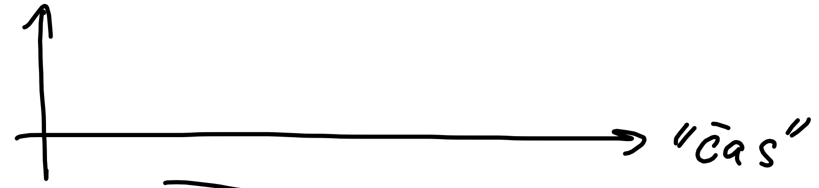
# Půlsemestrálka

ÚT 30.10. $9^{45} - 10^{45}$ tady, D0206 a A113.

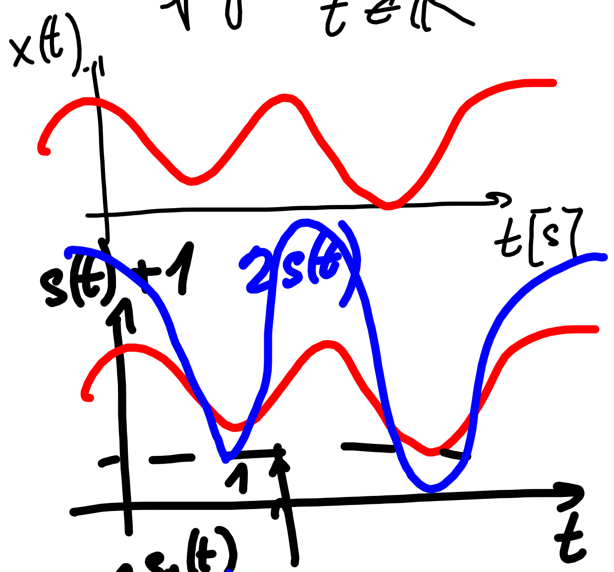
- první 4 "barevné" přednášky
- první 2 cvika.

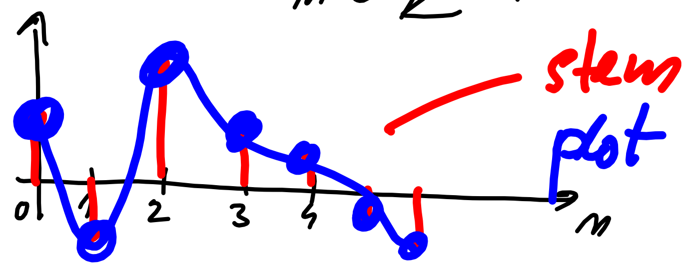
# Teor. úvod k sig



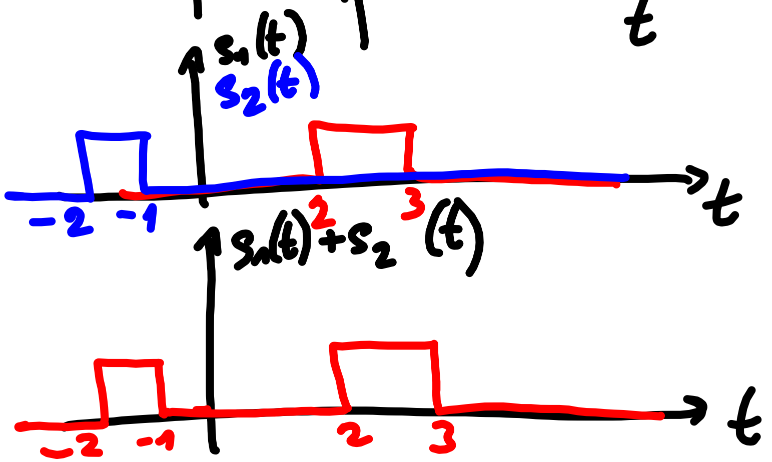

spojité $t \in \mathbb{R}$

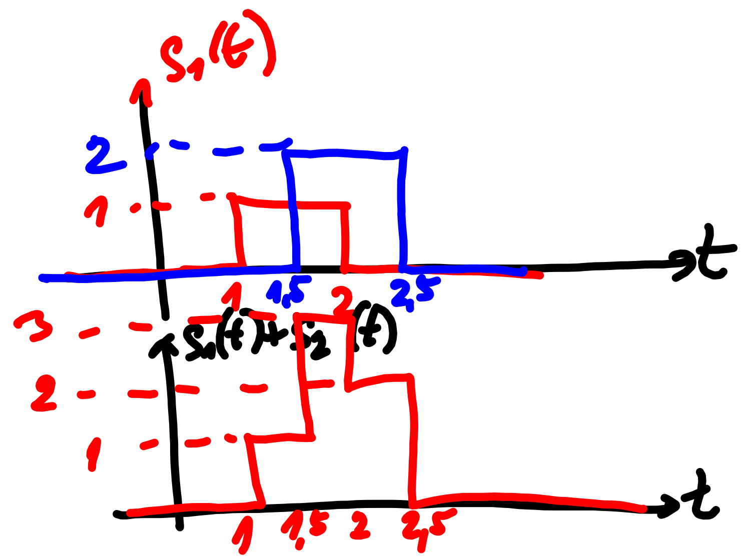
diskrétní (vzorkované) $n \in \mathbb{Z}$ (celá čísla)



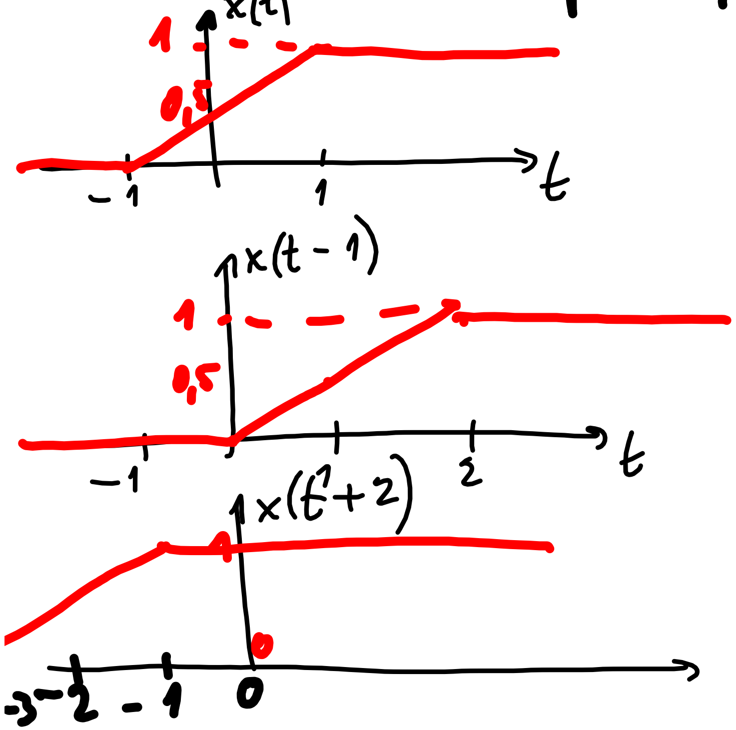




stem plot



# Modifikace (posuny) časové osy

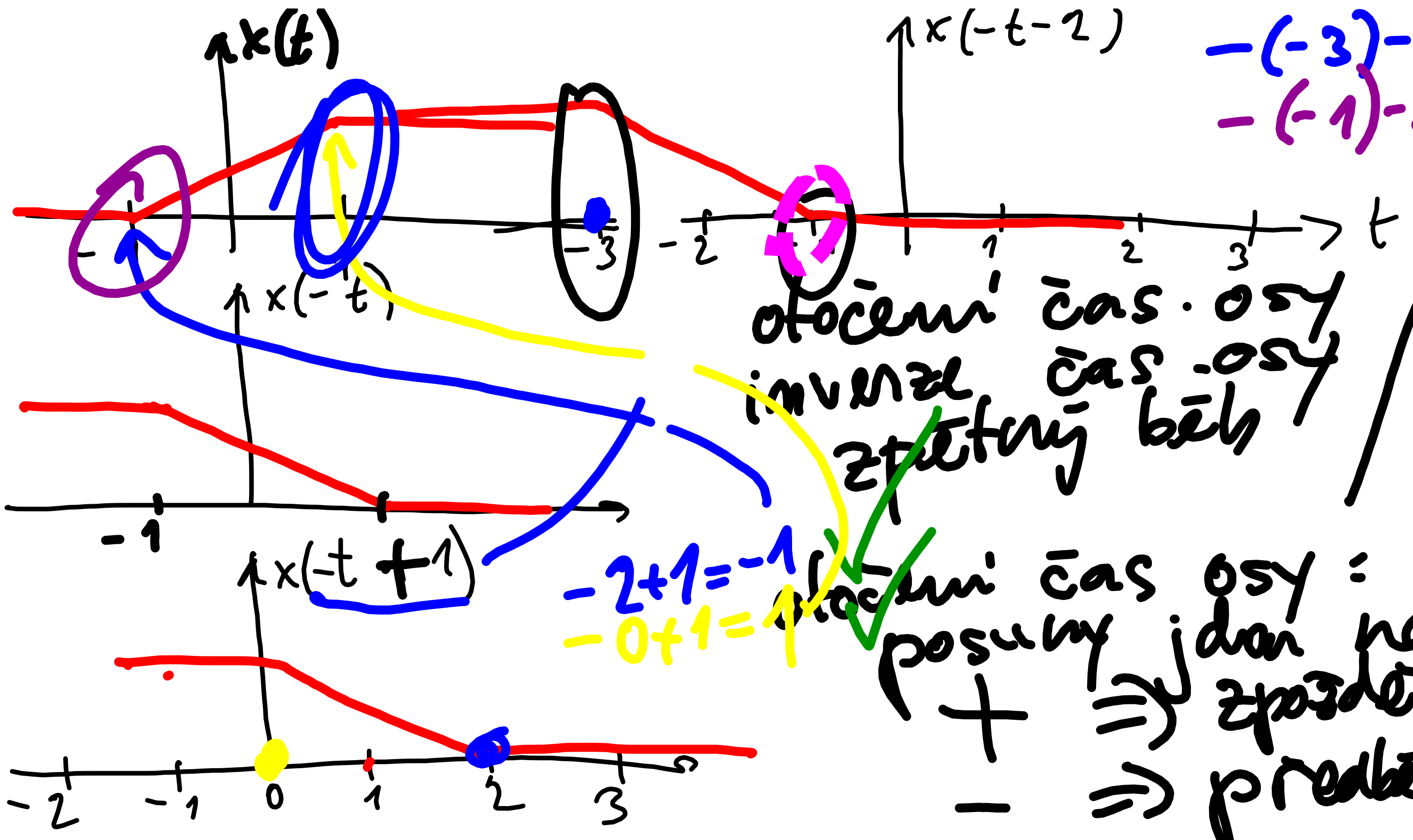



posun doprava
zpoždění / delay

posun doleva
předběhnutí / advance

$-(-3)-2=1$

$-(-1)-2=-1$




otočení čas. osy / reversing the time.

inverze čas. osy

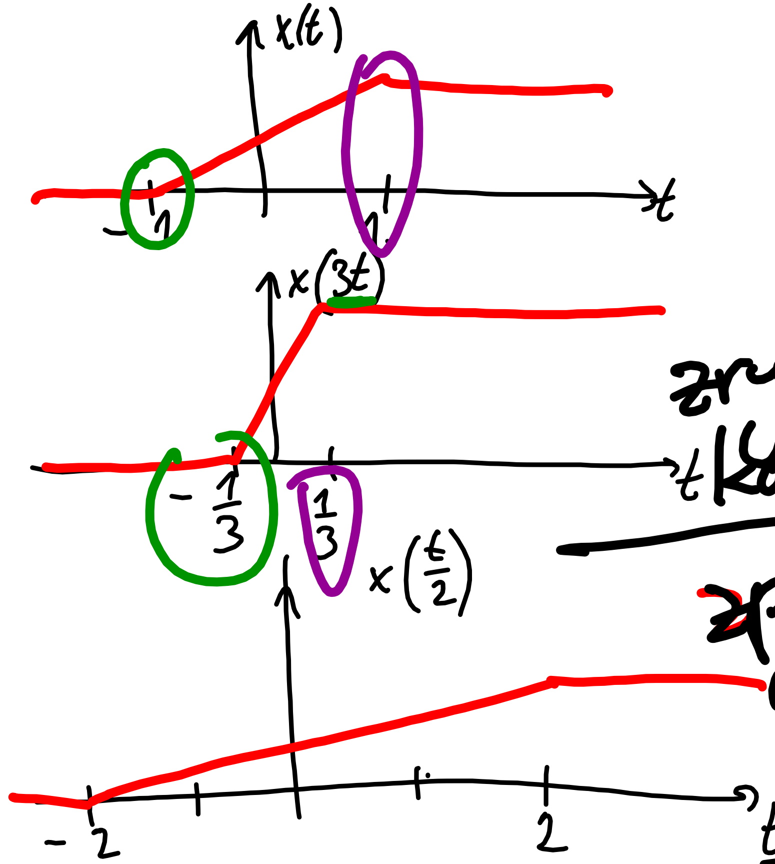
zpětný běh

otočení čas osy:

posuny jdou naopak

$+ \Rightarrow$ zpoždění

$- \Rightarrow$ předběh.

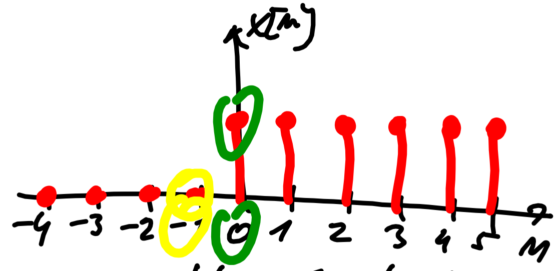



$3\left(-\frac{1}{3}\right) = -1$

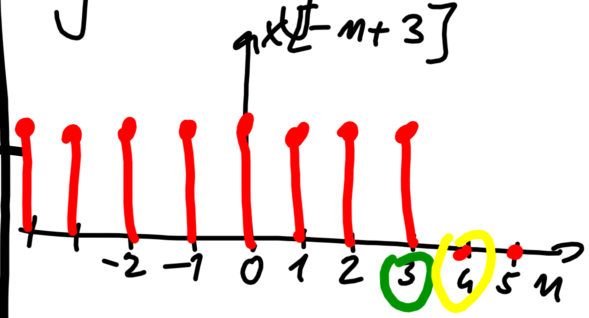
$3\left(\frac{1}{3}\right) = 1$

zrychlení
kontrakce

zpomalení
dilatace




jednotkový skok.




$-3+3=0$

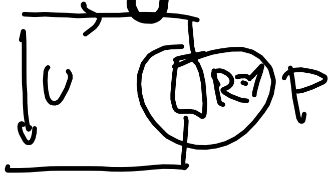
$-4+3=-1$

Celá čísla!

$x[n] \longrightarrow x\left[\frac{n}{2}\right]$ chybějící vzorky!

# Energie a výkon

$x(t)$




$P = U^2$

výkon = hodnota$^2$

$p(t) = x^2(t) = |x(t)|^2$

okamžitý výkon

$P$ $\quad E = P \cdot \Delta t$

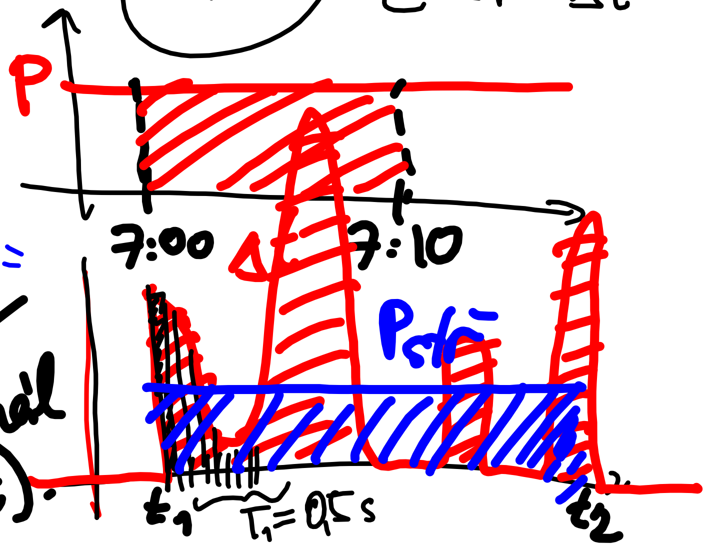
Energie ~~od času~~ $t_1$ do času $t_2$

$$E_{t_1,t_2} = \int_{t_1}^{t_2} p(t)\,dt = T_1 \sum_{n=n_1}^{n_2} p[n]$$

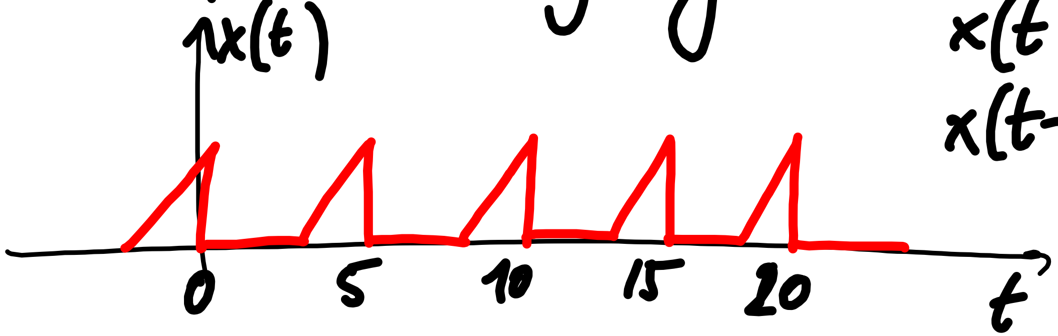
$$P_{stř_{t_1,t_2}} = \frac{E_{t_1,t_2}}{t_2 - t_1}$$

$$\int_{t_1}^{t_2} P_{stř}\,dt = P_{stř}(t_2 - t_1) = E_{t_1,t_2}$$

$\sqrt{P_{stř}}$ – efektivní hodnota na intervalu $t_1 \dots t_2$ dá stejný výkon jako pův. signál $x(t)$.



# Periodický signál




$x(t+5) = x(t)$

$x(t+15) = x(t)$

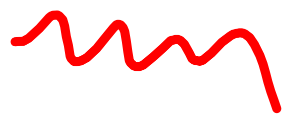
$\vdots$

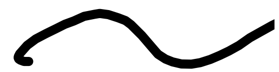
$T_1$ základní perioda

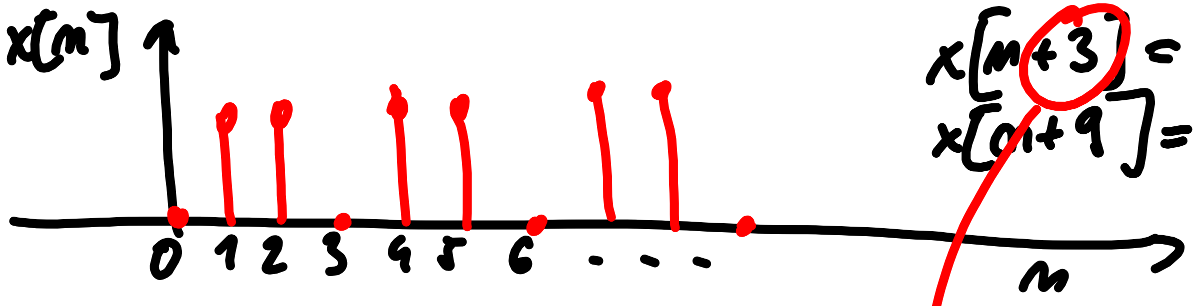
$f_1 = \frac{1}{T_1}$ [Hz] základní frekvence

$f_1 = 0{,}2$ Hz

$f_2$ .... vyšší frekv. komponenty.








$x[n+3] = x[n]$

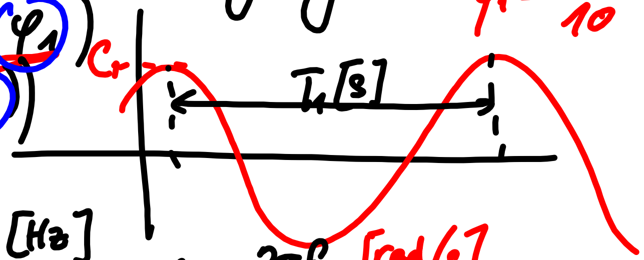
$x[n+9] = x[n]$

základní perioda $N_1$

# Cosinusovka / harmonický signál

$x(t) = C_1 \cos(\omega_1 t + \varphi_1)$ $\qquad \varphi_1 = -\frac{2\pi}{10}$

$C_1 \cos(\omega_1 (t + t_1))$




$\varphi_1 = \omega_1 \cdot t_1$ počáteční posunutí.

$f_1 = \frac{1}{T_1}$ [Hz] $\qquad \omega_1 = 2\pi f_1$ [rad/s]

$\boxed{\omega_1 = \frac{2\pi}{T_1}}$

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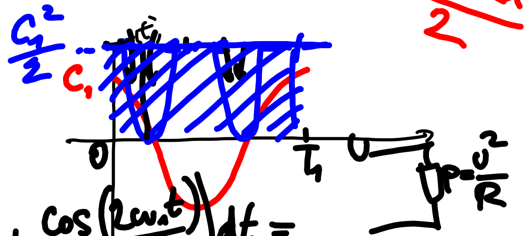
## střední výkon cosinusovky

$P_{stř\, t_1,t_2} = \frac{E_{t_1,t_2}}{t_2 - t_1} = \frac{E_{T_1}}{T_1} = \frac{1}{T_1}\int x^2(t)\,dt$ $\quad | \quad C_1 \cos(\omega_1 t)$

$T_1 C_1^2$ $\quad p(t)$

$(\cos x)^2 = \frac{1+\cos 2x}{2}$

$P_{stř} = \frac{1}{T_1}\int_0^{T_1} (C_1 \cos(\omega_1 t))^2\,dt =$

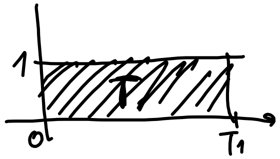


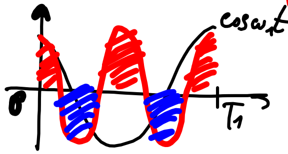

$= \frac{1}{T_1}\left(\frac{C_1^2}{2} \cdot T_1\right) = \frac{C_1^2}{2}$

$P_{stř} = \frac{1}{T_1}\int_0^{T_1} (C_1 \cos(\omega_1 t))^2\,dt = \frac{1}{T_1}\int C_1^2 \left(\frac{1 + \cos(2\omega_1 t)}{2}\right)dt =$

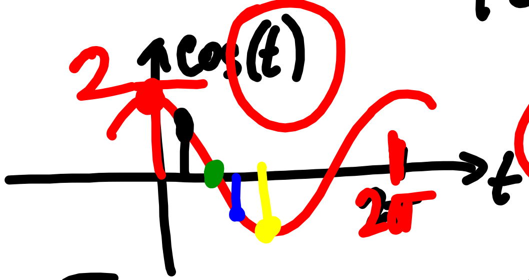
$= \frac{C_1^2}{T_1 \cdot 2}\left[\int_0^{T_1} 1\,dt + \int_0^{T_1} \cos(2\omega_1 t)\,dt\right] = \frac{C_1^2}{2 \cdot T_1}(T_1 + 0) = \frac{C_1^2}{2}$

$C_{ef} = \sqrt{P_{stř}} = \frac{C_1}{\sqrt{2}}$

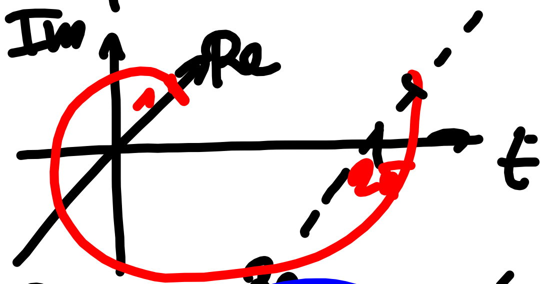


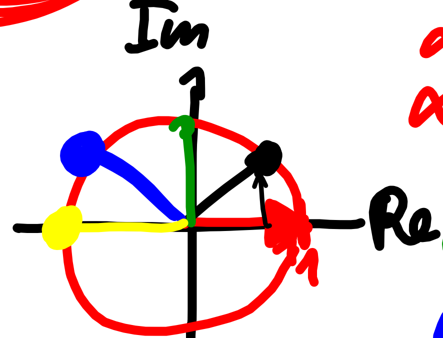



# Spek. analýza period. signálů se spoj. časem → Fourierova řada.

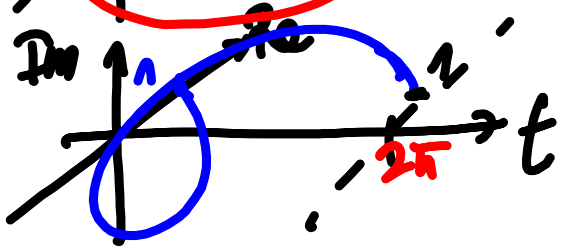


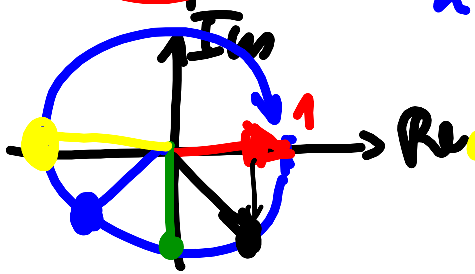

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$




| | |
|---|---|
| $\alpha = 0$ | $1 + 1 = 2$ |
| $\alpha = \frac{\pi}{4}$ | $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ |
| $\alpha = \frac{\pi}{2}$ | $0$ |
| $\alpha = \frac{3}{4}\pi$ | $-\sqrt{2}$ |
| $\alpha = \pi$ | $-2$ |



$$C_1\cos(\omega_1 t+\varphi_1)$$

$$\cos x=\frac{e^{jx}+e^{-jx}}{2}$$

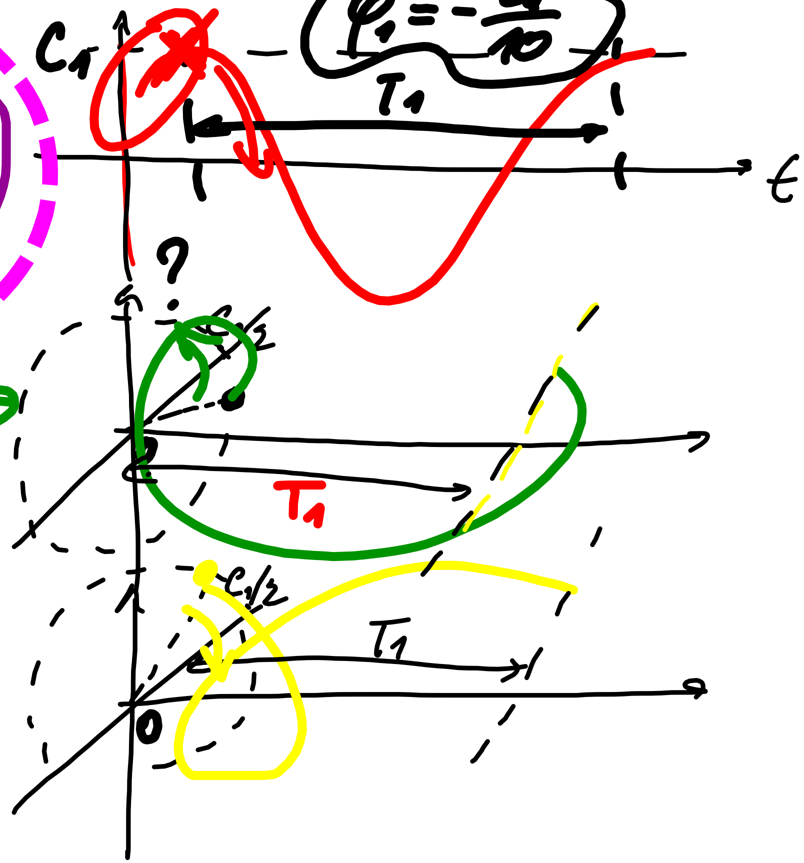
$$h^{f\bar{a}+g\bar{a}}=h^{f\bar{a}}\cdot h^{g\bar{a}}$$

$$\frac{C_1}{2}e^{j(\omega_1 t+\varphi_1)}+\frac{C_1}{2}e^{-j(\omega_1 t+\varphi_1)}=$$

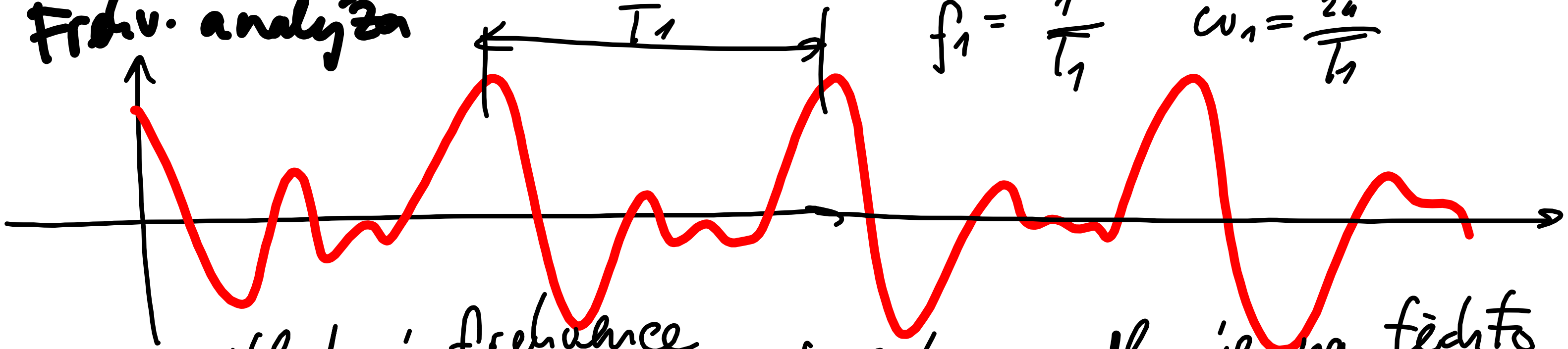
$$=\frac{C_1}{2}e^{j\varphi_1}e^{j\omega_1 t}+\frac{C_1}{2}e^{-j\varphi_1}e^{-j\omega_1 t}$$

start $\frac{C_1}{2}e^{j\varphi}$

start $\frac{C_1}{2}e^{-j\varphi}$



Frekv. analýza




$f_1 = \frac{1}{T_1}$ $\omega_1 = \frac{2\pi}{T_1}$

$\omega_1$ základní frekvence

$k\omega_1$ její násobky → kolik signálu je na těchto násobcích?

$$x(t) = \boxed{C_0} + \boxed{C_1 \cos(\omega_1 t + \varphi_1)} + C_2 \cos(2\omega_1 t + \varphi_2) + \dots + C_k \cos(k\omega_1 t + \varphi_k) + \dots$$

$\boxed{\text{Fourierova řada FR}}$

kolik?

jak je to posunuté

$$x(t) = \boxed{c_0} + \underbrace{c_1 e^{j\omega_1 t} + c_{-1} e^{-j\omega_1 t}} + c_2 e^{j2\omega_1 t} + c_{-2} e^{-j2\omega_1 t} + \dots + c_k e^{jk\omega_1 t} + c_{-k} e^{-jk\omega_1 t} + \dots$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$$

$c_1 = c_{-1}^*$ $c_2 = c_{-2}^*$ $c_k = c_{-k}^*$

$|c_1| = |c_{-1}|$ $|c_2| = |c_{-2}|$ $|c_k| = |c_{-k}|$

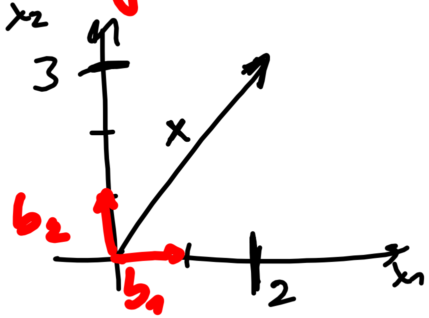
$C_1 = 2 \cdot |c_1|$ $C_2 = 2|c_2|$ $C_k = 2|c_k|$

$\varphi_1 = \arg c_1$ $\varphi_2 = \arg c_2$ $\varphi_k = \arg c_k$

$\varphi_1 = -\arg c_{-1}$ $\varphi_2 = -\arg c_{-2}$ $\varphi_k = -\arg c_{-k}$

$c_k$ ???

# Projekce do bázi (hledání podobnosti) korelace ...

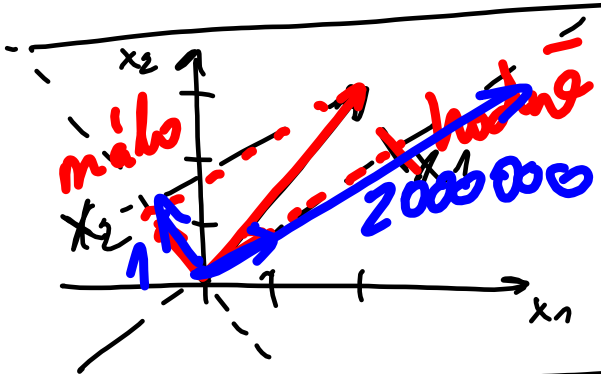



$x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ neznámý vektor

$b_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $b_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ báze (známe!)

$x_1 = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2$

$x_2 = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 3$




$b_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ $b_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$x_1 = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = 3{,}53$ $x_2 = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = 0{,}707$

$x = \begin{bmatrix} 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \end{bmatrix}$ $b_1 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

5,65

$y_n = b_n^T x$

$b_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

$b_2 = \begin{bmatrix} 1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$

ORTOGONALITA $i \neq j$ $b_i^T b_j = 0$

JEDNOTKOVOST $|b_i| = 1$