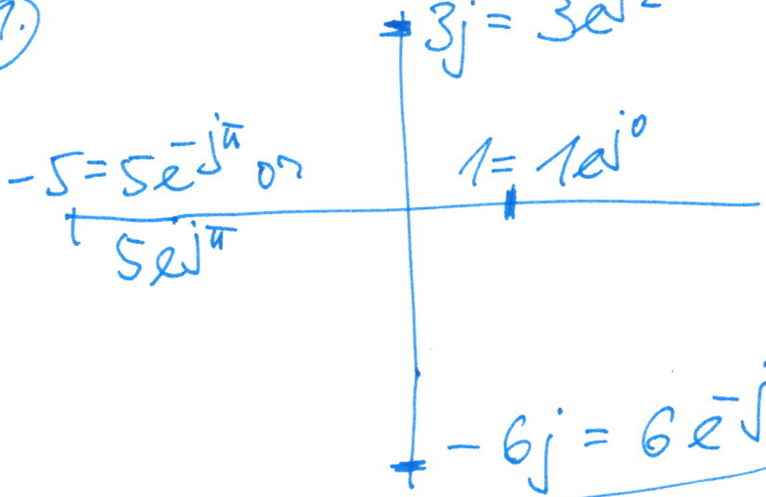
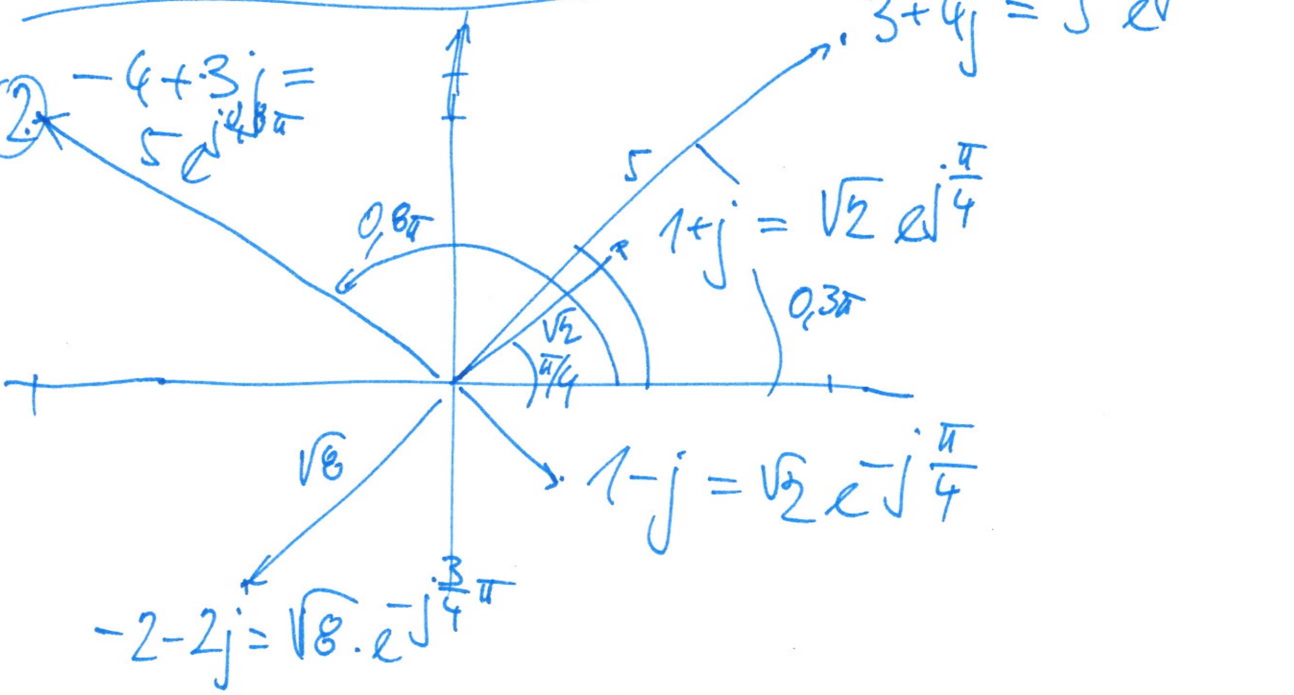
ISS exercise No. 2

1.



2.



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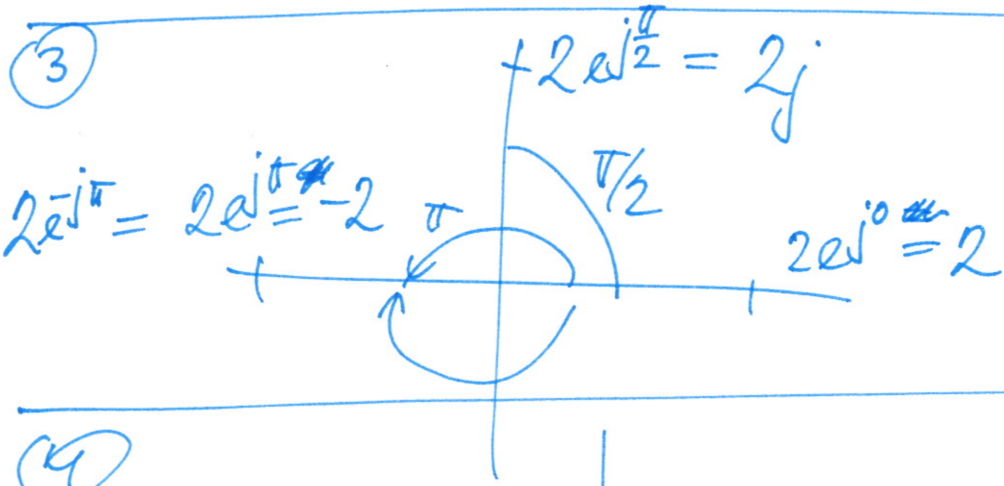

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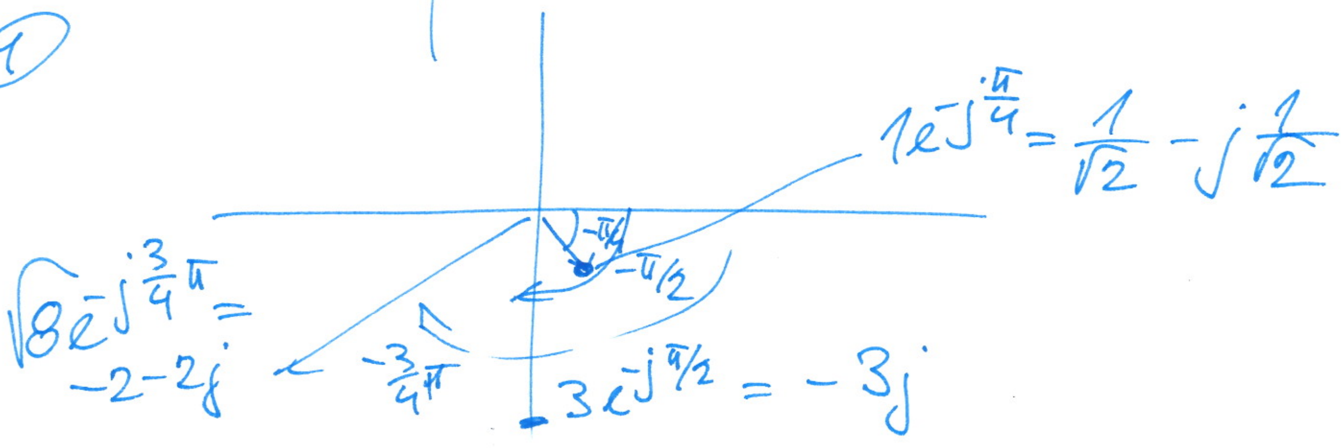

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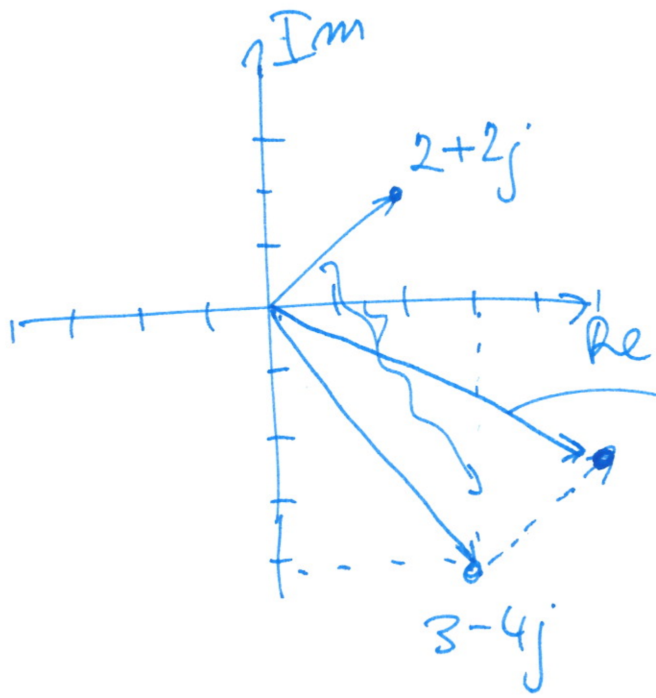
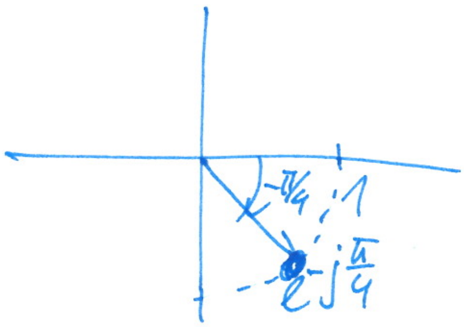


Adding real and imaginary parts

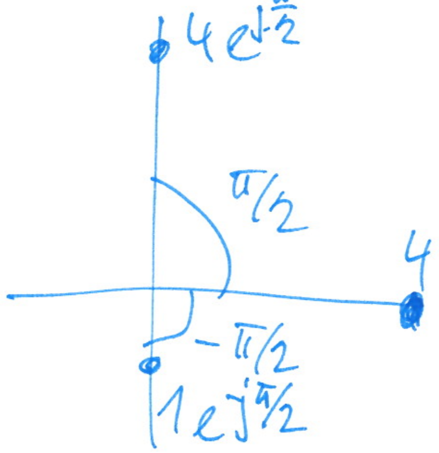
result: $5-2j$

6.

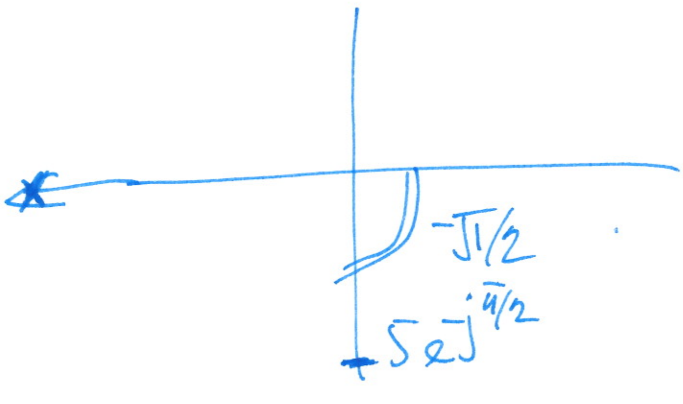


Multiplying magnitudes, adding angles.

$$1\cdot 1\cdot e^{j(0-\frac{\pi}{4})} = 1\cdot e^{-j\frac{\pi}{4}}$$

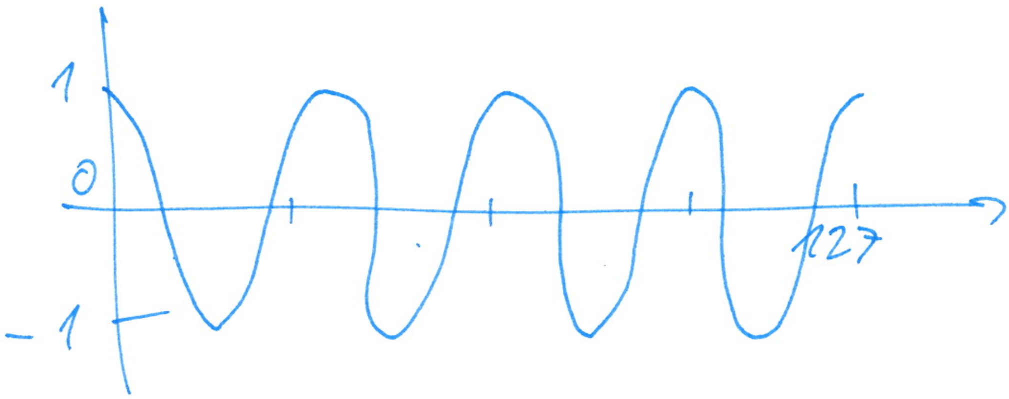


$$4\cdot e^{j\pi/2}\cdot 1e^{-j\pi/2} = 4$$

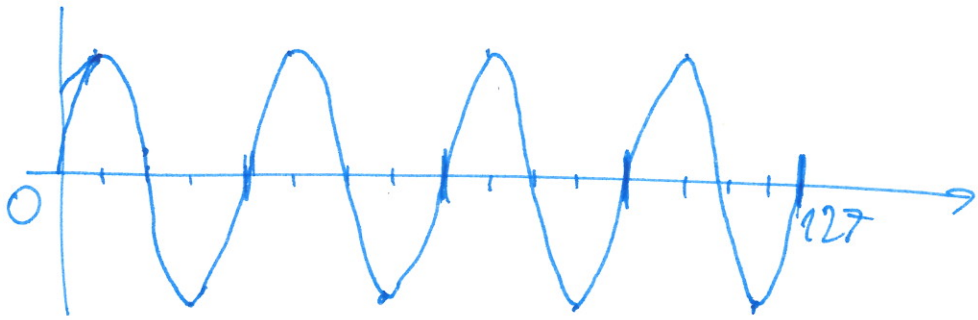


$$5\cdot e^{-j\pi/2}\cdot 5e^{-j\pi/2} =$$
$$25e^{j(-\frac{\pi}{2}-\frac{\pi}{2})} = 25e^{-j\pi} = -25$$

7.



8. $-\frac{\pi}{2} \approx$ delay of quarter a period.

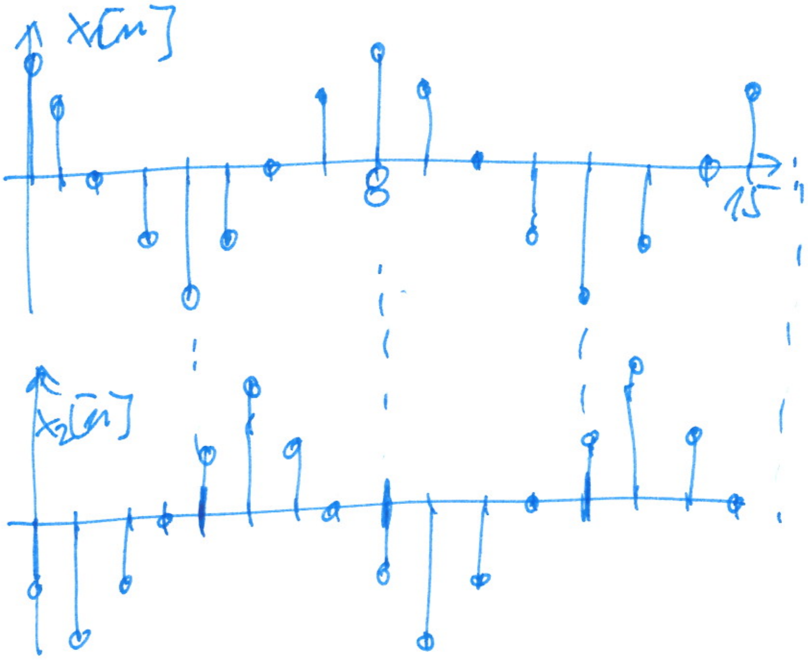


9. quarter period is $\frac{\frac{128}{4}}{4}$ samples ... 8.

$x_2[n] = x_1[n-8]$

10.

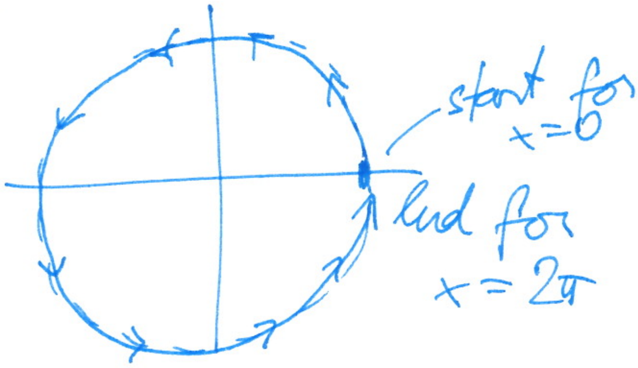
11.



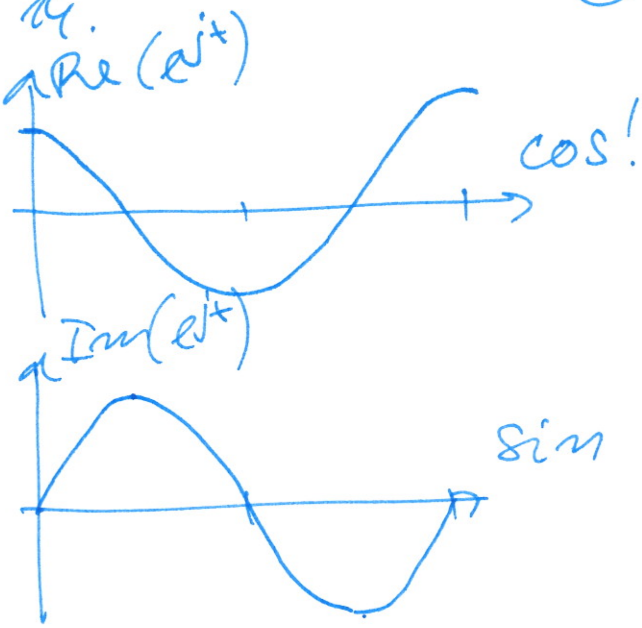
$+\frac{3}{4}\pi$ corresponds to $\frac{3}{8}$ of period advance, this is 3 samples.

12. $x_2[n] = x_1[n+3]$

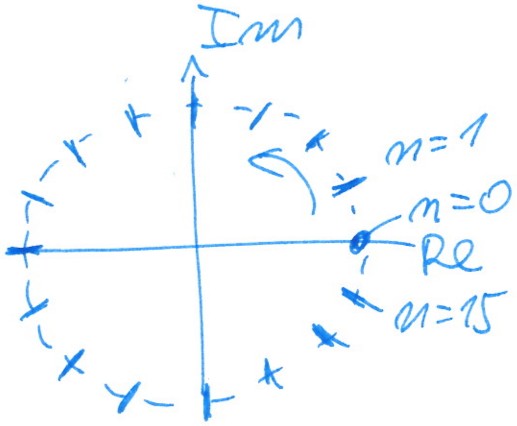
13.



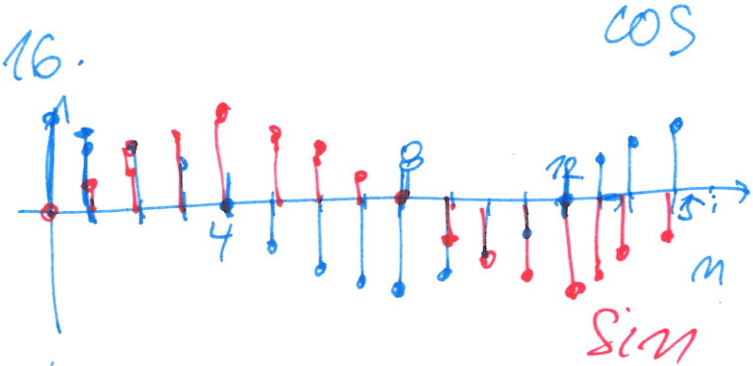
14.



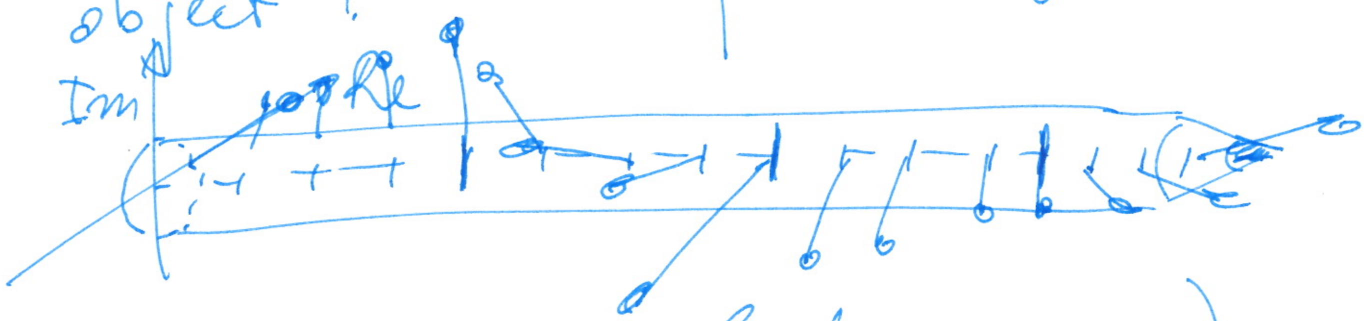
15.



16.

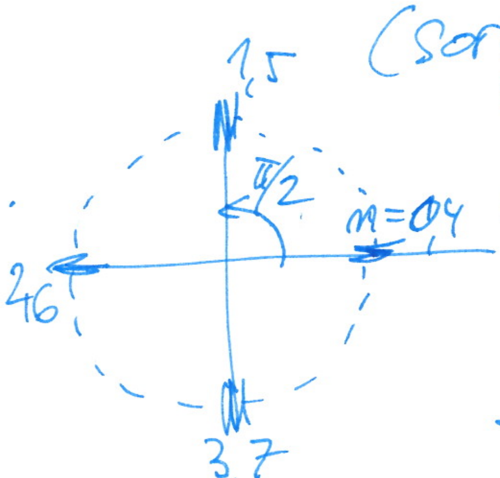


17. Show it on some ~~round allongated~~ object!

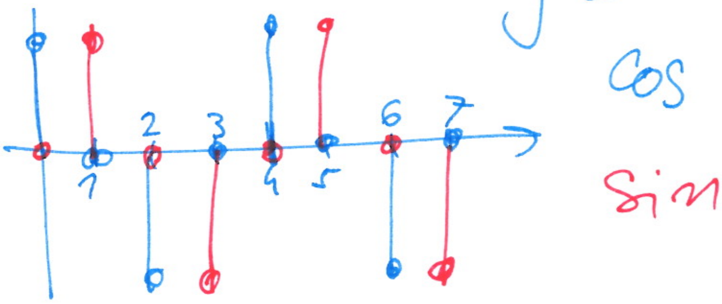


(sorry, ugly drawing...)

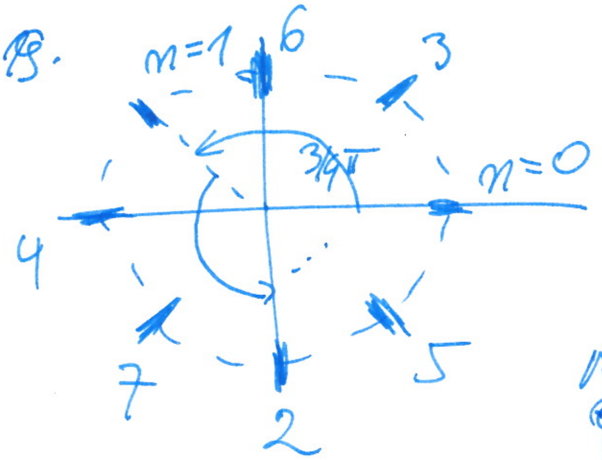
18.



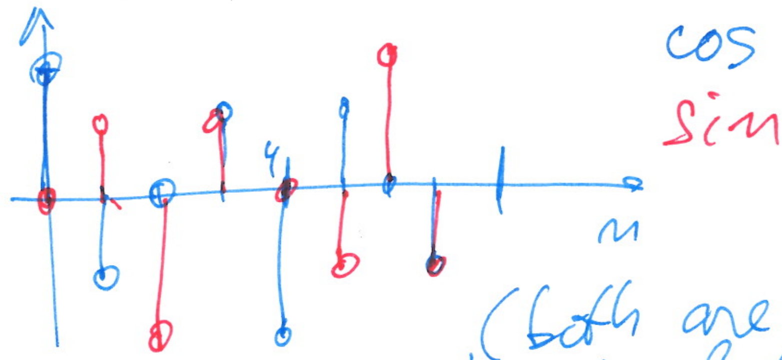
$e^{j 2\pi \frac{2}{8} n} = e^{j \frac{\pi}{2} n}$ – advancing by $\frac{\pi}{2}$!



19.



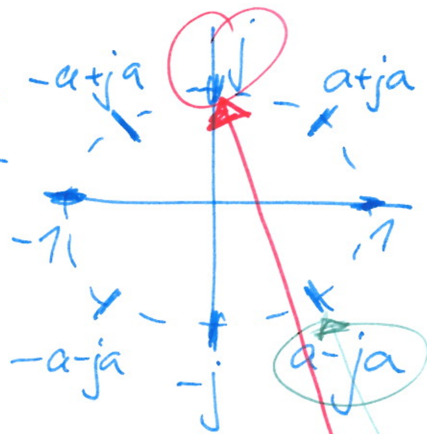
$e^{j2\pi\frac{3}{8}n} = e^{j\frac{3}{4}\pi n}$, advancing by $\frac{3}{4}\pi$



(both are a bit too fast!)

20.

n	0	1	2	3	4	5	6	7
$e^{\cdots}$	1	a+ja	j	-a+ja	-1	-a-ja	-j	a-ja
$e^{j\pi/2}e^{\cdots}$	j	-a+ja	-1	-a-ja	-j	a-ja	1	a+ja
$e^{-j\pi/4}e^{\cdots}$	a-ja	1	a+ja	j	-a+ja	-1	-a-ja	-j



21. When multiplied by $e^{j\pi/2}$, starting from here ~~21. When~~

22. When multiplied by $e^{-j\frac{\pi}{4}}$, starting from here

23, 24, 25 Similar, but turning in the opposite sense and starting from 1, -j and a+ja

n	0	1	2	3	4	5	6	7
$e^{\cdots}$	1	a-ja	-j	-a-ja	-1	-a+ja	j	a+ja
$e^{j\pi/2}e^{\cdots}$	-j	-a-ja	-1	-a+ja	j	a+ja	1	a-ja
$e^{j\pi/4}e^{\cdots}$	a+ja	1	a-ja	-j	-a-ja	-1	-a+ja	j

26. In the sums, the imaginary components
27.
28. will kill each other! You can also show it in the complex plane, where the two function start against each other (conjugated) and continue conjugated.

advance by $\frac{1}{4}$ period

n	0	1	2	3	4	5	6	7
$x_1[n]$	2	2a	0	−2a	−2	−2a	0	2a
$x_2[n]$	0	−2a	−2	−2a	0	2a	2	2a
$x_3[n]$	2a	2	2a	0	−2a	−2	−2a	0

delay by $\frac{1}{8}$ period

How much is 2a? $2\frac{1}{\sqrt{2}} = \sqrt{2}$

Expressing ~~the~~ it by one function:

$$x_1[n] = 2\cos\left(2\pi \frac{1}{8} n\right)$$

$$x_2[n] = 2\cos\left(2\pi \frac{1}{8} n + \frac{\pi}{2}\right)$$ ← advanced by $\frac{1}{4}$ period, so phase shift $+\frac{\pi}{2}$

$$x_3[n] = 2\cos\left(2\pi \frac{1}{8} n - \frac{\pi}{4}\right)$$ ← delay by $\frac{1}{8}$ period so phase shift $-\frac{\pi}{4}$

~~29. For example the last one...~~

$$x_3[n] = \frac{\cancel{2}\, e^{j\left(2\pi \frac{1}{8} n - \frac{\pi}{4}\right)} + e^{-j\left(2\pi \frac{1}{8} n - \frac{\pi}{4}\right)}}{\cancel{2}} =$$ we can use $e^{a+b} = e^a \cdot e^b$ =

$$= e^{-j\frac{\pi}{4}} e^{j2\pi \frac{1}{8} n} + e^{j\frac{\pi}{4}} e^{-j2\pi \frac{1}{8} n}$$ – proved ■