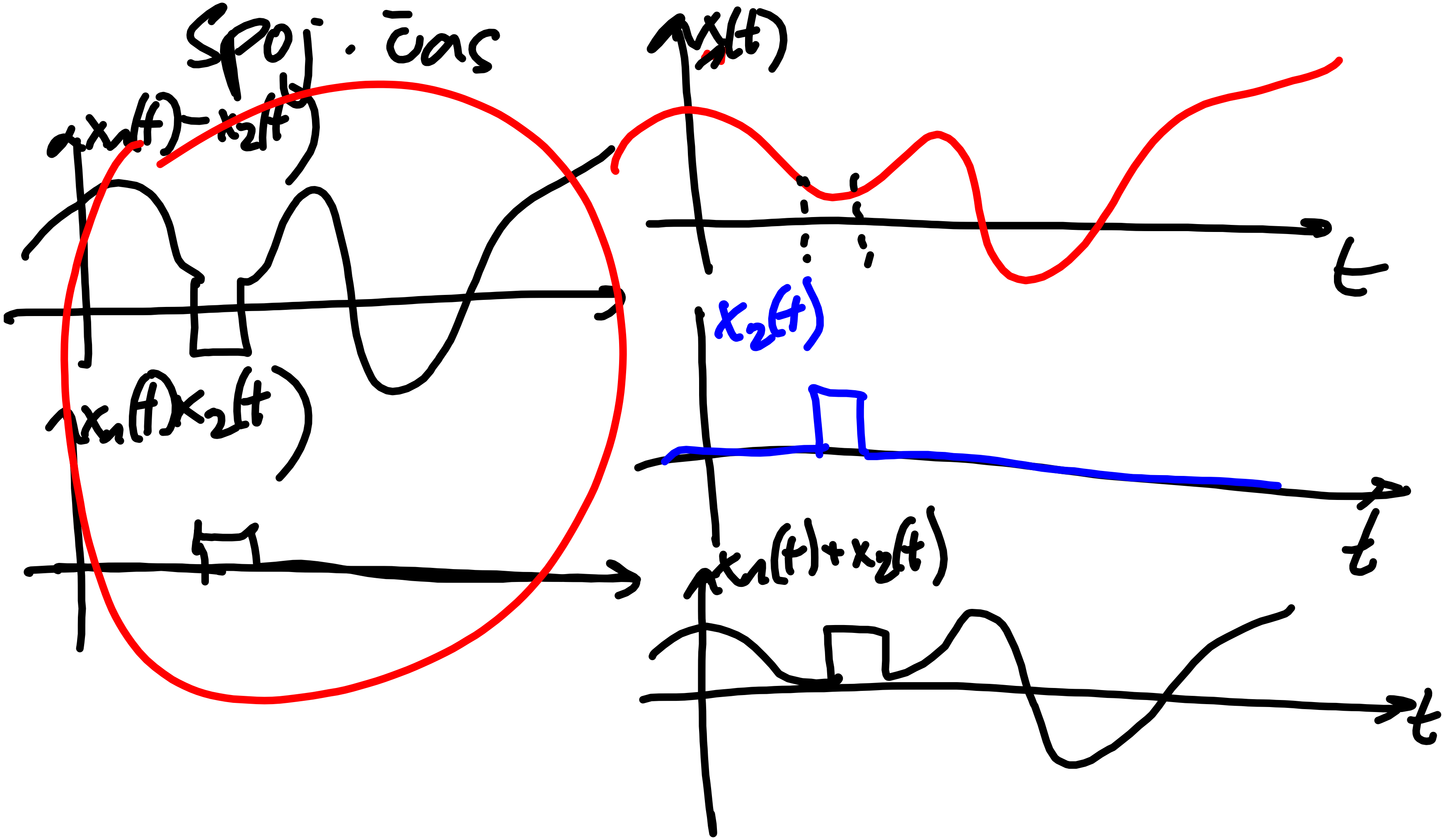
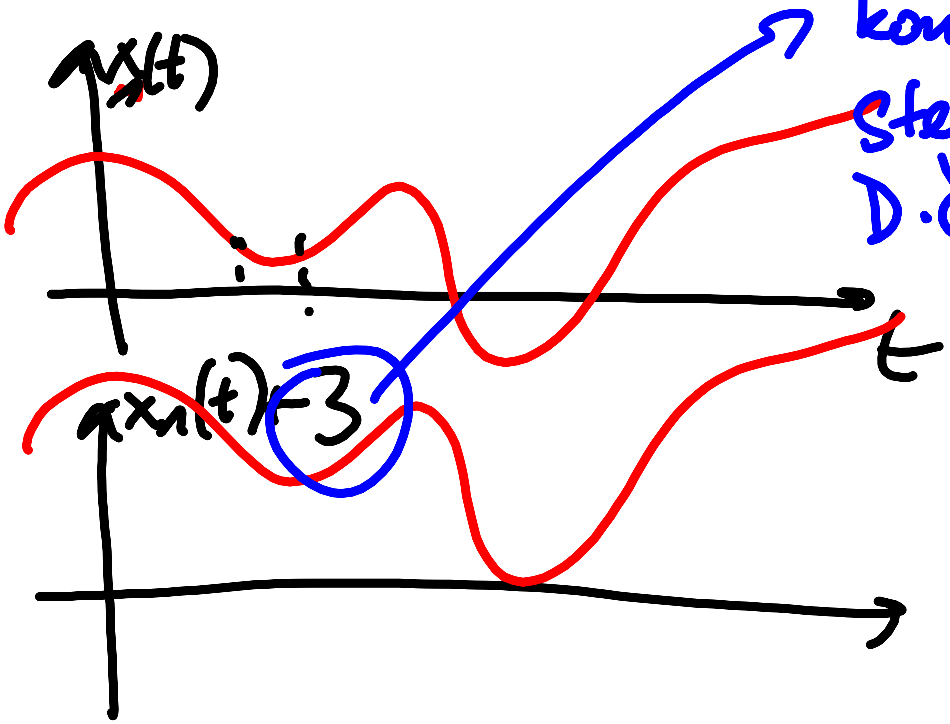
Spoj. čas



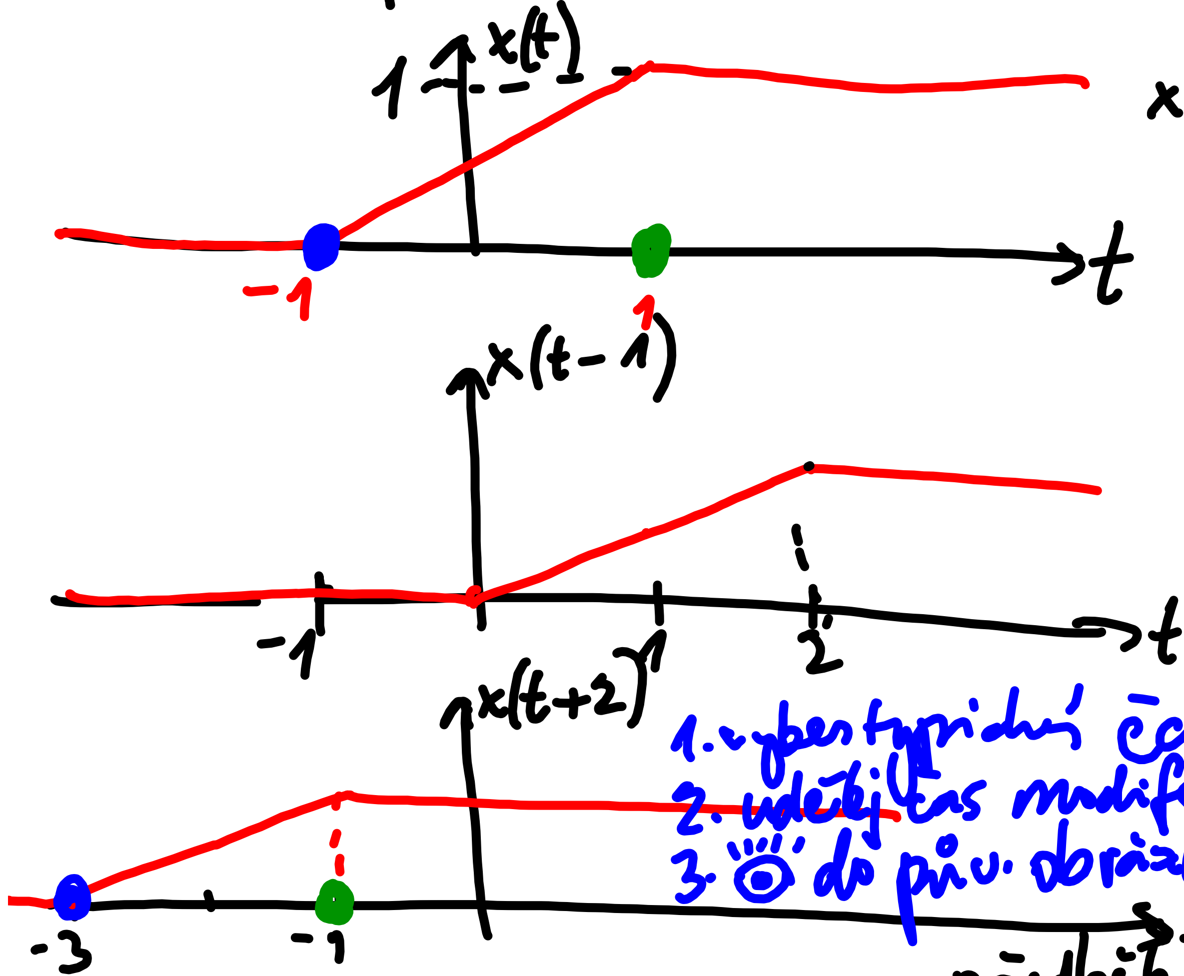


konstanta

stejnosměr. složka

D.C. component

Modifikace času



$$x(t) = \begin{cases} 0 & \text{pro } t < -1 \\ 0{,}5t + 0{,}5 & \text{pro } t \in [-1, 1] \\ 1 & \text{pro } t > 1 \end{cases}$$

$x(t - \tau)$

kladné

posun doprava / zpoždění / delay /

1. vyber typický čas
2. udělej čas modifikace
3. do pův. obrázku.

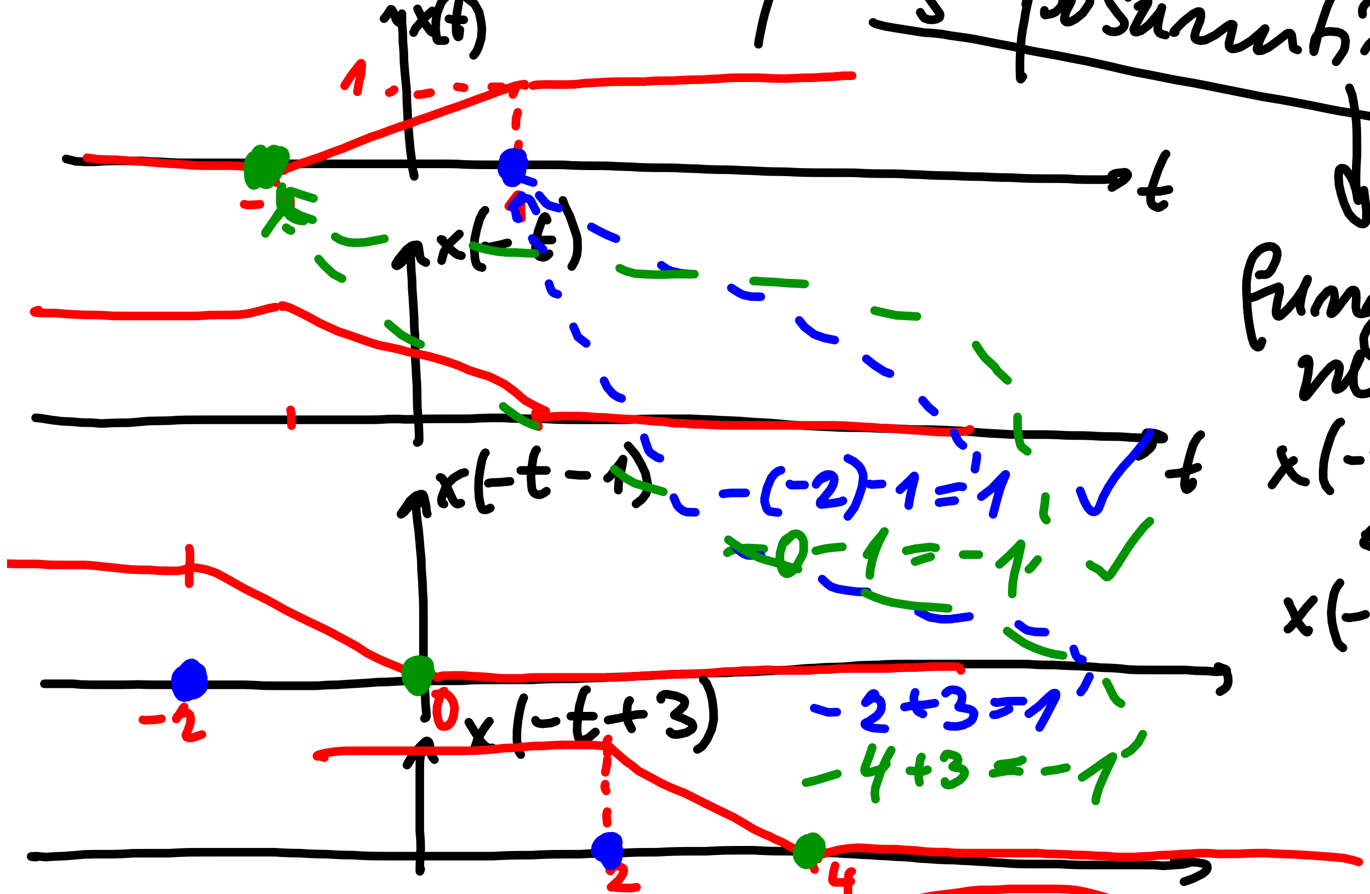
$x(t + \tau)$

kladné

posun doleva / předbíhání / advance.

otočení čas. osy

s posunutím



funguje naopak!!!

$x(-t-\tau)$ ←

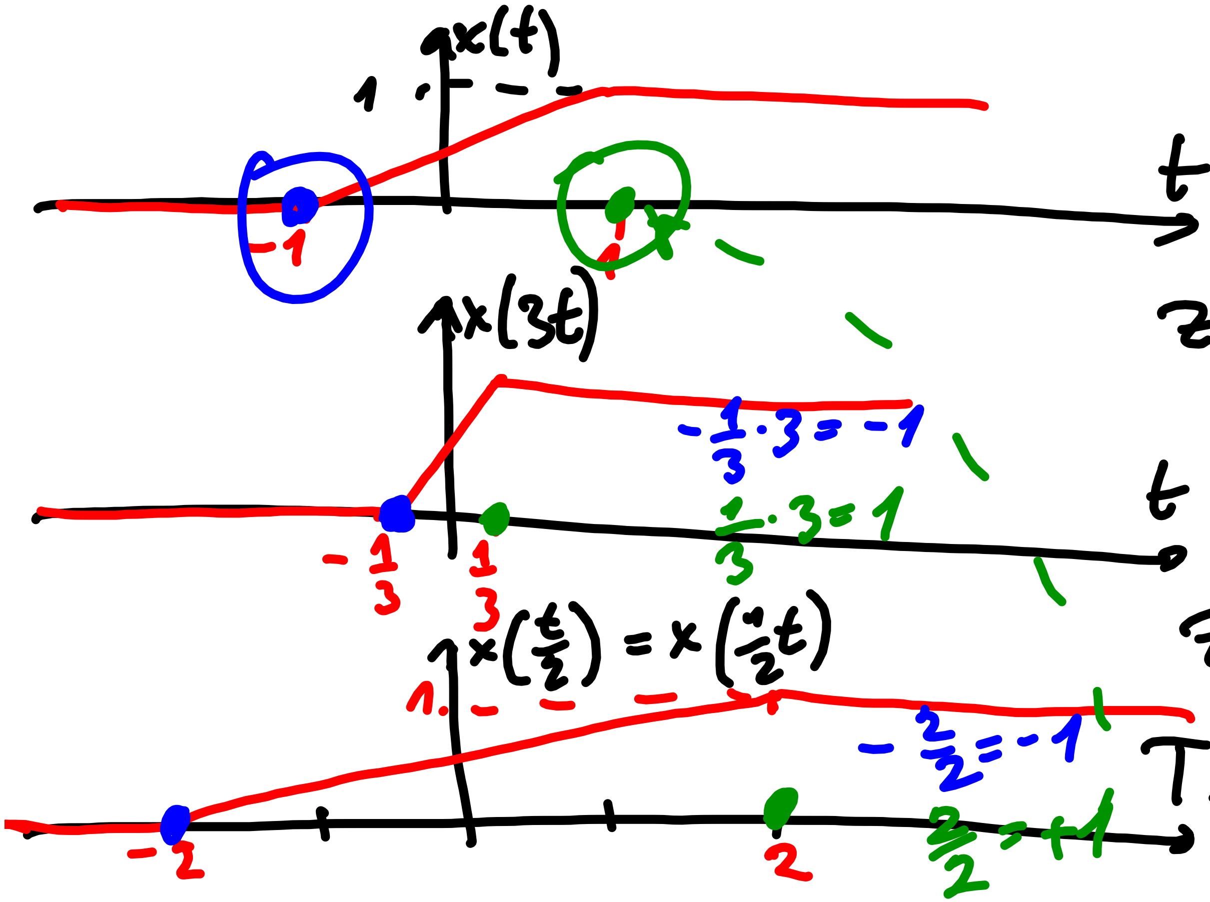
$x(-t+\tau)$ →

$$x[n]*h[n]=\sum_k x[k]\,h[n-k]$$

konvoluce

obrácení času

obrácení času s posunutím

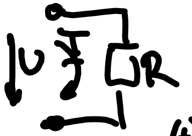


zrychlení
kontrakce času.

zpomalení
dilatace času.

TDNN
time-dilated NN

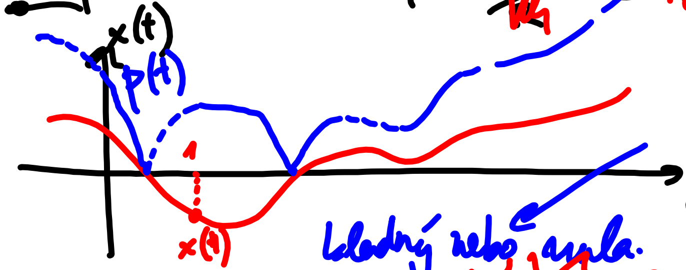
Energie výkon



$P = U \cdot I \qquad U = RI$

$P = \frac{U^2}{\cancel{R}} = I^2 \cancel{R}$

$P = X^2$ výkon $\rightarrow \mathbb{R}$



$p(t) = x^2(t)$ okamžitý výkon

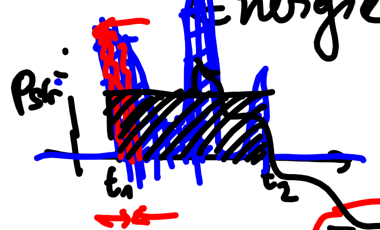
kladný nebo nula.

pokud $x(t) \in \mathbb{C}$ POZOR $p(t) = |x(t)|^2$

$p(t) = x(t) \cdot x^*(t)$

```
p = np.pow(np.abs(x), 2.0)
p = x * np.conj(x)
```

Energie od t_1 do t_2



$E_{t_1,t_2} = P(t_2 - t_1)$

$E_{t_1,t_2} = \int_{t_1}^{t_2} p(t)\, dt$

$P_{stř} = \frac{E_{t_1,t_2}}{t_2 - t_1}$

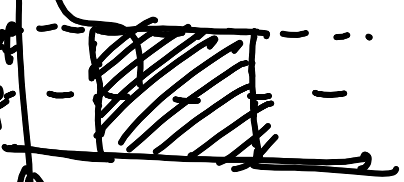
$\int_{t_1}^{t_2} P_{stř}\, dt = P_{stř}(t_2 - t_1)$

Efektivní hodnota

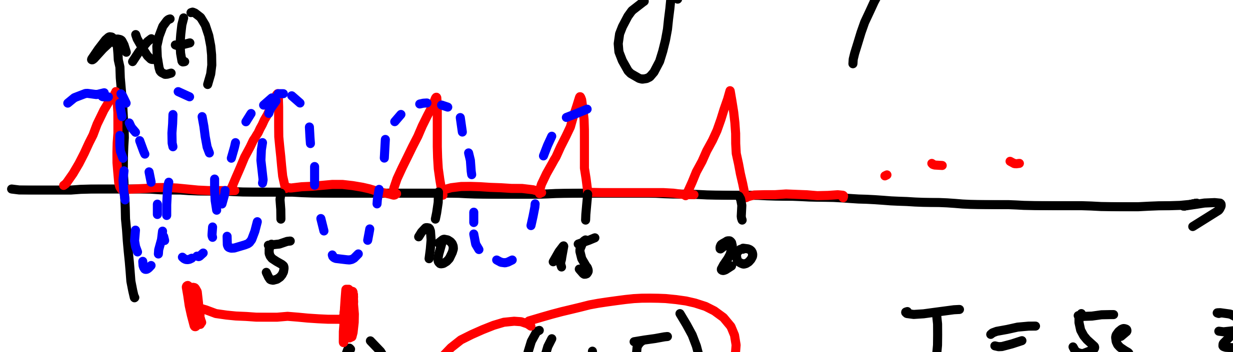
$\sqrt{P_{stř}} = X_{ef}$

na intervalu $t_1 \ldots t_2$ dá stejnou X_{ef} energii jako původní signál $x(t)$.

$P_{stř} = X_{ef}$



Periodické signály "se v čase opakují"



$$x(t) = x(t+5) = x(t+10) = x(t-15)$$

$T_1 = 5\,s$ základní perioda

$f_1 = \frac{1}{T_1}$ [Hz] základní frekvence

$\omega_1 = 2\pi f_1$ [rad/s] základní kruhová frekvence.

energie/výkon přes 1. periodu.

$$E_{T_1} = \int_{T_1} x^2(t)\,dt$$

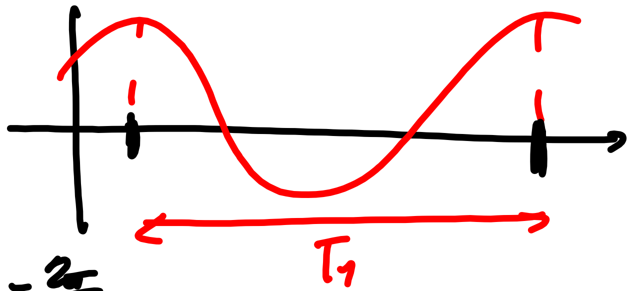
vybrat periodu libovolnou

$$P_{stř} = \frac{1}{T_1}\int_{T_1} x^2(t)\,dt$$

Cosinusovka

$x(t) = C_1\cos(\omega_1 t + \varphi_1)$

cokoliv rad rad/s $f_1\,[s] = \frac{1}{T_1}$ $\omega_1 = \frac{2\pi}{T_1}$ $\omega_1 = 2\pi f_1$



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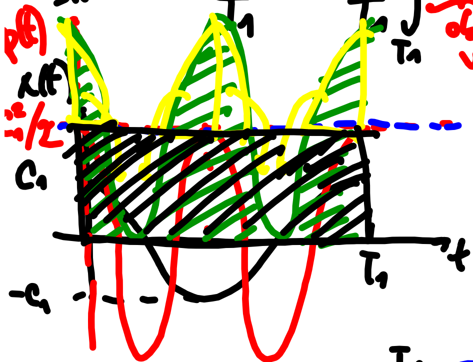
střední výkon cosinusovky $C_1\cos(\omega_1 t)$

$$P_{stř} = \frac{E_{T_1}}{T_1} = \frac{1}{T_1}\int_{T_1} x^2(t)\,dt = \frac{1}{T_1}\int_0^{T_1}\left[C_1\cos(\omega_1 t)\right]^2 dt =$$

okamžitý výkon!



$$\cos^2 x = \frac{1+\cos 2x}{2}$$

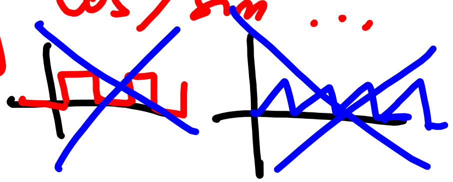
$$= \frac{1}{T_1}C_1^2\int_0^{T_1}\cos^2(\omega_1 t)\,dt =$$

$$= \frac{1}{T_1}C_1^2\int_0^{T_1}\frac{1+\cos(2\omega_1 t)}{2}\,dt =$$

$$= \frac{1}{T_1}\int_0^{T_1}\left[\frac{C_1^2}{2} + \frac{C_1^2}{2}\cos(2\omega_1 t)\right]dt = \frac{1}{\cancel{T_1}}\,\frac{C_1^2}{2}\cdot\cancel{T_1} = \frac{C_1^2}{2}$$

$$X_{ef} = \sqrt{P_{stř}} = \sqrt{\frac{C_1^2}{2}} = \boxed{\frac{C_1}{\sqrt{2}}}$$

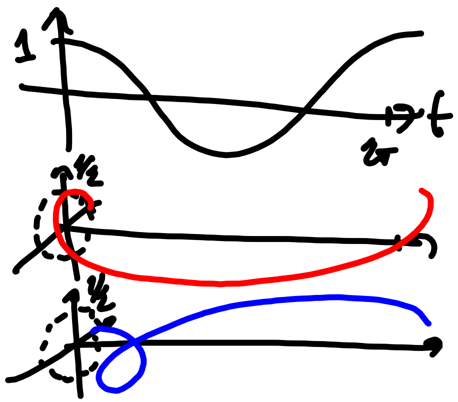
← pouze pro cos / sin !!!

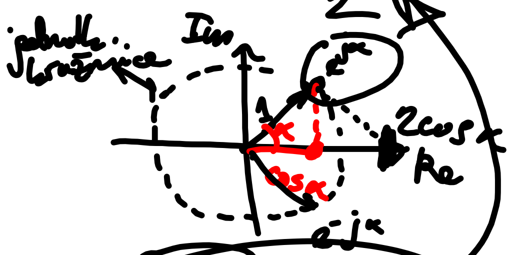


Rozklad cos na dvě komplexní exp.

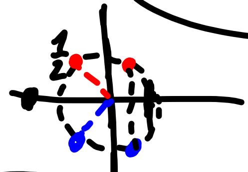
$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$x(t) = C_1\cos(\omega_1 t + \varphi_1)$$





$$2\cos\alpha = e^{j\alpha} + e^{-j\alpha}$$

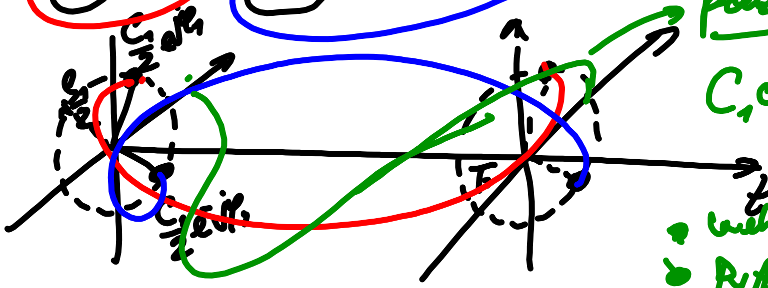


$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$x(t) = C_1\cos(\omega_1 t + \varphi_1)$$

$$x(t) = \frac{C_1}{2} e^{j(\omega_1 t + \varphi_1)} + \frac{C_1}{2} e^{-j(\omega_1 t + \varphi_1)} = \frac{C_1}{2} e^{j\varphi_1} e^{j\omega_1 t} + \frac{C_1}{2} e^{-j\varphi_1} e^{-j\omega_1 t}$$

$$e^{a+b} = e^a \cdot e^b$$

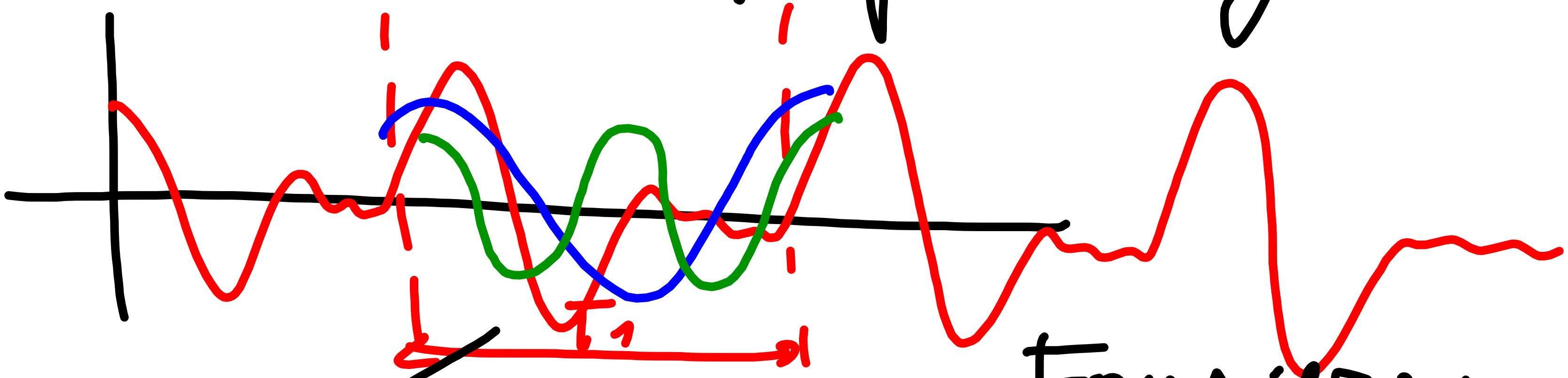


pouze reálné

$C_1\cos(\omega_1 t + \varphi_1)$

- web ISS
- Python k 1. přednášce (mat.)

Frekvenční analýza period. signálů



rozbít do sumy různě rychlých cosinusovek

3 otázky: → kde? (jaké frekvence?) → komplex. exp.
→ kolik?
→ jak posunuté?

Fourierova řada (Fourier series)

konstanta ss. složka D.C.

$$x(t) = C_0 + C_1 \underbrace{\cos(\omega_1 t + \varphi_1)} + C_2 \underbrace{\cos(2\omega_1 t + \varphi_2)} + \dots + C_k \underbrace{\cos(k\omega_1 t + \varphi_k)}$$

"syntezační vzorec"

harmonické
harmonicky vztažené cos ...

$$x(t) = c_0 + \underbrace{C_1 \cos(\omega_0 t + \varphi_1)} + \underbrace{C_2 \cos(2\omega_0 t + \varphi_2)} + \dots + \underbrace{C_k \cos(k\omega_0 t + \varphi_k)}$$

komplexní forma:

$$x(t) = \underbrace{c_{-k} e^{-jk\omega_0 t}} \dots \underbrace{c_{-2} e^{-j2\omega_0 t}} + \underbrace{c_{-1} e^{-j\omega_0 t}} + c_0 + \underbrace{c_1 e^{j\omega_0 t}} + \underbrace{c_2 e^{j2\omega_0 t} +} \dots + \underbrace{c_k e^{jk\omega_0 t}}$$

$\cos x = \frac{e^{jx} + e^{-jx}}{2}$

Převod R FŘ $\longleftrightarrow$ C FŘ

$|c_{-k}| = |c_k| = \frac{C_k}{2}$ $\quad \arg c_k = \varphi_k \quad \arg c_{-k} = -\varphi_k$

KDE? KOLIK? JAK POSUNUTÉ?

záporné kruhové frekv. ???

c_1 a c_{-1}, c_2 a c_{-2}, ... c_k a c_{-k} musí být k. sdružené.

stejné moduly / opačné arg $\Longrightarrow$

$$\boxed{x(t) = c_0 + \sum_{k=1}^{\infty} C_k \cos(k\omega_0 t + \varphi_k)}$$

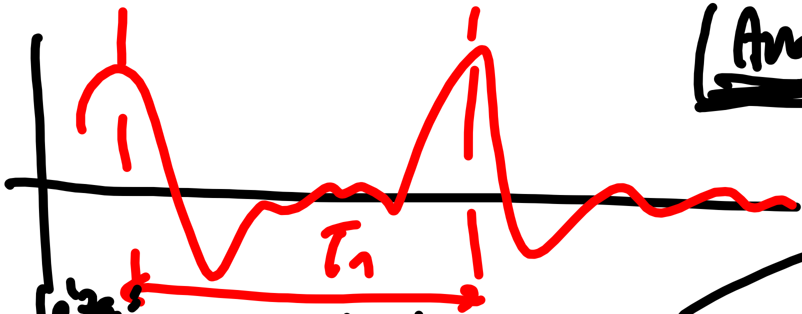
R FŘ

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}}$$

C FŘ

koeficienty FŘ

JAK ODHADNOUT koeficienty FŘ?



Syntéza:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$$

koeficienty

komponenty báze

Analýza (výpočet koef. FŘ.)

$$\boxed{c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt}$$

Projekce do bází

Hledání podobnosti -

Hledání korelace

$$\int_{T_1} x(t) \cdot b^*(t)$$

$e^{jk\omega_1 t}$ $e^{jk\omega_1 t}$

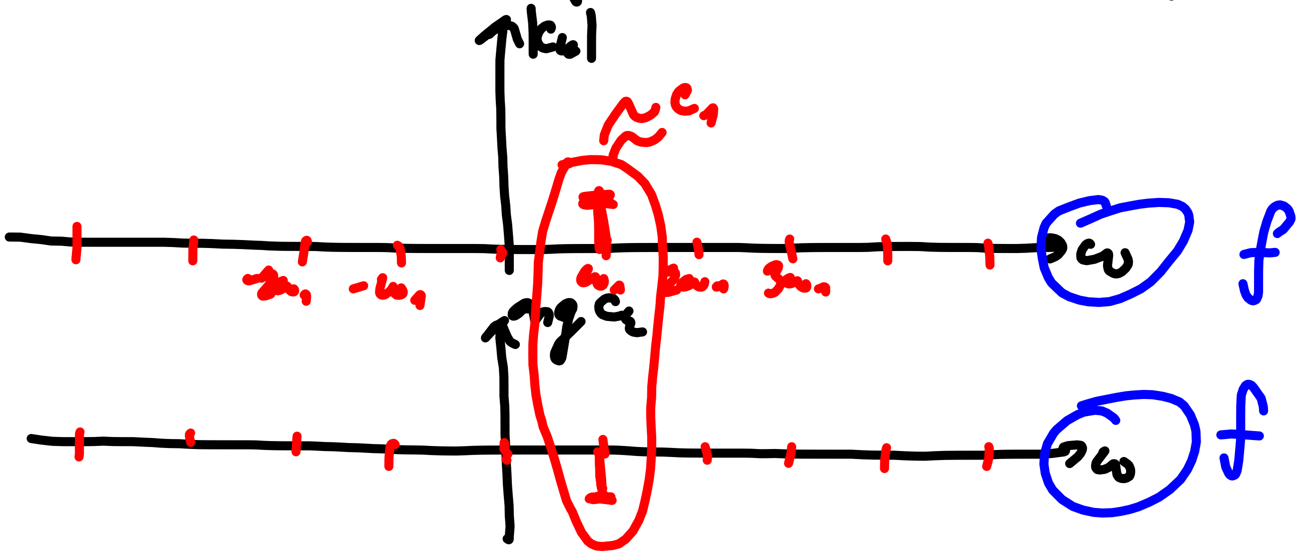
CHECK

ortonormálnost bází FŘ

$$\int_{T_1} e^{jk\omega_1 t}\, e^{-jl\omega_1 t}\, dt = \dots$$

$$\int_{T_1} |e^{jk\omega_1 t}|\, dt = T_1$$ ← BAD

Kreslení FŘ komplex. koef c_k vše frekvencí:

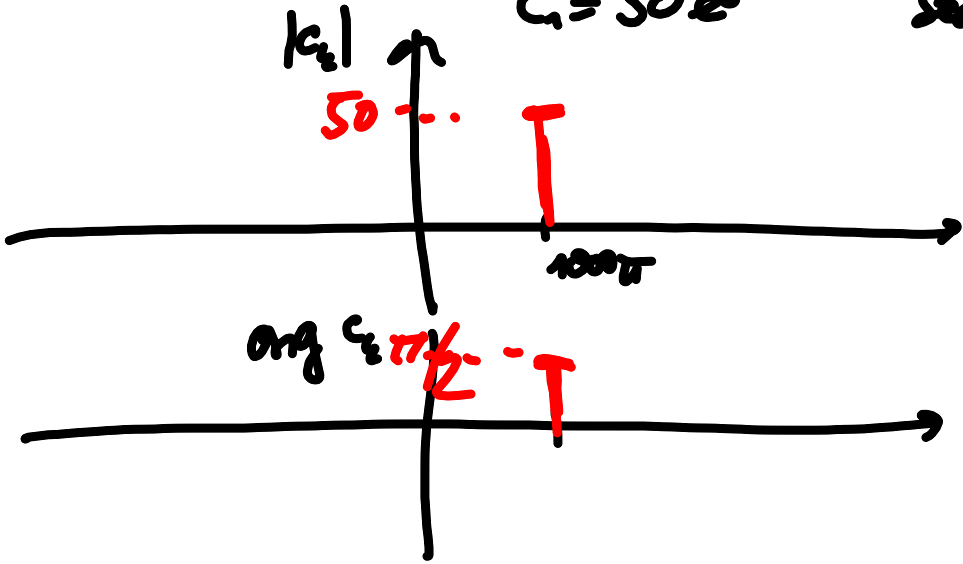


Příklady: $x(t) = 50\, e^{j(1000\pi t + \pi/2)} = 50\, e^{j\pi/2}\, e^{j1000\pi t} = c_1 e^{j1\omega_1 t}$

$\omega_1 = 1000\pi \qquad f_1 = 500\,Hz$

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t} = c_1 e^{j\omega_1 t}$

$c_1 = 50\, e^{j\pi/2}$ sedí na 1000π



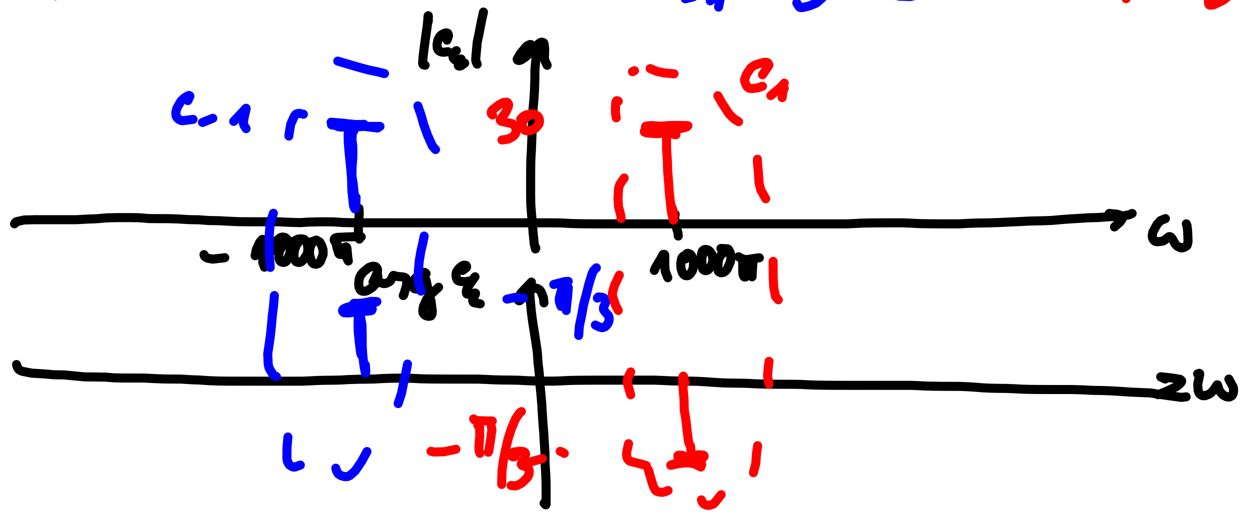
Př. 2: $x(t) = \underbrace{60\cos(1000\pi t - \pi/3)}$ $= \frac{60}{2} e^{-j\pi/3} e^{j1000\pi t} + \frac{60}{2} e^{j\pi/3} e^{-j1000\pi t}$

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$

$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$

$c_{-1} e^{-j1\omega_1 t} + c_1 e^{j1\omega_1 t}$

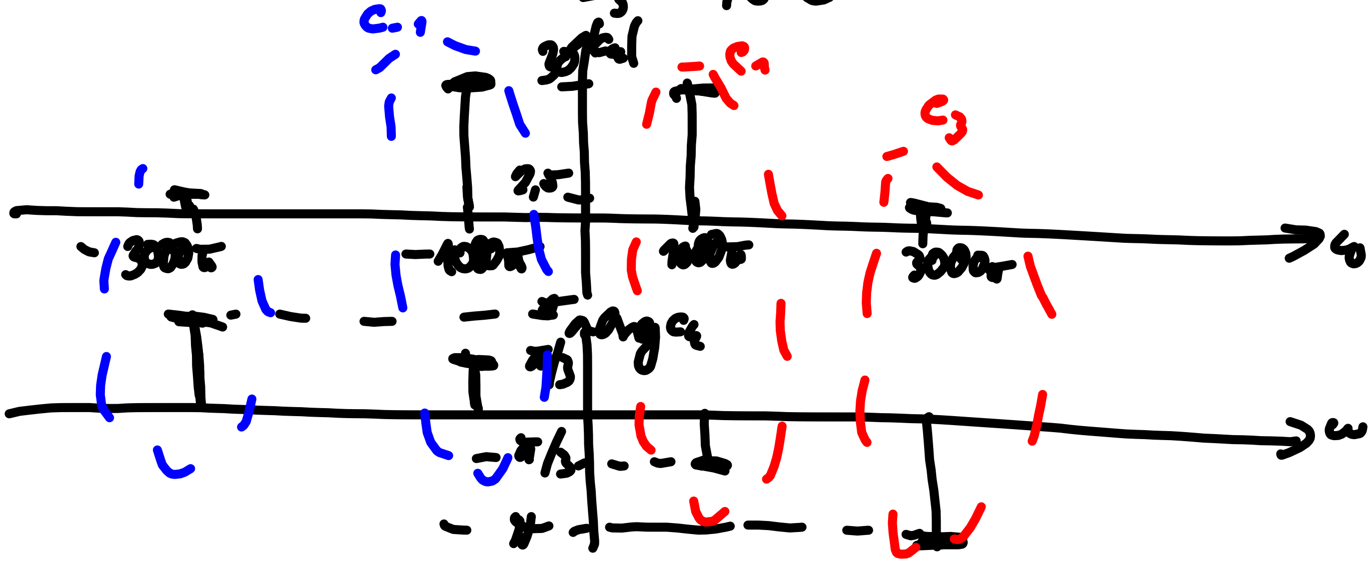
$c_{-1} = 30 e^{j\pi/3}$ $\quad c_1 = 30 e^{-j\pi/3}$



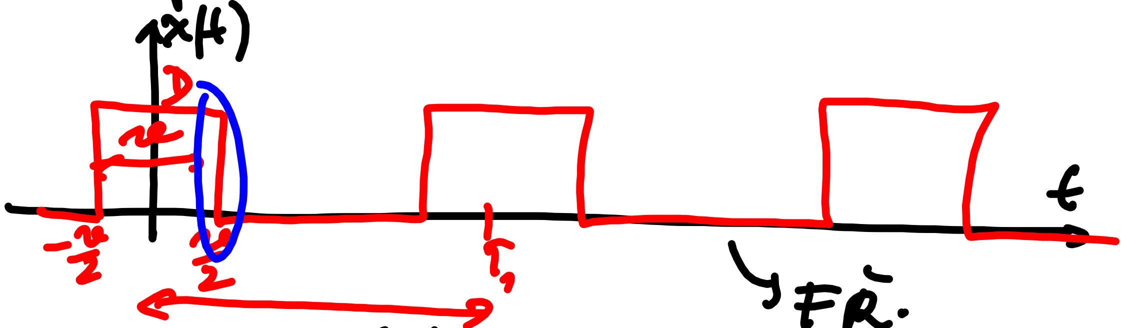
Př 3: $x(t) = 60\cos(\underbrace{1000\pi}_{\omega_1} t - \pi/3) + 5\cos(\underbrace{3000\pi}_{3\omega_1} t - \pi)$

$c_1 = 30 e^{-j\pi/3}$ $\qquad c_3 = 2{,}5 e^{-j\pi}$

$c_{-1} = 30 e^{+j\pi/3}$ $\qquad c_{-3} = 2{,}5 e^{+j\pi}$



FŘ periodického sledu obdélníkových impulsů.



↳ FŘ.

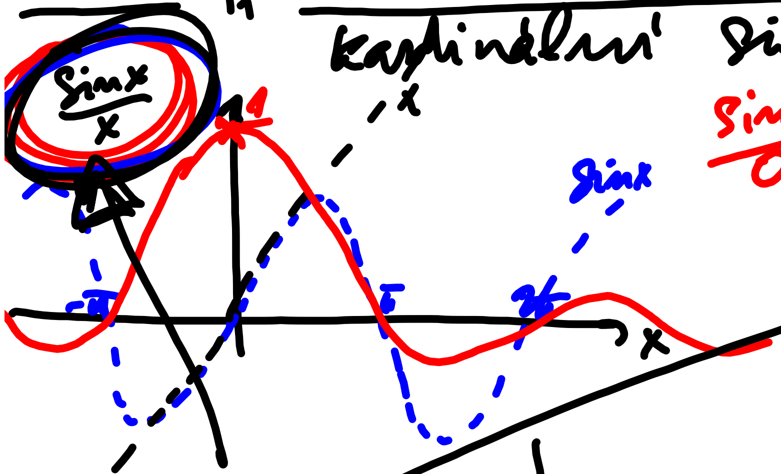
$\frac{\tau}{T_1}$ střída

duty cycle

$$c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$$

kardinální sinus

$\frac{\sin x}{x}$



$\frac{\sin 0}{0} = \frac{0}{0}$???

POZOR!

sinc(x) Matlab

np.sinc(x) numpy

$\frac{\sin \pi x}{\pi x}$

sox, ffmpeg

sinc(x)

$$\boxed{I(x) = \int_{-b}^{b} e^{+jxy}\,dy} = \begin{cases} \to x = 0 \\ \to x \neq 0 \end{cases}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$= \left[\frac{e^{+jxy}}{jx} \right]_{-b}^{b} = 2\,\frac{e^{jxb} - e^{-jxb}}{2jx} = 2b\,\frac{\sin xb}{xb} =$$

$$\boxed{2b\,\mathrm{sinc}\; bx}$$