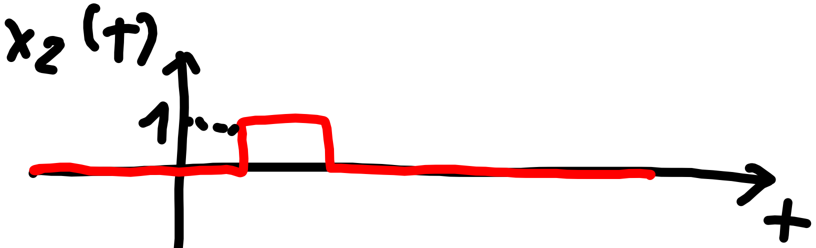
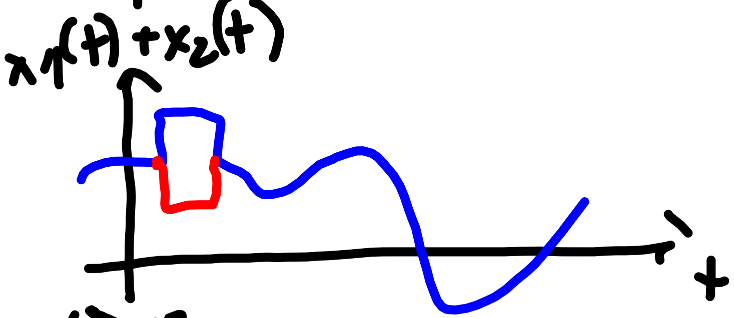
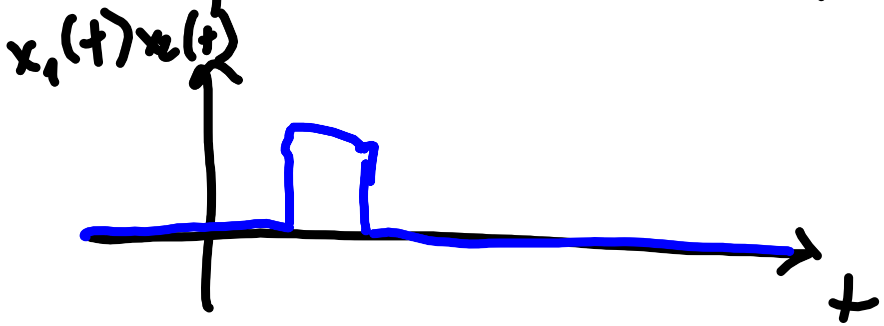
TODAY'S PLAN continuous signals

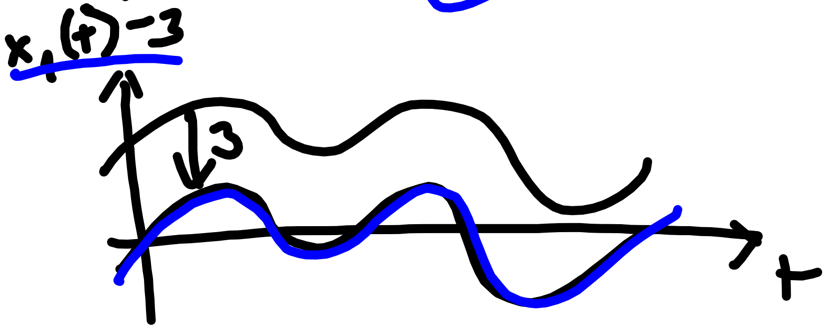
1. basic operations with signals
2. energy / power
3. periodic signals
4. spectral analysis


$x_1(t)$
t

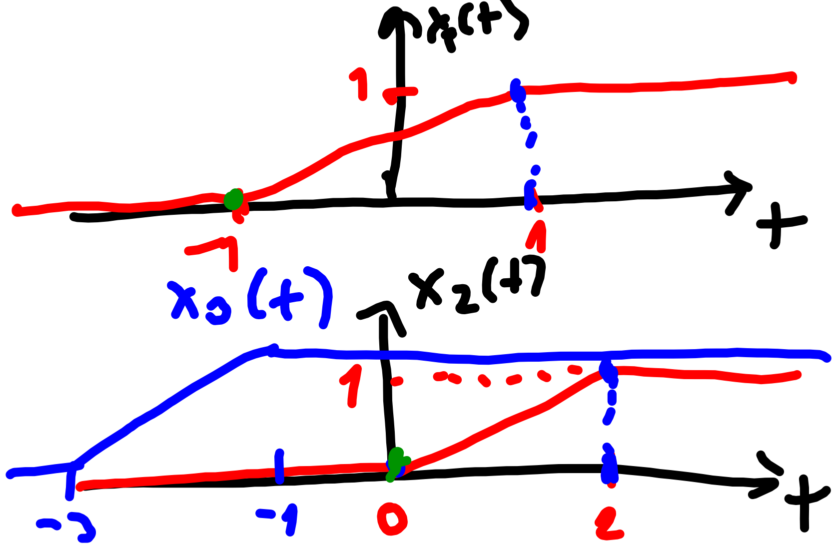

$x_2(t)$
1
t


$x_1(t)+x_2(t)$
t


$x_1(t)x_2(t)$
t


$x_1(t)-3$
3
t

Time modifications



$$x_1(t) = \begin{cases} 0 & \text{for } t < -1 \\ 0.5t + 0.5 & \text{for } t \in \langle -1, 1) \\ 1 & \text{for } t > 1 \end{cases}$$

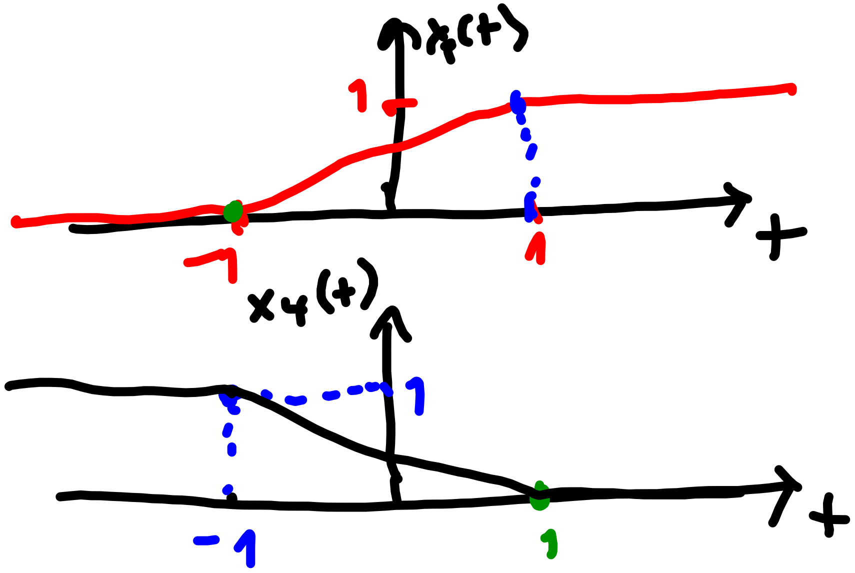
$$x_2(t) = x_1(t-1)$$

$$x_2(2) = x_1(2-1) = x_1(1)$$

$$x_3(t) = x_1(t+2)$$

$x(t+\tau)$ $\tau > 0$ SHIFT LEFT (ADVANCE)

$x(t-\tau)$ $\tau > 0$ SHIFT RIGHT (DELAY).



FLIPPING / REVERSING

$$x_4(t) = x_1(-t)$$

$$x_4(1) = x_1(-1)$$

$$x_4(-1) = x_1(--1)$$

$$x(-t) \neq -x(t)$$

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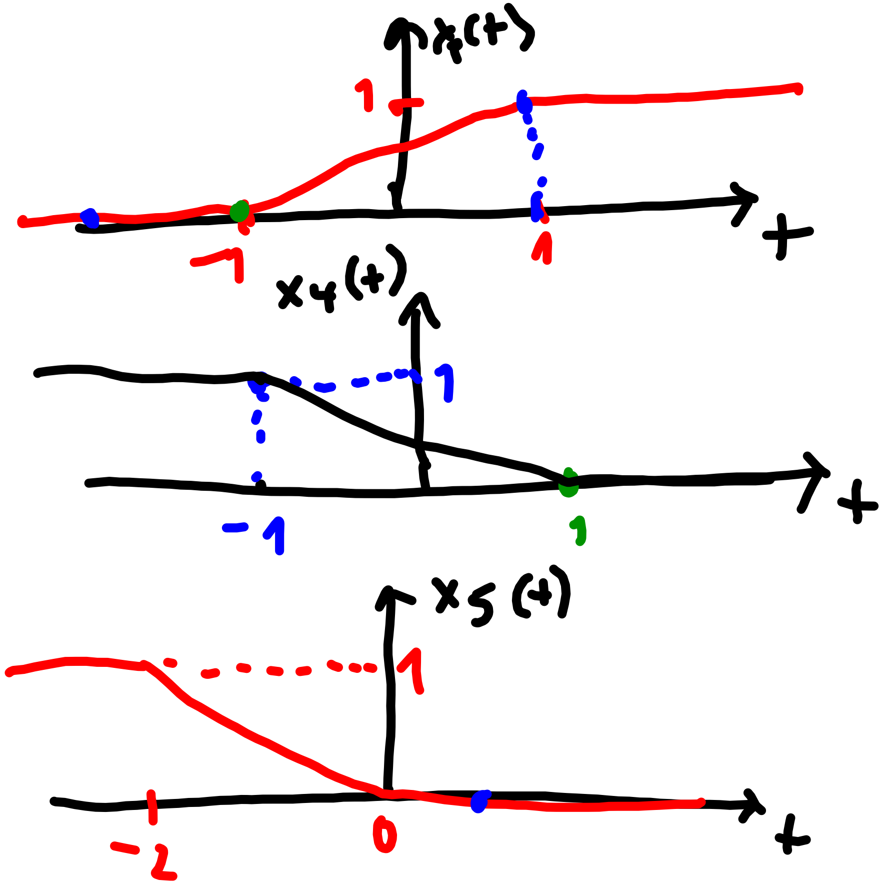


$x(-t+\tau)$ $\tau > 0$

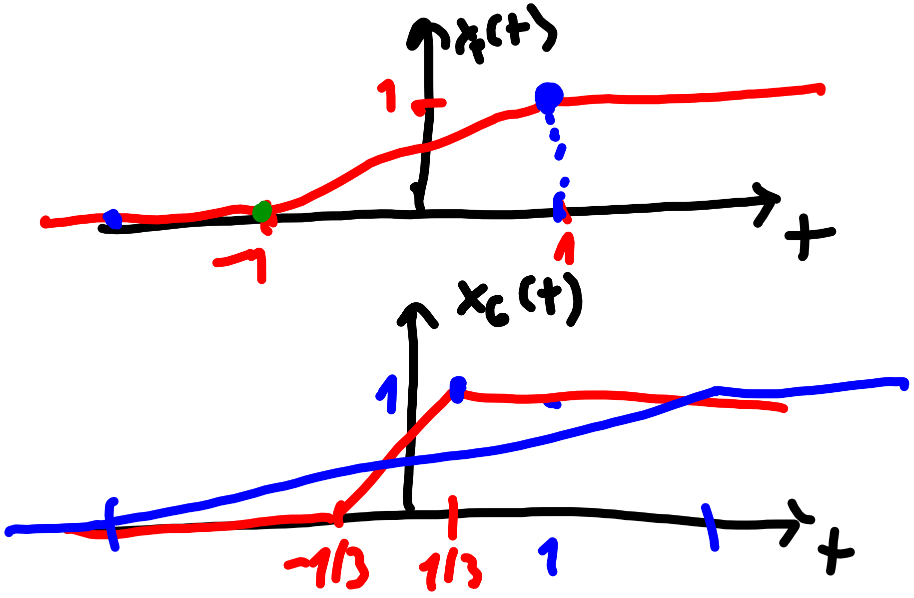
SHIFT RIGHT

$x(-t-\tau)$ $\tau > 0$

SHIFT LEFT

$x_5(t) = x_1(-t-1)$

$x_5(1) = x_1(-1-1)$



$x(kt)$ $k>1$ CONTRATION

$x(kt)$ $k<1$ DILATION

$x_6(t) = x_1(3t)$

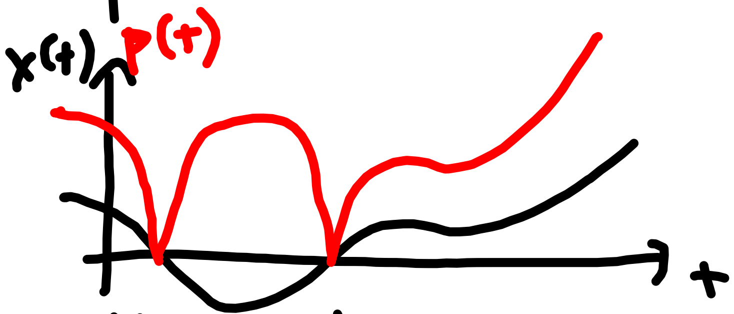
$x_6(1) = x_1(3 \cdot 1)$

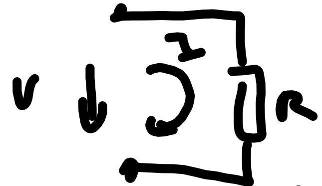
$x_7(t) = x_1(t/2)$

ENERGY / POWER

- instantaneous power (okamžitý výkon)
- energy
- mean power
- effective value

$$p(t) = |x(t)|^2$$





$$P = VI = \frac{V^2}{R} = I^2 R$$

$$\sim z^2 = z \cdot z$$

$$z = r e^{j\varphi}$$

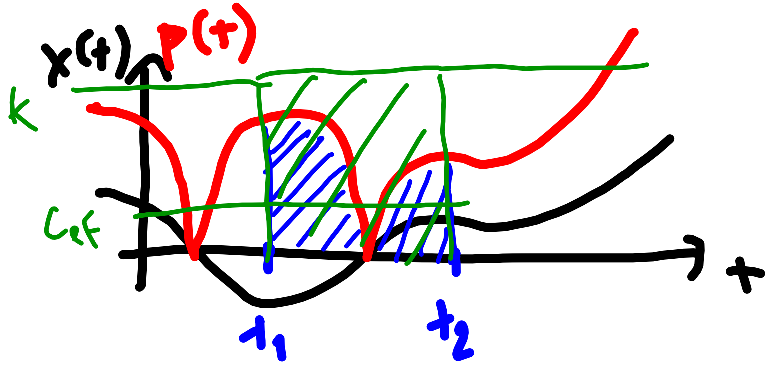
$$p(t) = |x(t)|^2 = x^2(t) \qquad x(t) \in \mathbb{R}$$

$$|x(t)|^2 \neq x^2(t) \qquad x(t) \in \mathbb{C}$$

$$|z|^2 = z \cdot z^* = r e^{j\phi} \cdot r e^{-j\phi} = r^2 e^0 = r^2$$

$$z^2 = z \cdot z = r e^{j\phi} \cdot r e^{j\phi} = r^2 e^{2j\phi}$$

ENERGY from t_1 to t_2



$$E_{t_1,t_2} = \int_{t_1}^{t_2} p(t)\,dt$$

$\lim_{t_1 \to -\infty,\ t_2 \to \infty}$

$$E_{t_1,t_2} = k(t_2 - t_1)$$

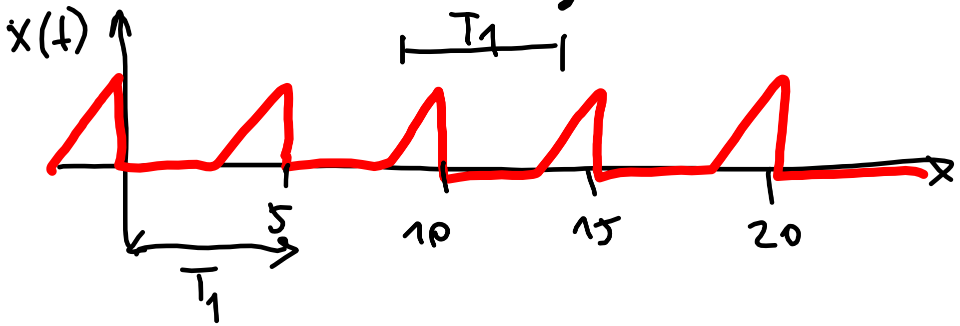
MEAN POWER (střední výkon)

$$P_{mean} = \frac{E_{t_1,t_2}}{t_2 - t_1}$$

EFFECTIVE VALUE

$$c_{ef} = \sqrt{P_{mean}}$$

PERIODIC SIGNALS



repeats in time

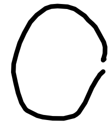
$\exists T \, \forall t \; x(t) = x(t + T)$

$x(5) = x(5 + 10)$

$T_1 = 5\,s$ fundamental period

$f_1 = \frac{1}{T_1}$ Hz fundamental freq.

$\omega_1 = 2\pi f_1 \; \frac{rad}{s}$ fundamental ang. freq.



$2\pi \; rad = 360^0$

energy

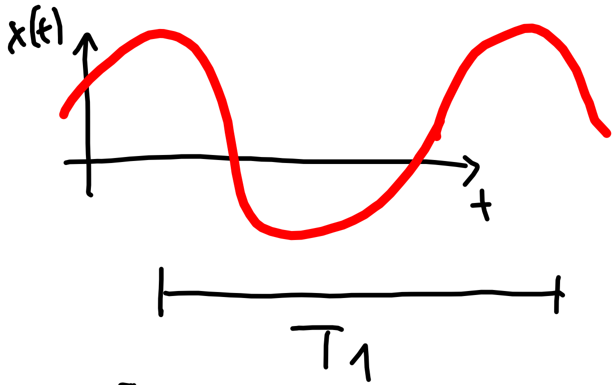
$E_{T_1} = \int_{T_1} x^2(t)\,dt$

mean power

$P_{mean} = \frac{1}{T_1} \int_{T_1} x^2(t)\,dt$

COSINE

$$x(t) = C_1 \cos(\omega_1 t + \varphi_1)$$



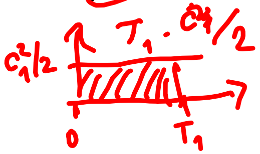
$$\omega_1 = \frac{2\pi}{T_1}$$

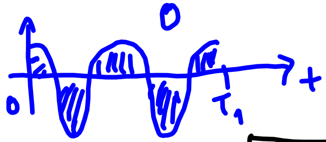
Mean power of cosine

$$P_{mean} = \frac{1}{T_1} \int_{T_1} x^2(t)\, dt =$$

$$= \frac{1}{T_1} \int_0^{T_1} [C_1 \cos(\omega_1 t)]^2 dt = \frac{1}{T_1} C_1^2 \int_0^{T_1} \cos^2(\omega_1 t)\, dt =$$

$$= \frac{1}{T_1} C_1^2 \int_0^{T_1} \frac{1+\cos(2\omega_1 t)}{2}\, dt = \frac{1}{T_1} \left[\underbrace{\int_0^{T_1} \frac{C_1^2}{2}\, dt}_{T_1 \cdot C_1^2/2} + \underbrace{\int_0^{T_1} \frac{C_1^2}{2} \cos(2\omega_1 t)\, dt}_{0} \right] =$$





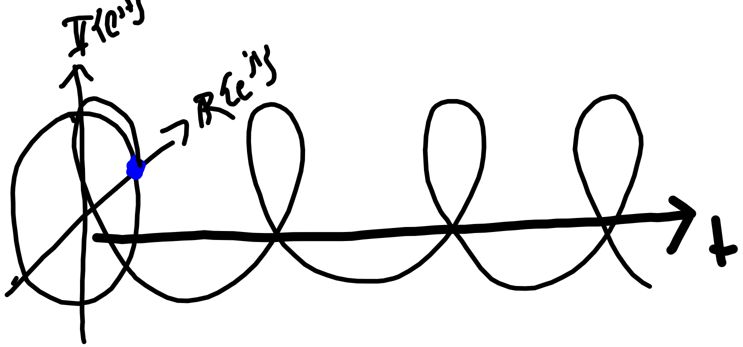
$$= \frac{1}{T_1} \cdot T_1 \cdot \frac{C_1^2}{2} = \underline{\underline{\frac{C_1^2}{2}}}$$

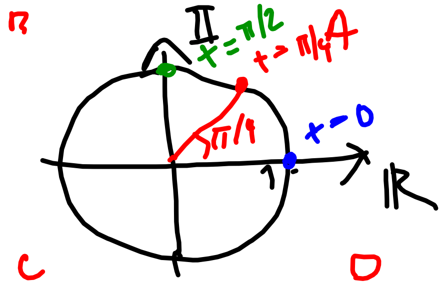
$$C_{ef} = \frac{C_1}{\sqrt{2}}$$

$$\boxed{\cos^2 \alpha = \frac{1+\cos 2\alpha}{2}}$$

COMPLEX EXPONENTIAL

$$x(t) = e^{jt}$$





$t = 0 \quad e^{j0} = 1$

$t = \pi/4 \quad e^{j\pi/4}$

$t = \pi/2 \quad e^{j\pi/2}$

$$\mathbb{R}\{e^{jt}\} = \cos(t)$$

$$\mathbb{I}\{e^{jt}\} = \sin(t)$$

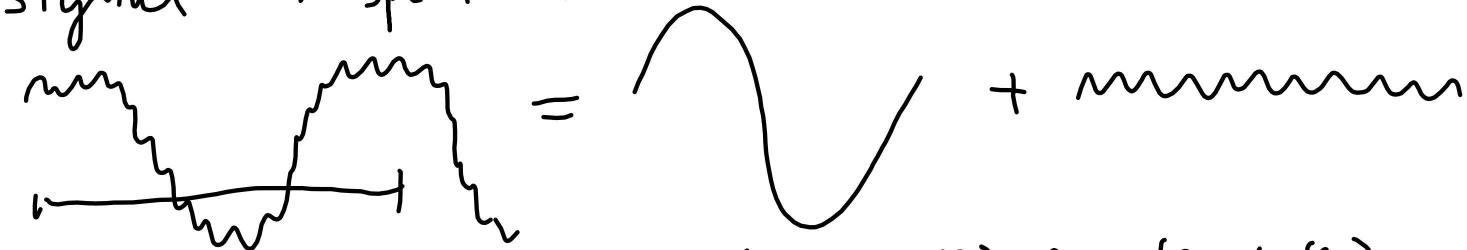
$$e^{-jt}$$

$$\underbrace{e^{jt}}_{\cos(t) + \cancel{j\sin(t)}} + \underbrace{e^{-jt}}_{\cos(t) - \cancel{j\sin(t)}} = 2\cos(t)$$

$$\boxed{\cos(t) = \frac{e^{jt} + e^{-jt}}{2}}$$

SPECTRAL ANALYSIS of cont. periodic signal:

- which freq. components and how much? $T_1 / F_1 / \omega_1$
  and how delayed?
- signal → spectrum



$$x(t) = C_0 + C_1\cos(\omega_1 t + \varphi_1) + C_2\cos(2\omega_1 t + \varphi_2) + C_3\cos(3\omega_1 t + \varphi_3) + \dots$$

$$10\cos(\omega_1 t + \pi/4) + 0\cos(2\omega_1 t) \dots + 0\cos(3\omega_1 t)$$

$$+ 0\cos(4\omega_1 t) + \underbrace{0.5}_{C_5}\cos(5\omega_1 t)$$

$$x(t) = \dots + c_{-2}e^{-j2\omega_1 t} + c_{-1}e^{-j\omega_1 t} + c_0 + c_1 e^{j\omega_1 t} + c_2 e^{j2\omega_1 t} + \dots$$

$$5e^{-j\pi/4}e^{-j\omega_1 t} + 5e^{j\pi/4}e^{j\omega_1 t} + 0.25\, e^{-j5\omega_1 t} + 0.25\, e^{j5\omega_1 t}$$

$$e^{a+b} = e^a e^b$$

$$\boxed{\cos(x) = \frac{e^{jx} + e^{-jx}}{2}}$$

$$C_1\cos(\omega_1 t + \varphi_1) = C_1\left(\frac{e^{j(\omega_1 t + \varphi_1)} + e^{-j(\omega_1 t + \varphi_1)}}{2}\right) =$$

$$= \underbrace{\frac{C_1}{2}e^{j\varphi_1}}_{c_1} e^{j\omega_1 t} + \underbrace{\frac{C_1}{2}e^{-j\varphi_1}}_{c_{-1}} e^{-j\omega_1 t}$$

$$x(t) = \dots + c_{-2} e^{-j2\omega_1 t} + c_{-1} e^{-j\omega_1 t} + c_0 + c_1 e^{j\omega_1 t} + c_2 e^{j2\omega_1 t} + \dots$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}}$$

synthesis formula of Fourier series (inverse Fourier series)

$$\boxed{c_k = \frac{1}{T_1} \int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt}$$

analysis formula of Fourier series (Forward Fourier series)

Ex: Compute FS of $x(t)$

$$x(t) = 50\, e^{j(1000\pi t + \pi/2)}$$

$$c_k = ?$$

$$\omega_1 = 1000\pi$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_1 t}}$$

$$x(t) = \underbrace{50\, e^{j\pi/2}}_{c_k}\, e^{j1000\pi t}$$

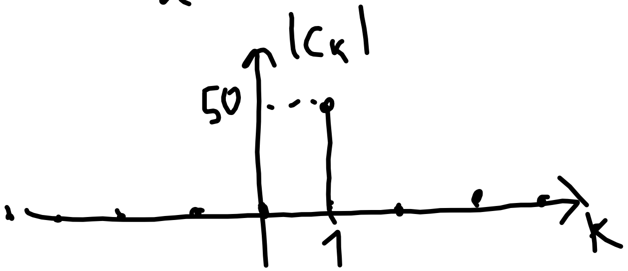
$$\boxed{c_k = \frac{1}{T_1} \int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt}$$

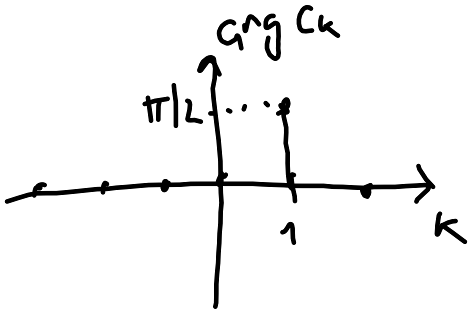
$$c_1 = 50\, e^{j\pi/2}$$

$$c_k = 0 \text{ for } k \neq 1$$

$$0 \cdot e^{j\phi}$$

$$re^{j\phi}$$





$$c_1 e^{j\phi} e^{j\omega_1 t}$$