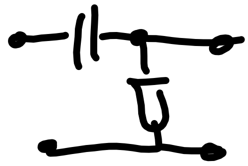
1. Systémy se spoj. časem
2. Vzorkování!

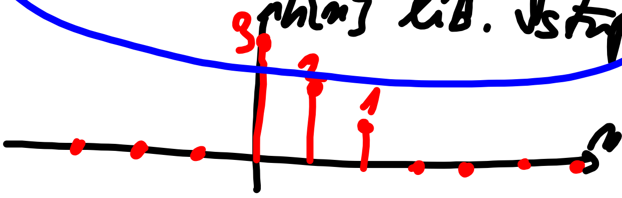


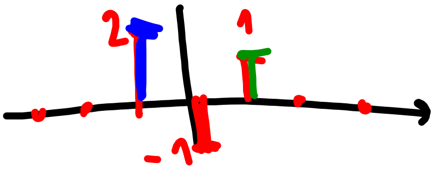
Systémy
- kauzalita
- linearita
- čas. invariance
- stabilita

} LTI

Impulsní odezva

Jak spočítat výstup pro lib. vstup?





?

n	-2	-1	0	1	2	3	4	5	6	7
x[n]		2	-1	1						
2·δ[n+1]		2								
→		6	4	2						
-1 δ[n]			-1							
→			-3	-2	-1					
1 δ[n-1]				1						
→				3	2	1				
y[n]		6	1	3	1	1				

$$y[1] = x[-1]h[2] + x[0]h[1] + x[1]h[0]$$

$$y[2] = \dots$$

$$y[3] = \dots$$

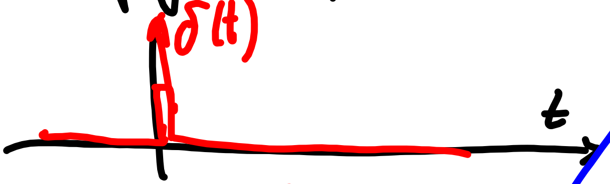
$$y[n] = x[n]\cdot h[0] + x[n-1]h[1] + x[n-2]h[2]$$

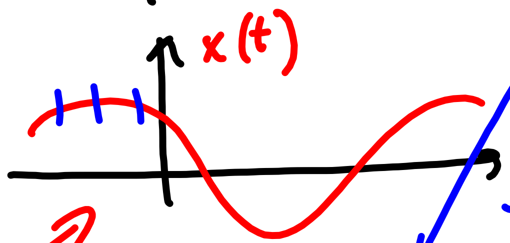
konvoluční suma

$$\boxed{x[n] * h[n]}$$

$$\boxed{y[n] = \sum_k x[n-k]h[k]}$$

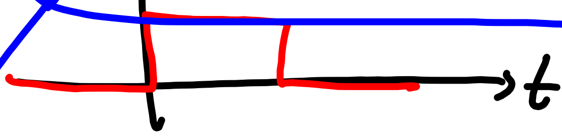
Spoj čas ?





h(t)

$y[n] = \sum_k x[n-k]\, h[k]$



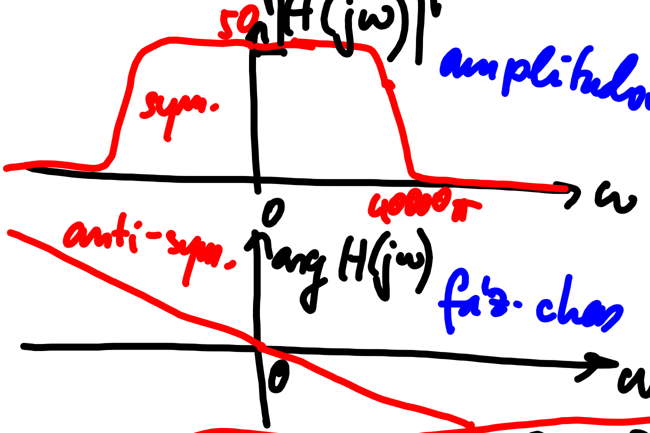
- rozdělit vstup na jednotkové impulsy
- proženu každý z nich systémem.
- sečti výstupy

$$\boxed{x(t) * h(t) = \int_\tau x(t-\tau)\, h(\tau)\, d\tau}$$

konvoluční integrál

$$\boxed{\int_\tau x(\tau)\, h(t-\tau)\, d\tau}$$

Frekvenční (kmitočtová) charakteristika.
Komplexní funkce

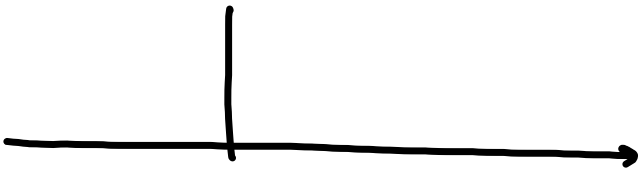


amplitudová char. – jak zesiluje / zeslabuje

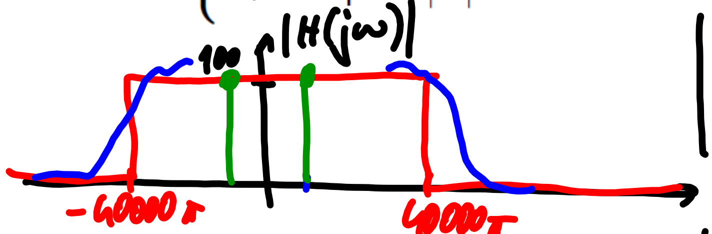
fáz. char. – jak posouvá

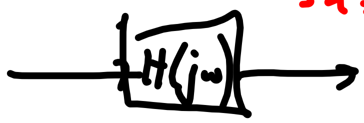
$$\boxed{H(j\omega) = FT\{h(t)\} = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt}$$

správně, ale ne zcela užitečný ....

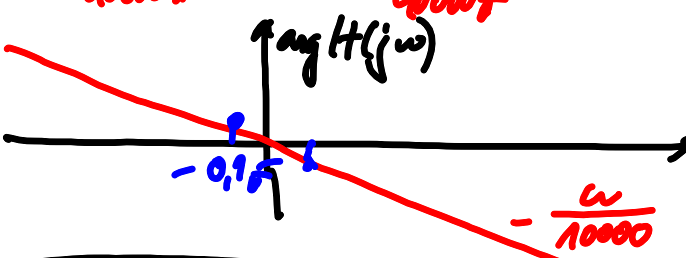


$$|H(j\omega)| = \begin{cases} 100 & \text{pro } 0 \le |\omega| \le 40000\pi \\ 0 & \text{pro } |\omega| > 40000\pi \end{cases} \qquad \arg H(j\omega) = -\frac{\omega}{100000}.$$





$\cos x = \frac{e^{jx} + e^{-jx}}{2}$



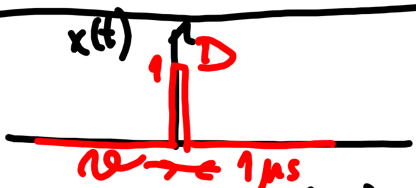
$x(t) = 5\, e^{j10000\pi t}$

$y(t) = 5 \cdot 100\, e^{j10000\pi t - 0{,}1\pi}$

$= 500\, e^{j10000\pi t - 0{,}1\pi}$

$x(t) = 5 \cdot \cos(10000\pi t) = 250\, e^{j10000\pi t - 0{,}1\pi} + 250\, e^{-j10000\pi t + 0{,}1\pi} =$

$= 500 \cos(10000\pi t - 0{,}1\pi)$



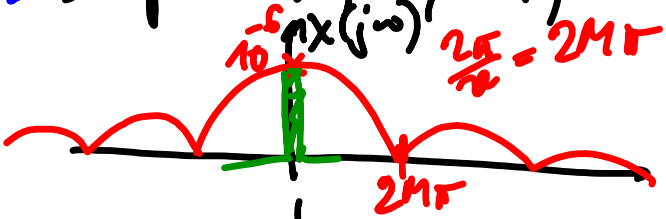
option 1: $y(t) = x(t) * h(t)$

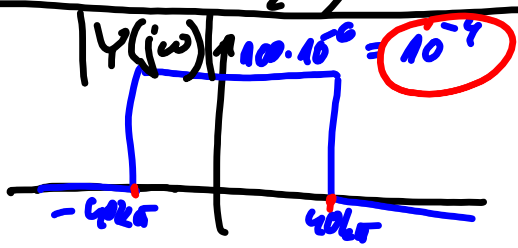
option 2: $Y(j\omega) = X(j\omega) \cdot H(j\omega)$ ✓

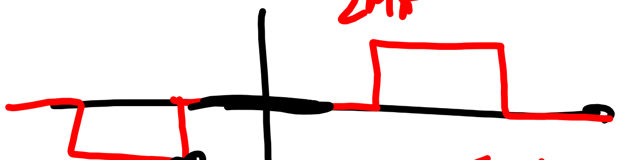
1. spočítat $X(j\omega)$
2. vynásobit $X(j\omega) \cdot H(j\omega)$
3. zpětná FT aby $x(t)$

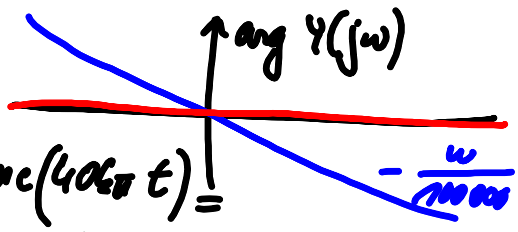
1. $X(\omega) = FT\{x(t)\} =$
$= D\vartheta\, \mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right)$

$\frac{2\pi}{\vartheta} = 2M\pi$









$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega)\, e^{+j\omega t}\, d\omega = \frac{10^{-4}}{2\pi}\, 40k\pi\, \mathrm{sinc}(40k\pi t) =$

$\int_{-b}^{b} e^{\pm j\omega y}\, dy = 2b\, \mathrm{sinc}(bx)$

$= 4\, \mathrm{sinc}(40k\pi t)$

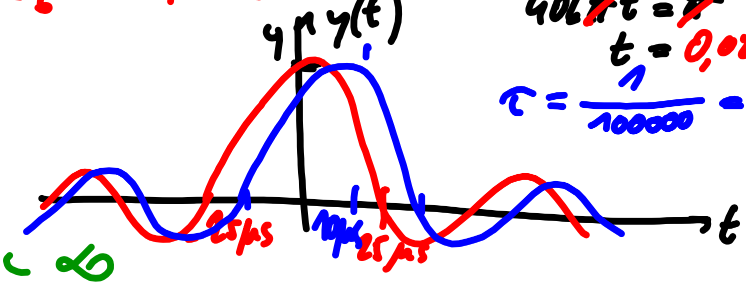
$40k\pi t = \pi$

$t = 0{,}025 \cdot 10^{-3} = 25\,\mu s$

$a(t) \to a(t-\tau)$

$A(j\omega) \to A(j\omega)\, e^{-j\omega\tau}$

$\tau = \frac{1}{100000} = 10\,\mu s$



Jak na kmitočtovou char. ???

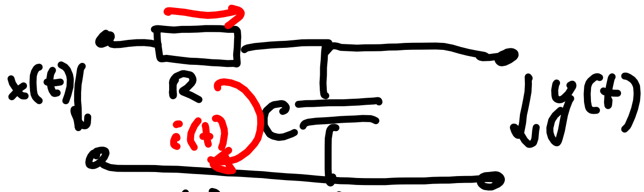
Teorie | **Příklad**

diferenciální rovnice

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

$$a_0 y(t) + a_1 \frac{dy(t)}{dt} = b_0 x(t)$$

$a_0 = 1$ $a_1 = RC$ $b_0 = 1$



$$i(t) = \frac{x(t) - y(t)}{R}$$

$$i(t) = C\frac{dy(t)}{dt}$$

$$x(t) - y(t) = RC\frac{dy(t)}{dt}$$

$$y(t) + RC\frac{dy(t)}{dt} = x(t)$$

Laplaceova transformace

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st}\, dt$$

komplexní proměn.

$$H(j\omega) = \int h(t) e^{-j\omega t}\, dt$$

$$x(t) \rightarrow X(s)$$
$$a\,x(t) \rightarrow a\,X(s)$$
$$\frac{dx(t)}{dt} \rightarrow s\,X(s)$$
$$\frac{d^k x(t)}{dt^k} \rightarrow s^k X(s)$$

$$Y(s) + RC\,s\,Y(s) = X(s)$$

Přenosová funkce — Transfer function

$$H(s) = \frac{Y(s)}{X(s)}$$

$$Y(s)(1 + RCs) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{1 + RCs}$$

$$\left[\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}\right] \mathcal{L}$$

$$\sum_{k=0}^{N} a_k Y(s) s^k = \sum_{k=0}^{M} b_k X(s) s^k$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$$

polynom

polynom.

Jak na frekv. char ???

$H(j\omega)$ je přenos. funkce $H(s)$ vyhodnocené pouze pro $s = j\omega$!!!

$$H(s) \longrightarrow H(j\omega)$$

$$H(\cancel{s}\,j\omega) = \frac{Y(\cancel{s}\,j\omega)}{X(\cancel{s}\,j\omega)} = \frac{\sum_{k=0}^{M} b_k (\cancel{s}\,j\omega)^k}{\sum_{k=0}^{N} a_k (\cancel{s}\,j\omega)^k}$$

b ... a ...

freqs(b, a, ...)

np.freqs...

$$H(s) = \frac{1}{1+RCs} =$$

$$H(j\omega) = \frac{1}{1+RCj\omega}$$

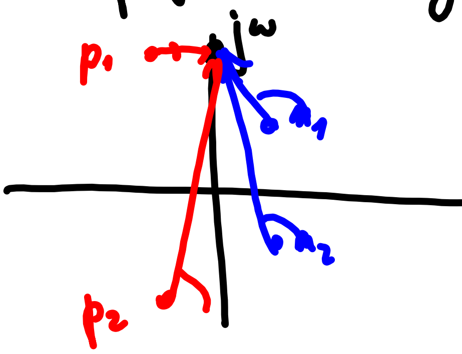
$$\boxed{\frac{1}{RC}} \cdot \frac{1}{(j\omega - (-\frac{1}{RC}))}$$

konst

$$H(s) = \frac{\sum b_k s^k}{\sum a_k s^k} = \text{konst.} \frac{(s-n_1)(s-n_2)\dots(s-n_M)}{(s-p_1)(s-p_2)\dots(s-p_N)}$$

n_1, n_2 .... kořeny čitatele - nulové body

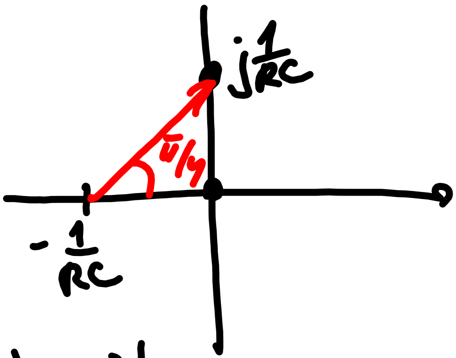
p_1, p_2 ... kořeny jmenovatele - póly



$$|H(j\omega)| = \text{konst} \frac{\text{součin délek modrých vektorů}}{\text{součin délek červených vektorů}}$$

arg $H(j\omega)$ = součet úhlů modrých vektorů
− součet úhlů červených vektorů

$$H(s) = \frac{1}{RC}\cdot\frac{1}{\left(s-\left(-\frac{1}{RC}\right)\right)}$$



$\omega = 0 \quad |H(j0)| = \frac{1}{RC}\cdot\frac{1}{\frac{1}{RC}} = 1$

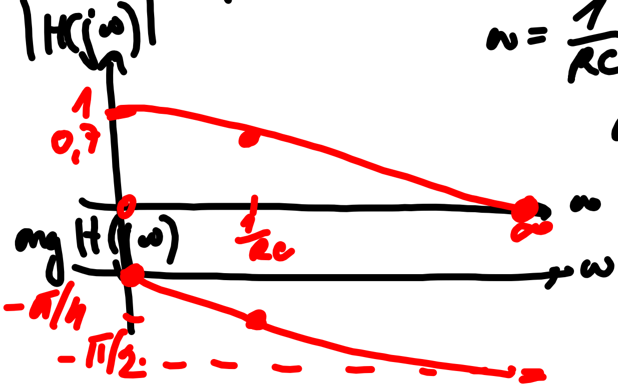
$ang = 0$

$\omega = \infty \quad |H(j\infty)| = \frac{1}{RC}\cdot\frac{1}{\infty} = 0$

$ang = -\frac{\pi}{2}$

$\omega = \frac{1}{RC} \quad |H(j\frac{1}{RC})| = \cancel{\frac{1}{RC}}\cdot\frac{1}{\sqrt{2}\,\cancel{\frac{1}{RC}}} = 0{,}7$

$ang = -\pi/4$



imp. odezva $h(t) \xrightarrow{FT} H(j\omega)$

schéma, obvod, fyz. simulace

↓

diferenciální rovnice

$$\sum_k a_k \frac{d y^k(t)}{dt^k} = \sum_k b_k \frac{d x^k(t)}{dt^k} \rightarrow \text{Laplaceova transf.}$$

$$\sum a_k Y(s) s^k = \sum b_k X(s) s^k$$

přenosová funkce

$$H(s) = \frac{\sum b_k s^k}{\sum a_k s^k} \xrightarrow[s = j\omega]{\text{frekv.}}$$

frekv. charakteristika $H(j\omega)$

↓ rozklad (faktorizace) polynomů

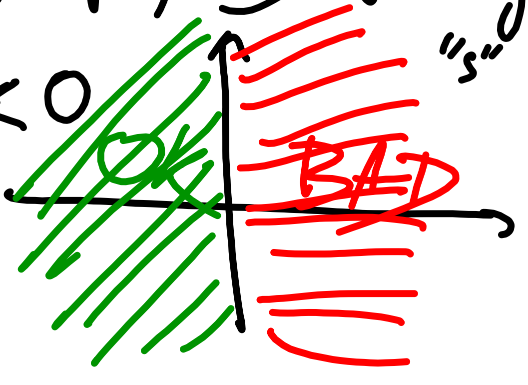
$$H(s) = \text{konst} \frac{\prod (s - n_k)}{\prod (s - p_k)}$$

nul. body, póly

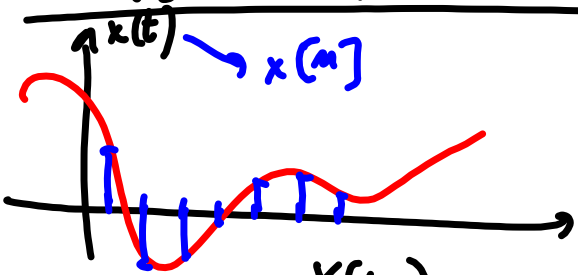
grafický "s"

Stabilita

$$\mathrm{Re}\{p_k\} < 0$$



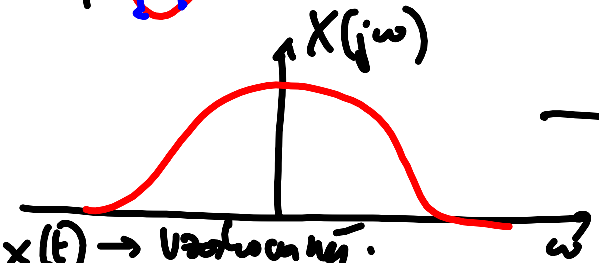
VZORKOVÁNÍ



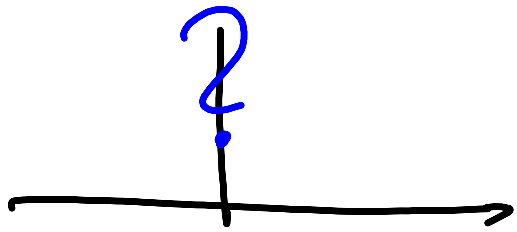
vzork. perioda T [s]

vzork. frekvence $F = \frac{1}{T}$ [Hz]

$\Omega = 2\pi F$ [rad/s]

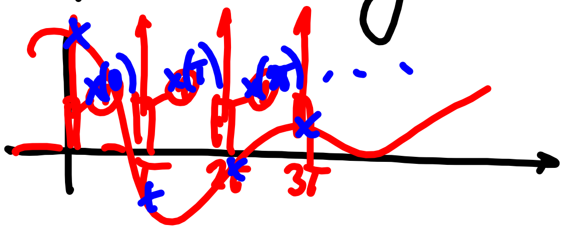


→



$x(t) \rightarrow$ vzorkovaný:

$s(t)$ vzorkovací signál – sekvence Diracových impulsů na násobcích T

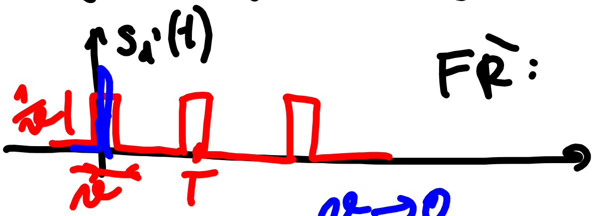


$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\boxed{x_s(t) = x(t)\,s(t)}$$ součin

→ spektrum ???

$$X_s(j\omega) = X(j\omega) * \underline{S(j\omega)}$$



FŘ: $c_k = \frac{D\vartheta}{T}\,\text{sinc}\left(\frac{\vartheta}{2}k\Omega\right) = \frac{1}{\cancel{\vartheta}}\frac{\cancel{\vartheta}}{T}\,\text{sinc}\left(\frac{\vartheta}{2}k\Omega\right) = \frac{1}{T}\,\text{sinc}(0\ \dots) = \frac{1}{T}$

$\vartheta \rightarrow 0$

~~FŘ~~ FT

$S(j\omega)$ c_k na násobcích ω_1

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi\,\delta(\omega - k\omega_1)$$