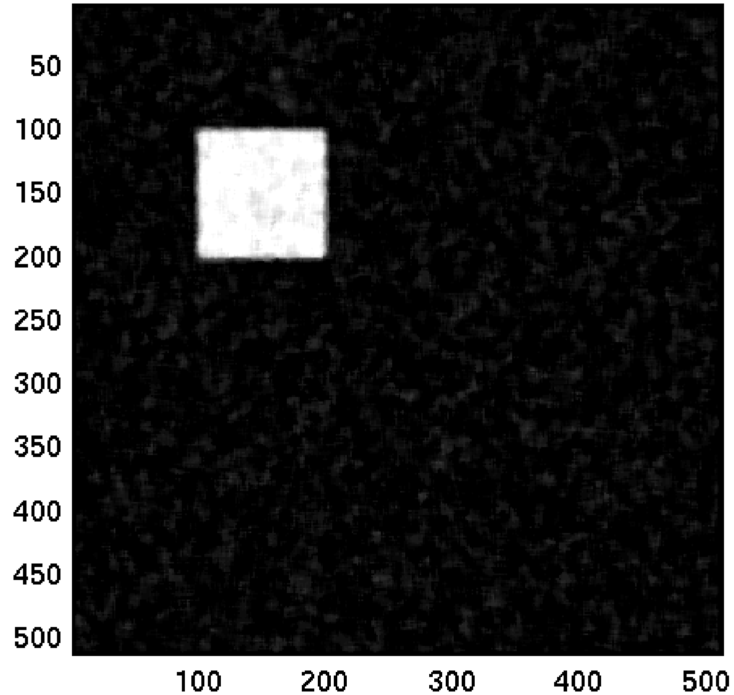


Odšumování II – mediánový filtr

$$y[k,l] = \text{median}_{k=-\frac{I-1}{2}\dots\frac{I-1}{2},\quad l=-\frac{J-1}{2}\dots\frac{J-1}{2}} x[k,l]$$

- **zoom noise_median.png**





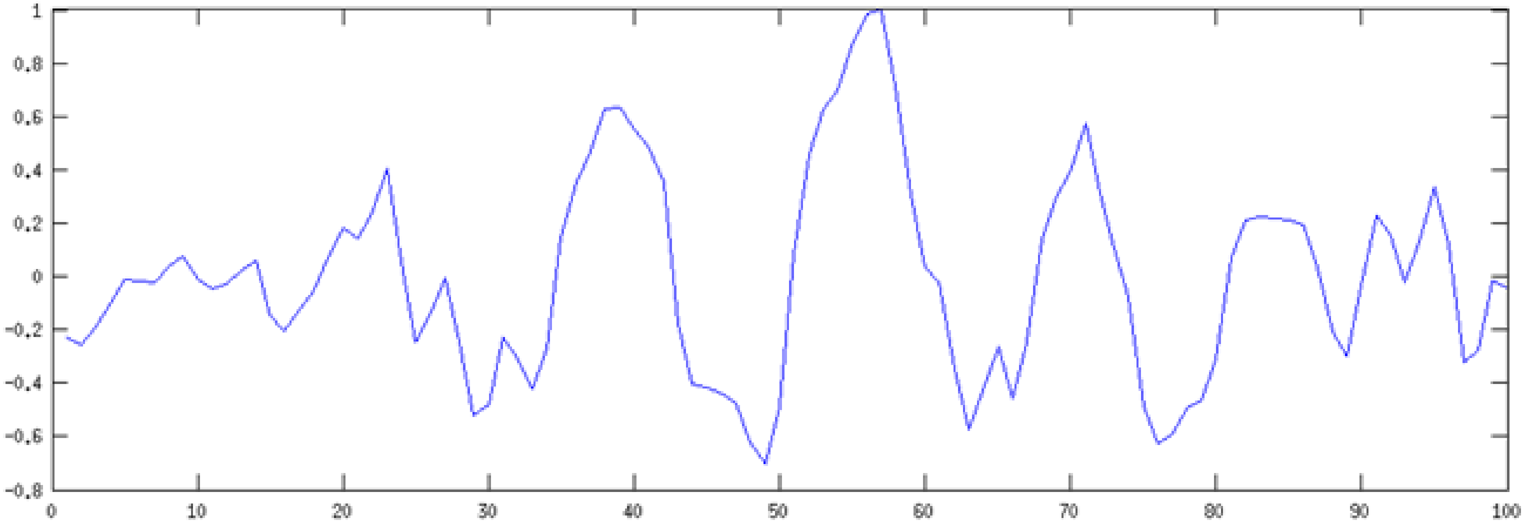
Spektrální analýza

- Úkol:

  Zjistit, co je v signálu na jakých frekvencích

- Proč ?
  - Vizualisace,
  - Výpočet parametrů (think of Facebook)
  - Filtrace (převod do spektrální oblasti a násobení tam může být efektivnější)
  - Kódování (think of JPEG)

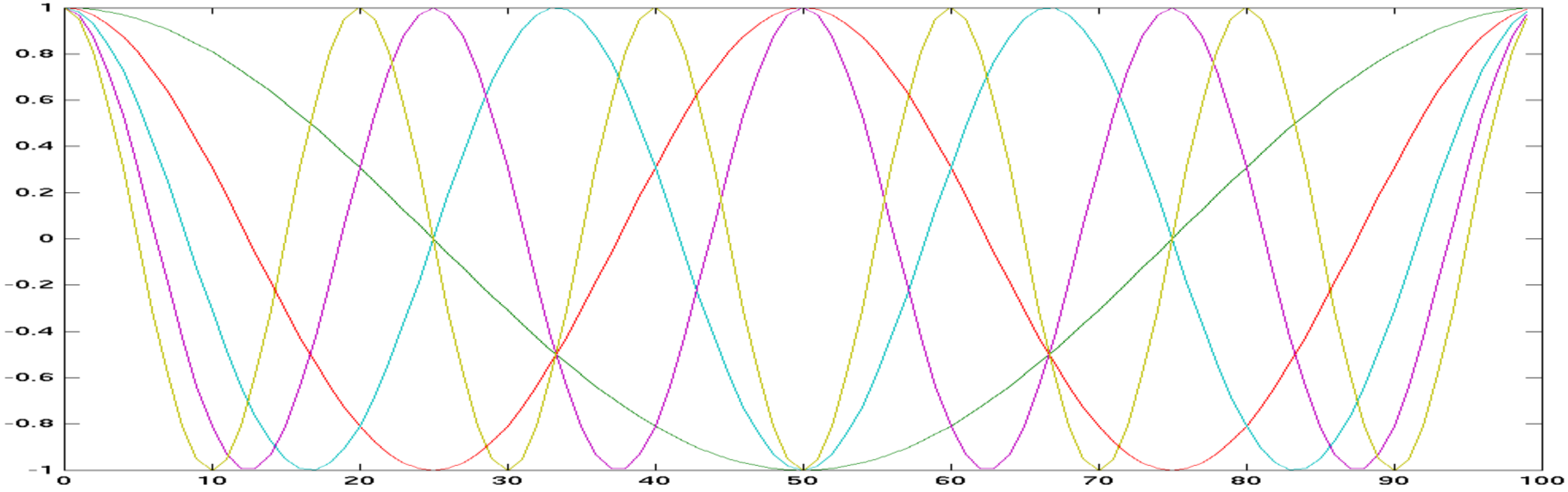
Opakování – 1D



- Co je v něm ?

$$c = \sum_{n=0}^{N-1} x[n]a[n]$$

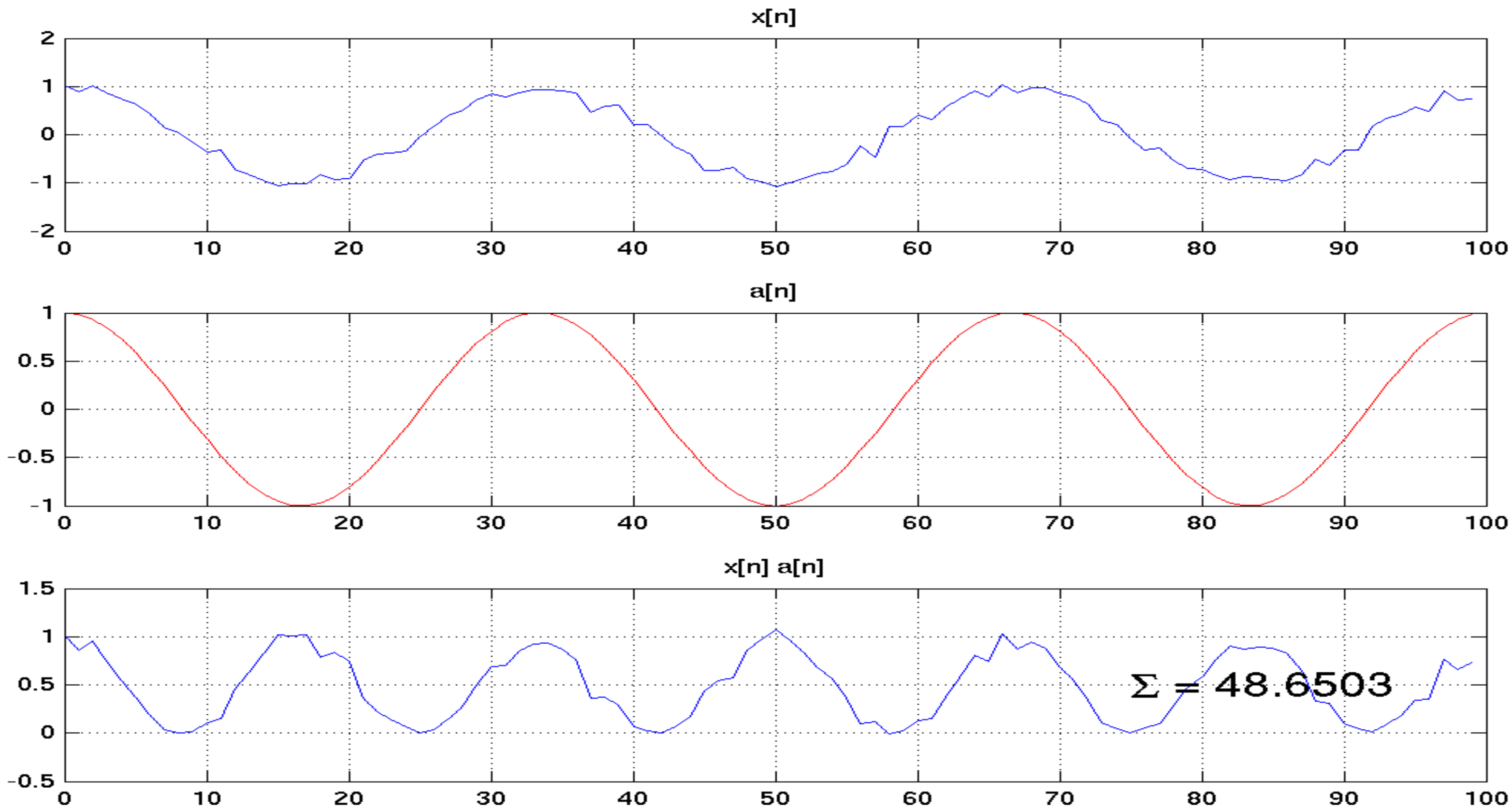
Opakování – 1D cosinové báze



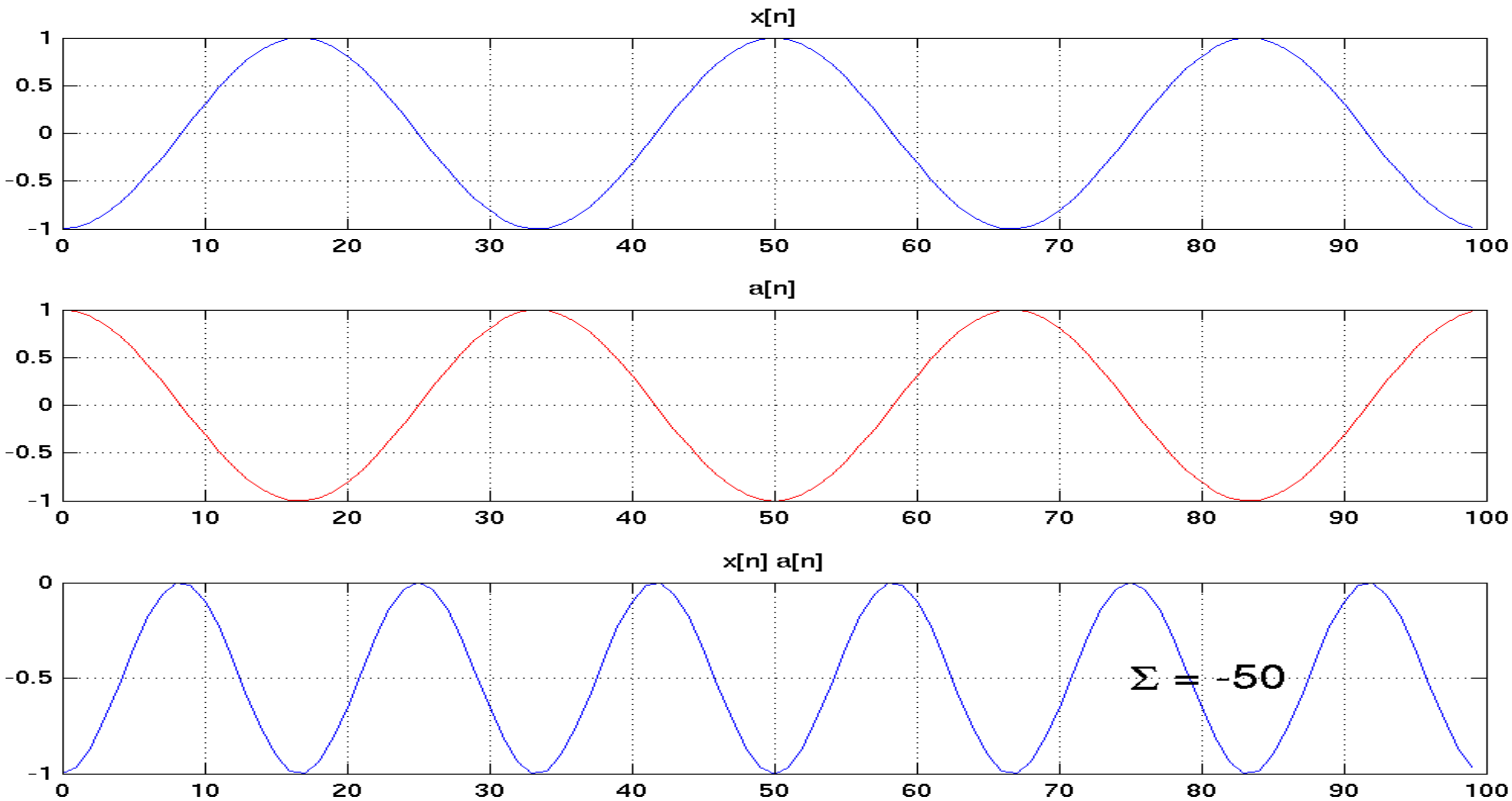
$$\cos(2\pi \frac{k}{N} n)$$

Problém s fází – „vlna“ signálu může začínat jinde než v nule …

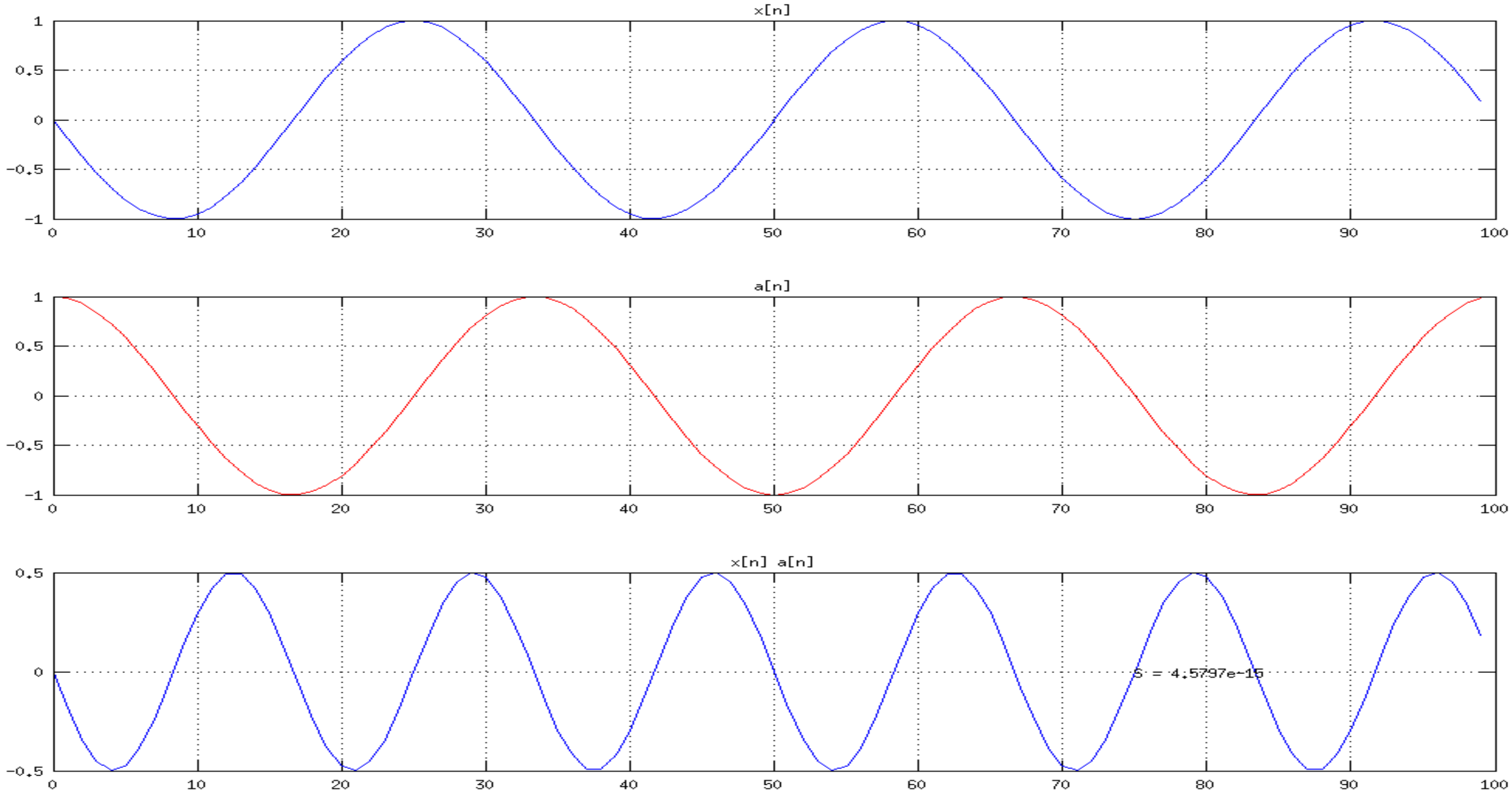
Tady dobře …



Tady také dobře …

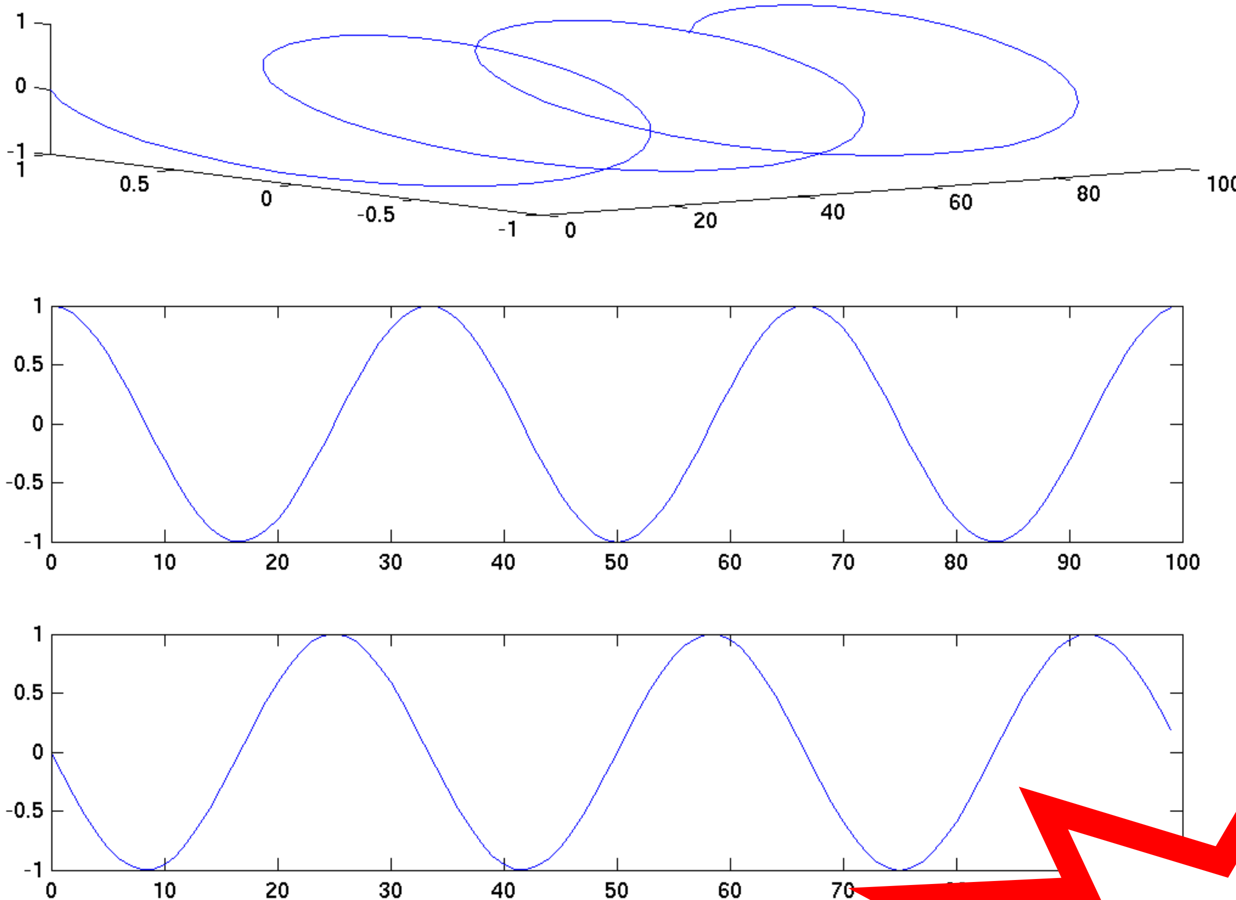


Aj aj ☹



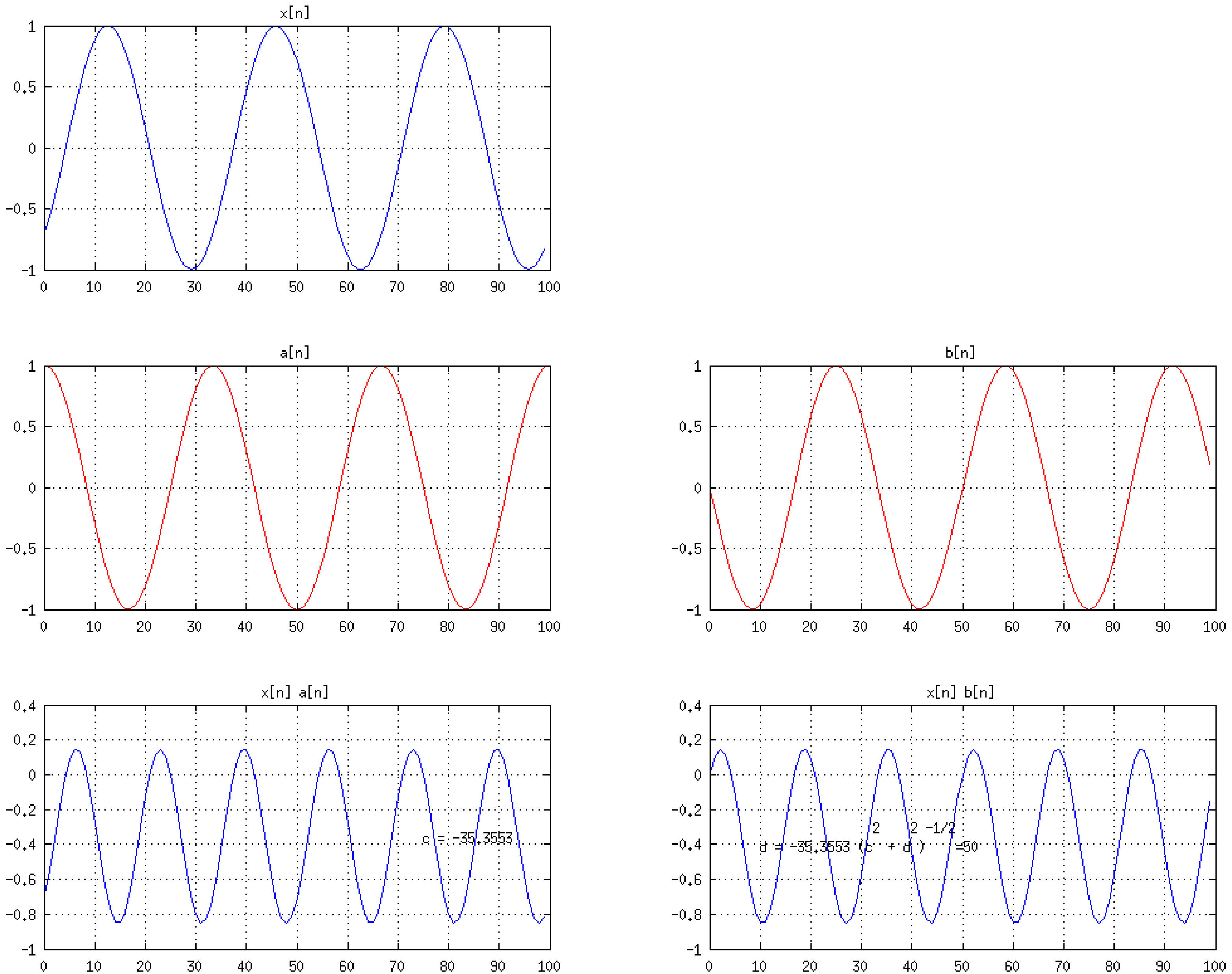
- Jak to že nula, když $\sin(x) = \cos(x - \frac{\pi}{2})$

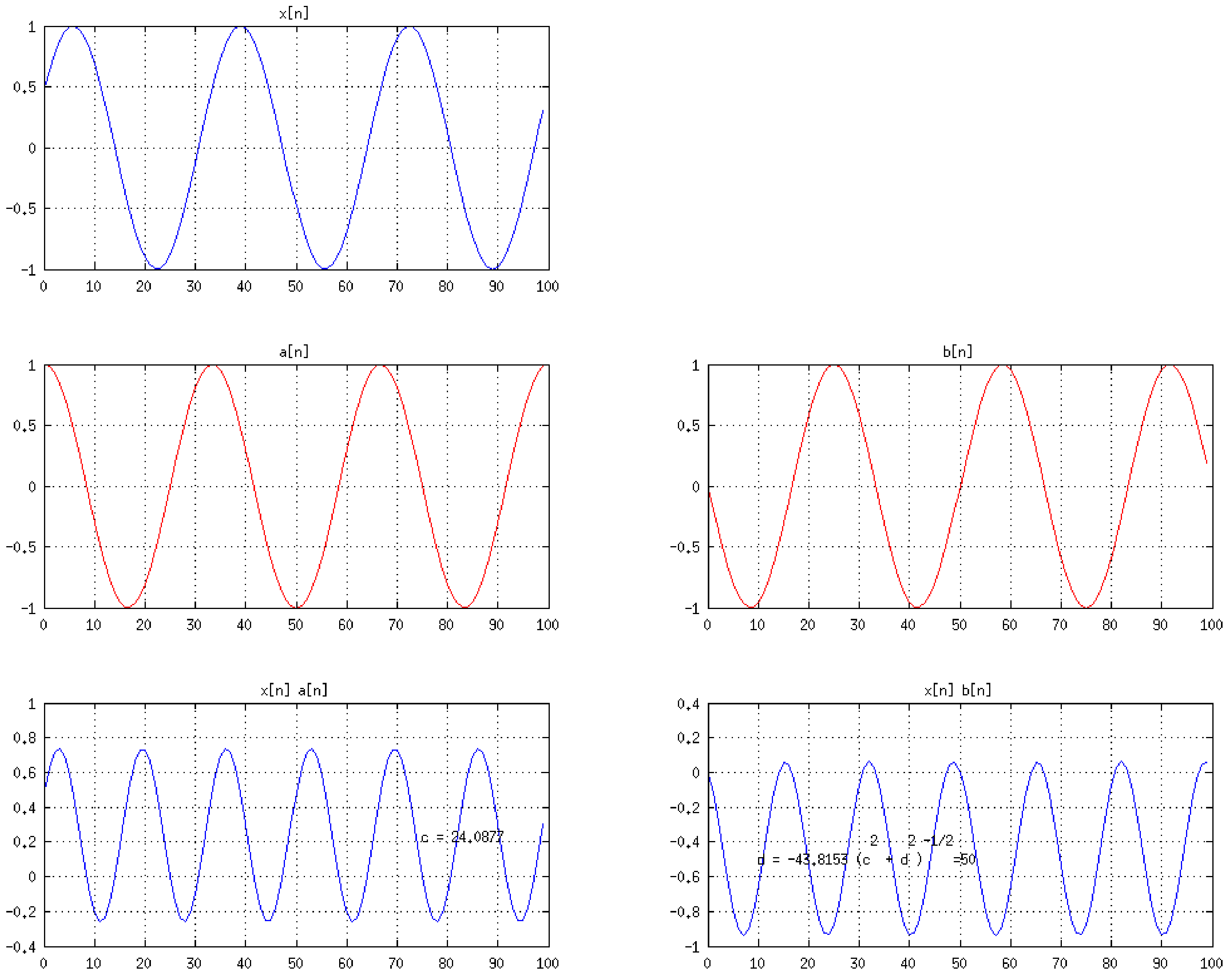
Komplexní exponenciály …



$$e^{jx} = \cos(x) + j\sin(x)$$

$$e^{-j2\pi\frac{k}{N}n} = \cos(2\pi\frac{k}{N}n) - j\sin(2\pi\frac{k}{N}n)$$


x[n]
a[n]
b[n]
x[n] a[n]
c = -35.3553
x[n] b[n]
d = -35.3553 (c^2 + d^2)^-1/2 =50


x[n]
a[n]
b[n]
x[n] a[n]
x[n] b[n]
c = 24.0877
2 2 -1/2
d = -43.8153 (c + d) =50

1D ultimate result - DFT

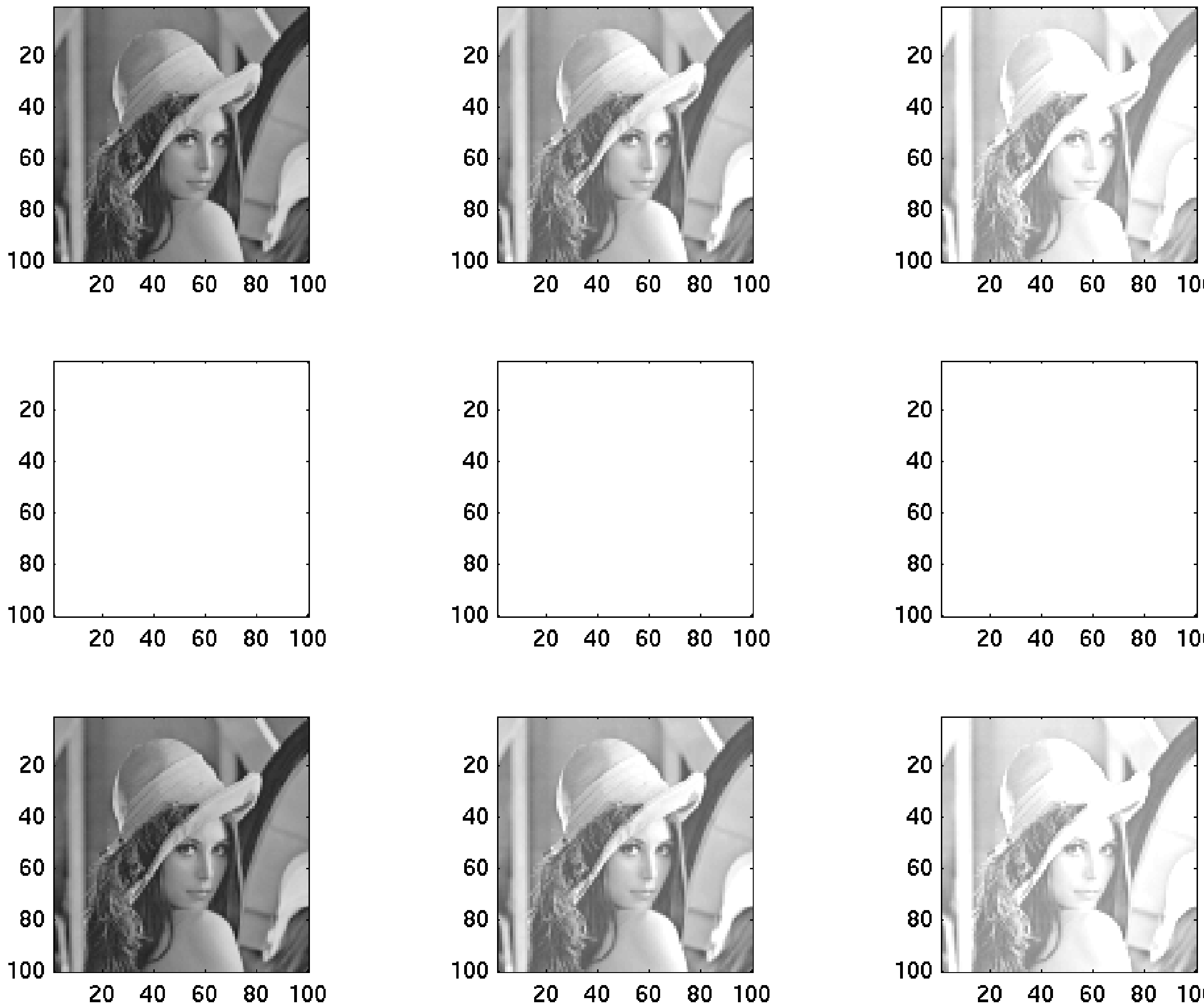
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{k}{N} n}, \quad k = 0 \ldots N-1$$

Teď konečně 2D

- Korelace = určování podobnosti = projekce do bází …
- **To už jsme jednou viděli, je to furt to samý … ANO, JE !**

$$c = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} x[k,l] a[k,l]$$

Analyzační signál = d.c. $a[k,l] = 1$

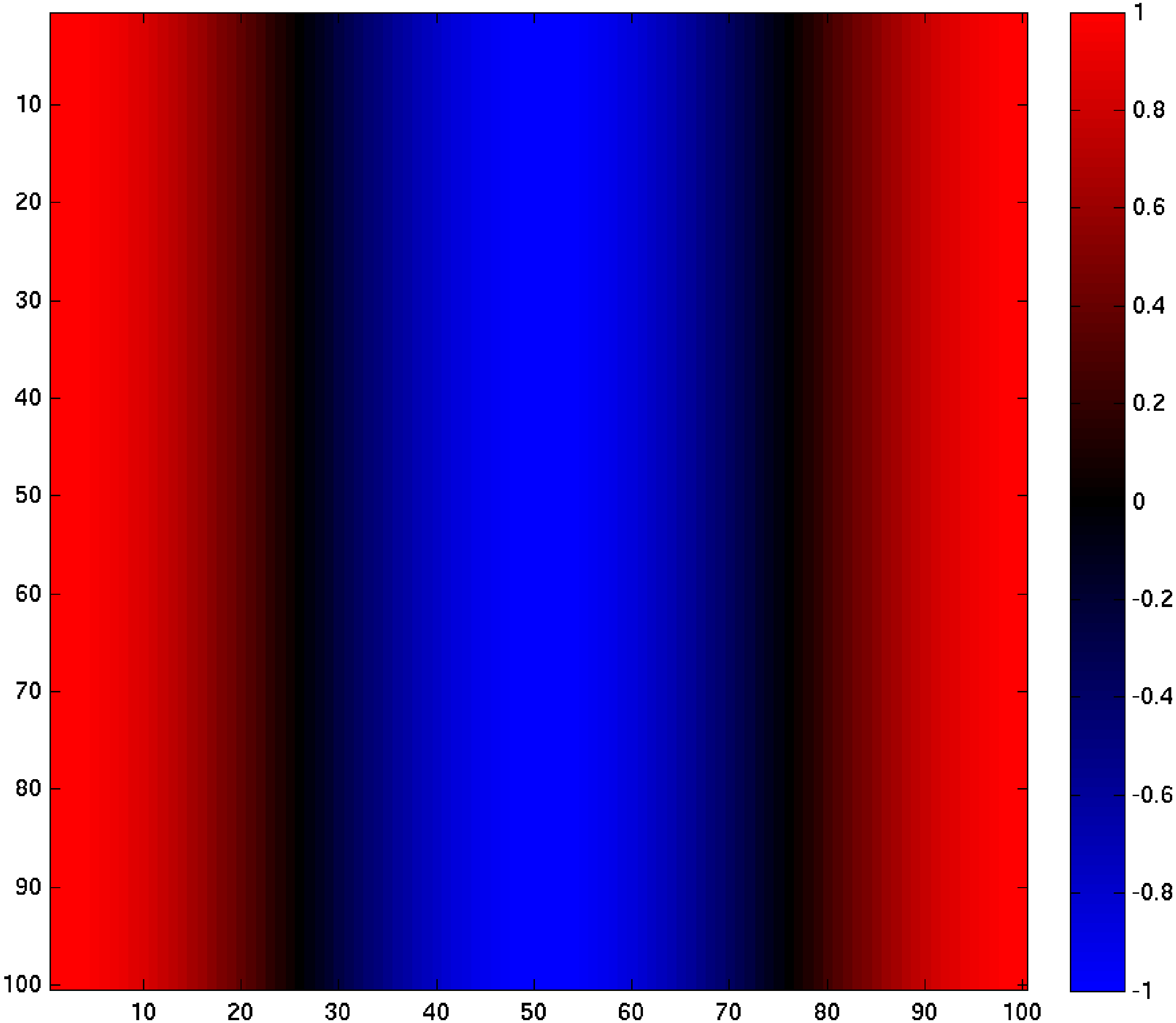


c=4.8287e+03 c=6.8287e+03 c=8.8287e+03

Analyzační signál – horizontální cosinusovka

$$a[k,l] = \cos(2\pi \frac{1}{100} l)$$

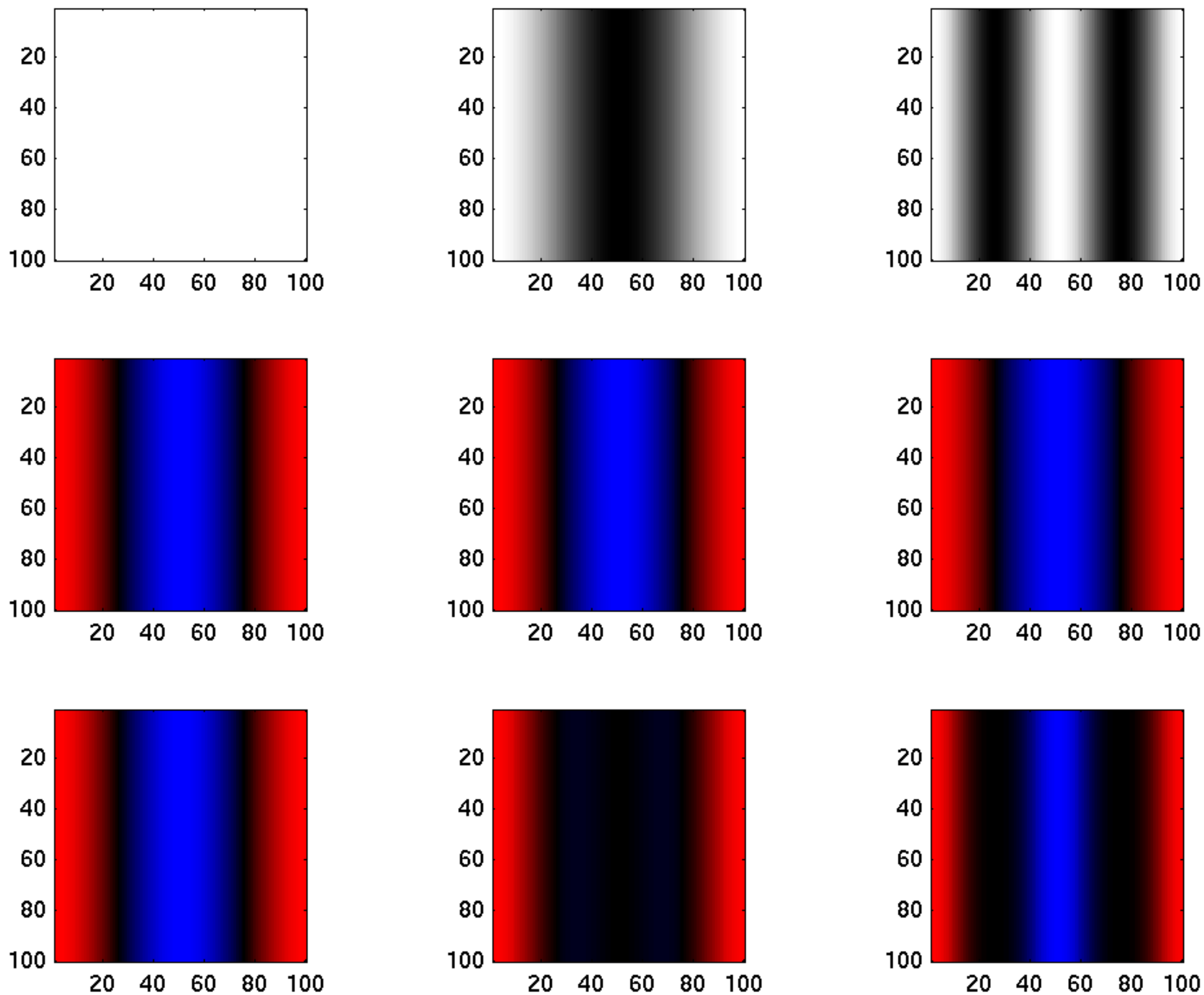
- Mění se pouze v jednom směru.
- Budeme potřebovat zobrazovat i záporné hodnoty



Geeks: jak se toto dělá v Matlabu ??

Analýza pro

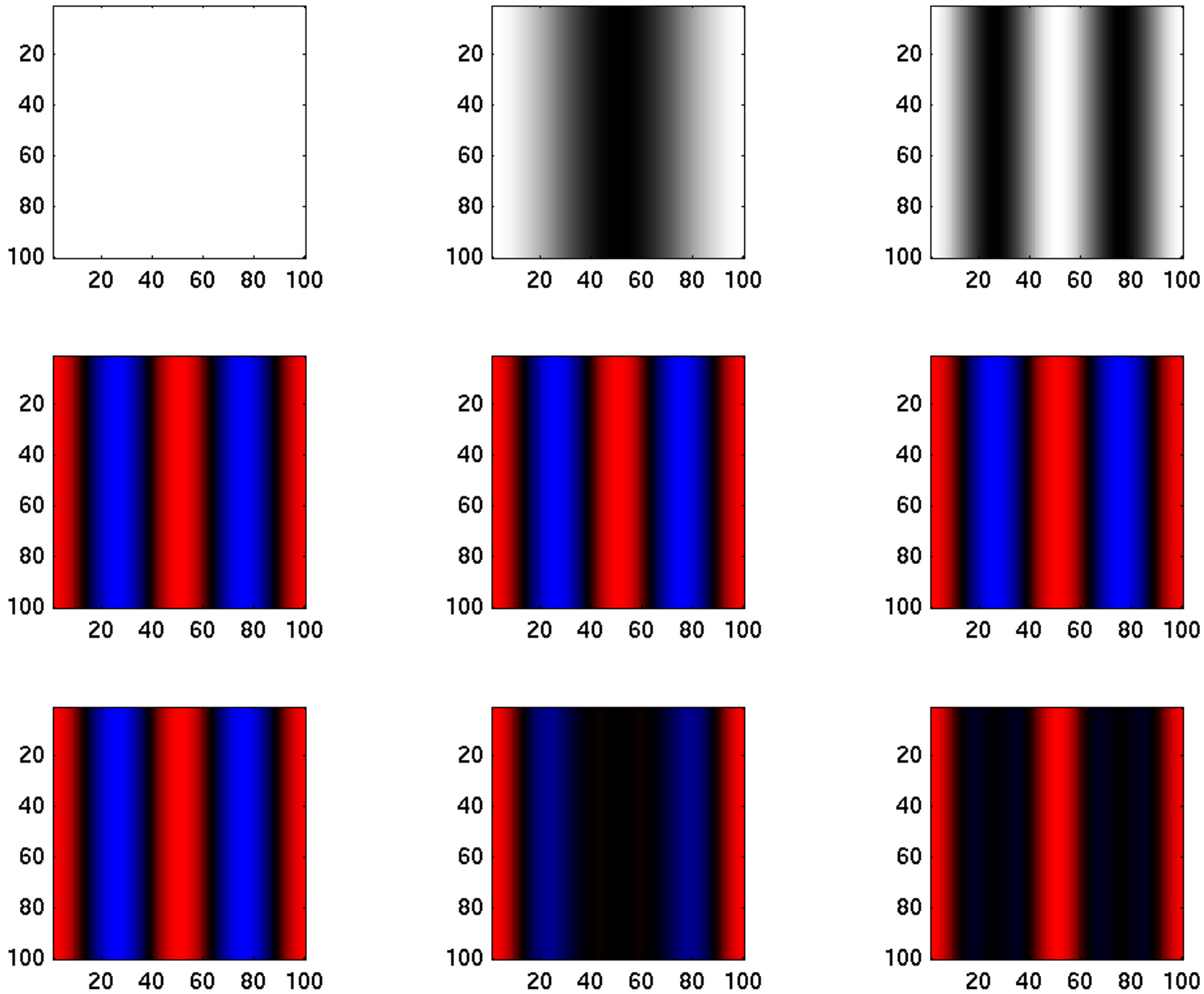
$$a[k,l] = \cos(2\pi \frac{1}{100} l)$$



c=4.5475e-13 c=2500 c=3.4106e-13

2 x rychlejší horizontální cos

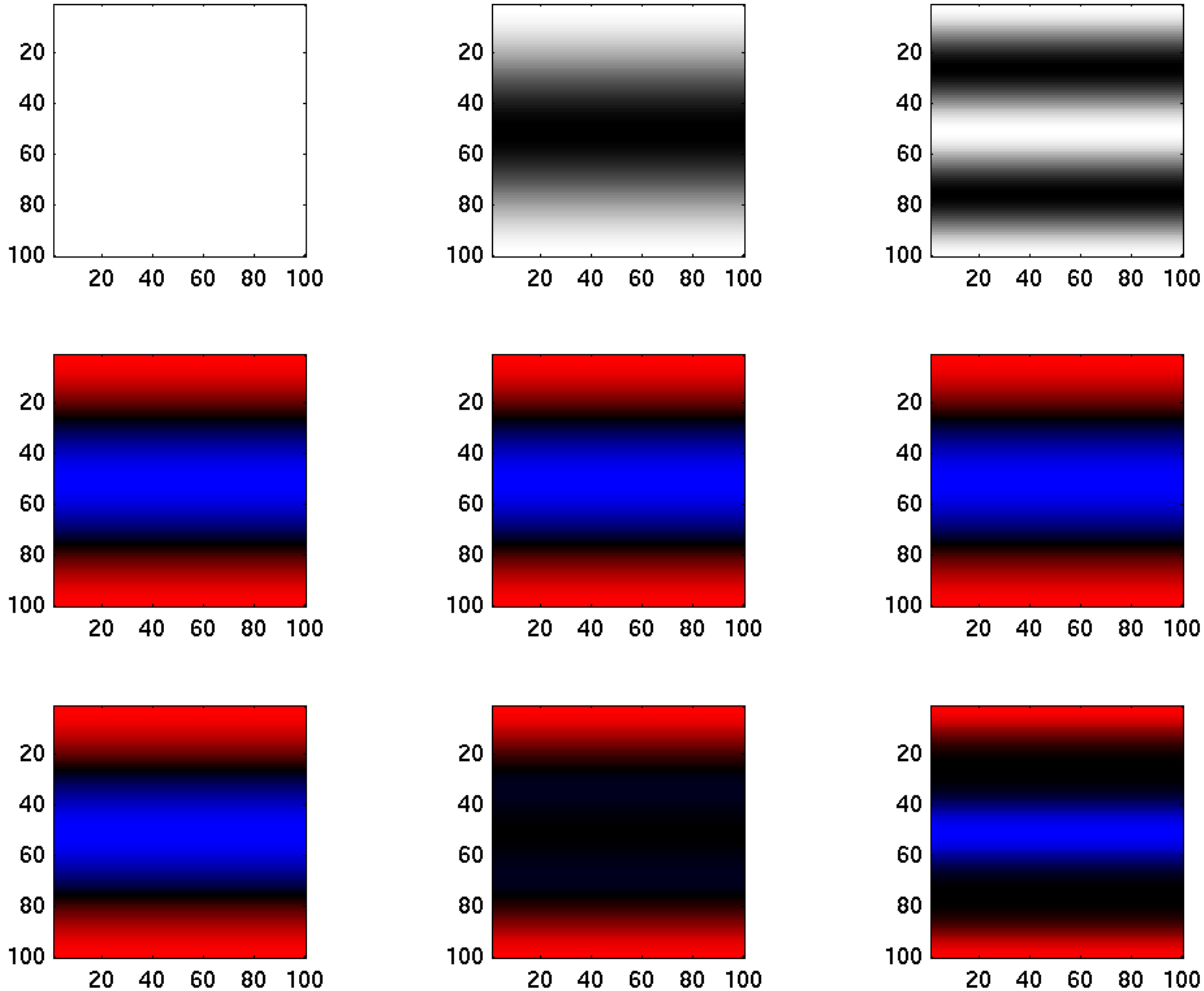
$$a[k,l] = \cos(2\pi \frac{2}{100} l)$$



c=2.9843e-13 c=1.5064e-12 c=2.5000e+03

Vertikální cos

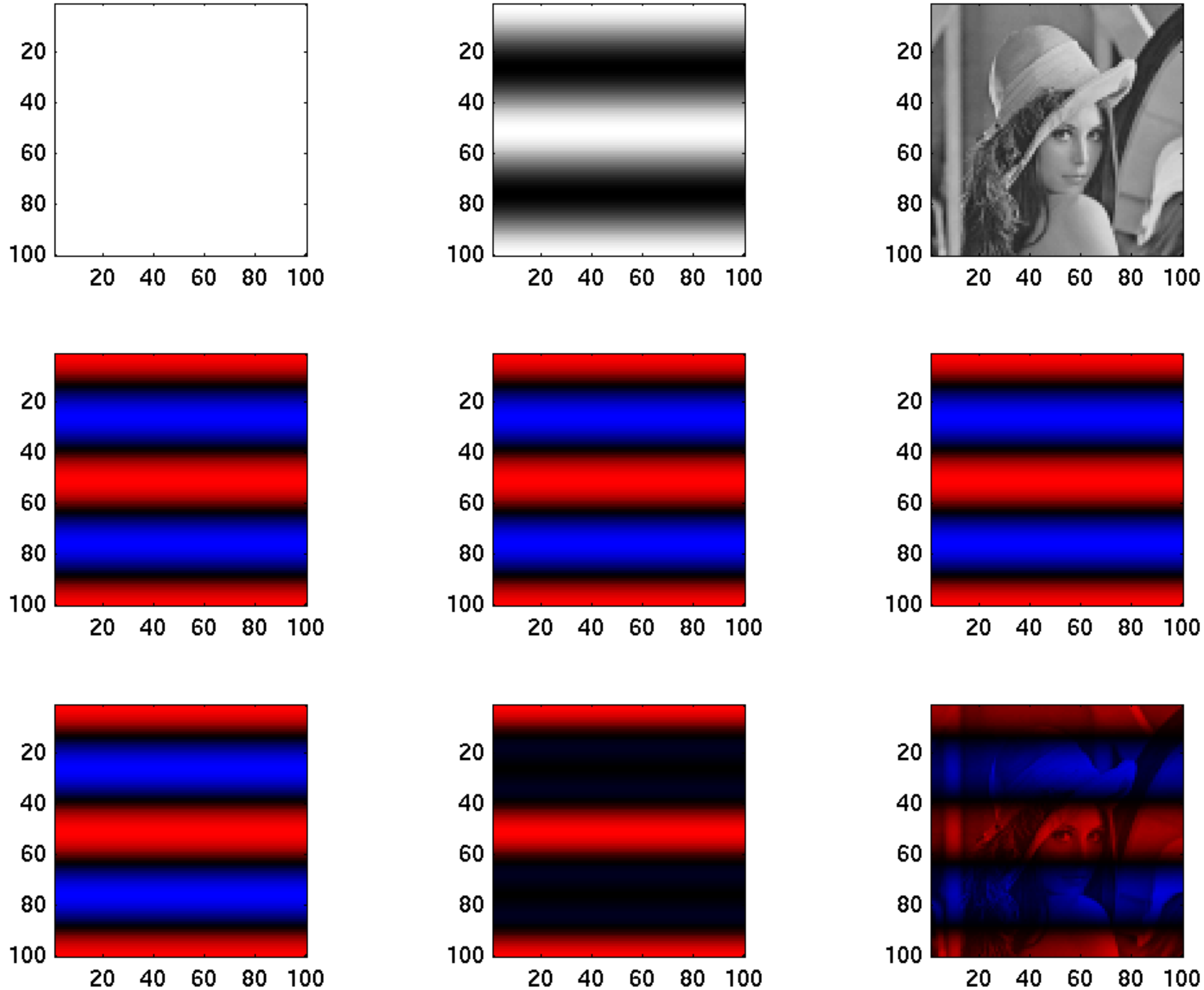
$$a[k,l] = \cos(2\pi \frac{1}{100} k)$$



c= -7.7716e-14 c=2.5000e+03 c=6.9944e-13

2 x rychlejší vertikální cos

$$a[k,l] = \cos(2\pi \frac{2}{100} k)$$

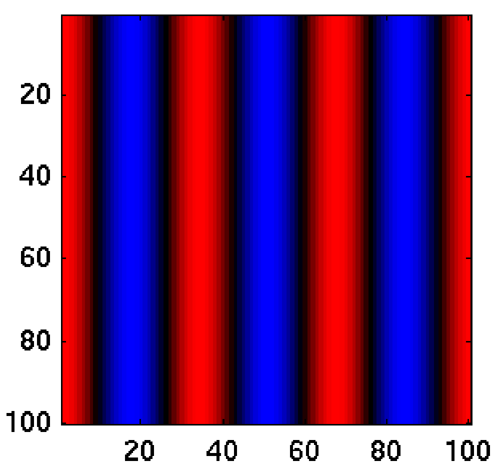


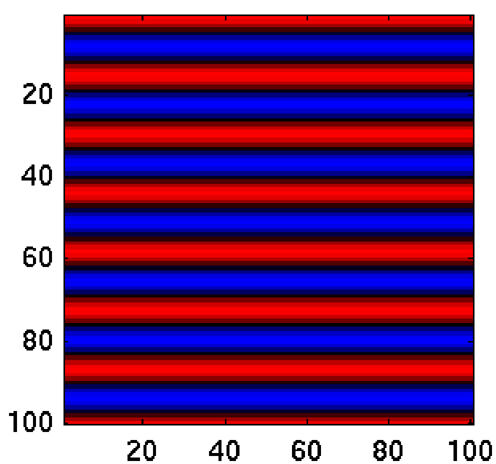
c= 4.9960e-13 c=2.5000e+03 c=-65.4694

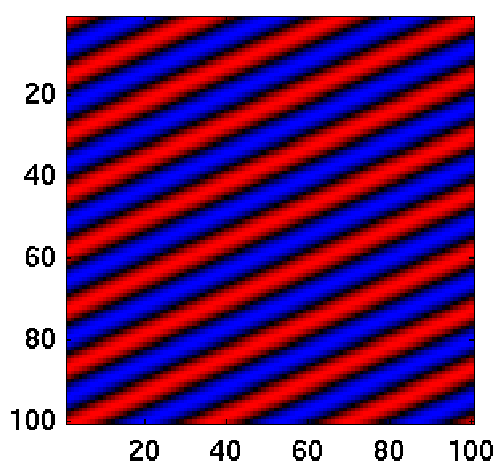
Mix obou směrů …

$$a[k,l] = \cos(2\pi \frac{3}{100} l) \qquad a[k,l] = \cos(2\pi \frac{7}{100} k)$$

$$a[k,l] = \cos\left[2\pi(\frac{7}{100}k + \frac{3}{100}l)\right]$$







Zobecnění

$$X[m,n] = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} x[k,l] \cos\left[2\pi(\frac{m}{K}k + \frac{n}{L}l)\right]$$

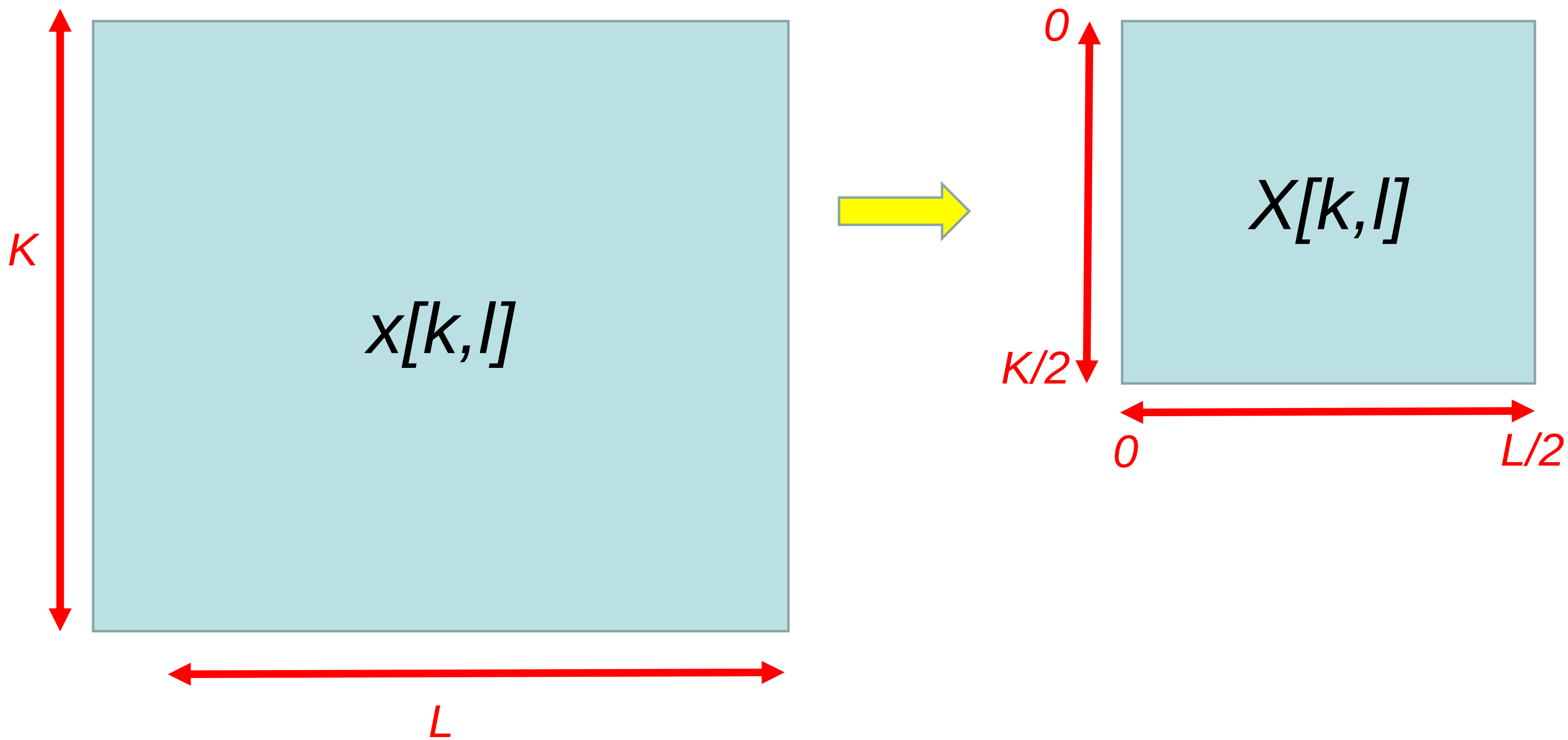
- *m/K* – svislá frekvence
- *n/L* – vodorovná frekvence

Rozumné rozsahy frekvencí ?

- *m/K* a *n/L*

  0 … ½ => OK

  > ½ => hm…



Ještě o frekvencích …

- 1D

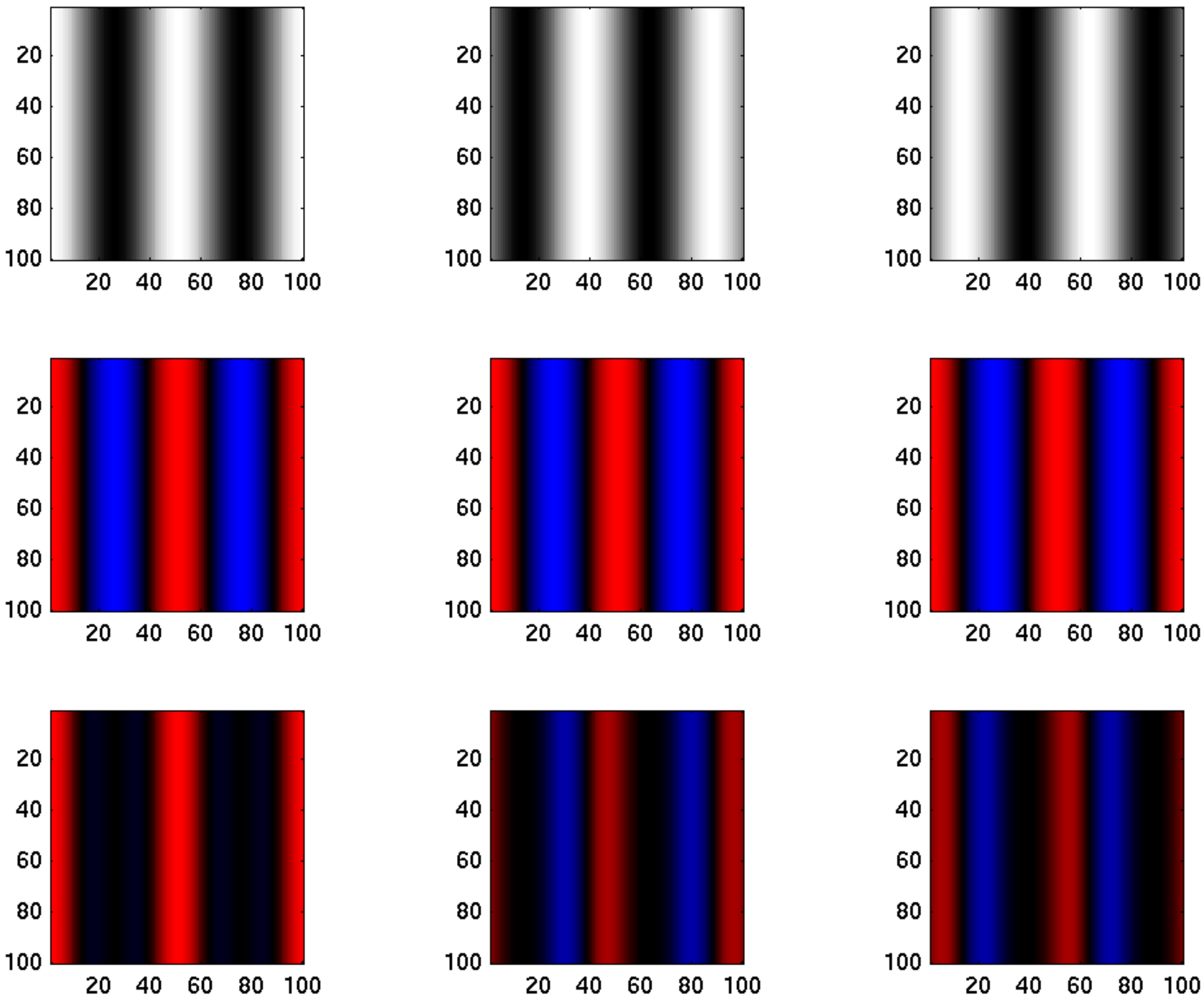
$$Hz = \frac{1}{s} \qquad F_s = \frac{\#samples}{s} \qquad f_{norm} = \frac{f_{skut}}{F_s} \qquad f_{skut} = \frac{k}{N} F_s$$

- 2D

$$dpi = \frac{1}{inch} \qquad F_s = \frac{\#pixels}{inch} \qquad f_{norm} = \frac{f_{skut}}{F_s}$$

$$f_{skut,vert} = \frac{m}{K} F_s, \quad f_{skut,horiz} = \frac{n}{L} F_s$$

Fáze – opět problém ☹



c=2.5000e+03 c=9.9476e-13 c=-5.6843e-14

Řešení – komplexní exponenciály (opět)

$$X[m,n] = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} x[k,l] e^{-j\left[2\pi\left(\frac{m}{K}k + \frac{n}{L}l\right)\right]}$$

Opakování, co je co …

- *k,l* jsou počitadla pixelů (vstup)
- *m,n* jsou počitadla frekvencí (výsledek)
- *m/K* je normalizovaná vertikální frekvence
- *n/L* je normalizovaná horizontální frekvence

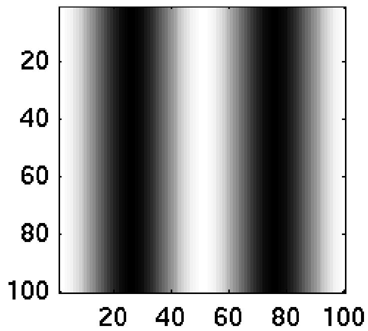
Jak si to představit I.

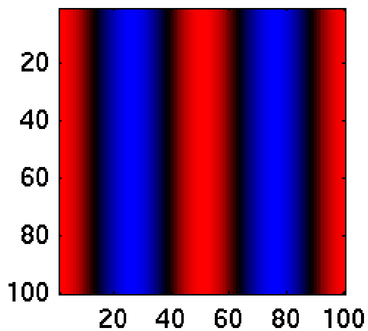
$$X[0,1] = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} x[k,l] e^{-j\left[2\pi\left(\frac{0}{K}k + \frac{1}{L}l\right)\right]} = \ldots$$

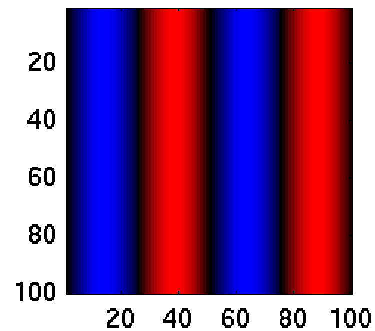
Jak si to představit II.

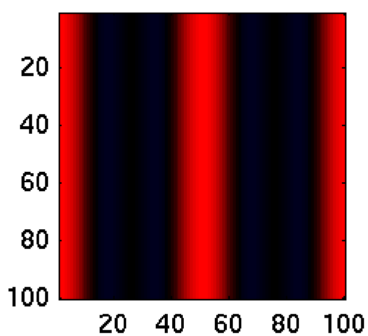
$$X[3,0] = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} x[k,l] e^{-j\left[2\pi\left(\frac{3}{K}k + \frac{0}{L}l\right)\right]} = \dots$$

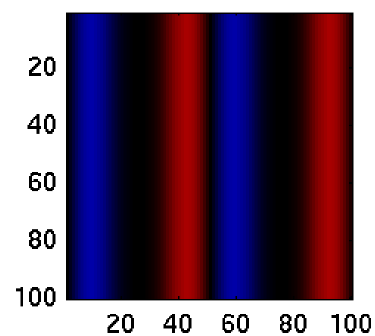
Korelace s *x[n]*









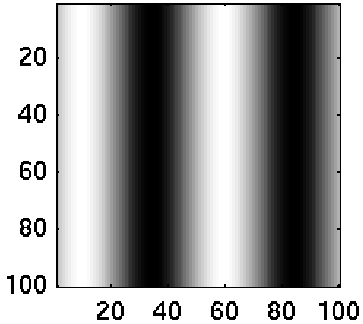


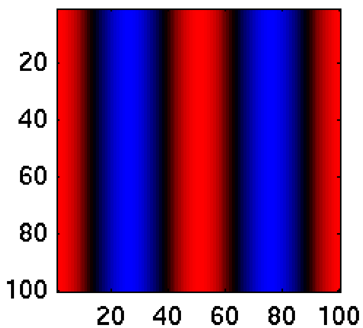
Re(c) = 2500

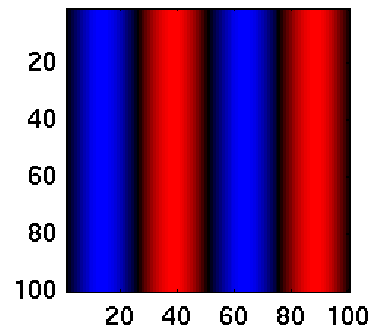
Im(c) = 0

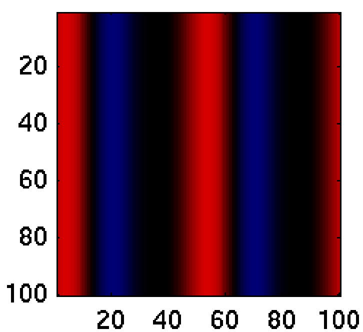
|c| = 2500

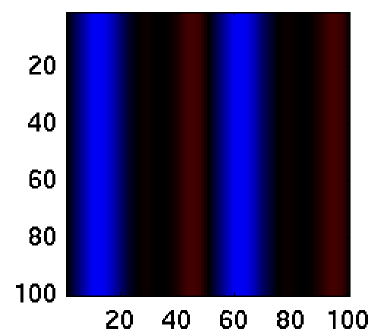
Korelace s *x[n]*









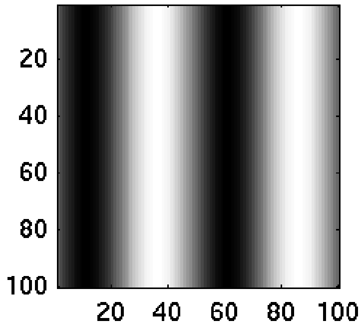


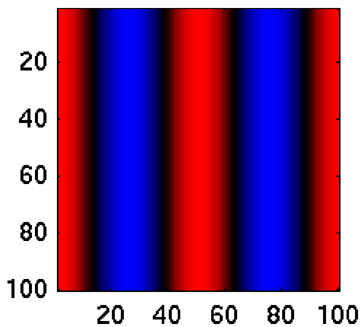
Re(c) = 1250

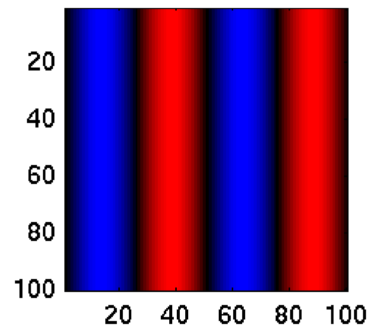
Im(c) = -2165

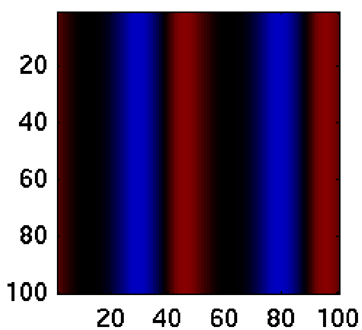
|c| = 2500

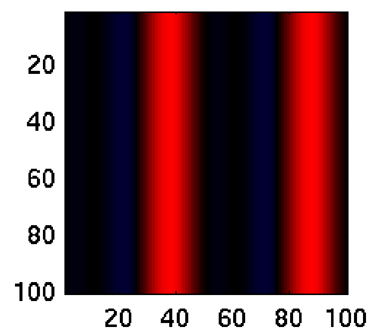
Korelace s *x[n]*









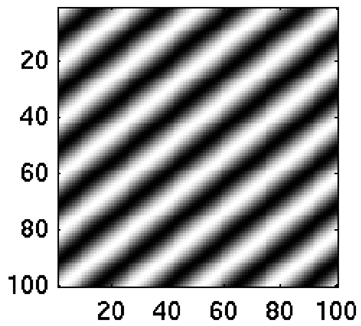


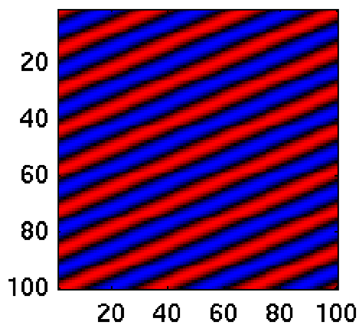
Re(c) = -772

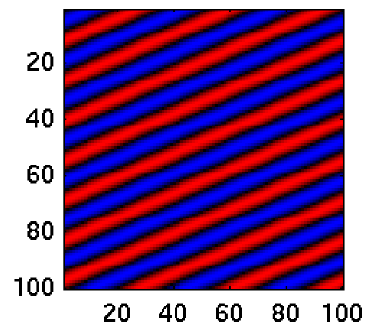
Im(c) = 2378

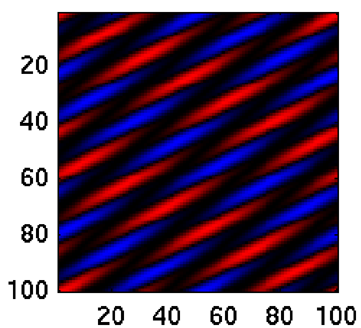
|c| = 2500

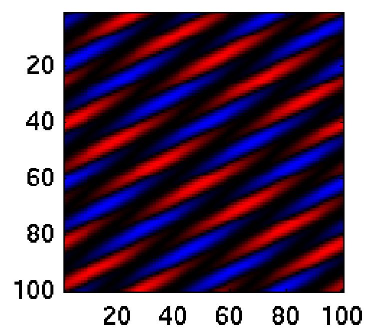
Změna v obou směrech…*X[7,3]*







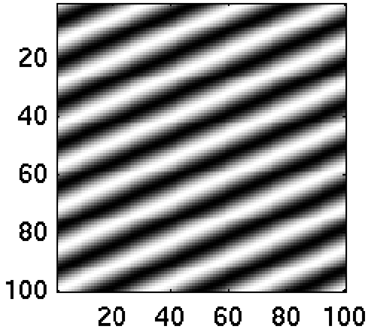


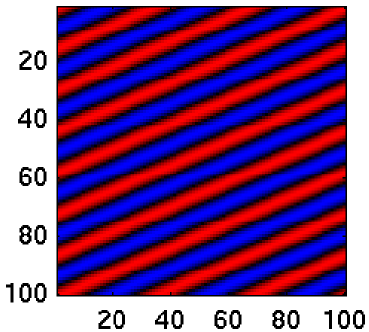


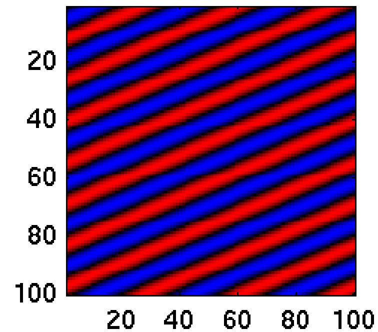
Re(c) = 0 Im(c) = 0

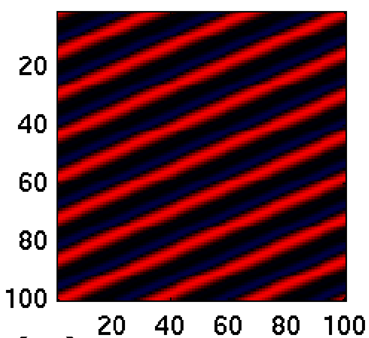
|c| = 0 … nepodobný

Změna v obou směrech…*X[7,3]*

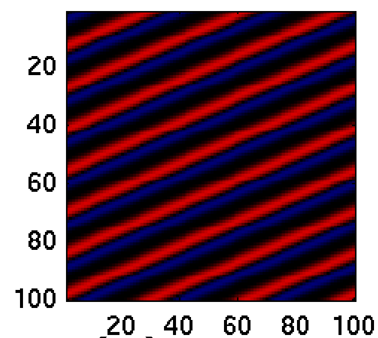








Re(c) = 2165



Im(c) = 1250

|c| = 2500 … podobný

2D DFT pomocí 2 x 1D DFT

$$X[m,n] = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} x[k,l] e^{-j2\pi\left(\frac{mk}{K}+\frac{nl}{L}\right)} =$$

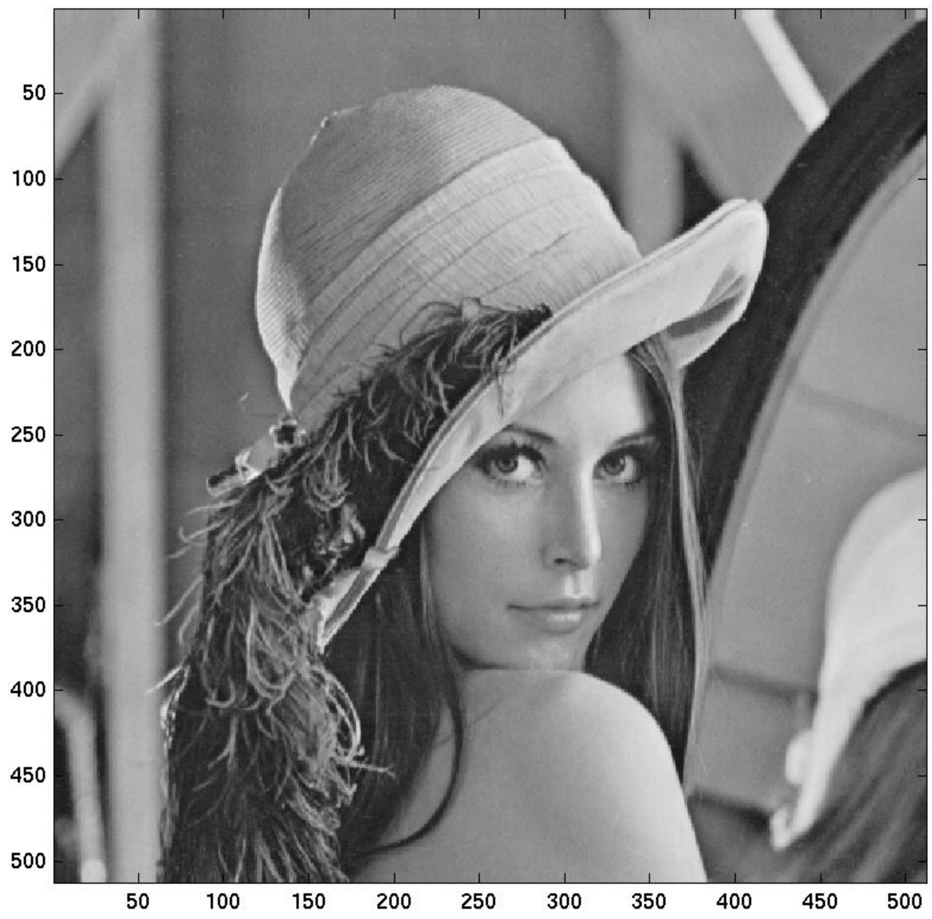
$$= \sum_{k=0}^{K-1} e^{-j2\pi\frac{mk}{M}} \sum_{l=0}^{L-1} x[k,l] e^{-j2\pi\frac{nl}{L}}, \quad \ldots \quad \text{nebo naopak}$$

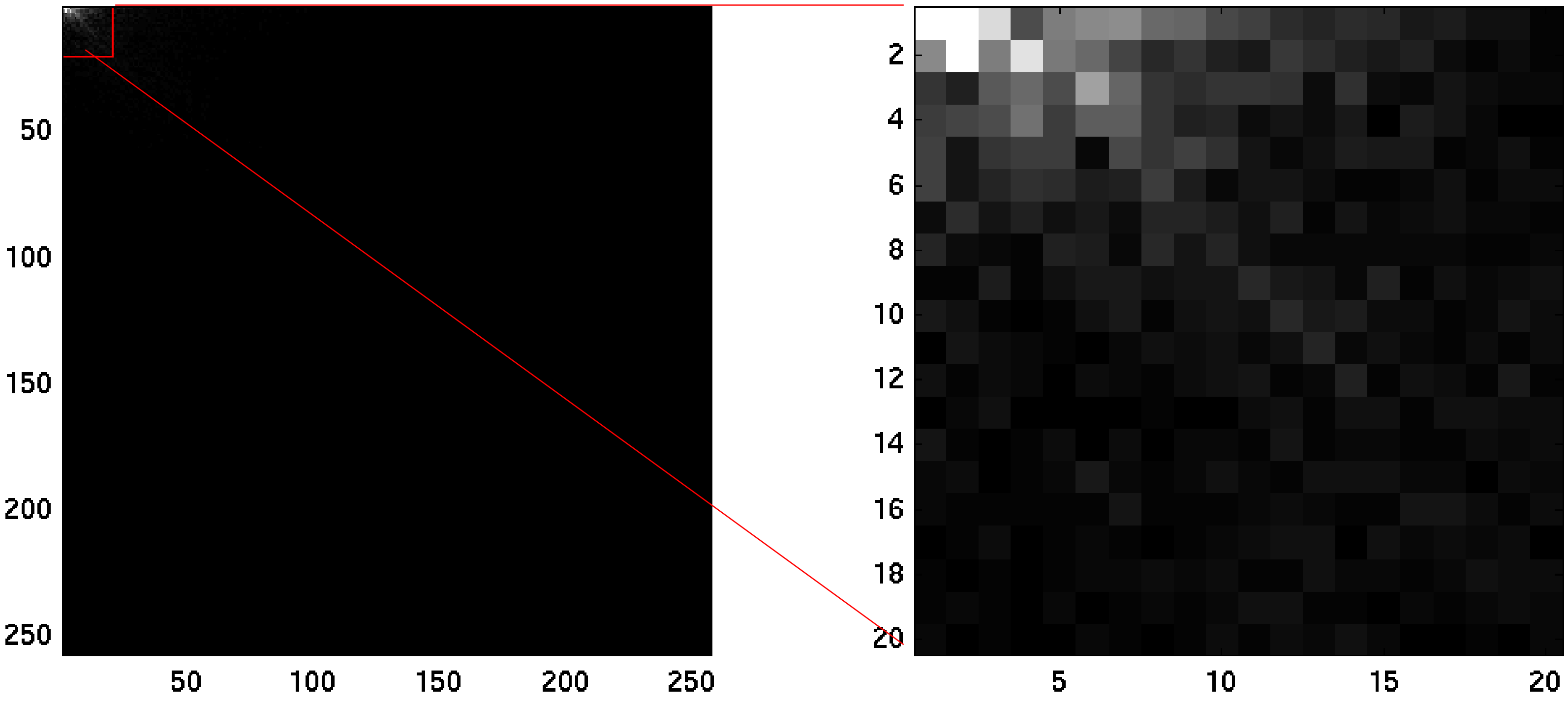
- takže

$$2DDFT\{x[k,l]\} =$$

$$= 1DDFT_{sloupce}\{1DDFT_{radky} x[k,l]\} \quad \ldots \quad \text{nebo naopak}$$

2D DFT pro skutečný signál

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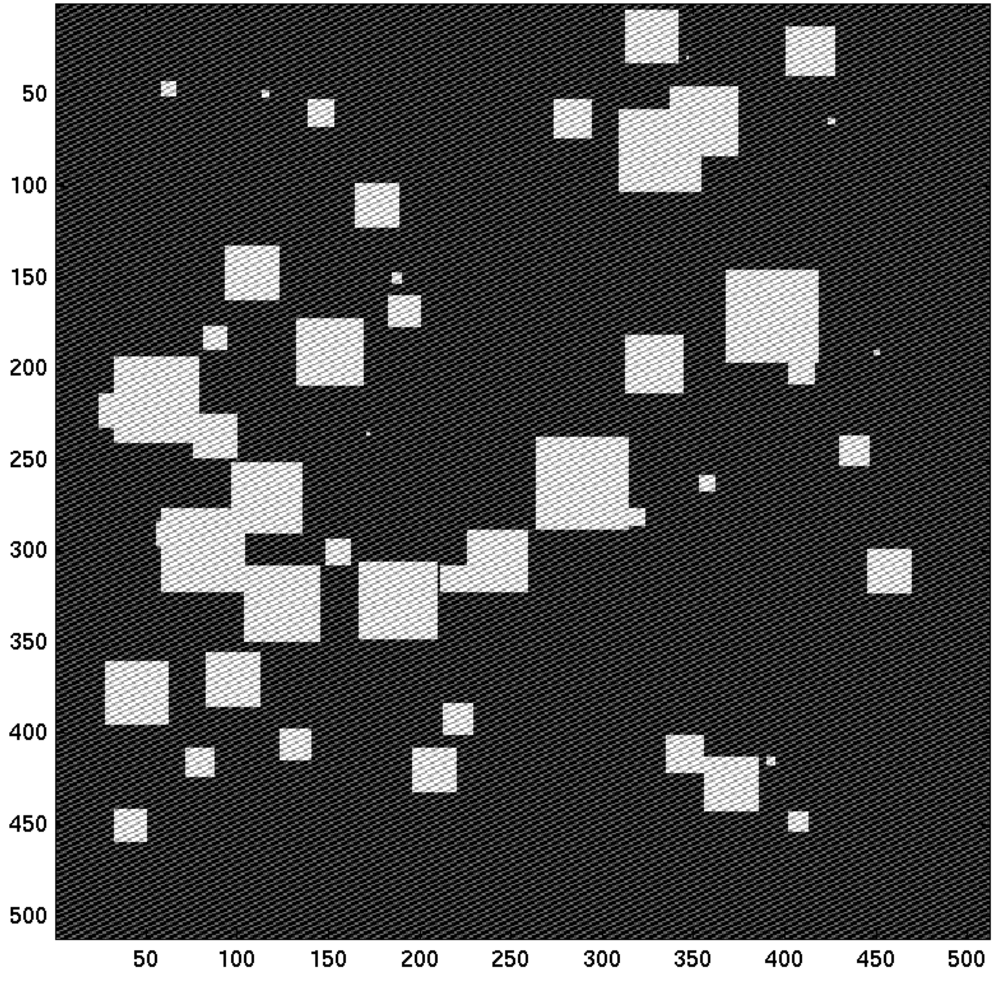
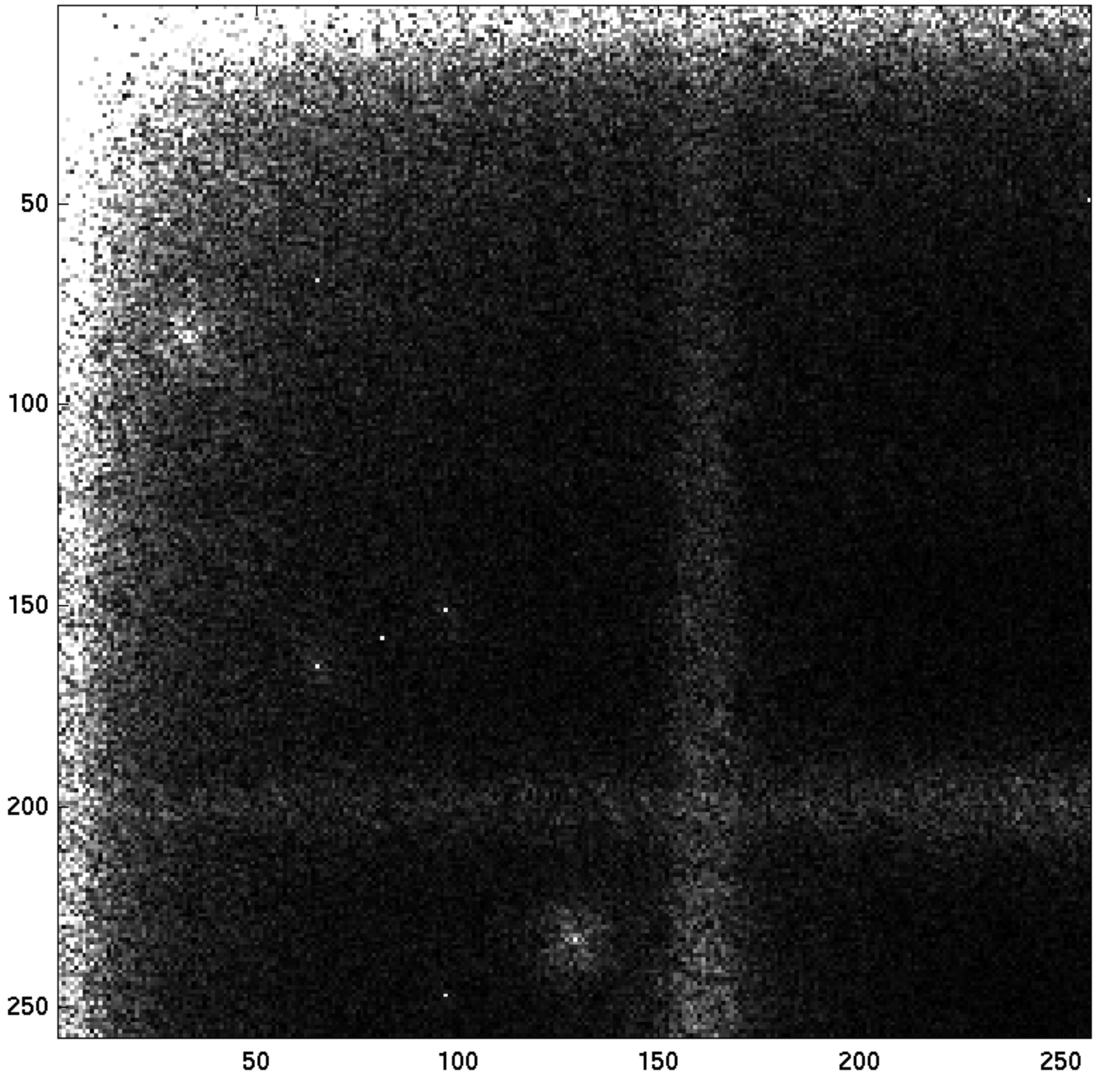
Něco s více vyššími frekvencemi …

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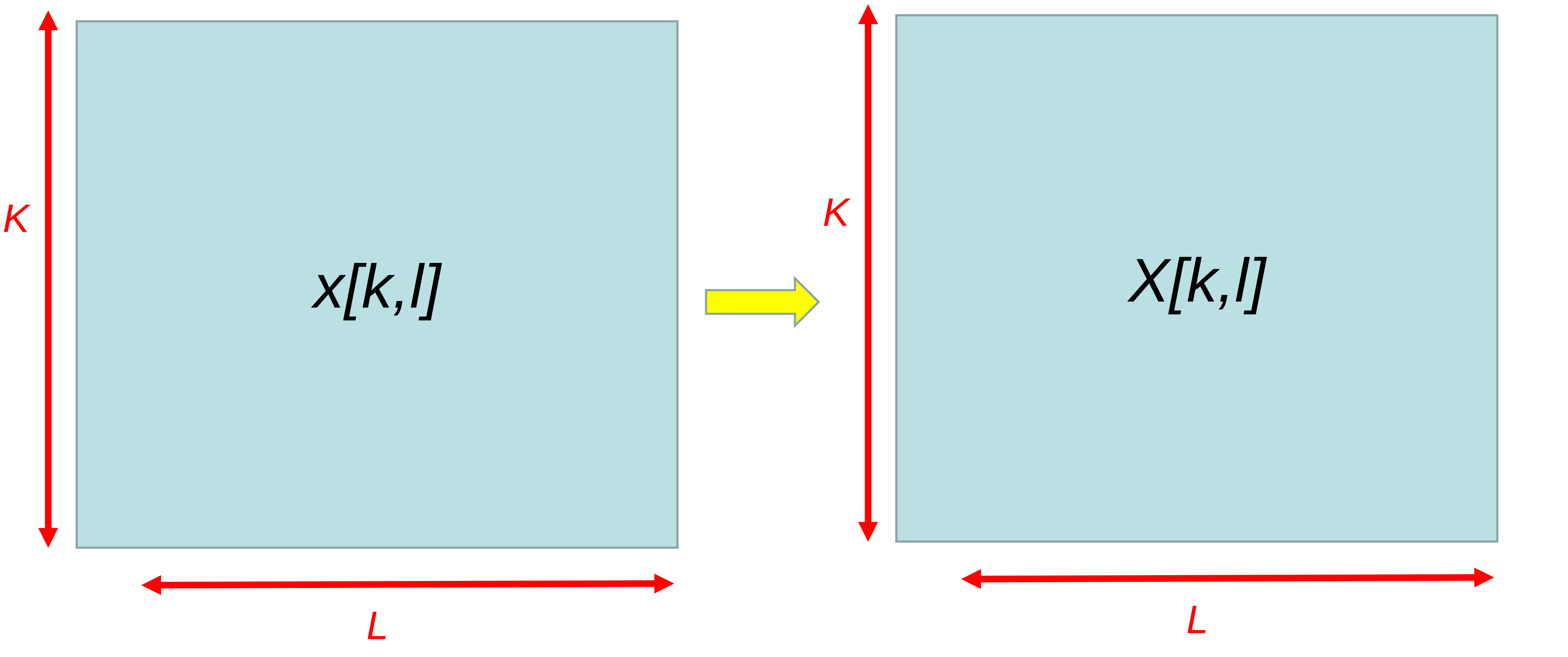
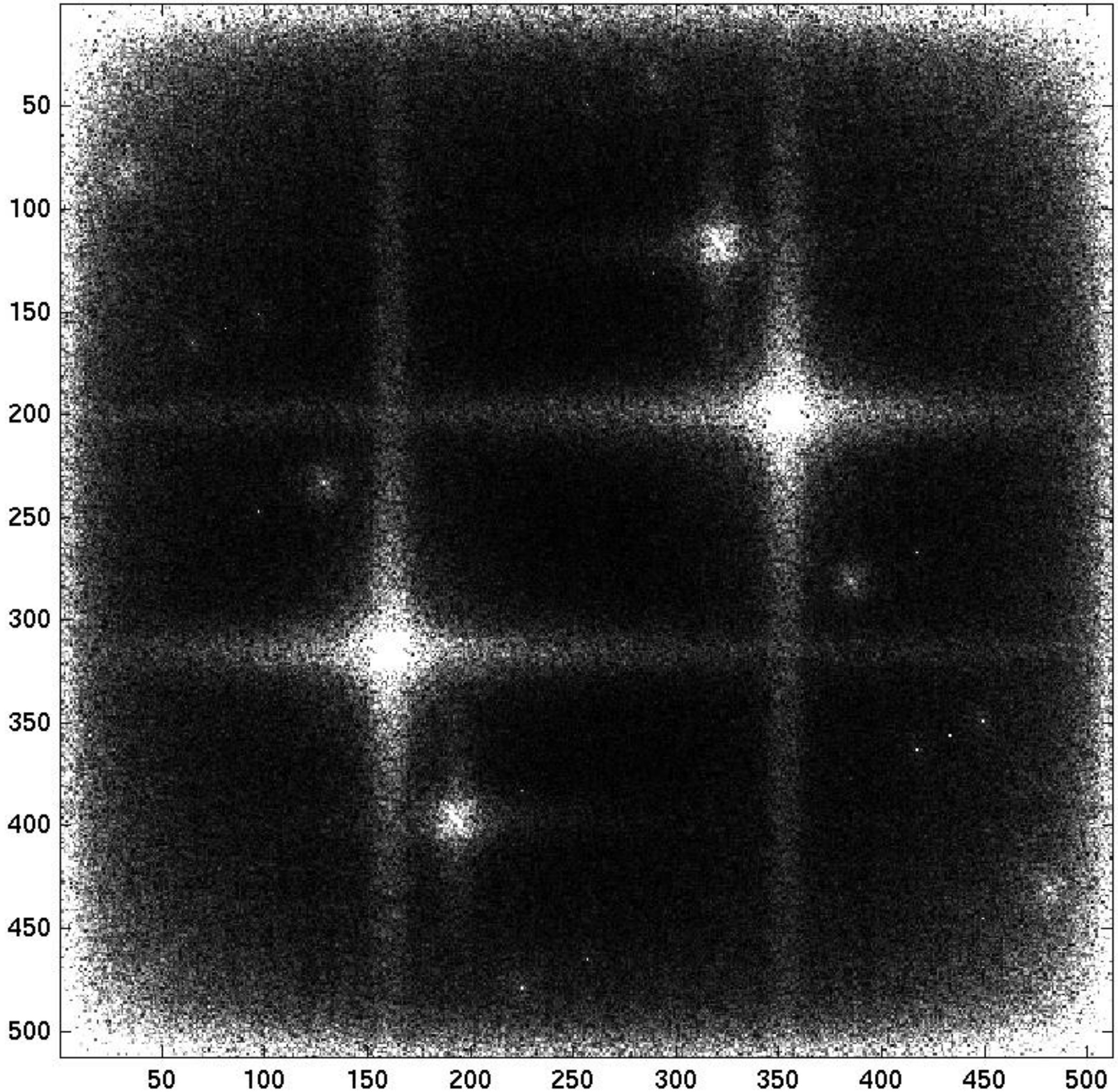
DFT produkuje *K x L* bodů !

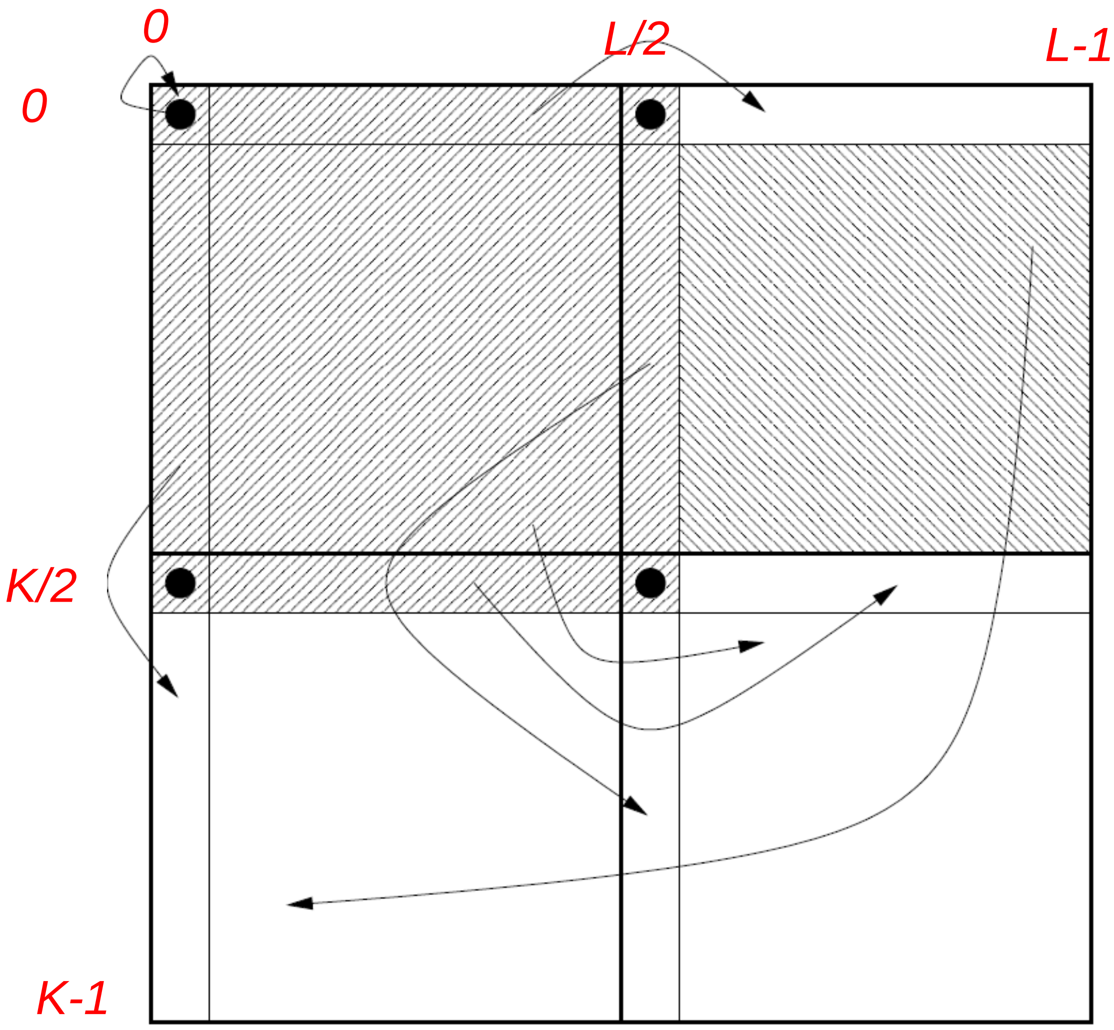


Celý výsledek *k=0…K-1, l=0…L-1*



Symetrie ?

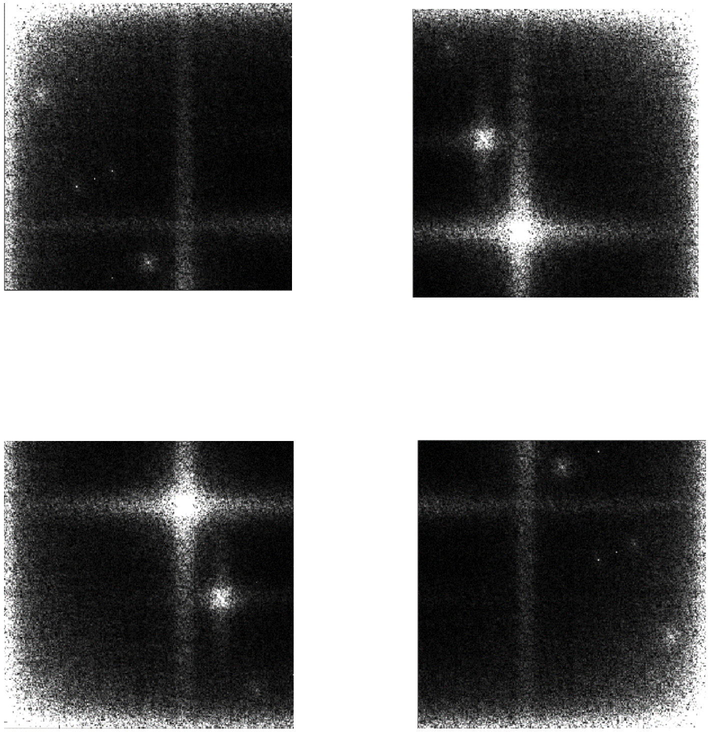
$$X[m,n] = X^{\star}[K-m, L-n]$$

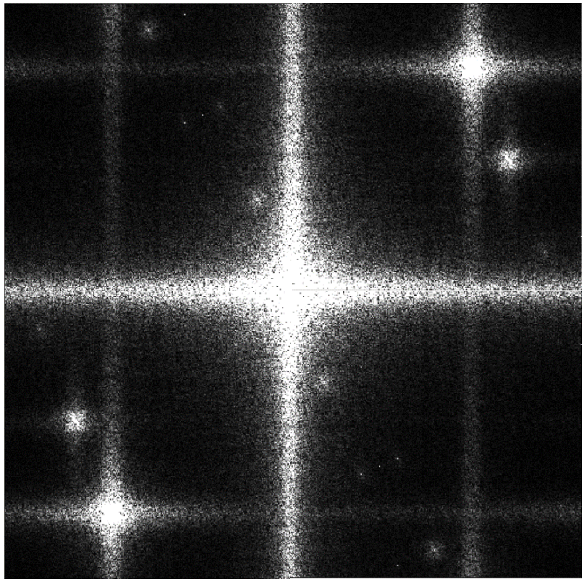


Zdroj: http://www-personal.umich.edu/~hyunjinp/notes/n-dft.pdf

Přeskládání …

- Nízké frekvence uprostřed

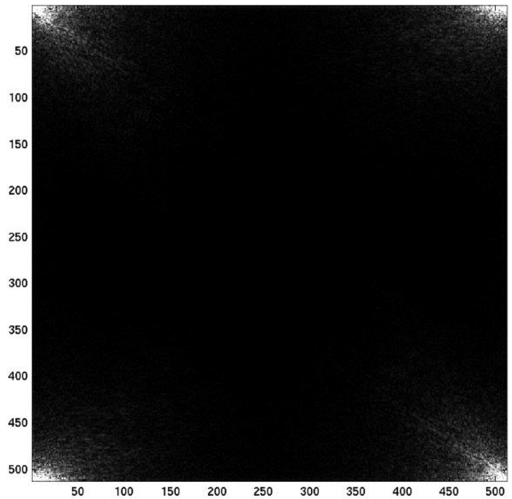




Zpětná 2D DFT

$$x[k,l] = \frac{1}{KL} \sum_{m=0}^{K-1} \sum_{n=0}^{L-1} X[m,n] e^{+j2\pi\left(\frac{mk}{K} + \frac{nl}{L}\right)}$$

Hraní s frekvencemi …



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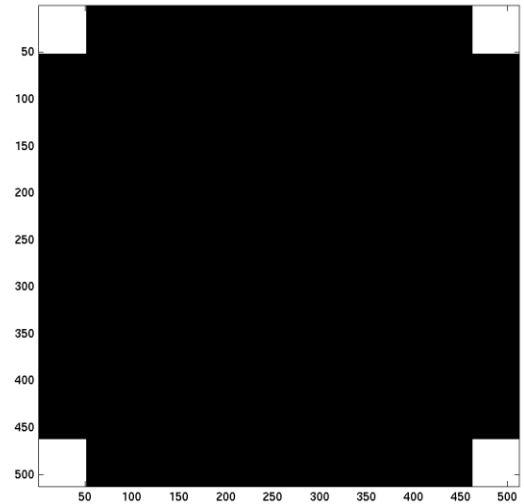

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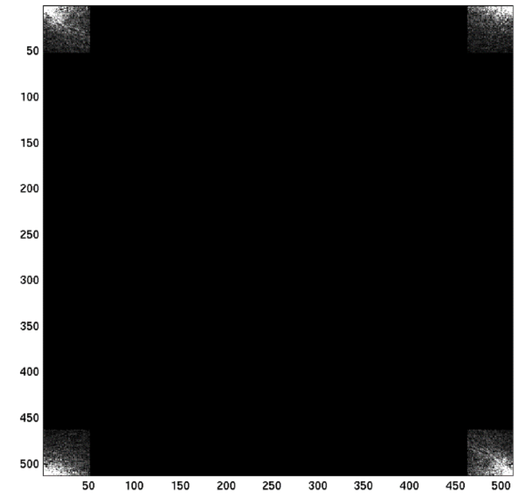

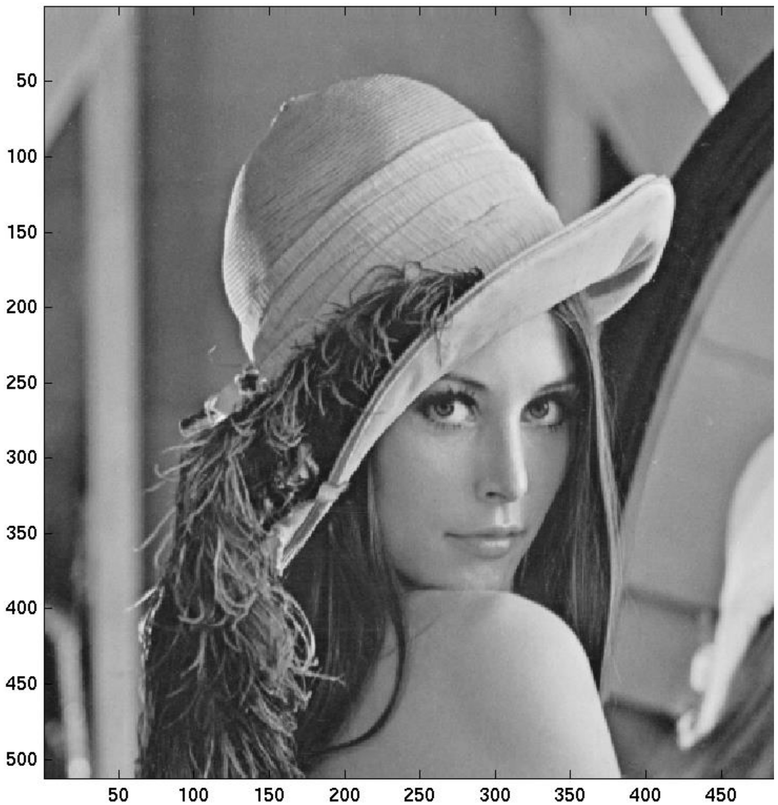
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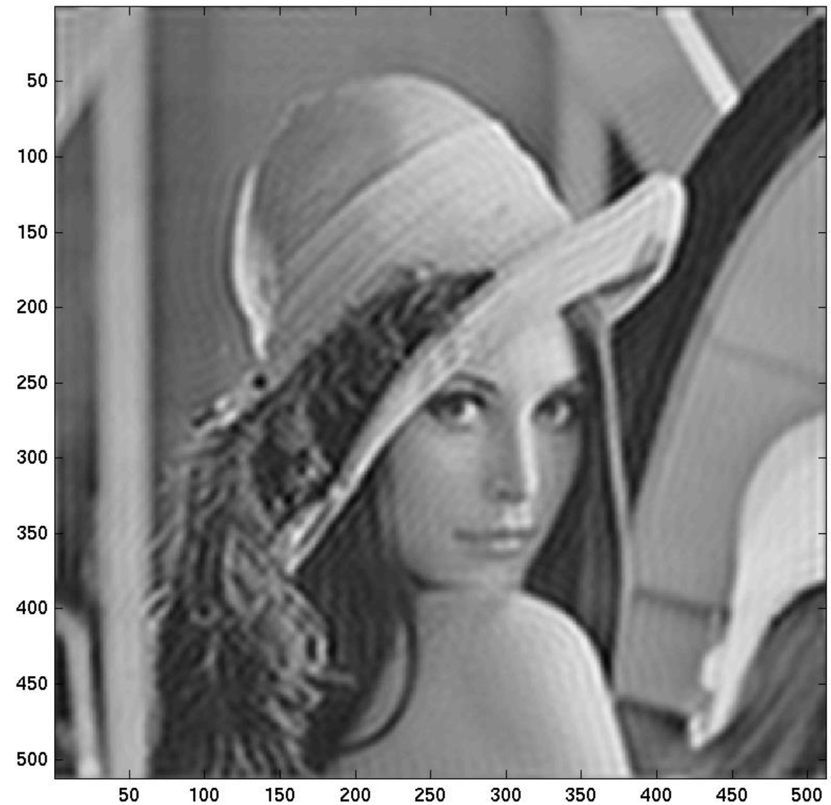
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DCT

Proč ?

- Komplexní koeficienty jsou otravné
- Symetrie v obrázku jsou otravné
- Pro obrázek $K \times L$ chceme $K \times L$ reálných čísel.

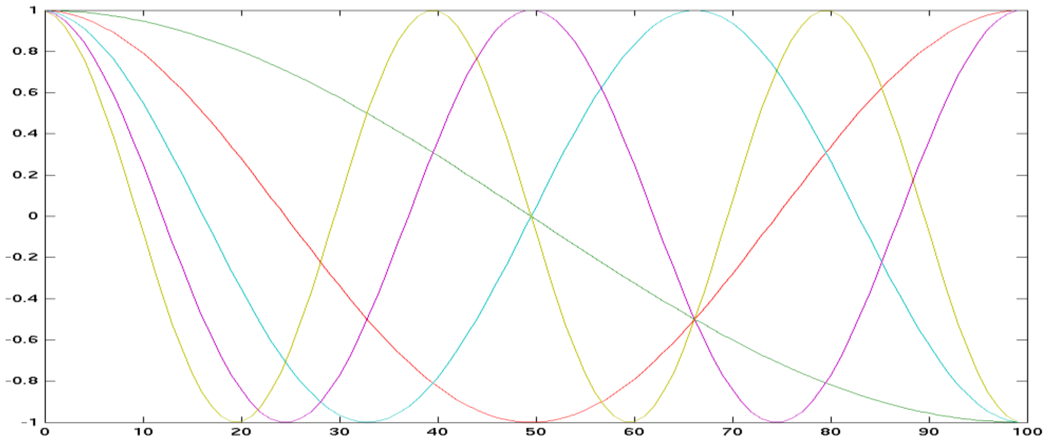
1D DCT

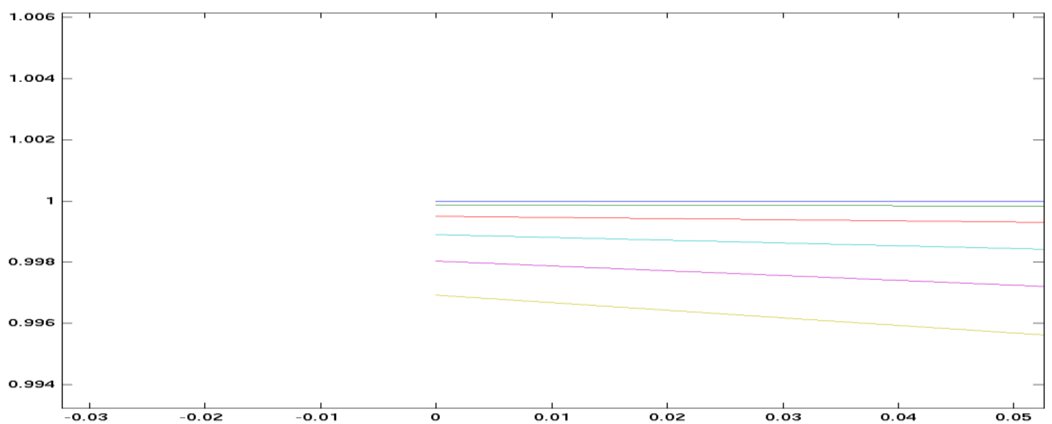
$$X[k] = \sum_{n=0}^{N-1} x[n] \cos\left[\frac{\pi}{n}(n+\frac{1}{2})k\right]$$

+ normalisace koeficientů …

$$X[0] \ldots \times \sqrt{\frac{1}{N}}$$

$$X[1 \ldots N-1] \ldots \times \sqrt{\frac{2}{N}}$$

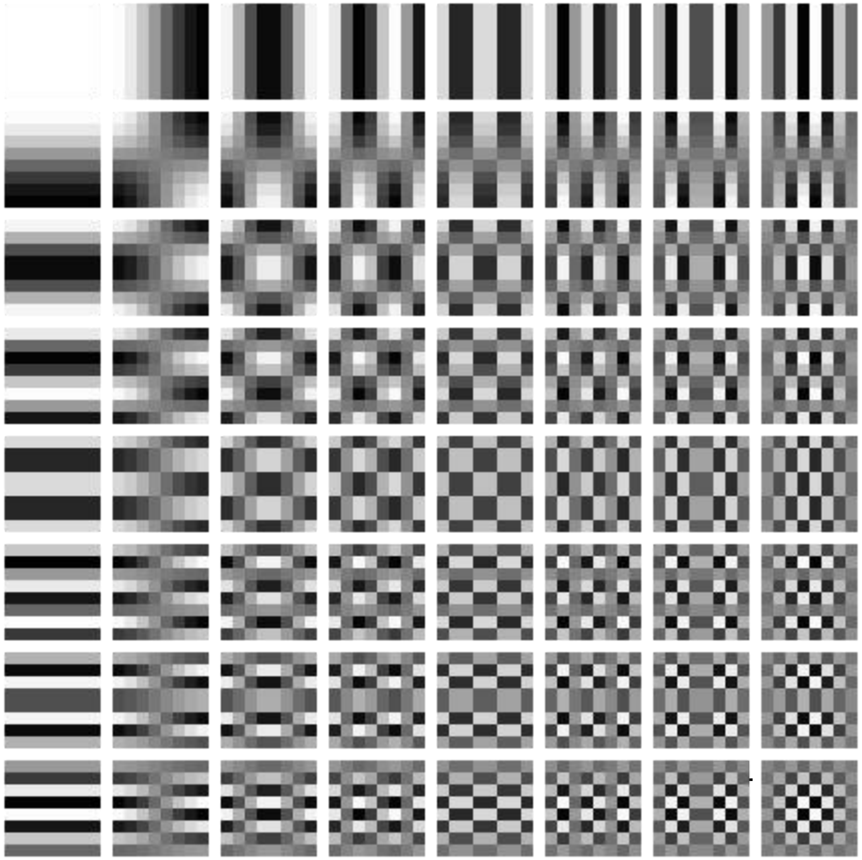




2D DCT báze

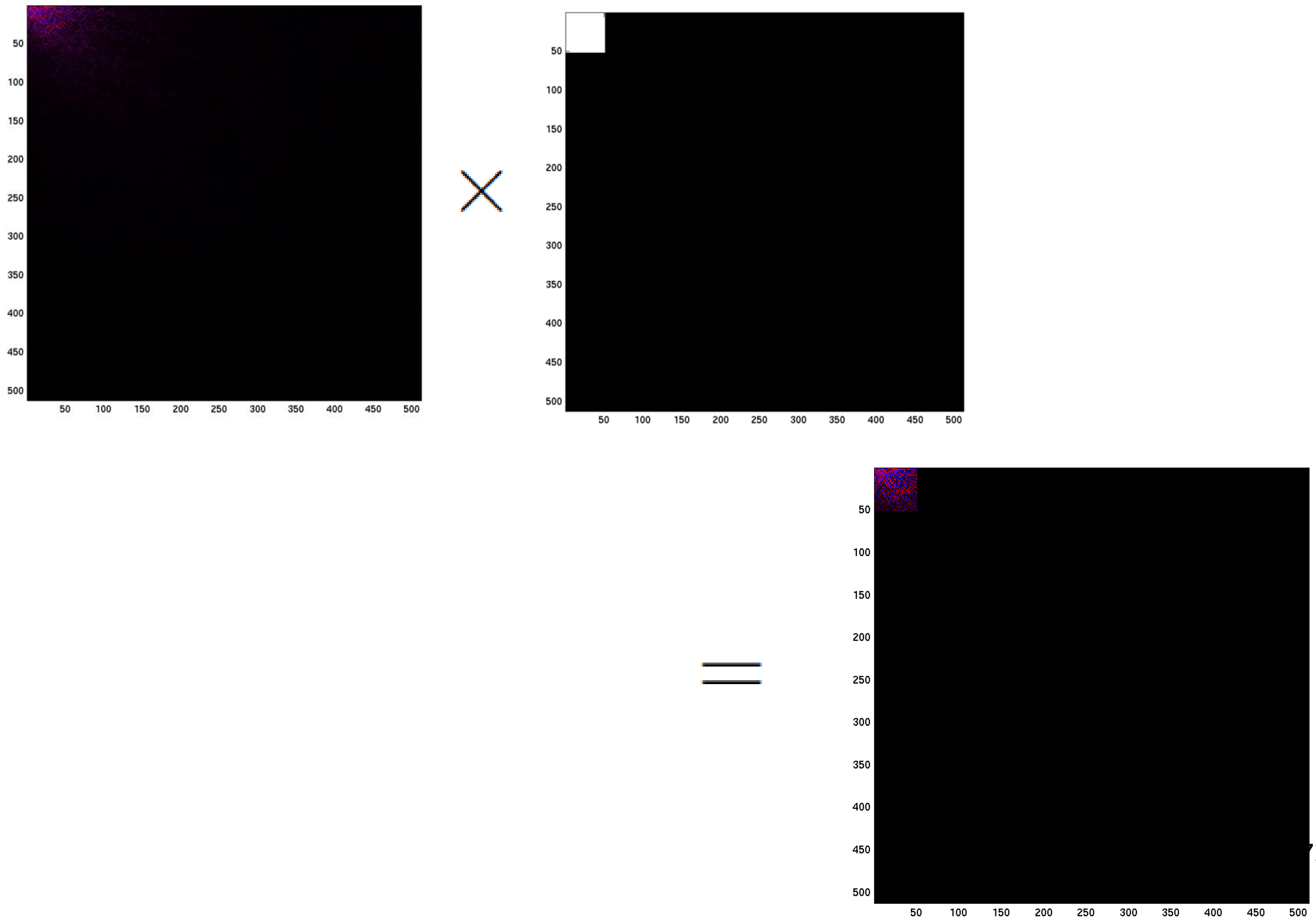
$$X_{k_1,k_2} = \sum_{n_1=0}^{N_1-1} \left(\sum_{n_2=0}^{N_2-1} x_{n_1,n_2} \cos\left[\frac{\pi}{N_2}\left(n_2+\frac{1}{2}\right)k_2\right]\right) \cos\left[\frac{\pi}{N_1}\left(n_1+\frac{1}{2}\right)k_1\right]$$

$$= \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_{n_1,n_2} \cos\left[\frac{\pi}{N_1}\left(n_1+\frac{1}{2}\right)k_1\right] \cos\left[\frac{\pi}{N_2}\left(n_2+\frac{1}{2}\right)k_2\right].$$



Zdroj:
https://en.wikipedia.org/wiki/Discrete_cosine_transform#Multidimensional_DCTs

Filtrování v DCT

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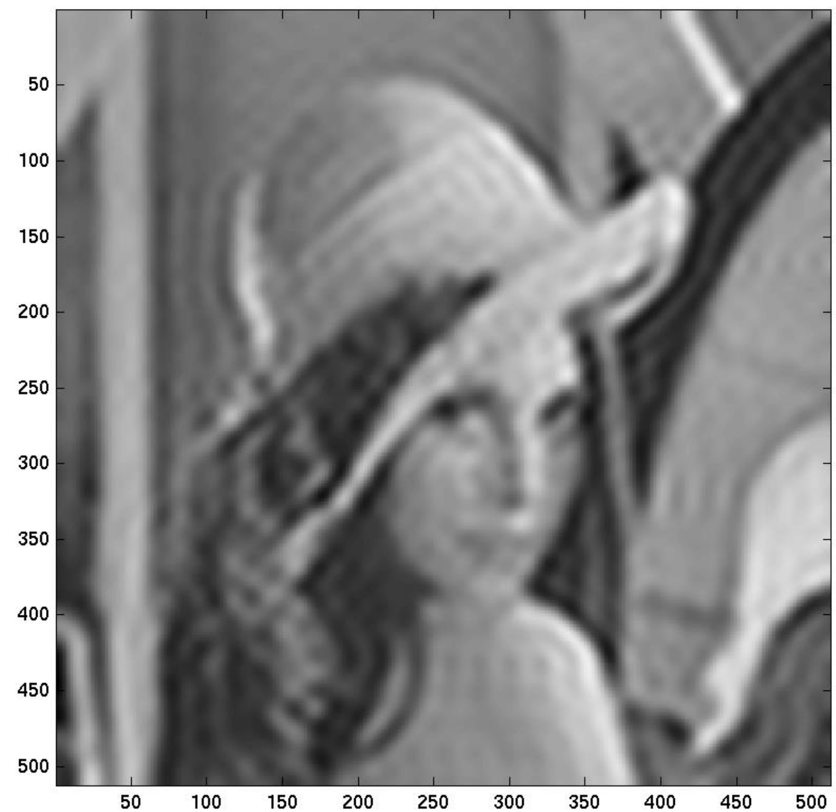
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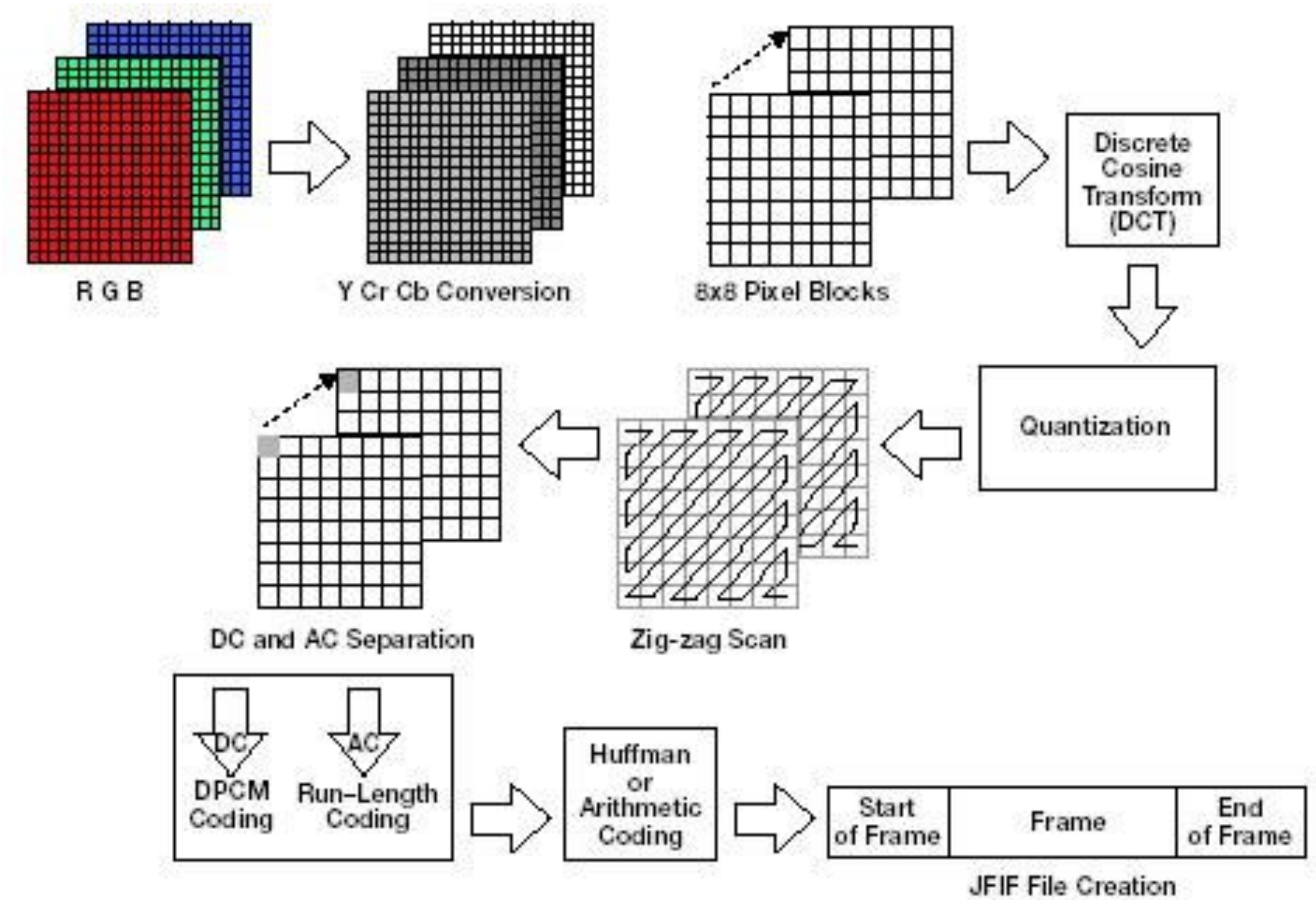
JPEG



- Zdroj: http://www.eetimes.com/document.asp?doc_id=1225736
- Více v grafických předmětech a 2. cvičení.

SUMMARY

- Obraz je 2D signál
- Filtrace
  - Konvoluce, analogie s 1D FIR filtry
  - IIR v obrázcích nepotkáte
- Jezdím s maskou po obrázku, co pod ni padne, vynásobím s koeficienty a sčítám.

SUMMARY II.

- Frekvenční analýza 2D
  - Stejně jako u 1D – projekce do bází
  - Obrazové frekvence mají svůj význam.
  - Cos báze jsou dobré cvičení, ale nestačí
- 2D DFT
  - Projekce/podobnost/korelace s komplexními exponenciálami
  - Koeficienty jsou komplexní, modul a argument
  - Ve výsledné DFT matici je spousta symetrií
  - Ve spektru se dá filtrovat.

SUMMARY III.

- DCT
  - 2x „pomalejší“ báze než DFT
  - Trochu složitější definice
  - Produkuje reálné koeficienty, nízké frekvence (jen!) na začátku
  - Využití v JPEG

TO BE DONE

- Určení frekvenční charakteristiky 2D filtru
- Jak je to přesně se symetriemi 2D DFT ?
- Jak je přesně definována 1D a 2D DCT a proč je tam ten půlvzorkový posuv ?
- Jak je to s barvami (barevné modely, atd).
- Jak funguje rozpoznávání xichtů na FB ?
  – Hint: https://research.facebook.com/researchers/684639631606527/yann-lecun/

The END