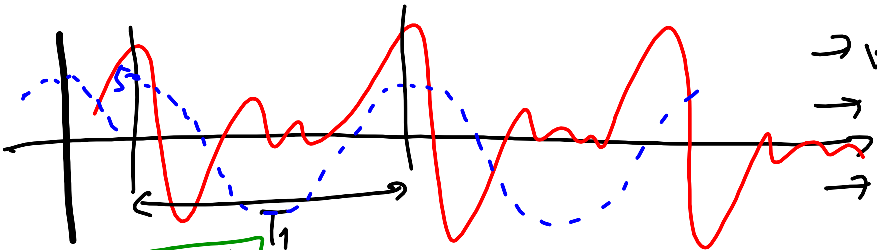
Half-semestral exam - půlsemestrálka

Wed 30th October 9^{45} - 10^{45}

D105/D0206 not ~~Friday~~!!!!

assignment in Czech.

- 1st 2 lectures
- 1st 2 num. exercises.



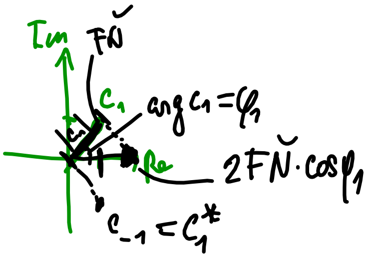
→ which frequencies? $f_1 = \frac{1}{T_1}$

→ how much? 5

→ how shifted? $-\frac{\pi}{2}$

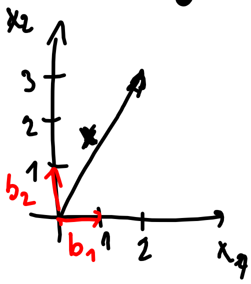
$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k \cdot \underbrace{e^{jk\omega_1 t}}_{\text{basis}}} = \underbrace{c_0}_{\substack{\text{constant} \\ \text{"DC" component}}} + \underbrace{c_1 e^{j1\omega_1 t} + c_{-1} e^{-j1\omega_1 t}}_{2|c_1|\cos(\omega_1 t + \arg c_1)} + \underbrace{c_2 e^{j2\omega_1 t} + c_{-2} e^{-j2\omega_1 t}}_{2|c_2|\cos(2\omega_1 t + \arg c_2)} + \dots + \underbrace{c_k e^{jk\omega_1 t} + c_{-k} e^{-jk\omega_1 t}}_{2|c_k|\cos(k\omega_1 t + \arg c_k)} + \dots$$

Fourier series "synthesis" formula.

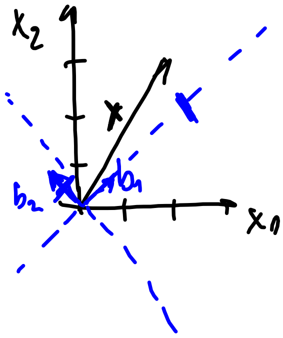


PROJECTION TO BASES ??

$c_i = b_i^T x$ dot product



$c_1 = [1\ \ 0]\begin{bmatrix}2\\3\end{bmatrix} = 2$ $\qquad c_1 = [0\ \ 1]\begin{bmatrix}2\\3\end{bmatrix} = 3$



$b_1 = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$ $\qquad c_1 = \left[\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right]\begin{bmatrix}2\\3\end{bmatrix} = 3{,}53$

$b_2 = \begin{bmatrix}-\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$ $\qquad c_2 = \left[-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right]\begin{bmatrix}2\\3\end{bmatrix} = 0{,}7$

ORTHOGONALITY

$[1\ \ 0]\begin{bmatrix}0\\1\end{bmatrix} = 0$

$\left[\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right]\begin{bmatrix}-\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = -\frac{1}{2} + \frac{1}{2} = 0$

NORMALITY

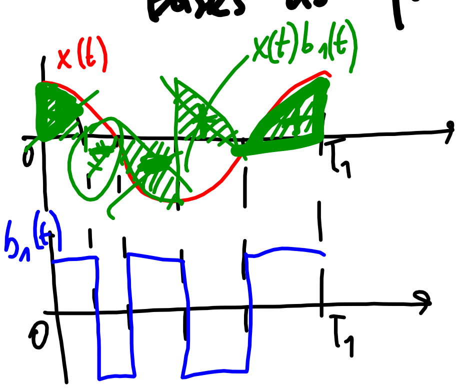
$\|b\| = 1$ $\qquad \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

ORTHONORMAL set of bases

$x = [3\ 2\ 1\ 0\ 1\ 2\ 3\ 4]^T$ $\qquad b_1 = \frac{1}{\sqrt{8}}[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]^T$

$c_1 = b_1^T \cdot x = 5{,}6.$

Bases as functions!



$$c_1 = \int_0^{T_1} x(t)\, b_1(t)\, dt$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{jk\omega_1 t}}$$

coefficients bases

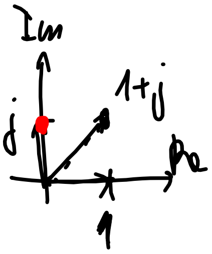
$$\boxed{c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt}$$

Fourier series analysis

T_1 ?? $0 \ldots T_1$? ✓

$-\frac{T_1}{2} \ldots \frac{T_1}{2}$? ✓

$1000T_1 \ldots 1001T_1$? ✓

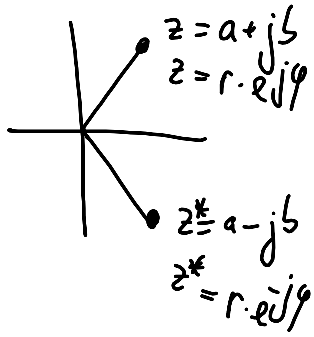


$\mathrm{Re}\big((1+j)\cdot 1\big) = 1$ ✓

$\mathrm{Re}\big((1+j)\cdot j\big) = -1$ ✗

j^{-1}

$\mathrm{Re}\big((1+j)(-j)\big) = 1$ ✓



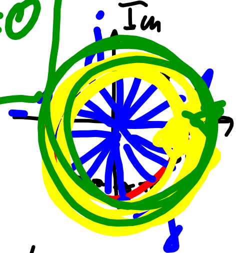
ORTHOGONAL?

$$\int_0^{T_1} e^{jk\omega_1 t} \cdot e^{-jl\omega_1 t}\, dt = \boxed{\int_0^{T_1} e^{j(k-l)\omega_1 t}\, dt = 0} \qquad k \neq l \qquad e^a \cdot e^b = e^{a+b}$$

$k - l = 1$ $\int_0^{T_1} e^{j\omega_1 t}\, dt = 0$

$t = \frac{T_1}{8}$

$e^{j\frac{2\pi}{T_1}\cdot\frac{T_1}{8}} = e^{j\frac{\pi}{4}}$



⇒ ARE ORTHO!

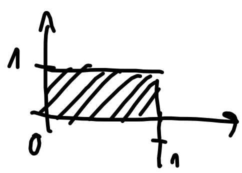
$$\int_0^{T_1} e^{j7\omega_1 t}\, dt = 0 \qquad \int_0^{T_1} e^{-j3\omega_1 t}\, dt = 0$$

NORMAL? $\|b\| = \sum b_i^2$

$$\|e^{jk\omega_1 t}\| = \int_0^{T_1} \underbrace{|e^{jk\omega_1 t}|^2}_{1} dt = \int_0^{T_1} 1\, dt$$

$$|e^{j\,\text{anything}}| = 1$$

$$= T_1$$



COSINE

$x(t) = 5\cos(\underbrace{1000\pi}_{\omega_1} t + \frac{\pi}{4})$

F. S.

$\cancel{c_k = \frac{1}{T_1}\int_0^{T_1} x(t)\, e^{-jk\omega_1 t}\, dt}$

$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$

$x(t) = \frac{5}{2} e^{j(1000\pi t + \pi/4)} + \frac{5}{2} e^{-j(1000\pi t + \pi/4)} =$

$= \frac{5}{2} e^{j\pi/4} \underbrace{e^{j1000\pi t}}_{e^{j1\omega_1 t}} + \frac{5}{2} e^{-j\frac{\pi}{4}} \underbrace{e^{-j1000\pi t}}_{e^{j(-1)\omega_1 t}}$

$c_1 = 2.5\, e^{j\pi/4}$ $\qquad c_{-1} = 2.5\, e^{-j\pi/4}$

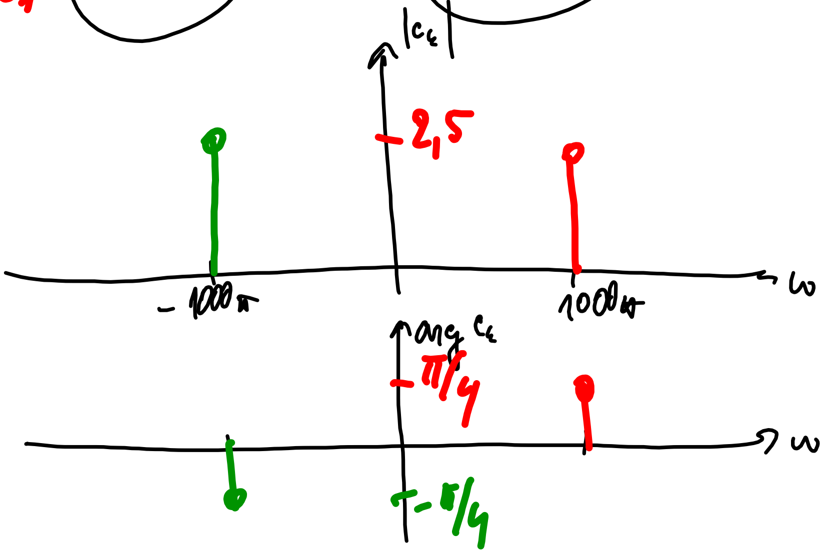
$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}\, dt$

$c_1 e^{j\omega_1 t} + c_{-1} e^{-j\omega_1 t}$

$e^{a+b} = e^a e^b$

$f_1 = 500\,\text{Hz}$

$T_1 = 2\,\text{ms}$



$$x(t) = 5\cos\left(1000\pi t + \pi/4\right) + 2\cos\left(3000\pi t - \frac{\pi}{2}\right)$$

$c_1 = 2{,}5\, e^{j\pi/4}$

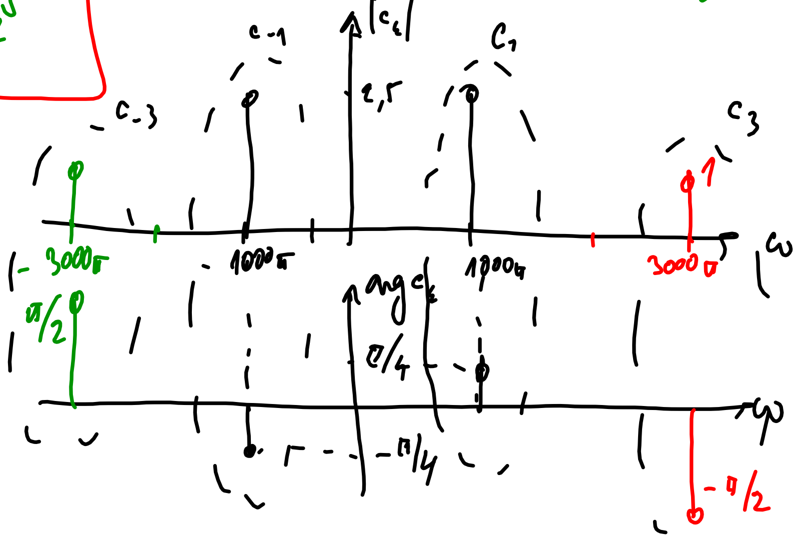
$c_{-1} = 2{,}5\, e^{-j\pi/4}$

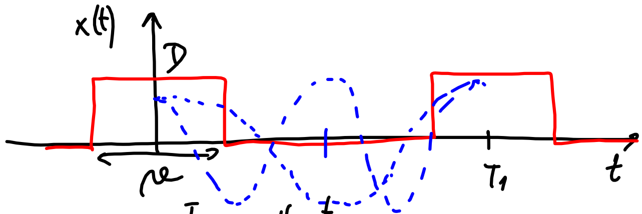
$c_3 = 1\cdot e^{-j\pi/2}$

$c_{-3} = 1\, e^{j\pi/2}$

Same

$$\underbrace{\frac{2}{2}e^{-j\pi/2}}_{c_3} e^{j3\cdot 1000\pi t} + \underbrace{\frac{2}{2}e^{+j\pi/2}}_{c_{-3}} e^{-j3\cdot 1000\pi t}$$





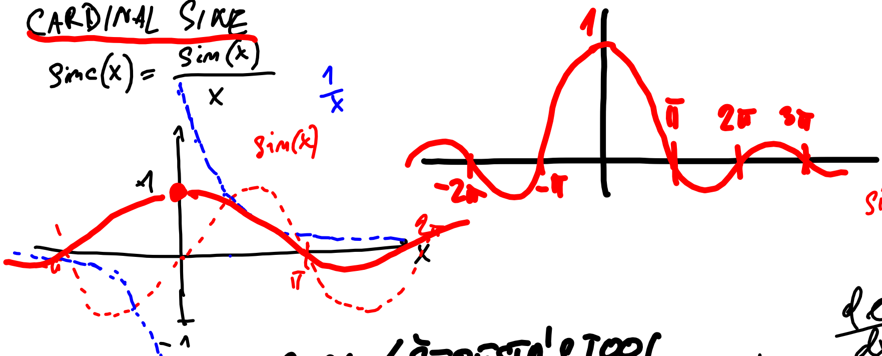
Series of rectangles...

$$c_k = \frac{1}{T_1}\int_0^{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$$

PREPARATION

CARDINAL SINE

$$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$$



$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

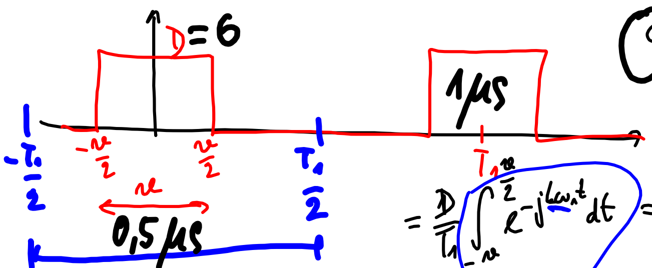
$$\frac{d e^x}{dx} = e^x$$

EULER'S INTEGRAL / ŠEBESTA'S TOOL

$$\int_{-b}^{b} e^{jxy}\, dy = \left[\frac{e^{jxy}}{jx}\right]_{-b}^{b} = \frac{e^{jxb}}{jx} - \frac{e^{-jxb}}{jx} = \frac{2}{x}\,\frac{e^{jxb} - e^{-jxb}}{2j} = \frac{b\,2}{b\,x}\sin(xb) = 2b\,\mathrm{sinc}(bx)$$

$$\frac{e^{jxy}}{dy} = e^{jxy}\cdot jx$$

multiplication!



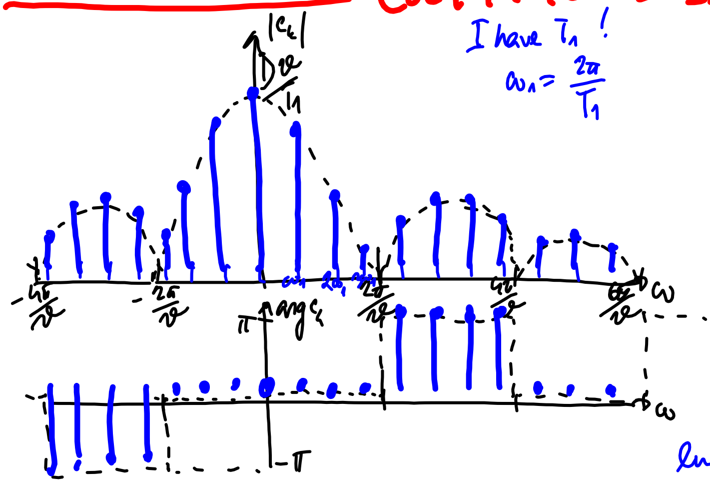
$$c_k = \frac{1}{T_1}\int_{-T_1/2}^{T_1/2} x(t)\, e^{-jk\omega_1 t}\, dt = \frac{D}{T_1}\int_{-\vartheta/2}^{\vartheta/2} e^{-jk\omega_1 t}\, dt = \frac{D}{T_1}\,\vartheta\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right)$$

$\frac{\vartheta}{T_1}$ střída
duty cycle

COEFFICIENTS BY HAND

I have T_1!

$$\omega_1 = \frac{2\pi}{T_1}$$



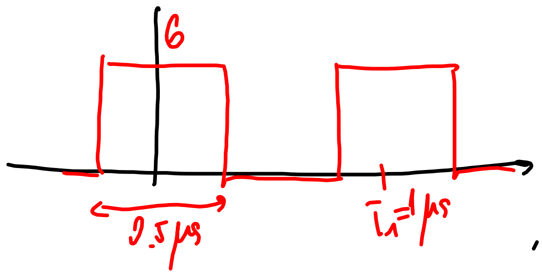
Step one: draw aux function for all frequencies

$$\frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right)$$

$$\frac{\vartheta}{2}\omega = \pi \quad ?$$

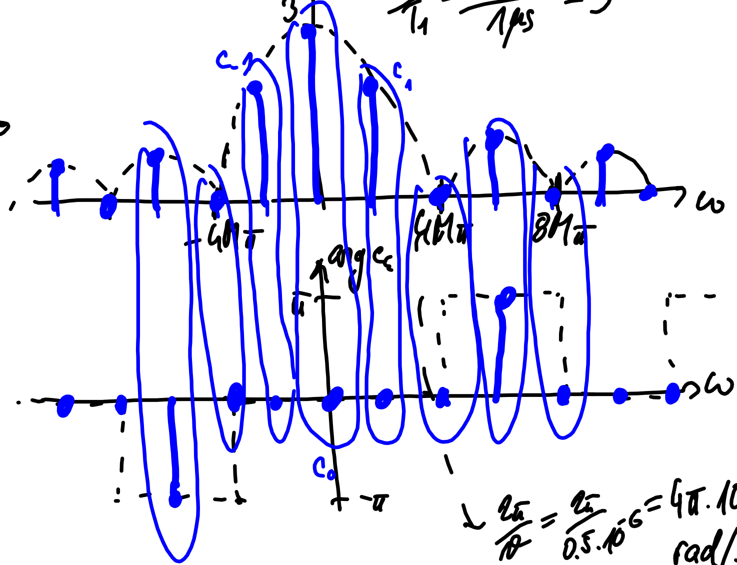
$$\omega = \frac{2\pi}{\vartheta}$$

Step two: shoot coefficients under the aux. function



$$\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{1\cdot10^{-6}} = 2M\pi$$

$$|c_k| \quad \frac{D\,m}{T_1} = \frac{6\cdot 0.5\mu s}{1\mu s} = 3$$



$$\frac{2\pi}{D} = \frac{2\pi}{0.5\cdot10^{-6}} = 4\pi\cdot10^{6} \text{ rad/s}$$

$4M\pi$

$5 = 5^*$

$-5 = (-5)^*$ $\quad 5e^{j\pi}$ $\quad 5e^{-j\pi}$