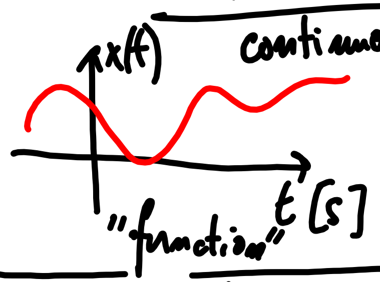
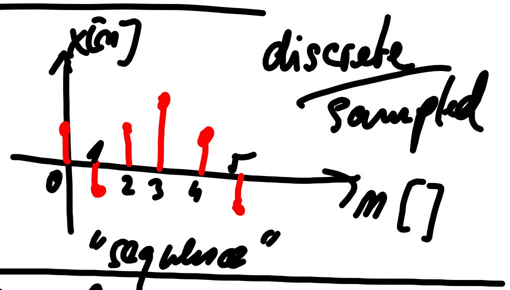
MATH INTRO





Operations with 2 signals

$x_1(t)$
$x_2(t)$

$$y(t) = x_1(t) + x_2(t)$$
$$y(t) = x_1(t) - x_2(t)$$
$$y(t) = x_1(t)\,x_2(t)$$

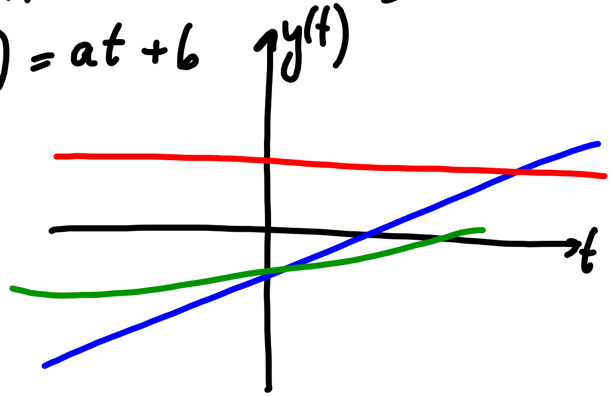
do the operation for all times!

$x_1[n]$
$x_2[n]$

$$y[n] = x_1[n] + x_2[n]$$
$$= x_1[n] - x_2[n]$$
$$= x_1[n] \cdot x_2[n]$$

LINEAR FUNCTIONS

$$y(t) = at + b$$



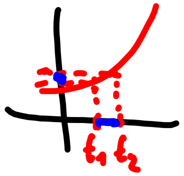
DERIVATIVES & INTEGRALS

$$x(t) = t^2 + 2t + 3$$

DERIVATIVE in a given time $t_1 = 0$

1. Analytic solution $\frac{dx(t)}{dt} = 2t + 2$

$$\left.\frac{dx(t)}{dt}\right|_{t_1=0} = 2$$



2. Numerically $\left.\frac{dx(t)}{dt}\right|_{t_1} \approx \frac{x(t_2) - x(t_1)}{t_2 - t_1}$

Integration – finite integral from t_1 to t_2

$t_1 = 0$ and $t_2 = 1$

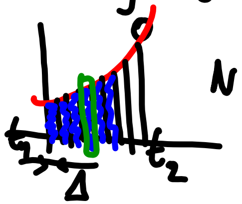
1. Analytically $\int_{t_1}^{t_2} x(t)\,dt = \left[x_p(t) \right]_{t_1}^{t_2} = x_p(t_2) - x_p(t_1)$

$$x(t) = t^2 + 2t + 3$$

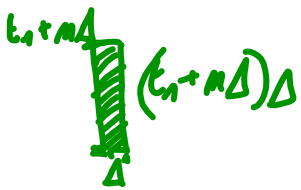
$$\int \ldots = \left[\frac{t^3}{3} + t^2 + 3t \right]^1 = \frac{1}{3} + 1 + 3 - 0 = \underline{4\tfrac{1}{3}}$$

2. Numerically!

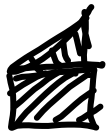
$$\int_{t_1}^{t_2} x(t)\,dt \approx \Delta \sum_{n=0}^{N-1} x(t_1 + n\Delta)$$



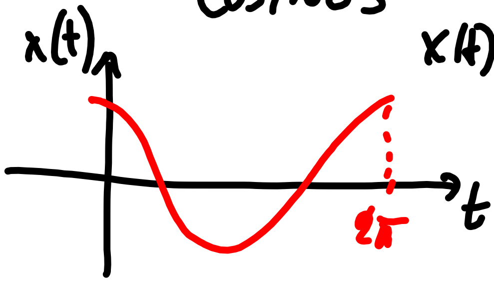
N "noodles"



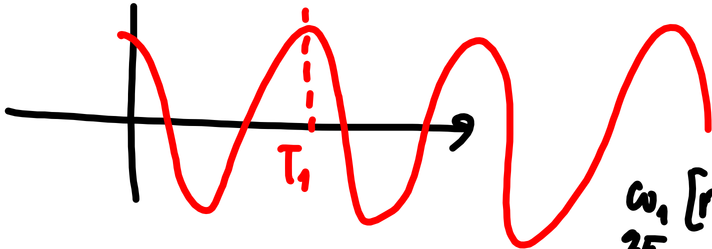
3. "piggy" = 3+1,5 = 4,5



COSINES



$x(t)=\cos(t)$



T_1 [sec] period

$x(t)=\cos\left(2\pi \frac{t}{T_1}\right)$

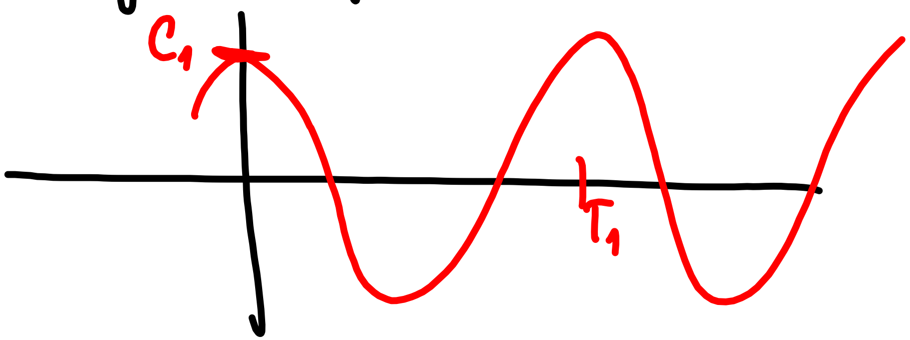
ω_1 [rad/s] angular frequency

$\omega_1 = \frac{2\pi}{T_1}$

$f_1 = \frac{1}{T_1}$ normal freq. [Hz = $\frac{1}{s}$]

$x(t)=C_1\cos(\omega_1 t+\varphi_1)$

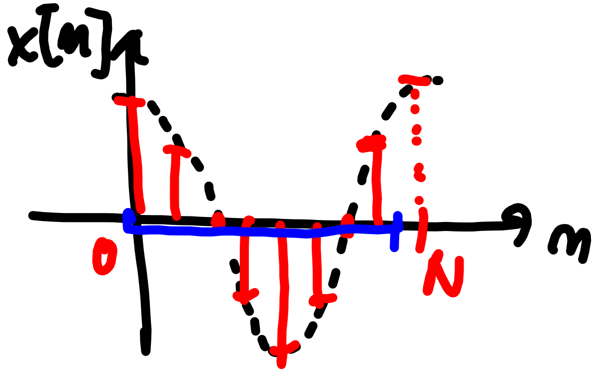
magnitude ang. freq. initial phase [rad]



$\varphi_1>0 \Rightarrow$ left

$\varphi_1<0 \Rightarrow$ right

COSINE IN DISCRETE TIME



$$x[n] = \cos\left(2\pi \frac{n}{N}\right)$$

$\omega_1 = \frac{2\pi}{N}$ normalized angular frequency [rad]

$f_1 = \frac{1}{N}$ normalized frequency [ ]

period of N
0 ... N-1

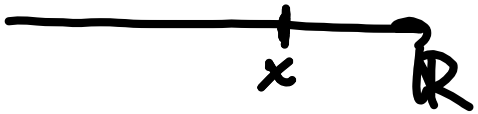
$$x[n] = C_1 \cos(\underbrace{\omega_1 n}_{k\frac{2\pi}{N}n} + \varphi_1)$$

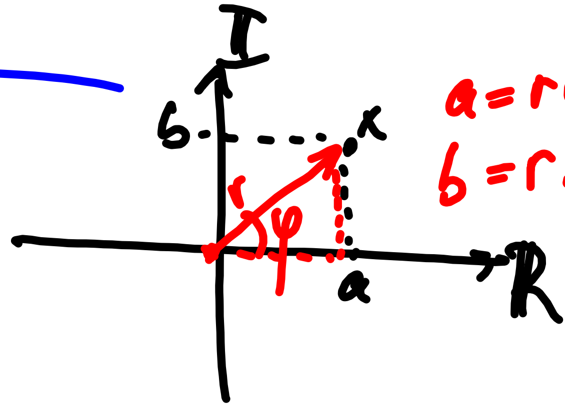
mag.

norm. ang. freq [rad]

init. phase [rad]

COMPLEX NUMBERS



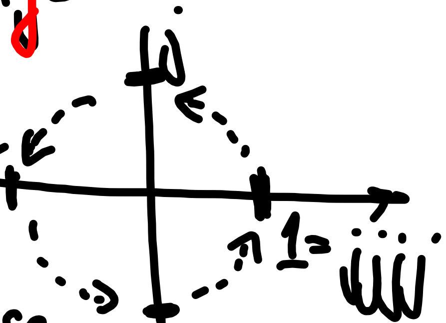


$a = r\cos\varphi$
$b = r\sin\varphi$

i
$x = a + jb$

$j = \sqrt{-1}$

1 j j j j



Polar coordinates
r – modulus, abs. value, magnitude
φ – argument, phase, angle

$x = r\cos\varphi + j\sin\varphi$

$r = \sqrt{a^2+b^2}$
$\varphi = \operatorname{arc\,tan}\frac{b}{a}$

phi = np.angle(x)

Exponential form
$x = re^{j\varphi}$

~~$r\angle\varphi$~~

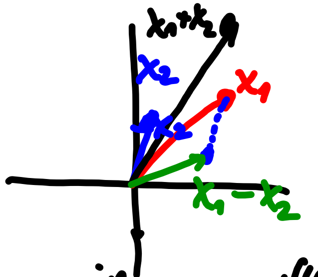
Operations

$x_1 = a_1 + jb_1$

$x_2 = a_2 + jb_2$

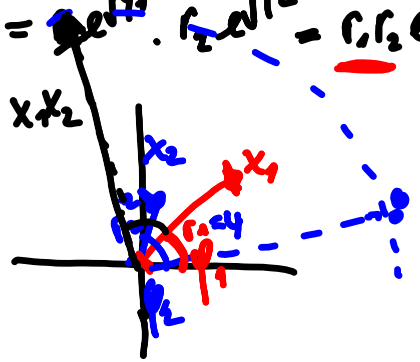
$x_1 + x_2 = a_1 + a_2 + j(b_1 + b_2)$

$x_1 - x_2 = a_1 - a_2 + j(b_1 - b_2)$



$$\boxed{e^a \cdot e^b = e^{a+b}}$$

$$x_1 x_2 = r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2} = r_1 r_2 e^{j(\varphi_1+\varphi_2)}$$

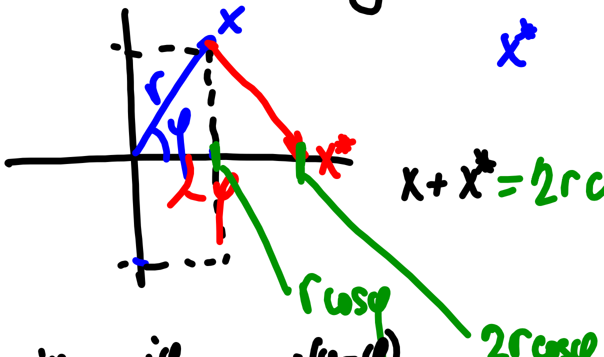


COMPLEX CONJUGATION

$x = a + jb = re^{j\varphi}$

$x^* = a - jb = re^{-j\varphi}$

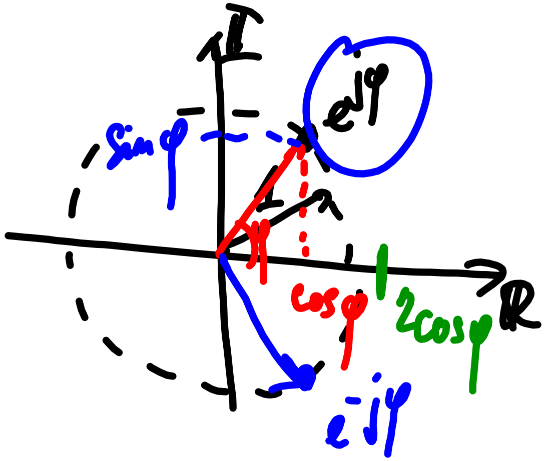
x^*



$x + x^* = 2r\cos\varphi$

$$x \cdot x^* = re^{j\varphi} \cdot re^{-j\varphi} = r^2 \underbrace{e^{j(\varphi-\varphi)}}_{1} = r^2 = |x|^2$$

UNIT CIRCLE

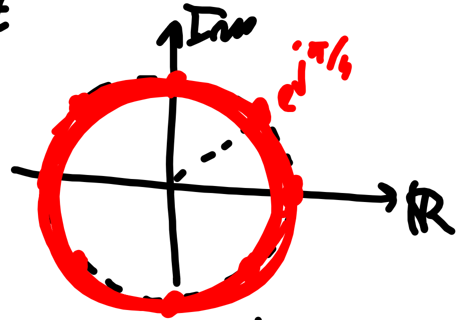


$e^{j\varphi}$

$$e^{j\varphi}+e^{-j\varphi}=2\cos\varphi$$

$$\boxed{\cos\varphi=\frac{e^{j\varphi}+e^{-j\varphi}}{2}}$$

$x(t)=e^{jt}$

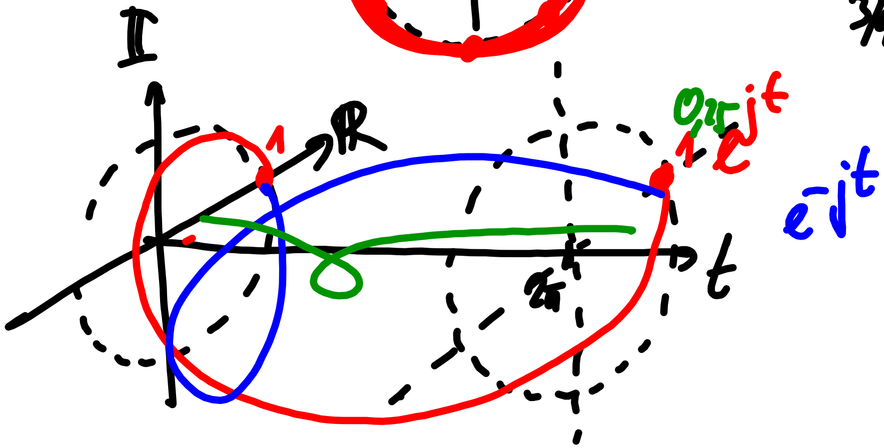


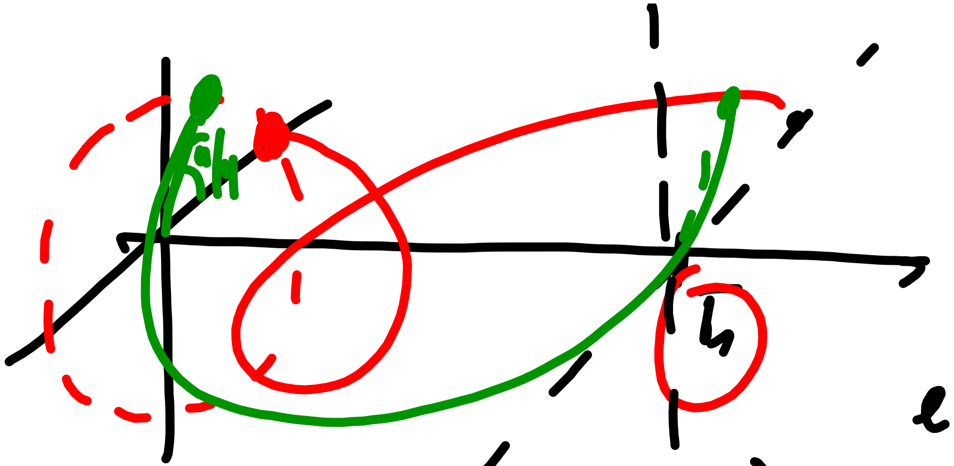
$t=0 \qquad e^{0}=1$

$t=\frac{\pi}{4} \qquad e^{j\pi/4}$

$\pi/2$

$3/4\pi$ . . .





$$x(t) = e^{j2\pi\frac{t}{T_1}} = e^{j(\omega_1 t + \varphi_1)}$$

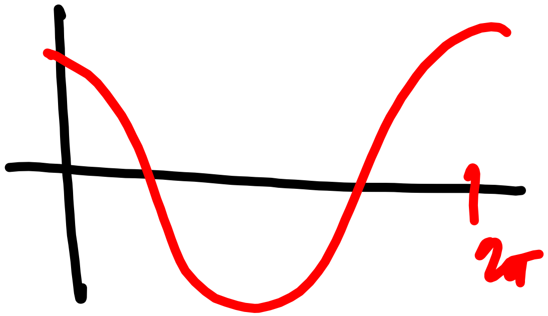
$$e^{a+b} = e^a \cdot e^b$$

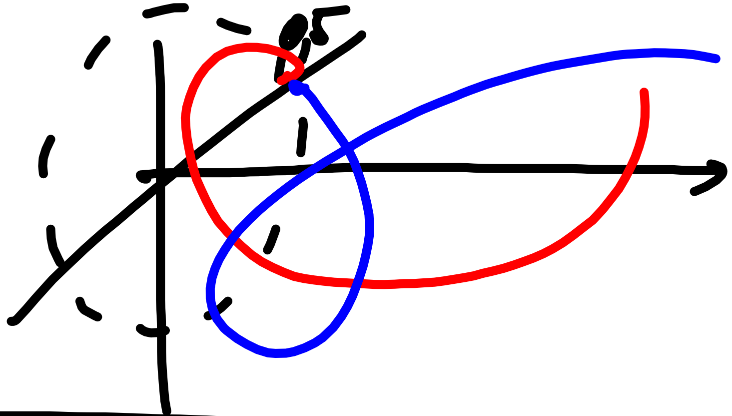
$$x(t) = C_1 e^{j(\omega_1 t + \varphi_1)} = C_1 e^{j\varphi_1} \cdot e^{j\omega_1 t}$$

complex constant

Decomposing cos into 2 complex exps.

$$\cos x = \frac{e^{jx}+e^{-jx}}{2}$$

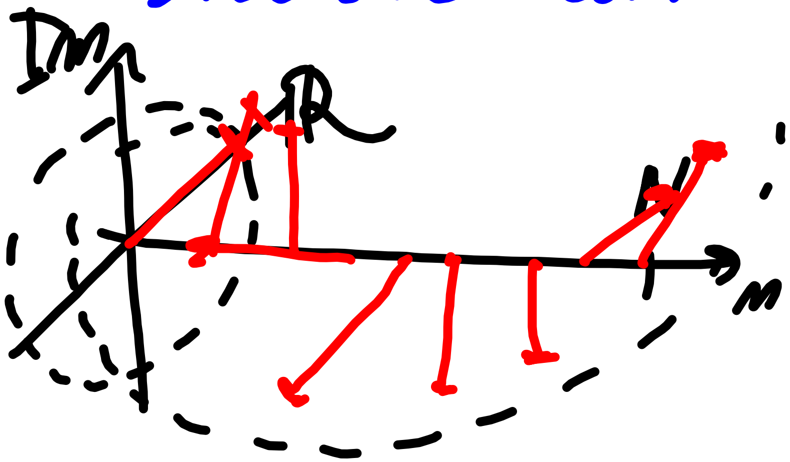




$$x(t) = C_1\cos(\omega_1 t+\varphi_1) = \frac{C_1}{2}e^{j(\omega_1 t+\varphi_1)} + \frac{C_1}{2}e^{-j(\omega_1 t+\varphi_1)} =$$

$$= \boxed{\frac{C_1}{2}e^{j\varphi_1}}\, e^{j\omega_1 t} + \boxed{\frac{C_1}{2}e^{-j\varphi_1}}\, e^{-j\omega_1 t}$$

DISCRETE COMPLEX EXP



$$x[n] = e^{j 2\pi \frac{n}{N}}$$

ω_1 norm. ang. frequency

$$x[n] = C_1 e^{j(\omega_1 n + \varphi_1)} = C_1 e^{j\varphi_1} e^{j\omega_1 n}$$

Disc. cos into compl. exps:

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$x[n] = C_1 \cos(\omega_1 n + \varphi_1) =$$

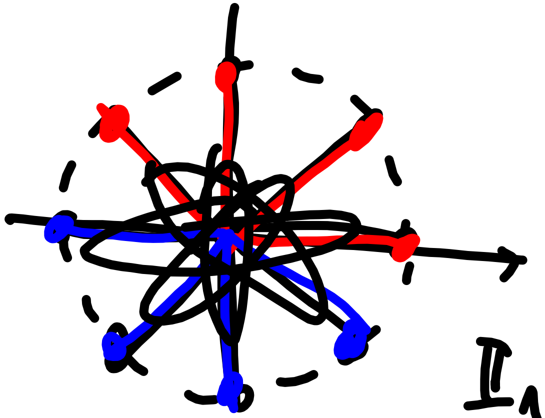
$$= \frac{C_1}{2} e^{j(\omega_1 n + \varphi_1)} + \frac{C_1}{2} e^{-j(\omega_1 n + \varphi_1)} =$$

$$= \frac{C_1}{2} e^{j\varphi_1} e^{j\omega_1 n} + \frac{C_1}{2} e^{-j\varphi_1} e^{-j\omega_1 n}$$

Sum / integral of 1 period of compl. exp.

$x[n] = e^{j\frac{2\pi}{N}n}$

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n} = \emptyset.$$



$N=8$

$\omega_0 = \frac{2\pi}{T_0}$

$$\int_{T_1} e^{j\omega_0 t}\,dt = \int_{T_1/2} 0\,dt = \emptyset$$