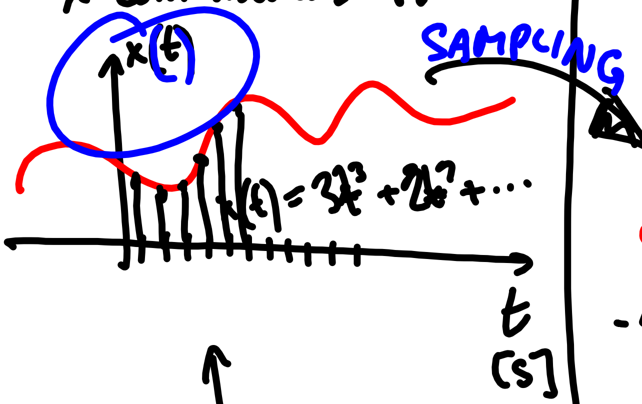
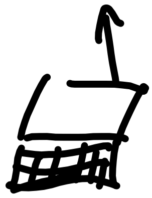
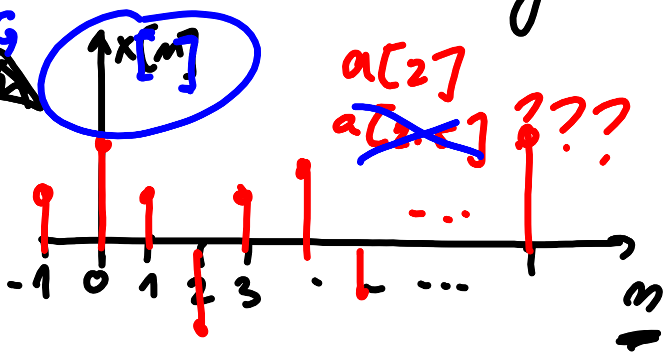
SIGNALS

functions
≈ continuous-time





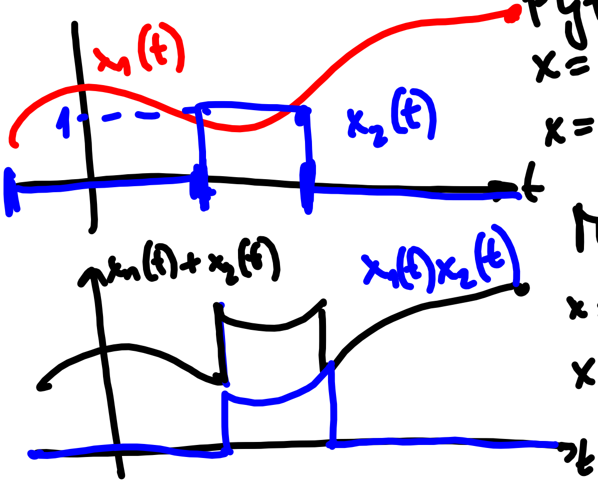
sequences of numbers
≈ discrete-time signals



n	-1	0	1	2	3	....
x[n]	1	2	1	-2	1	2

Operations with signals

$x_1(t)$ $x_2(t)$



Python

```
x = x1 + x2
x = x1 * x2
```

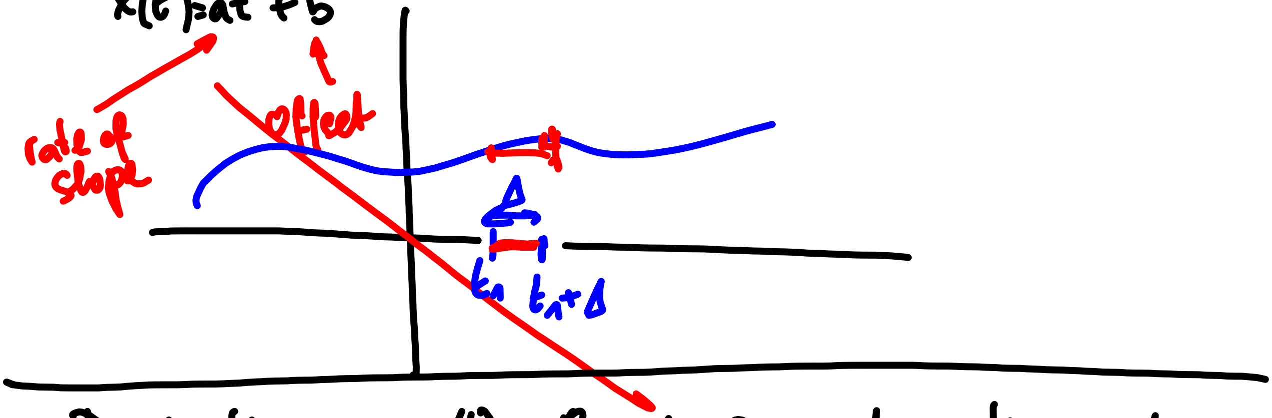
element-by-element product!

Matlab

```
x = x1 + x2;
x = x1.* x2;
```

Linear function

$x(t) = at + b$



Derivation

$x(t) = t^2 + 2t + 3$

Analytically: $\frac{dx(t)}{dt} = 2t + 2$

Numerically: $\left.\frac{dx(t)}{dt}\right|_{t_1} = \frac{x(t_1 + \Delta) - x(t_1)}{\Delta}$

derivations at

$t_a = 0$ $\quad 2 \cdot 0 + 2 = 2$

$t_b = 1$ $\quad 2 \cdot 1 + 2 = 4$

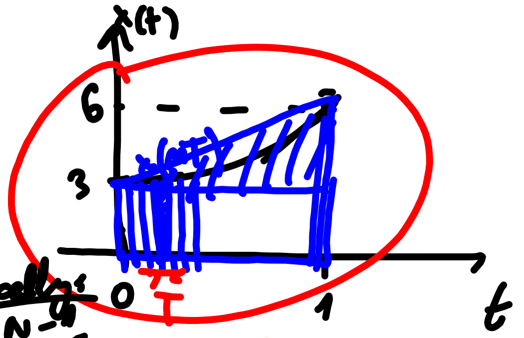
2.05!

4.05

Integration

$x(t) = t^2 + 2t + 3$

$$\int_0^1 x(t)\,dt$$



analytically:

$$\left[\frac{t^3}{3} + t^2 + 3t\right]_0^1 =$$

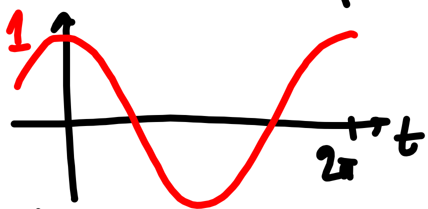
$$= \left(\frac{1}{3} + 1 + 3\right) - (0 + 0 + 0) = \underline{4{,}33}$$

numerically:

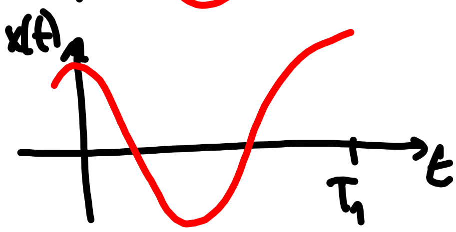
$$\sum_{n=0}^{N-1} x(nT)\,T \qquad N = \frac{1}{T}$$

approximate: $1 \cdot 3 + \frac{1 \cdot 3}{2} = 4{,}5$

Cosine function



$x(t) = \cos t$



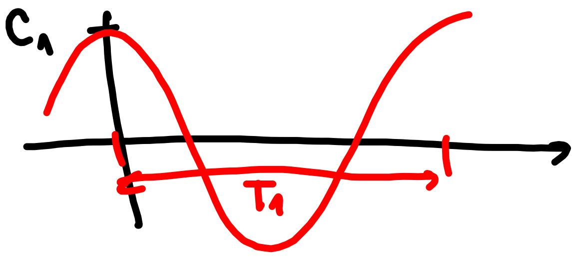
$x(t) = \cos\left(2\pi \frac{t}{T_1}\right)$

$T_1 = \frac{1}{50}\,s$

circular
angular frequency

$\frac{1}{T_1} = f_1$ [Hz]

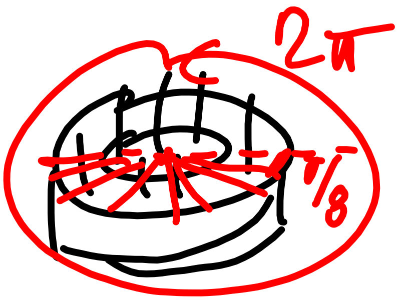
$\frac{2\pi}{T_1} = 2\pi f_1 = \omega_1$ [rad/s]



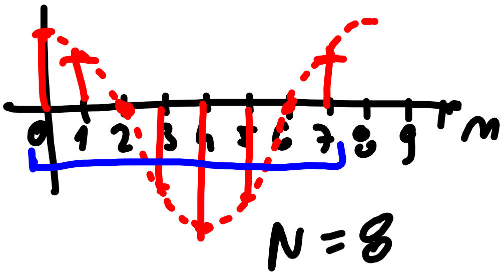
$x(t) = C_1 \cos(\omega_1 t + \varphi_1)$

any unit — rad/s — rad

$\pi/2$



Discrete cosine



$N = 8$

plot a[8]

$x[n] = C_1 \cos(\omega_1 n + \varphi_1)$

anything [rad] [rad]

N samples

$x[n] = \cos\left(2\pi \frac{n}{N}\right)$

$\frac{1}{N} = f_1$ [ ] no unit!

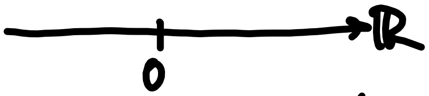
normalized frequency

$\frac{2\pi}{N} = 2\pi f_1$ [rad] $= \omega_1$

normalized angular frequency

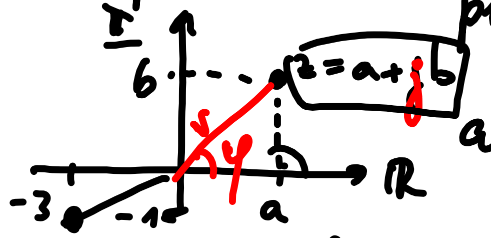
Complex numbers

Real



$j = \sqrt{-1}$ comple unit.

Complex



complex plane.

composite form

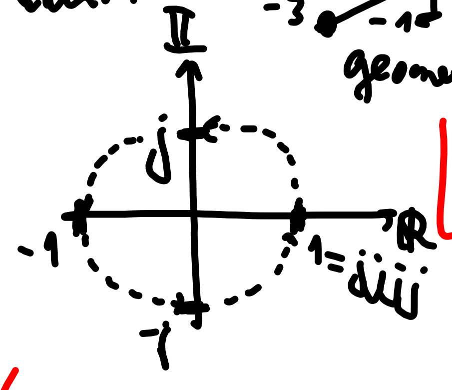
geometrical form...

$$\boxed{\begin{aligned} a &= r\cos\varphi \\ b &= r\sin\varphi \end{aligned}}$$

Math: $z = a + ib$

Eng. $z = a + jb$

$1\, j\, j\, j\, j \dots$



$r = \sqrt{a^2 + b^2}$ ✓

$\varphi = \arctan \frac{b}{a}$ ✗

mp. angle (z)

angle (z)

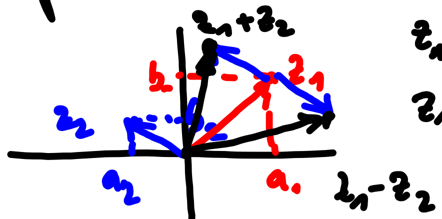
r - abs. value (magnitude)

φ - angle (phase)

Exponential form:

$z = r e^{j\varphi}$ $\quad z = r \cdot \exp(j\varphi)$ $\quad z = r \measuredangle \varphi$

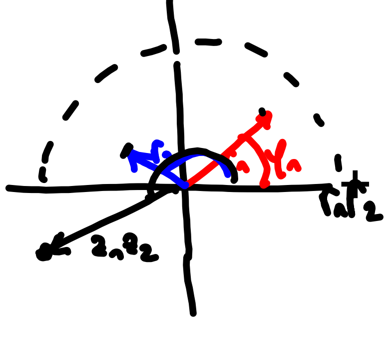
Operations with complex numbers.



$z_1 + z_2 = a_1 + a_2 + j(b_1 + b_2)$

$z_1 - z_2 = a_1 - a_2 + j(b_1 - b_2)$

$e^a \cdot e^b = e^{a+b}$

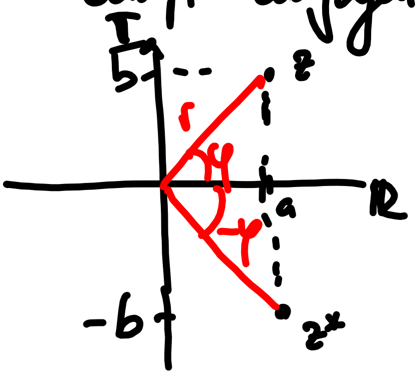


$z_1 \cdot z_2 = \cancel{(a_1 + jb_1)(a_2 + jb_2) = a_1 a_2 + ja_1 b_2 + jb_1 a_2 - b_1 b_2}$

$= r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2} = r_1 r_2 e^{j(\varphi_1 + \varphi_2)}$

$\frac{z_1}{z_2} = \frac{r_1 e^{j\varphi_1}}{r_2 e^{j\varphi_2}} = \frac{r_1}{r_2} e^{j\varphi_1} \cdot e^{-j\varphi_2} = \frac{r_1}{r_2} e^{j(\varphi_1 - \varphi_2)}$

Complex conjugation



$z = a + jb = re^{j\varphi}$

$z^* = a - jb = re^{-j\varphi}$

$z + z^* = 2a$

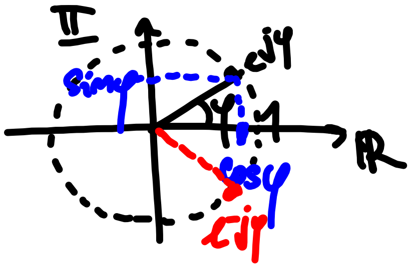
$zz^* = re^{j\varphi} \cdot re^{-j\varphi} = r^2 e^{j\underbrace{(\varphi-\varphi)}_{0}} = r^2$

z square of magnitude

np.pow(np.abs(z), 2.0)

z * np.conj(z)

Unit circle

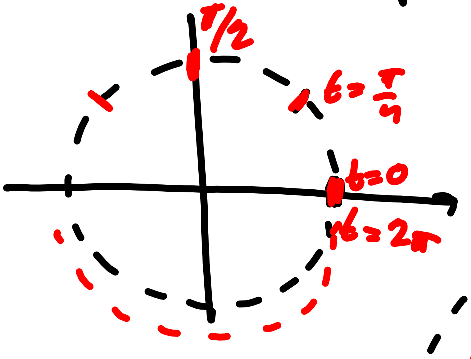


$|e^{j\varphi}| = 1$

$e^{j\varphi} + e^{-j\varphi} = 2\cos\varphi$

$$\boxed{\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}}$$

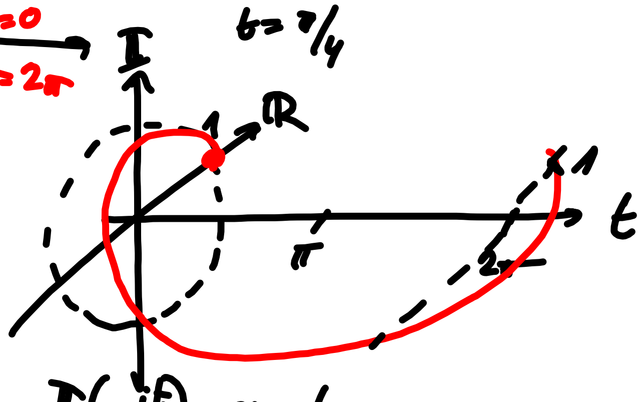
Complex exponentials functions!



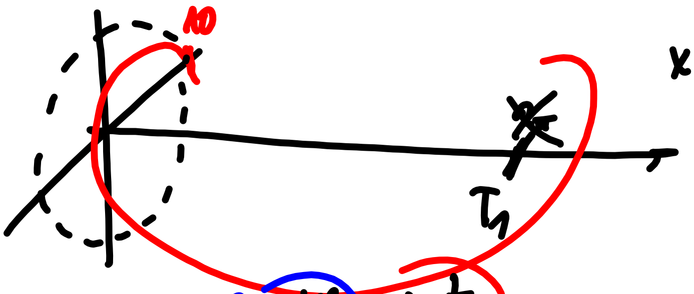
$x(t) = e^{jt}$

$t=0$

$t=\pi/4$



$\mathrm{Re}(e^{jt}) = \cos t$ $\quad$ $\mathrm{Im}(e^{jt}) = \sin t$



$x(t) = C_1 e^{j 2\pi \frac{t}{T_1}}$ $\quad$ e^{a+b}

$= C_1 e^{j(\omega_1 t + \varphi_1)} =$

$= C_1 e^{j\varphi_1} e^{j\omega_1 t}$

constant! $\quad$ function