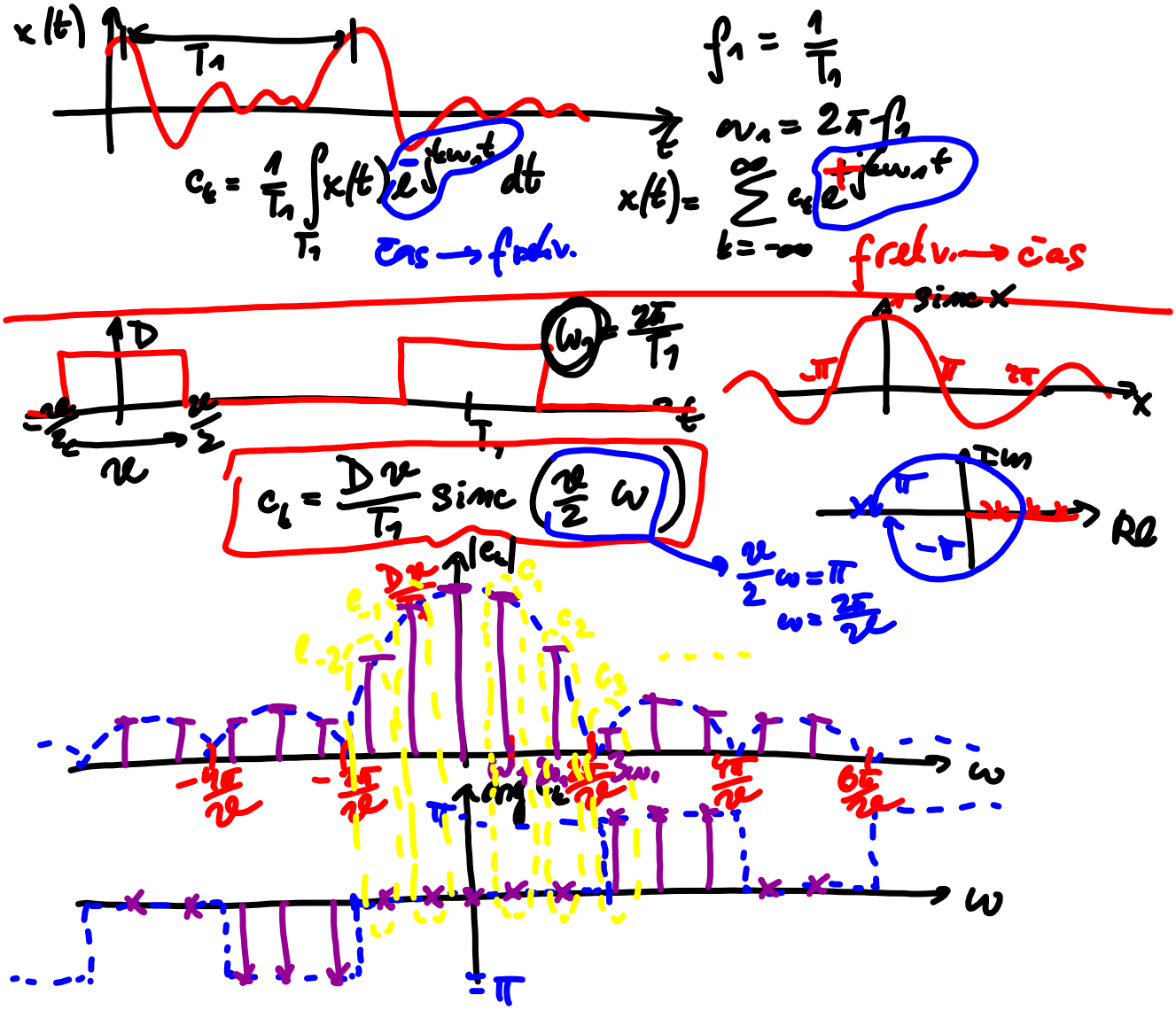
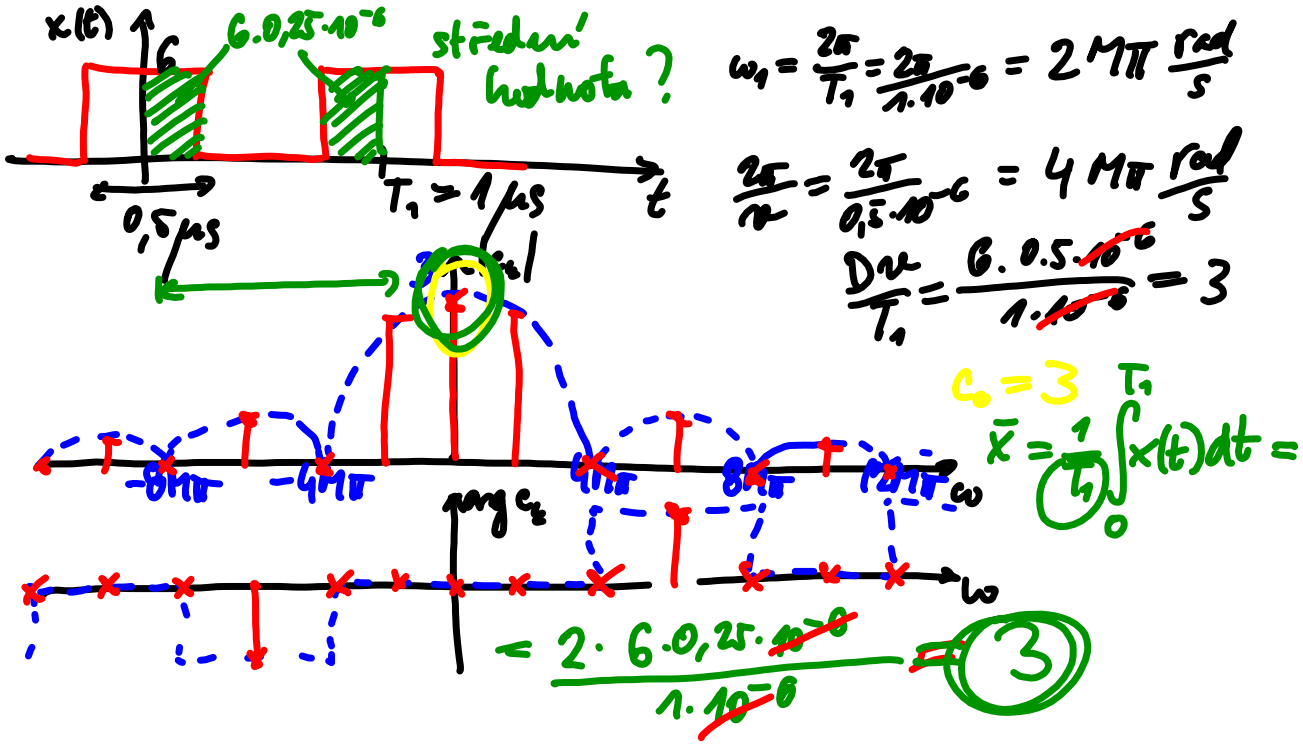

x(t)
T1
t
$f_1 = \frac{1}{T_1}$
$\omega_1 = 2\pi f_1$
$c_k = \frac{1}{T_1}\int_{T_1} x(t) e^{-jk\omega_1 t} dt$
$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{+jk\omega_1 t}$
čas → frekv.
frekv. → čas
D
$\omega_1 = \frac{2\pi}{T_1}$
$c_k = \frac{D\nu}{T_1} \mathrm{sinc}\left(\frac{\nu}{2}\omega\right)$
sinc x
Im
Re
$\frac{\nu}{2}\omega = \pi$
$\omega = \frac{2\pi}{\nu}$
|c_k|
ω



$\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{1\cdot 10^{-6}} = 2M\pi \frac{rad}{s}$

$\frac{2\pi}{\tau} = \frac{2\pi}{0{,}5\cdot 10^{-6}} = 4M\pi \frac{rad}{s}$

$\frac{D\omega}{T_1} = \frac{6\cdot 0.5\cdot 10^{-6}}{1\cdot 10^{-6}} = 3$

$c_0 = 3$

$\bar{x} = \frac{1}{T_1}\int_0^{T_1} x(t)dt =$

$= \frac{2\cdot 6\cdot 0{,}25\cdot 10^{-6}}{1\cdot 10^{-6}} = 3$

$x(t)$

$$c_k = \frac{1}{T_1}\int x(t) e^{-j k\omega_1 t}\,dt$$

$x(t-\tau)$

$$c'_k = \frac{1}{T_1}\int x(t-\tau) e^{-jk\omega_1 t}\,dt = \quad r = t-\tau,\ t = r+\tau$$

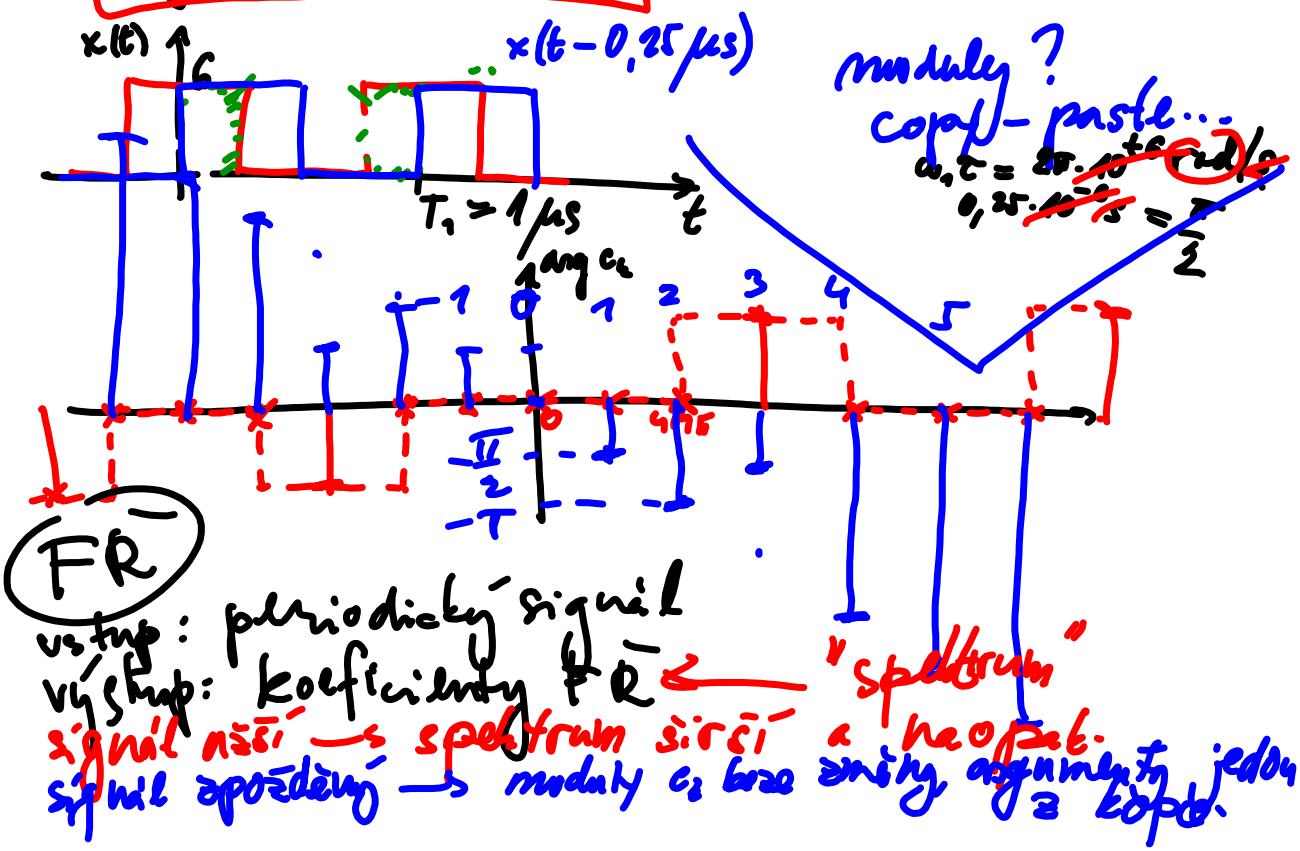
$$= \frac{1}{T_1}\int x(r) e^{-jk\omega_1 (r+\tau)}\,dr = \quad e^{-jk\omega_1 r}\cdot e^{-jk\omega_1 \tau}$$

$$= e^{-jk\omega_1\tau}\cdot \underbrace{\frac{1}{T_1}\int x(r) e^{-jk\omega_1 r}\,dr}_{c_k} =$$

$$= e^{-jk\omega_1\tau}\cdot c_k \qquad \left|e^{-jk\omega_1\tau}\right| = 1$$

$$|c'_k| = |c_k|$$

$$\arg c'_k = -k\omega_1\tau + \arg c_k$$



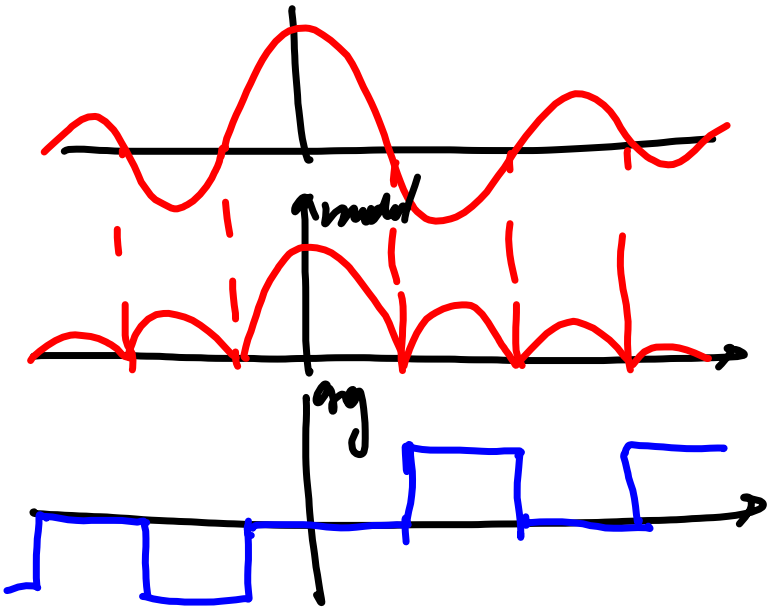
FR

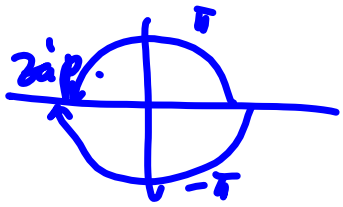
vstup: periodický signál

výstup: koeficienty FŘ ← "spektrum"

signál nižší → spektrum širší a naopak.

signál zpožděný → moduly c_k beze změny, argumenty jedna z kopie.


arg


π
2α
-π

Fourierova transformace F.T.

F.Ř. – periodické

neperiodické → ?

$$c_k = \frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} x(t)\, e^{-jk\omega_1 t}\, dt$$

$T_1 \to \infty$

$T_1 = \frac{2\pi}{\omega_1}$

$\frac{1}{T_1} = \frac{\omega_1}{2\pi}$

$\frac{1}{T_1} = \frac{d\omega}{2\pi}$

$\omega_1 = \frac{2\pi}{T_1} \to d\omega$

$k\omega_1 \to \omega$

$c_k \to dc$

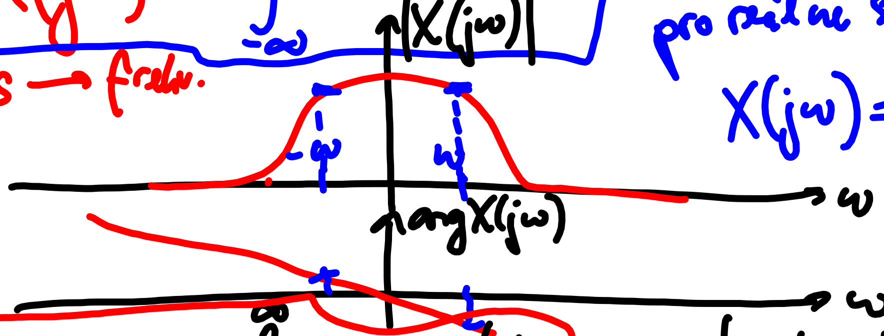
$$2\pi \frac{dc}{d\omega} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

F.T. $X(j\omega)$ spektrální funkce

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

čas → frekv.

pro reálné signály $X(j\omega) = X^*(-j\omega)$

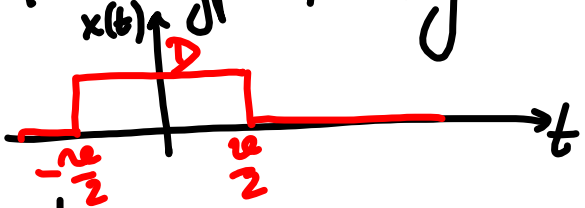


$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

frekv. → čas.

výstup = nějaká konstanta · vstup $e^{\pm j}$ — čas. op. frekv.

F.T. typických signálů



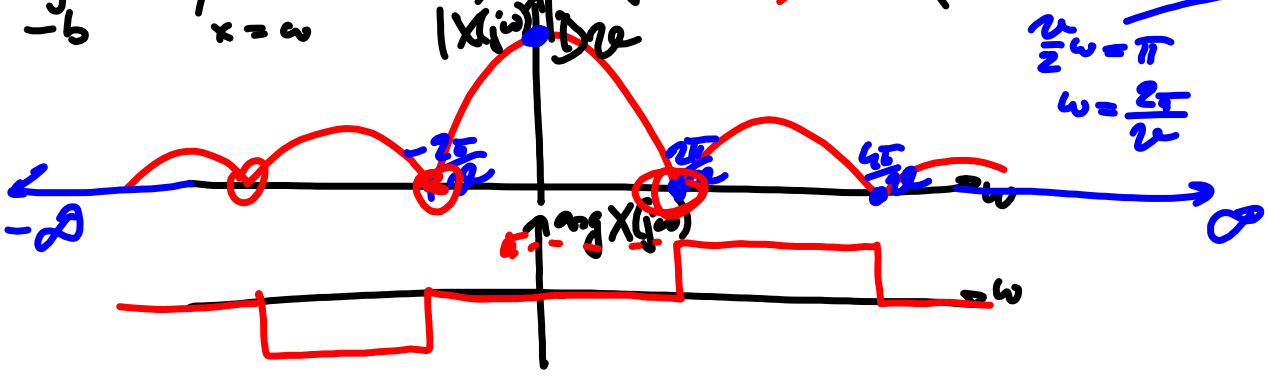
$$X(j\omega) = D\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\,dt$$

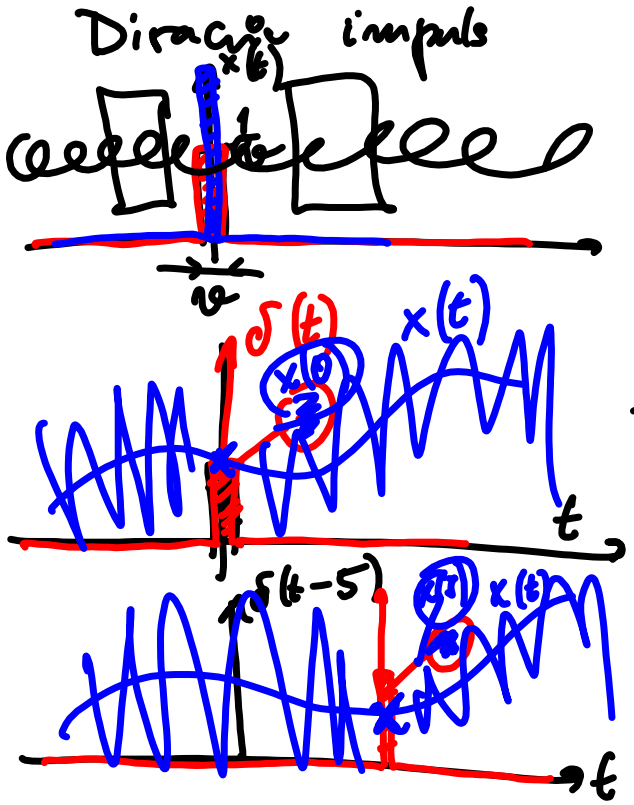
$$\int_{-b}^{b} e^{\pm jxy}\,dy = 2b\,\mathrm{sinc}(bx) \qquad x = \omega$$

$$X(j\omega) = D\frac{\tau}{2}\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right) = D\tau\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$$

$\frac{\tau}{2}\omega = \pi$

$\omega = \frac{2\pi}{\tau}$





$$\int_{-\infty}^{\infty} x(t)\,dt = 1$$

1

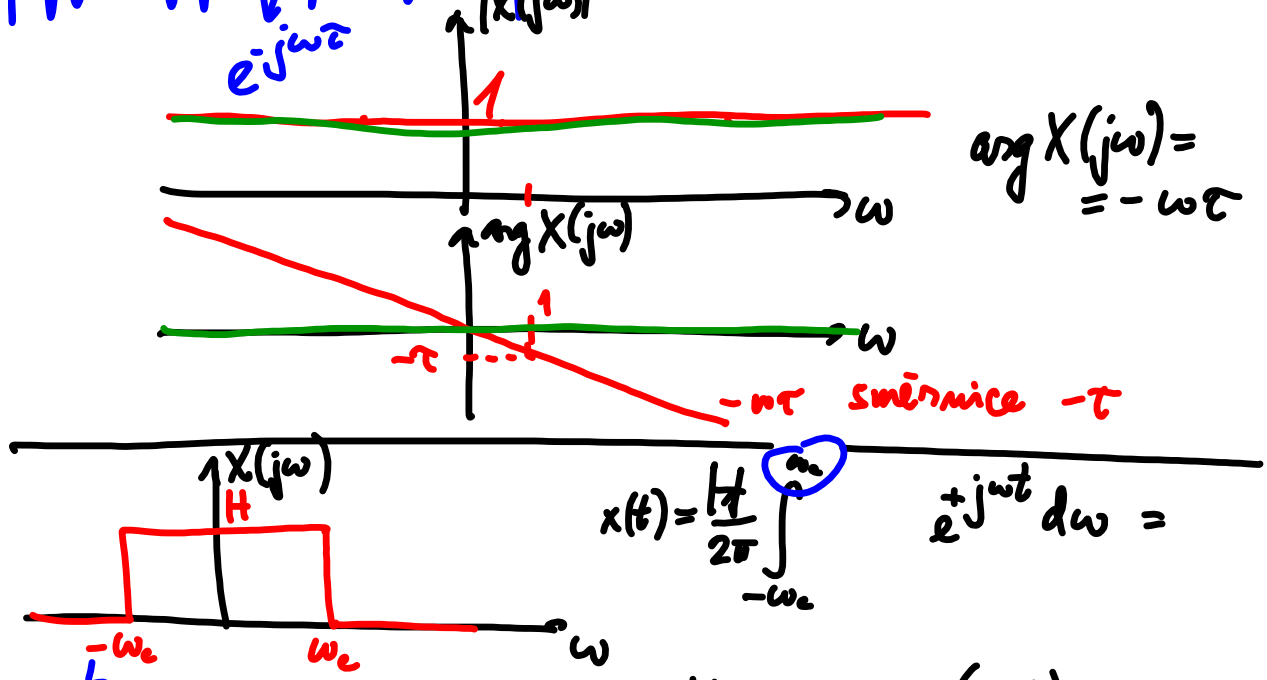
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

vzorkovací schopnost...

$$\int_{-\infty}^{\infty} \delta(t)\,x(t)\,dt = x(0)$$

$$\int_{-\infty}^{\infty} \delta(t-5)\,x(t)\,dt = x(5)$$

F.T.: $X(j\omega) = \int_{-\infty}^{\infty} \delta(t-\tau) e^{-j\omega t} dt = e^{-j\omega\tau}$ (ch.s.k.j.k)



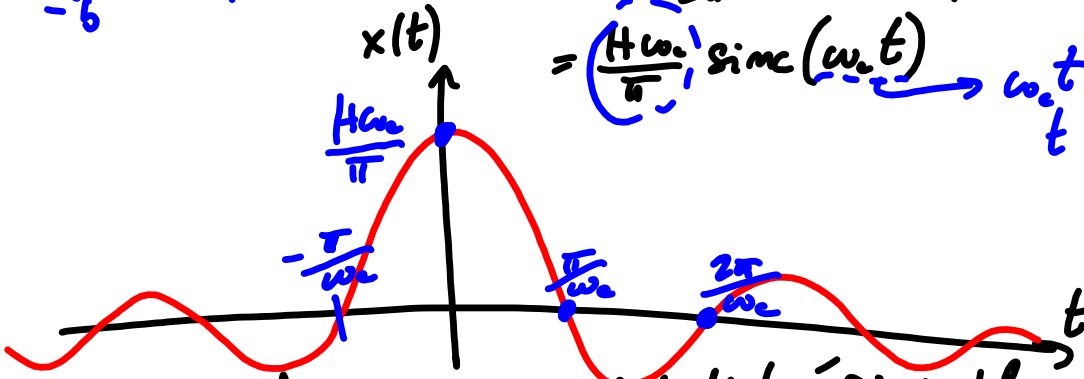
$\arg X(j\omega) = -\omega\tau$

$x(t) = \frac{H}{2\pi} \int_{-\omega_c}^{\omega_c} e^{+j\omega t} d\omega =$

$\int_{-b}^{b} e^{\pm jxy} dy = 2b\,\mathrm{sinc}(bx)$

$= \frac{H}{2\pi} 2\omega_c \,\mathrm{sinc}(\omega_c t) =$

$= \frac{H\omega_c}{\pi} \mathrm{sinc}(\omega_c t)$ → $\omega_c t = \pi$, $t = \frac{\pi}{\omega_c}$



F.T. vstupem je neperiodický signál

výstupem je spoj. funkce definovaná pro všechny frekvence.

Jinak dtto F.Ř.:

$c_k = c_{-k}^*$

$x(t) \to c_k$

$x(t-\tau) \to c_k e^{-jk\omega_0\tau}$

$X(j\omega) = X^*(-j\omega)$

$x(t) \to X(j\omega)$

$x(t-\tau) \to X(j\omega) e^{-j\omega\tau}$

nemění modul posouvá argumenty

Systémy

kauzalita
linearita
časová invariance
time invariance
stabilita
} LTI