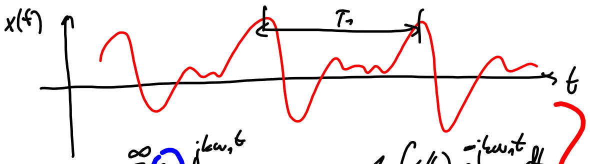
Půlsemestrálka

středa 30.10. první hodina přednášky

8:00 – 9:00 : D105, D0207, E105

→ první 2 přednášky

→ první 2 num. cvičení.



$f_1 = \frac{1}{T_1}$ [Hz]

$\omega_1 = 2\pi f_1 = \frac{2\pi}{T_1}$ [rad/s]

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$

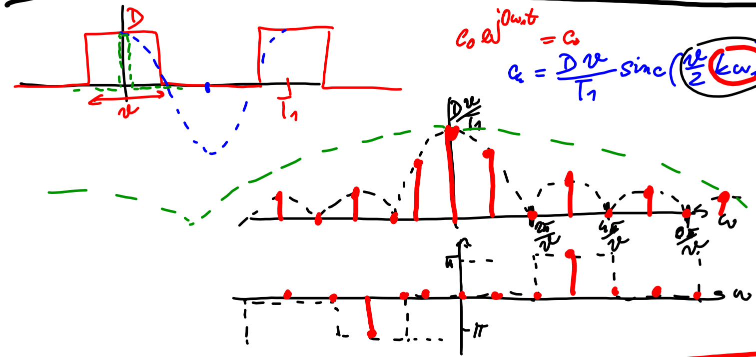
SYNT. VZOREC

$c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$

ANAL. VZOREC

Fourierova řada

SPEKTRUM!



$c_0 e^{j0\omega_1 t} = c_0$

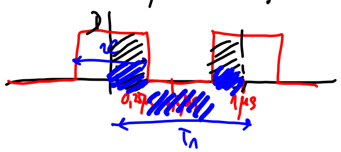
$c_k = \frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right)$

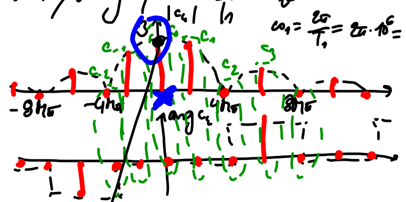
$D = 6 \quad T_1 = 1\,\mu s \quad \vartheta = 0{,}5\,\mu s$

Střída / duty cycle $\frac{\vartheta}{T_1}$

$\frac{2\pi}{\vartheta} = 4\pi\cdot 10^6 = 4\pi M$

$\omega_1 = \frac{2\pi}{T_1} = 2\pi\cdot 10^6 = 2\pi M$





c_0 odpovídá střední hodnotě?

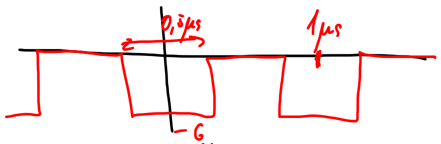
$\bar{x} = \frac{1}{T_1}\int_{T_1} x(t)\, dt = \frac{D\vartheta}{T_1}$

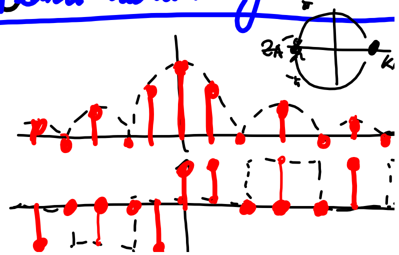
$\frac{D\vartheta}{T_1}$

$x(t) = \sum_{k=\infty}^{\infty} c_k e^{jk\omega_1 t}$

Posuny střední hod. signálu?

- pouze změna c_0. c_0 = střední hodnota signálu.

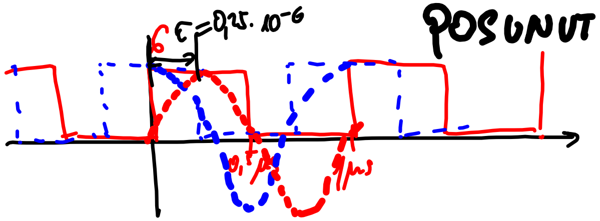


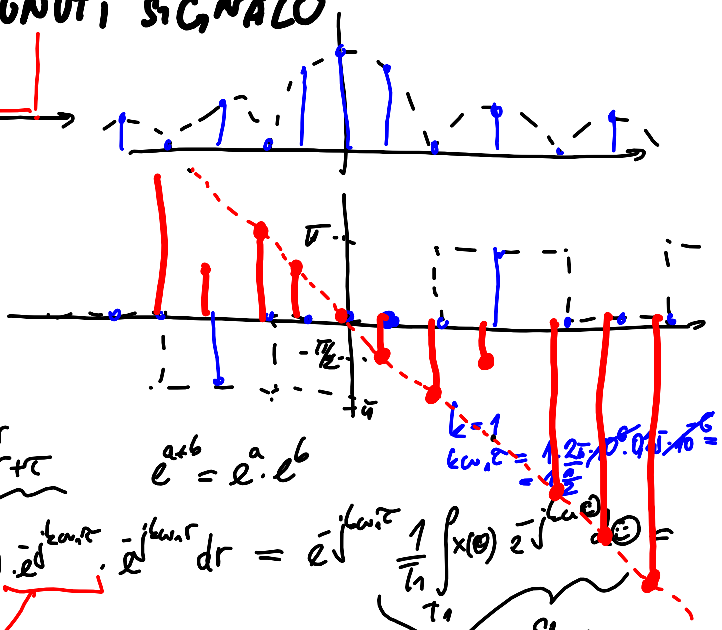


$x(t) \to -x(t)$

$c_k \to -c_k$

POSUNUTÍ SIGNÁLU





$$c_{k\,\text{pův}} = \frac{1}{T_1}\int_{T_1} x(t)\,e^{-jk\omega_1 t}\,dt$$

$$x(t) \longrightarrow x(t-\tau)$$

$t - \tau = r$

$t = r + \tau$

$e^{a+b} = e^a \cdot e^b$

$$c_k = \frac{1}{T_1}\int_{T_1} x(\underbrace{t-\tau}_{r})\,e^{-jk\omega_1 t}\,dt = \frac{1}{T_1}\int_{T_1} x(r)\,e^{-jk\omega_1 (r+\tau)}\,dr = \frac{1}{T_1}\int_{T_1} x(r)\,e^{-jk\omega_1 r}\cdot e^{-jk\omega_1 \tau}\,dr = e^{-jk\omega_1\tau}\,\underbrace{\frac{1}{T_1}\int_{T_1} x(\theta)\,e^{-jk\omega_1\theta}\,d\theta}_{c_{k\,\text{pův}}} =$$

$$= e^{-jk\omega_1\tau}\cdot c_{k\,\text{pův}}$$

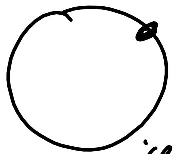
$k=1$

$k\omega_1\tau = 1\cdot\frac{2\pi}{\dots}\cdot 0.25\cdot 10^{-6} = \frac{\pi}{2}$

$\tau = 0.25\cdot 10^{-6}$

$$|c_k| = |c_{k\,\text{pův}}|\cdot|e^{-jk\omega_1\tau}| = |c_{k\,\text{pův}}|$$

$$\arg c_k = \arg c_{k\,\text{pův}} + \arg e^{-jk\omega_1\tau} = \arg c_{k\,\text{pův}} - k\omega_1\tau$$



$r\cdot e^{j\varphi}$

FŘ - summary

c frekv. analýza, koef F.Ř. c spektrum.

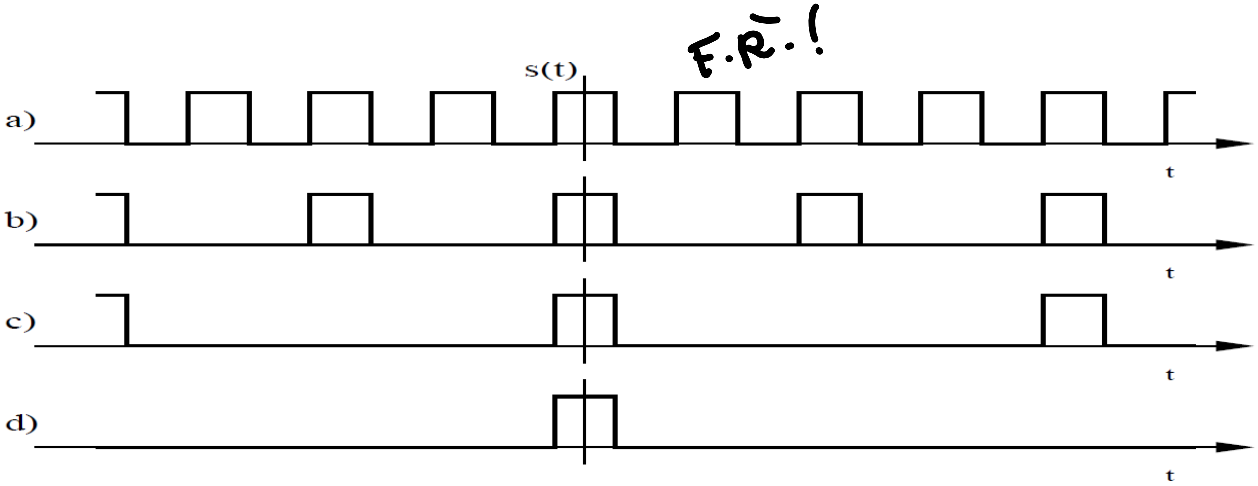
vstup: periodický signál výstup: koeficienty

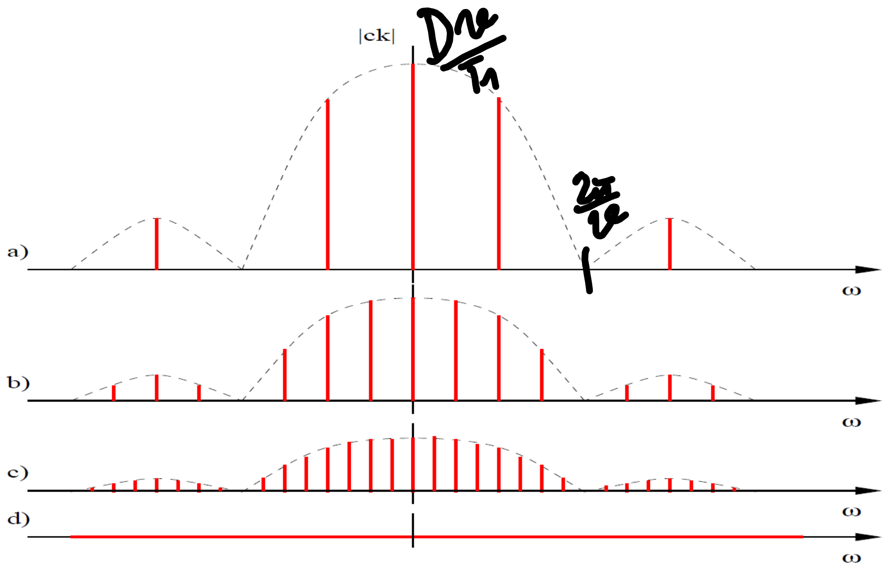
užší signál → širší spektrum a naopak.

zpožděný signál → moduly stejné, ale argumenty jedou z kopce.

Fourierova transformace

Frekv. analýza neperiodických signálů





MASÁŽ F.Ř. č. 2

$$c_k = \frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} x(t)\, e^{-jk\omega_1 t}\, dt$$

$T_1 \to \infty$

$T_1 = \frac{2\pi}{\omega_1}$ $\qquad \frac{1}{T_1} = \frac{\omega_1}{2\pi}$

$\omega_1 = \frac{2\pi}{T_1} \to d\omega$

$\frac{1}{T_1} = \frac{d\omega}{2\pi}$

$c_k \to dc$

$k\omega_1 ? \to \omega$

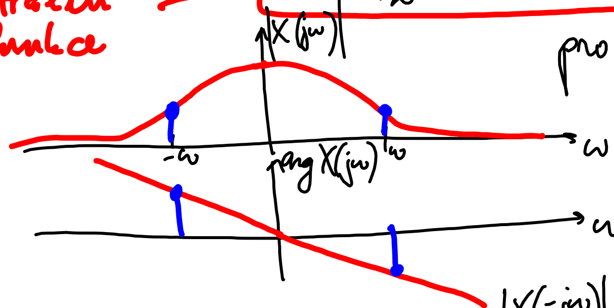
$$dc = \frac{d\omega}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\cancel{2\pi \frac{dc}{d\omega}} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Spektrální funkce

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

Fourierova transformace



pro reálný signál $x(t) \in \mathbb{R}$

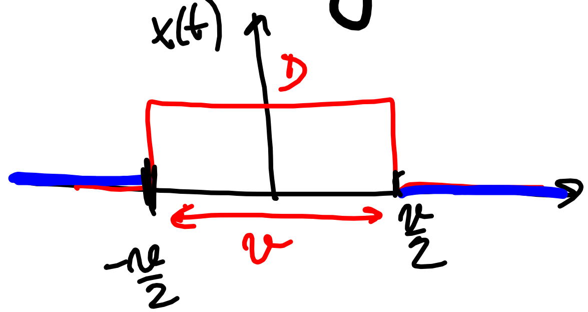
$X(-j\omega) = X^*(j\omega)$

$|X(-j\omega)| = |X(j\omega)|$

$\arg X(-j\omega) = -\arg X(j\omega)$

	$t \to f$	$f \to t$	
period.	$c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$	$x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{+jk\omega_1 t}$	F.Ř.
neper.	$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$	$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\, d\omega}$	F.T.

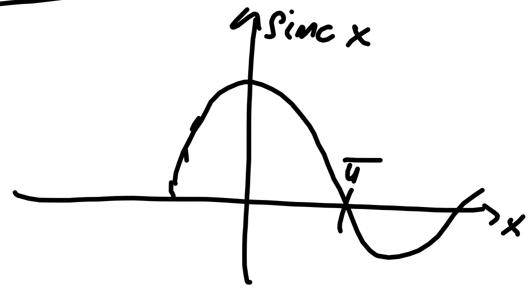
F.T. typických signálů



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = D \int_{-\frac{\vartheta}{2}}^{\frac{\vartheta}{2}} e^{-j\omega t}\, dt =$$

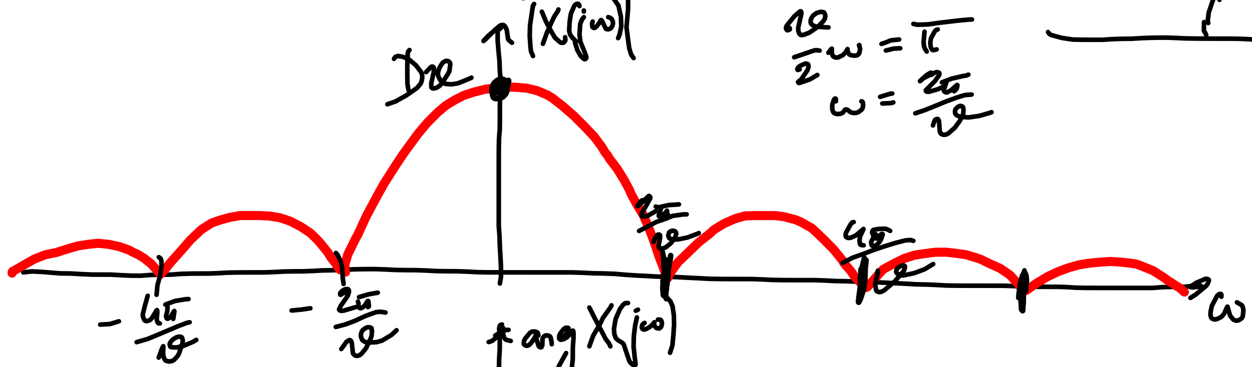
Š.P.: $\int_{-b}^{b} e^{\pm jxy}\, dy = 2b\, \mathrm{sinc}(bx)$

$$= D\, 2 \cdot \frac{\vartheta}{2}\, \mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right) = \underline{D\vartheta\, \mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right)}$$

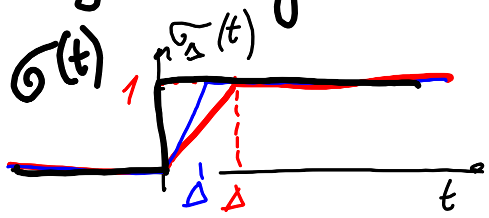


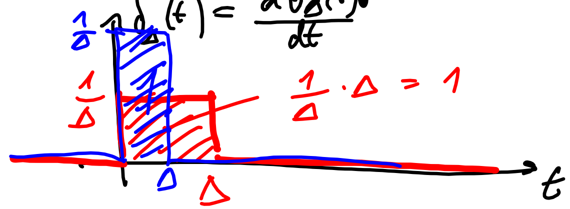
$\frac{\vartheta}{2}\omega = \pi$

$\omega = \frac{2\pi}{\vartheta}$

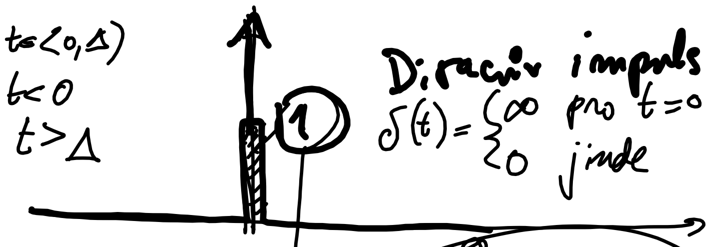


Jednotkový skok a Diracův impuls.





$$\sigma(t) = \lim_{\Delta \to 0} \begin{cases} \frac{1}{\Delta} & \text{pro } t \in \langle 0, \Delta) \\ 0 & \text{pro } t < 0 \\ 1 & \text{pro } t > \Delta \end{cases}$$

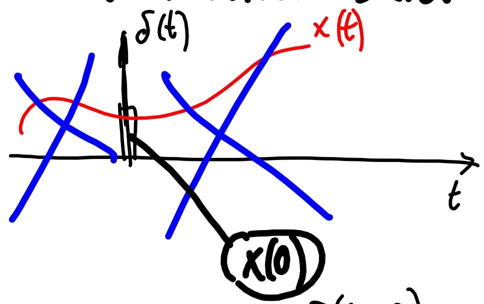


Diracův impuls

$$\delta(t) = \begin{cases} \infty & \text{pro } t = 0 \\ 0 & \text{jinde} \end{cases}$$

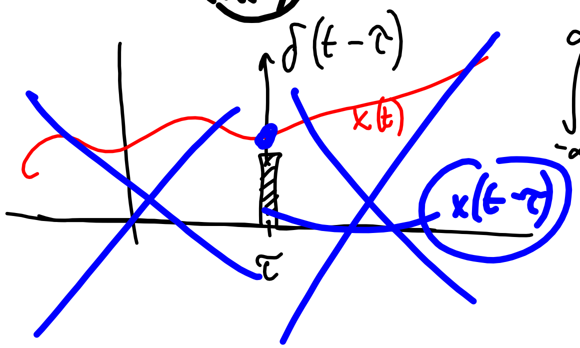
a mocnost $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

VZORKOVACÍ SCHOPNOST D.I.



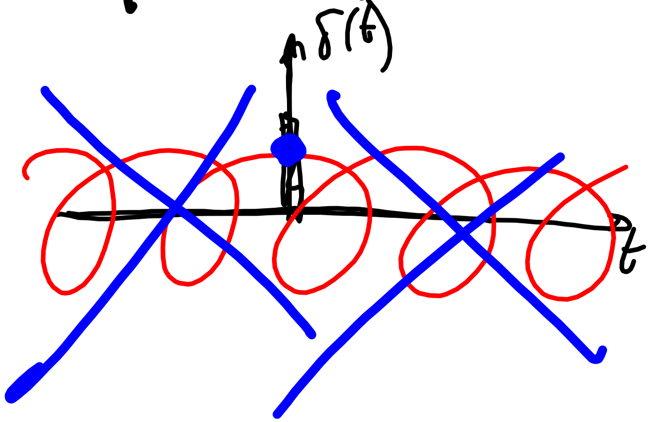
$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$

"přečte"
"navzorkuje"
1 hodnotu signálu.



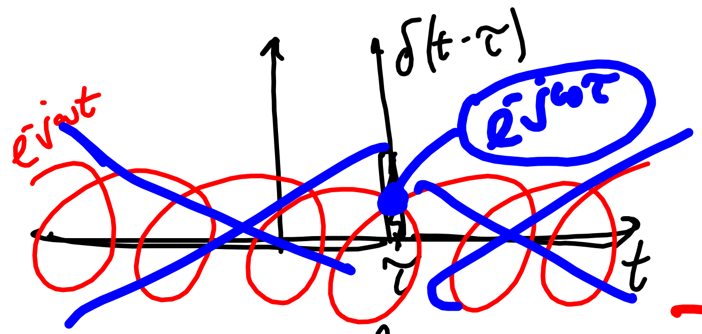
$$\int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,dt = x(t-\tau)$$

F.T. Diracova impulsu.

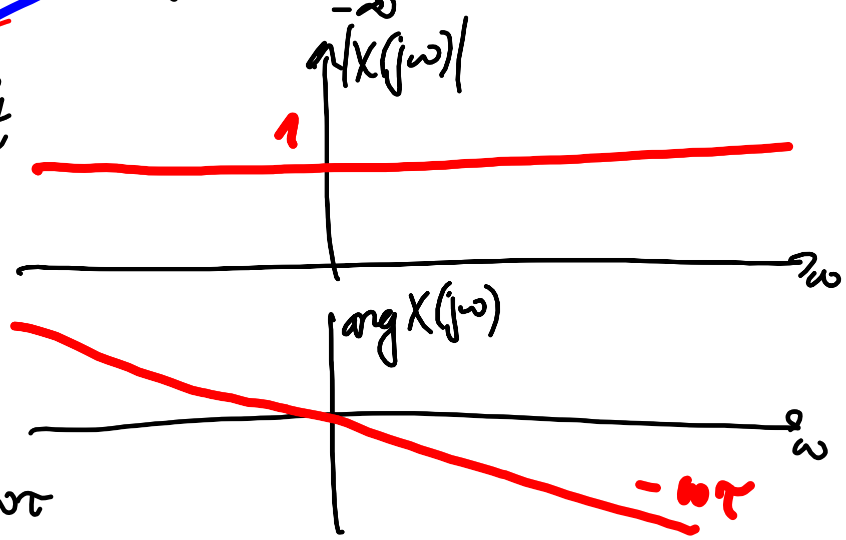


$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, \underbrace{e^{-j\omega t}}\, dt = 1$$

$$e^{-j\omega 0} = e^{0} = 1$$



$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t-\tau)\, e^{-j\omega t}\, dt = e^{-j\omega\tau}$$



Posunutí signálu:

$$x(t) \longrightarrow x(t-\tau)$$

$$X(j\omega) \longrightarrow X(j\omega)\, e^{-j\omega\tau}$$

modul se nezmění!

k argumentu se přičte funkce $-\omega\tau$