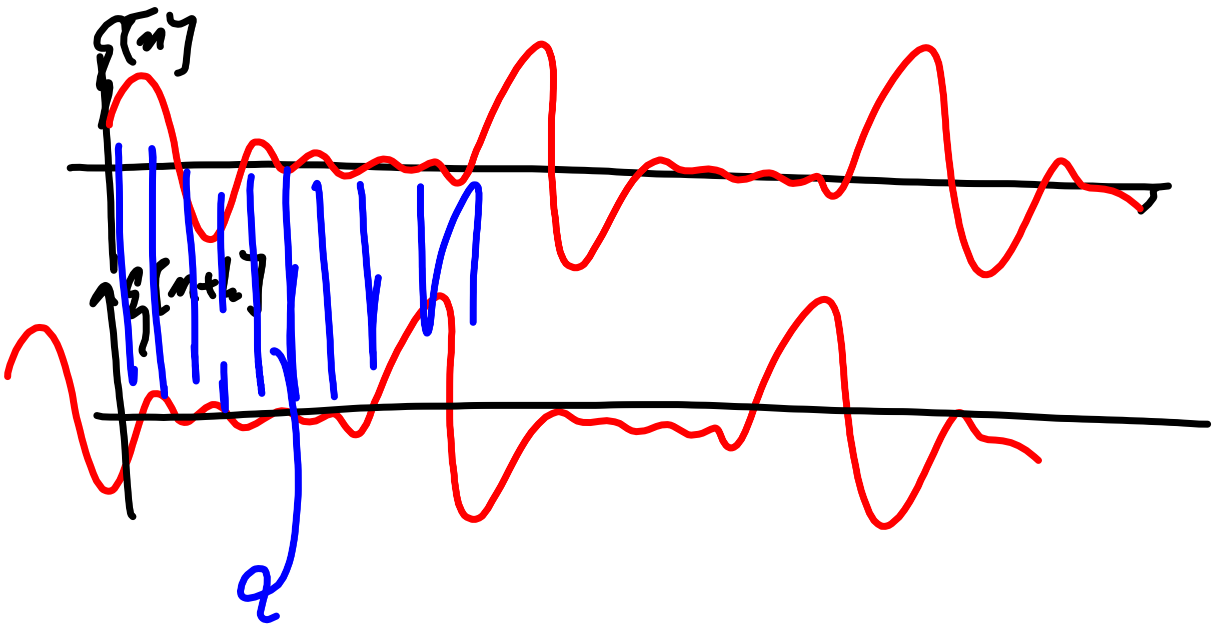
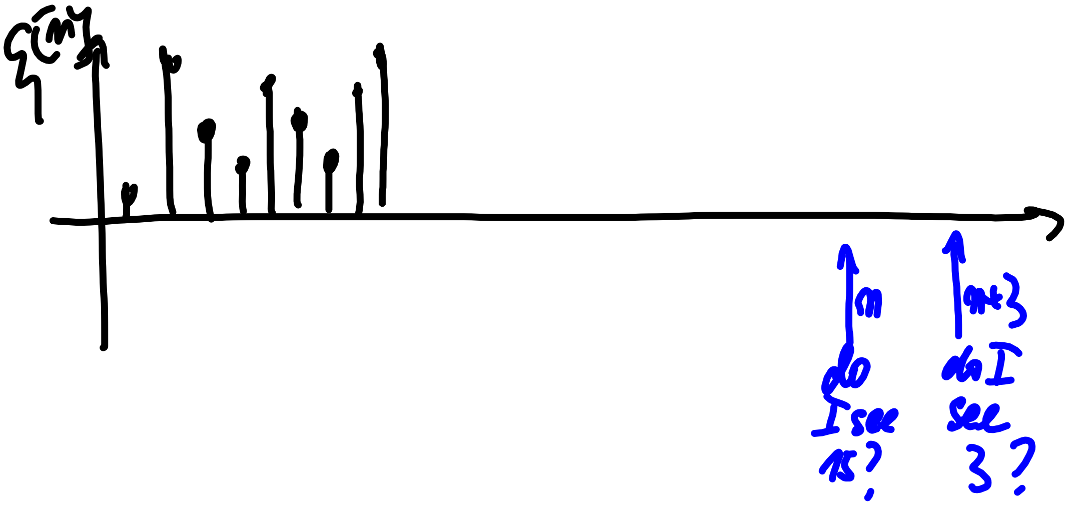


$$\frac{1}{N}\sum$$

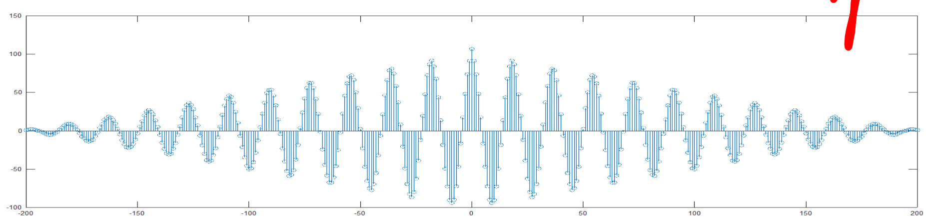

x[n]
n
do I see 15?
n+3
do I see 3?

Estimation of correlation coefficients

$$R[k] = \frac{1}{N}\sum_{n=0}^{N-1} \xi[n]\,\xi[n+k]$$

biased estimate
vychýlený odhad

np. xcorr

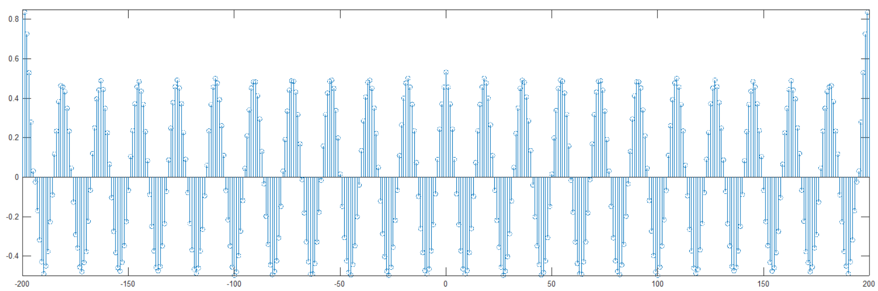


Unbiased estimate nevychýlený odhad

$$R[k] = \frac{1}{N-|k|}\sum_{n=0}^{N-1} \xi[n]\,\xi[n+k]$$

xcorr(x, 'unbiased')

beware of values for $|k|$ approaching N!
unreliable values divided by to small $N-|k|$!



Correlation coeffs. vs. power vs. variance

avg. power

$$P = \frac{1}{N}\sum_{n=0}^{N-1}|\xi[n]|^2$$

corr. coeffs.

$$R[k] = \frac{1}{N}\sum_{n=0}^{N-1}\xi[n]\xi[n+k]$$

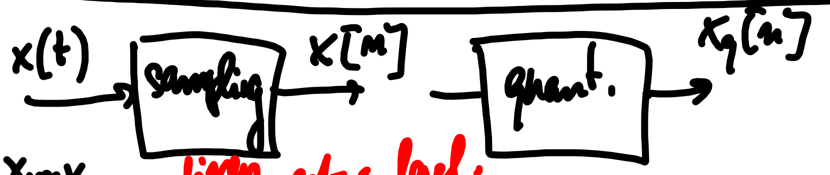
$$P = R[0]$$

Estimation of variance: $D = \frac{1}{N}\sum_{n=0}^{N-1}(\xi[n]-\cancel{a})^2$

common random signal have $a=0$: $\boxed{D = P} = R[0]$

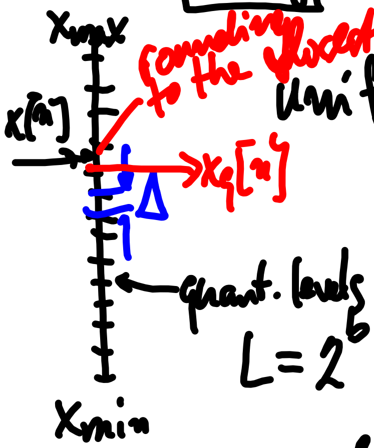
if not: $a \neq 0$: $D + a^2 = P = R[0]$

Quantization



← limited number of bits

$b = 8,\ 16$



uniform quantization

$\Delta = \text{const.}$

$$\Delta = \frac{x_{max} - x_{min}}{L}$$

$L = 2^b$

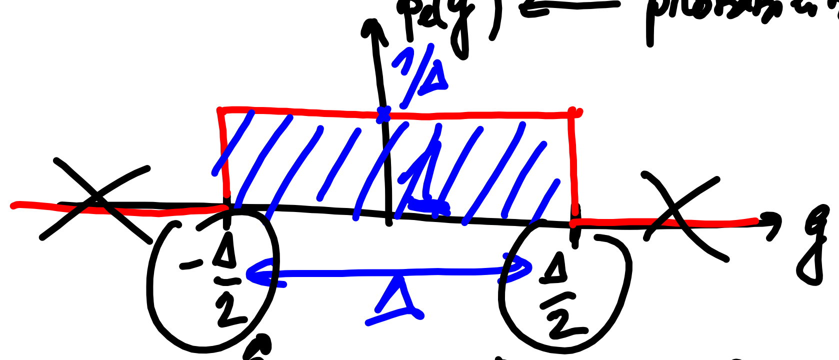
Determine the power of noise (distortion) caused by the quantization

error:

$$e[n] = x[n] - x_q[n]$$

which values can $e[n]$ have ???

$p_e(g)$ ← probability density of error signal



$$P_e = D = \int_{-\infty}^{\infty} p_e(g)(g - \cancel{a})^2\, dg = \int p_e(g)\, g^2\, dg = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} g^2\, dg =$$

power = variance

$a = 0$

$$= \frac{1}{\Delta}\left[\frac{g^3}{3}\right]_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{1}{\Delta}\left[\frac{\frac{\Delta^3}{8}}{3} + \frac{+\frac{\Delta^3}{8}}{3}\right] = \frac{1}{\cancel{\Delta}}\,\frac{\Delta^{\cancel{3}\,2} + \Delta^3}{24} = \frac{\Delta^2}{12}$$

How much does the q. error harm the signal?

$$SNR = 10\log_{10}\frac{P_s}{P_e}$$

P_s — power of useful signal

P_e — power of bad signal

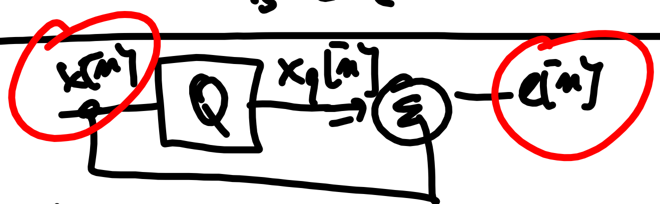
Signal to noise ratio

[dB] deciBell.

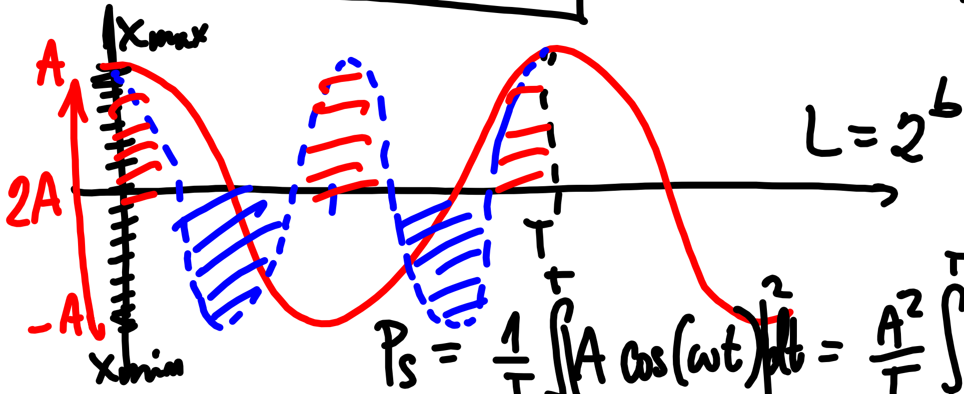
$P_s = 100\,P_e$ $SNR = 20$ dB

$P_s = P_e$ $SNR = 0$

$P_s < P_e$ SNR negative



Cosine fully exploiting the dym. range of quantizer.



$L = 2^b$

$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$

$$P_s = \frac{1}{T}\int_0^T \left(A\cos(\omega t)\right)^2 dt = \frac{A^2}{T}\int_0^T \cos^2(\omega t)\,dt =$$

$$= \frac{A^2}{2T}\int_0^T 1\,dt + \frac{A^2}{2T}\int_0^T \underline{\cos(2\omega t)}\,dt = \frac{A^2}{2\cancel{T}}\cdot\cancel{T} + 0 = \frac{A^2}{2}$$

$L = 2^b$

$$SNR = 10\log_{10}\frac{P_s}{P_e} = 10\log_{10}\frac{\frac{A^2}{2}}{\frac{\Delta^2}{12}} = 10\log_{10}\frac{6\cancel{A^2}}{\left(\frac{2\cancel{A}}{L}\right)^2} = 10\log_{10}\frac{3}{2}L^2 =$$

$$= 10\log_{10}\frac{3}{2}\left(2^b\right)^2 = 10\log_{10}\frac{3}{2} + 10\log_{10}2^{2b} = \underline{10\log_{10}\frac{3}{2}} + \underline{20b}\,\underline{\log_{10}2} =$$

$\log ab = \log a + \log b$

$$= \underline{1.76 + 6b}$$

constant

+1 bit +6 dB

−1 bit −6 dB