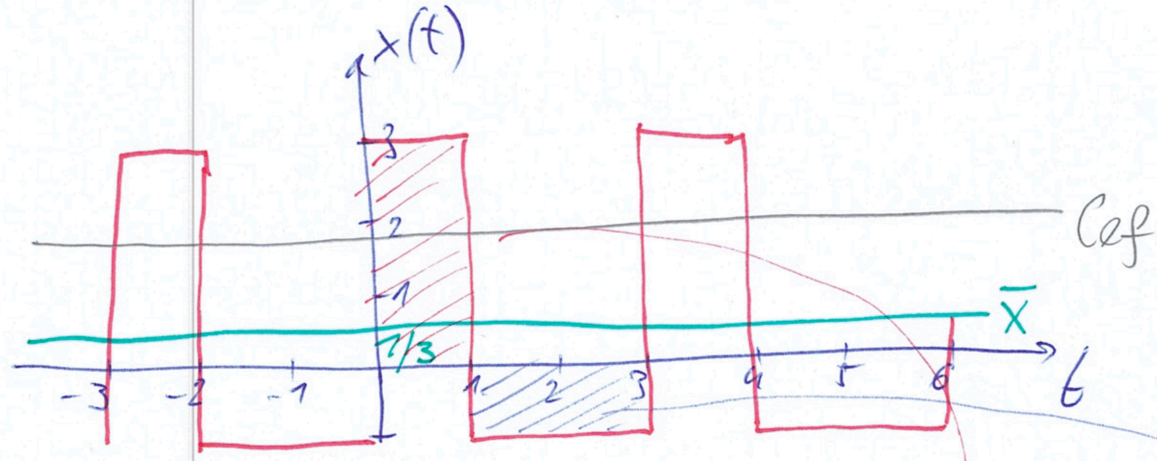
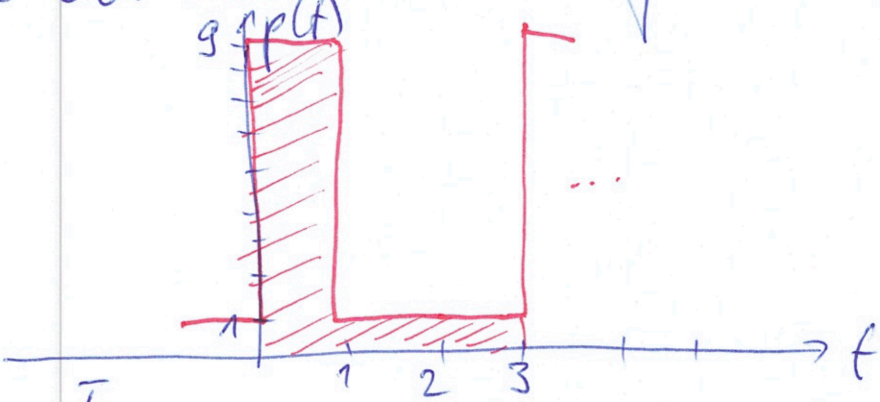
1.



2. Mean value: $\bar{x} = \frac{1}{T_1}\int_0^{T_1} x(t)\,dt = \frac{1}{3}\left(3\cdot 1 + 2\cdot(-1)\right) = \underline{\underline{\frac{1}{3}}}$

3. We need instantaneous power $p(t) = x^2(t)$



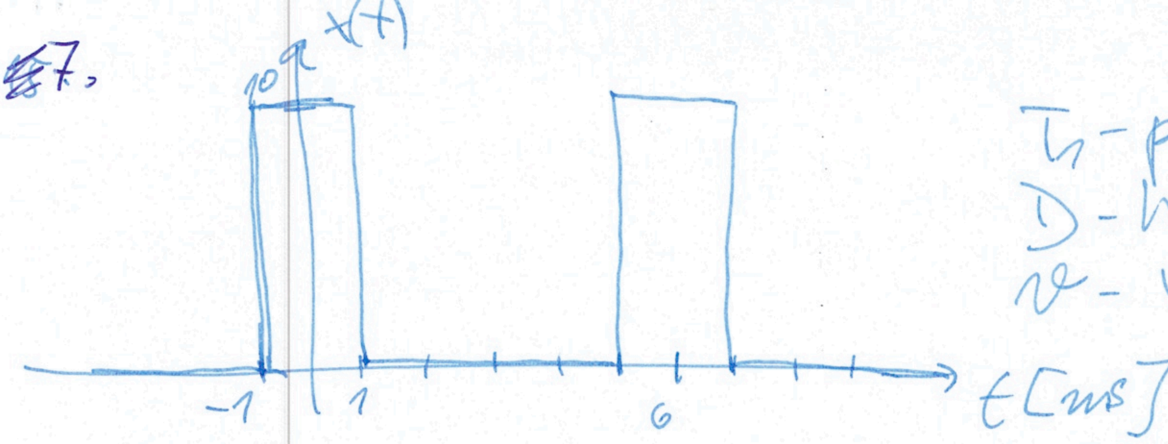
$$E = \int_0^{T_1} p(t)\,dt = 9 + 2 = \underline{\underline{11}}$$

4. $P = \frac{E}{T_1} = \frac{11}{3} = \underline{\underline{3,66}}$

5. $C_{ef} = \sqrt{P} = \sqrt{3,66} = 1,91$

6. See above the green and black lines.
Different, because negative signal decreases the mean value, but contributes to the power.

7.



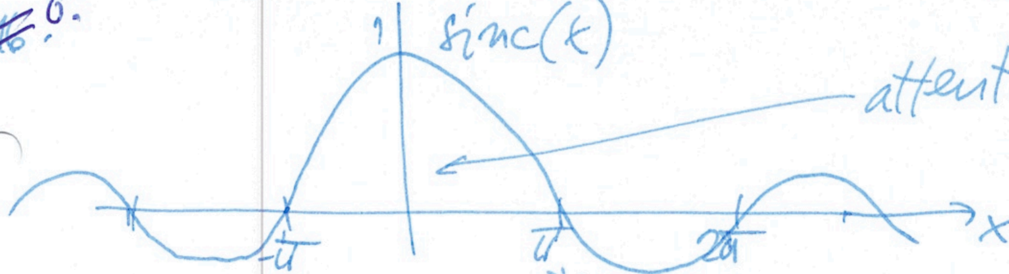
T_1 – period $6\cdot10^{-3}$

D – height 10

τ – width $2\cdot10^{-3}$

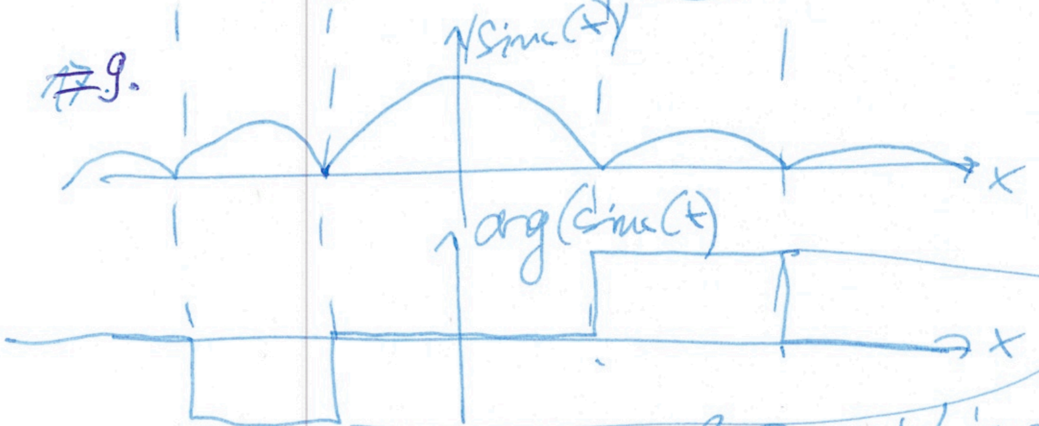
$\frac{\tau}{T_1}$ = "duty cycle"

8.



attention! Twice the width of the other "lobes" !!!

9.



can be any odd multiple of π, picking $+\pi/-\pi$ for having it "beautiful".

10. Only **blue** part first !!!

It is $\frac{D\tau}{T_1}\cdot \mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$

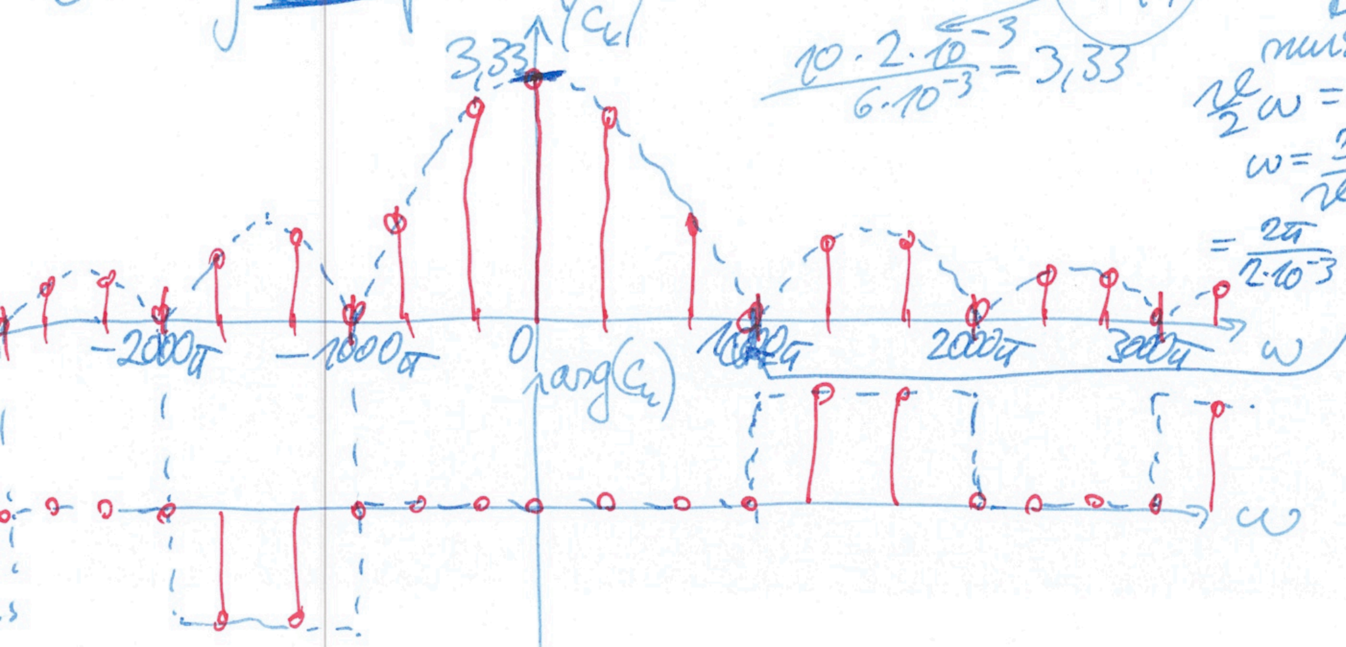
$\frac{10\cdot 2\cdot 10^{-3}}{6\cdot 10^{-3}} = 3{,}33$

must be π

$\frac{\tau}{2}\omega = \pi$

$\omega = \frac{2\pi}{\tau} =$

$= \frac{2\pi}{2\cdot 10^{-3}} = 1000\pi$ rad/s

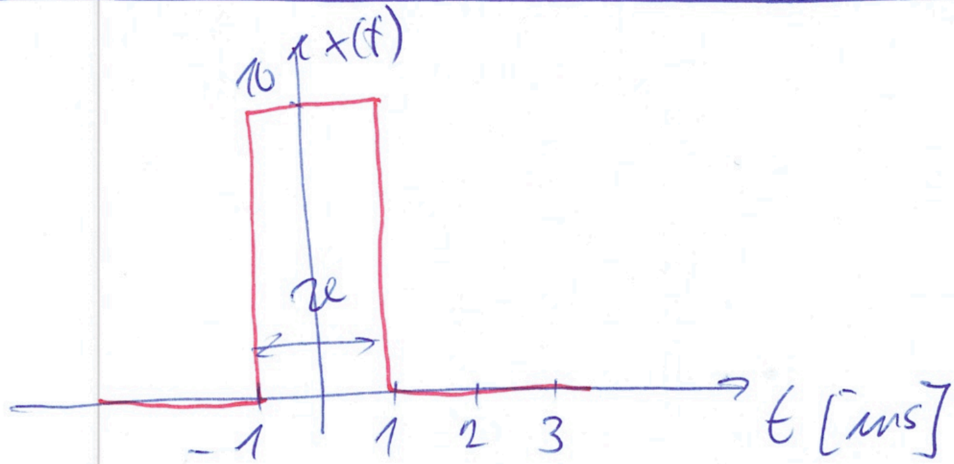


11. $\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{6 \cdot 10^{-3}} = 333{,}3\pi$ rad/s

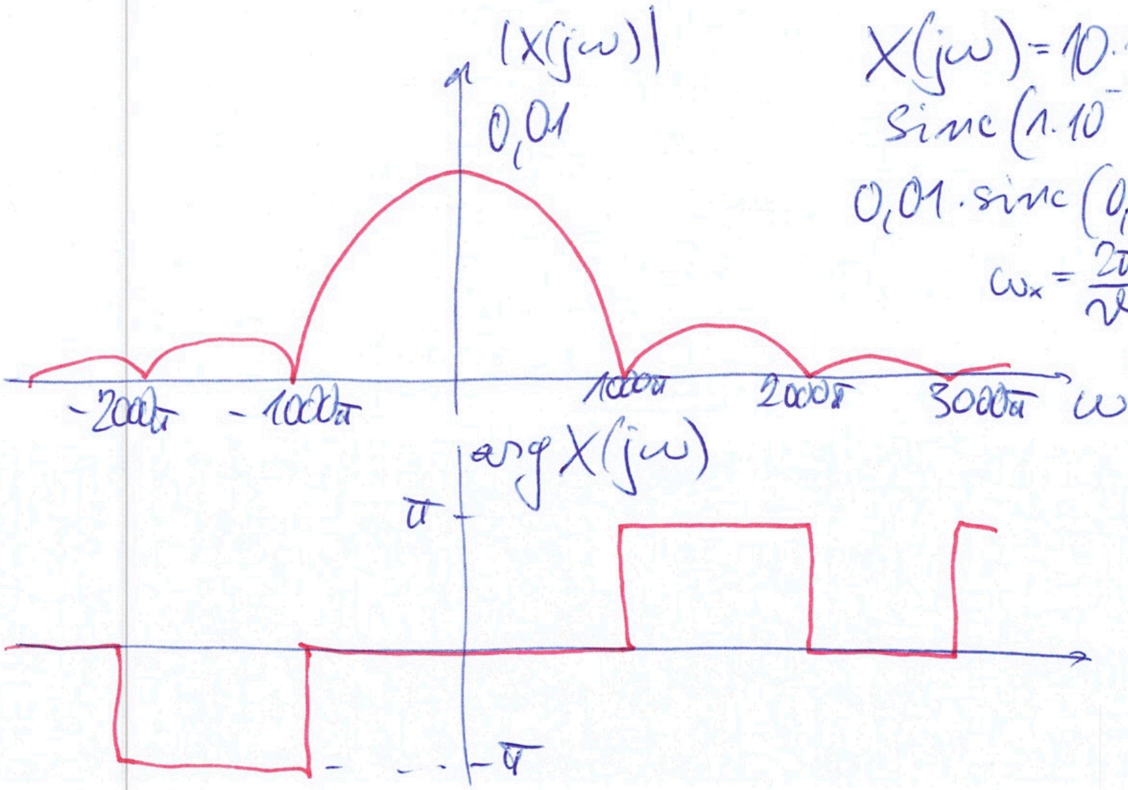
Drawing the red coefficients c_k at the multiples of $333{,}3\pi$.

For $c_3, c_6, c_9 \ldots, c_{-3}, c_{-6}, c_{-9}, \ldots$ the phase can be zero, π, $-\pi$ or anything else, as the magnitude is zero. We set it to zero.

12.



13.



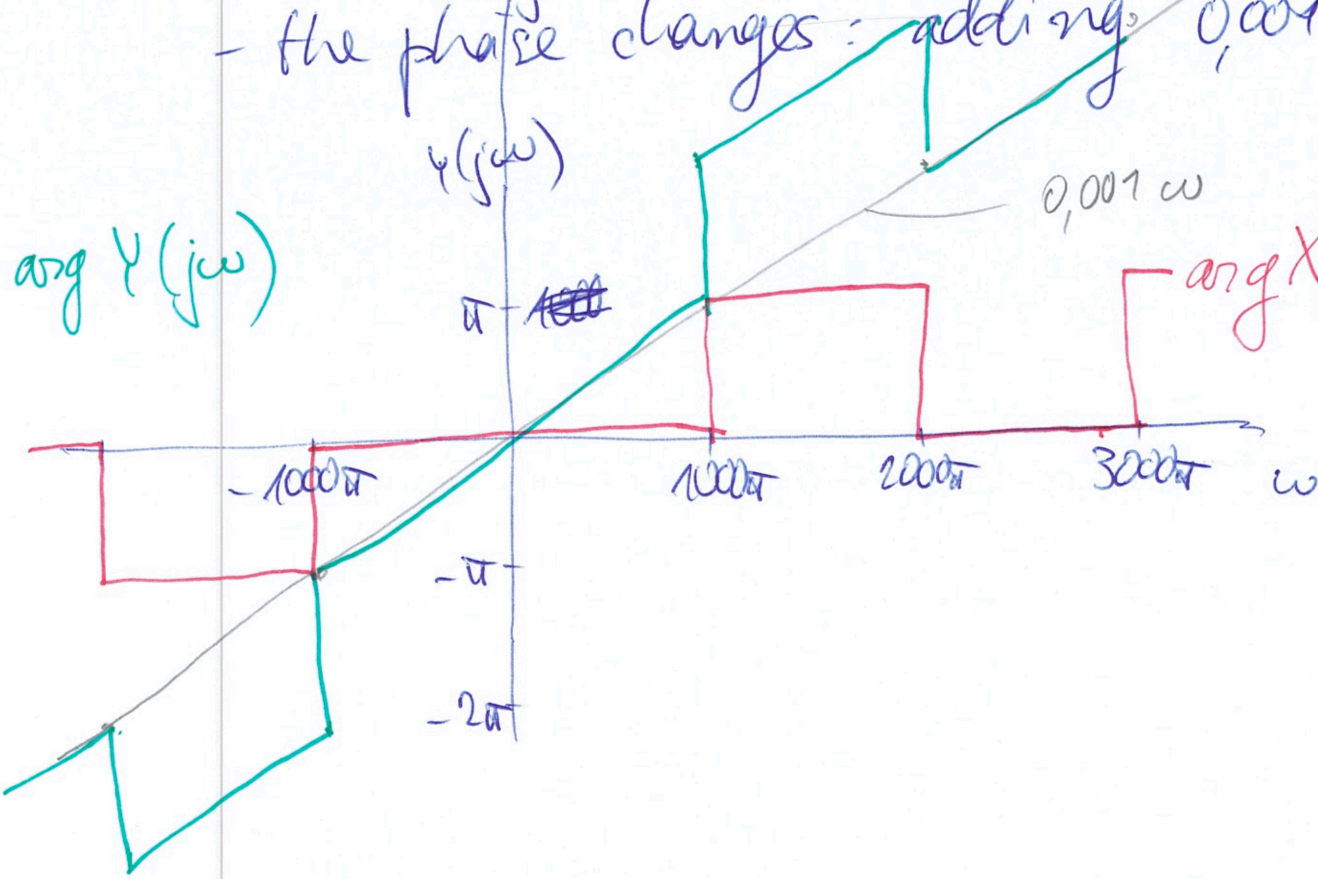
$X(j\omega) = 10 \cdot 1 \cdot 10^{-3} \cdot \text{sinc}(1 \cdot 10^{-3}\omega) = 0{,}01 \cdot \text{sinc}(0{,}001\,\omega)$

$\omega_x = \frac{2\pi}{\tau} = \frac{2\pi}{2 \cdot 10^{-3}} = 1000\pi$

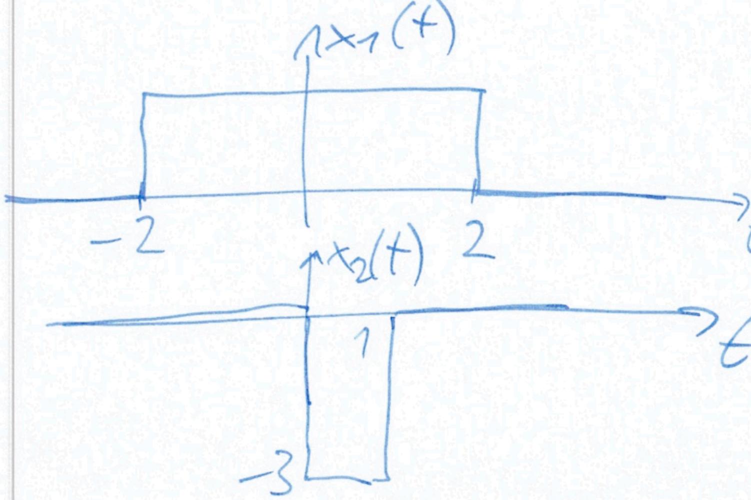
14. $y(t) = x(t + 1ms)$

$\underline{Y(j\omega)} = \underline{X(j\omega)} \cdot \underline{e^{+j\omega 0{,}001}}$

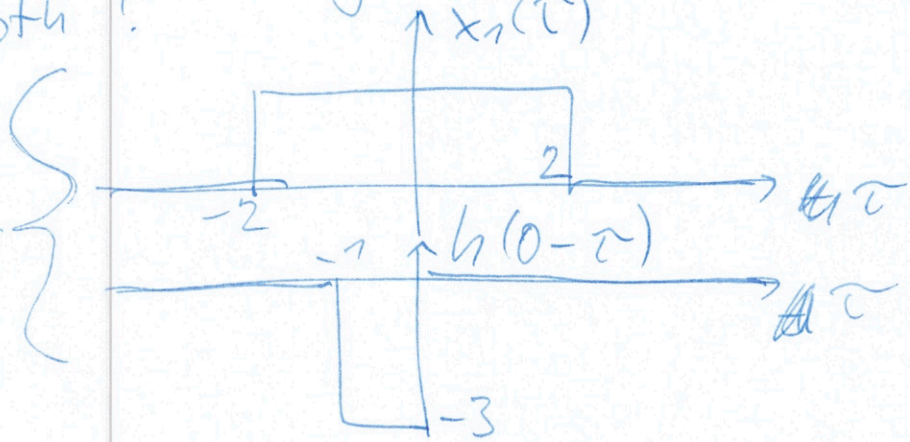
- the magnitude does not change
- the phase changes: adding $0{,}001\omega$



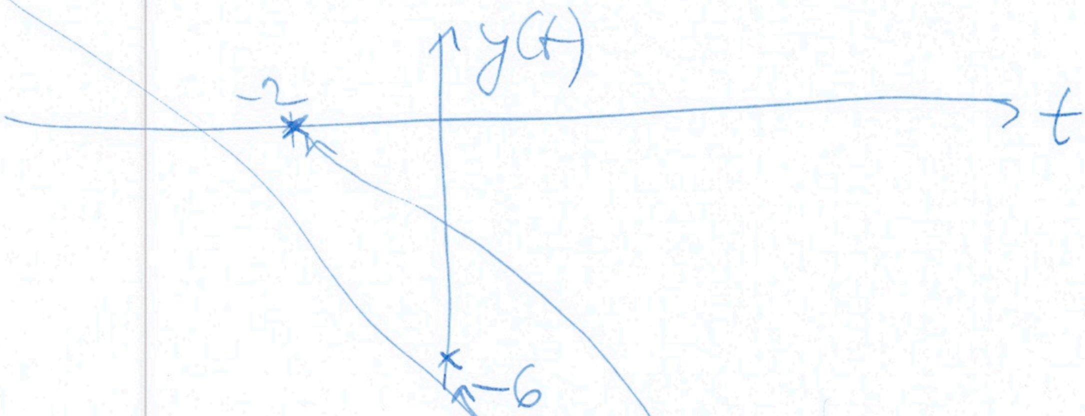
15



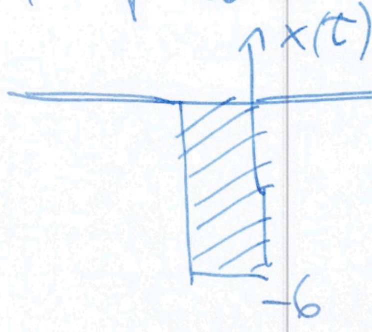
16 Selecting the 1st variant but divide students in group and let them pick both!



17

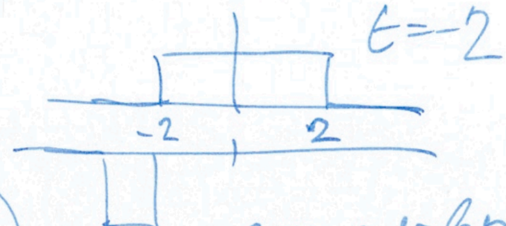


the product of signals

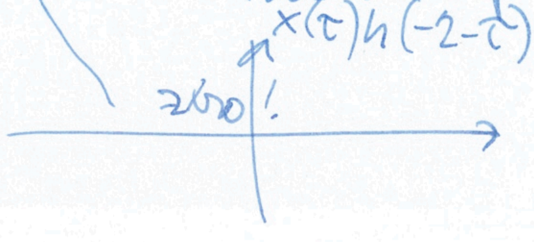


the integral is -6

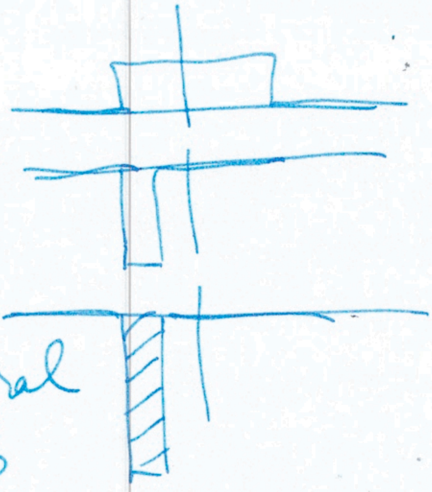
18



no overlap!



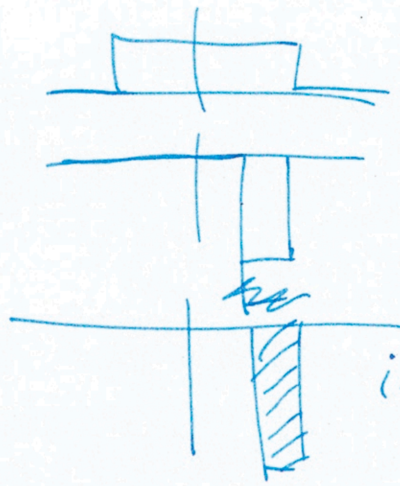
19



$t = -1$

integral $= -6$

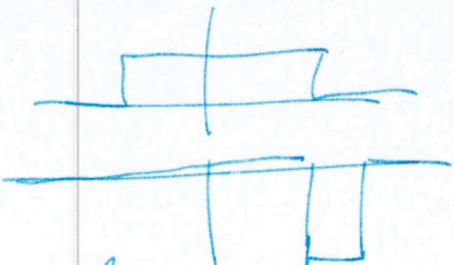
20



$t = 2$

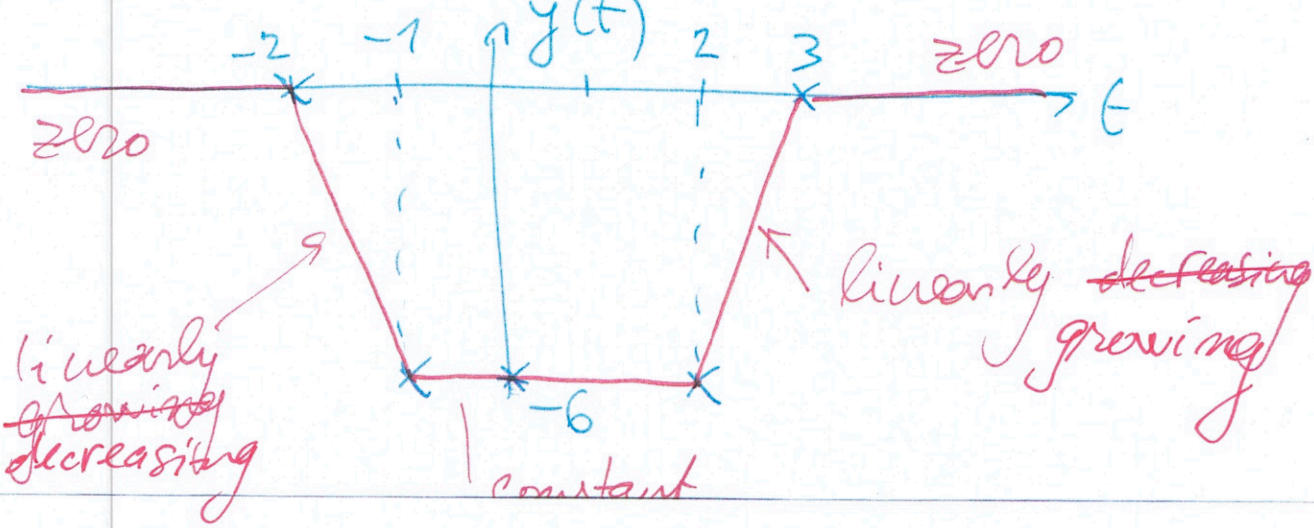
integral $= -6$

21 $t = 3$



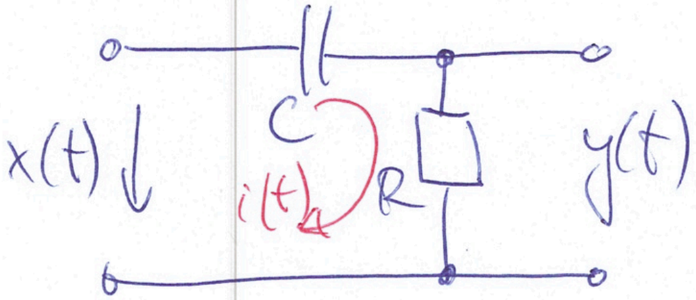
no overlap $\Rightarrow$ zero.

22



Lab #3 Continuous-time systems

We will study the behavior of the following circuit:



(1) Find differential equation describing the circuit!

23 We'll use current $i(t)$ that can be determined in two ways:

$$\left.\begin{aligned} i(t) &= \frac{y(t)}{R} \\ i(t) &= C\,\frac{d(x(t)-y(t))}{dt} \end{aligned}\right\} \text{this is the same current!}$$

24 $$y(t) = RC\,\frac{dx(t)}{dt} - RC\,\frac{dy(t)}{dt}$$

(2) Do its Laplace transform and find
25. the transfer function of the circuit:

"Vocabulary" of LT:

$$x(t) \longrightarrow X(s)$$

$$a\,x(t) \longrightarrow a\,X(s)$$

That's all we need!

$$\frac{dx(t)}{dt} \longrightarrow s\,X(s)$$

$$Y(s) = RCs\,X(s) - RCs\,Y(s)$$

... let's try to group everything depending on $Y(s)$ on the ~~righ~~ left, $X(s)$ on the right ...

$$Y(s)\left[1 + RCs\right] = X(s)\,RCs$$

Now, find $H(s)$ that is defined as $\frac{Y(s)}{X(s)}$

26.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{RCs}{1 + RCs}$$

27. What are the coefficients b_k, a_k?

$$H(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$$

$b_0 = 0 \quad b_1 = RC \quad M = 1$

$a_0 = 1 \quad a_1 = RC \quad N = 1$

28. Convert it to the form including zeros and poles

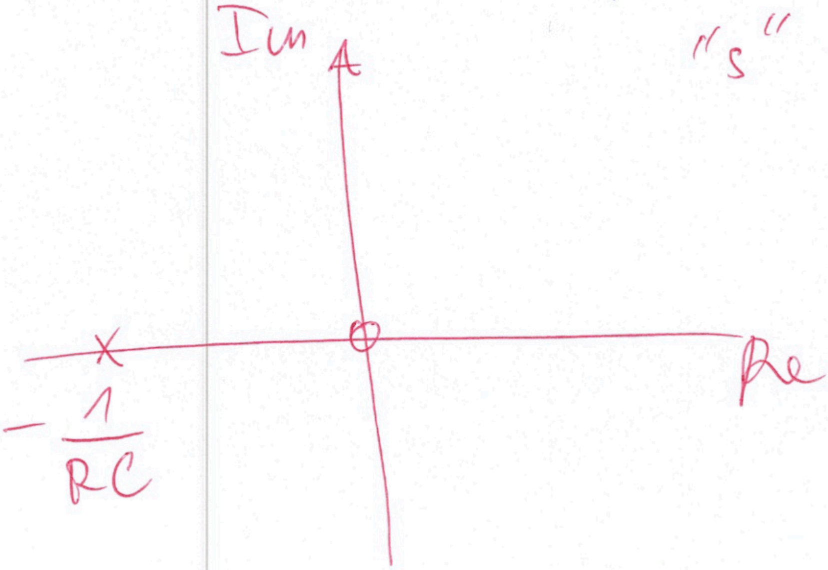
$$H(s) = \frac{b_M}{a_N} \cdot \frac{\prod_{k=1}^{M}(s - n_k)}{\prod_{k=1}^{N}(s - p_k)} = \frac{RC\,(s - 0)}{RC\left(s - \left(-\frac{1}{RC}\right)\right)} =$$

$$= \frac{s - 0}{s - \left(-\frac{1}{RC}\right)}$$

0 is the only zero

$-\frac{1}{RC}$ is the only pole.

29. Draw them into the "s" plane and check stability of the circuit ...



Real part of $-\frac{1}{RC}$ smaller than zero $\Rightarrow$ <u>stable</u> :-)

30. Determine the frequency response, pay particular attention to

$\omega = 0{,}000001$

$\omega = \frac{1}{RC}$

$\omega = \infty$

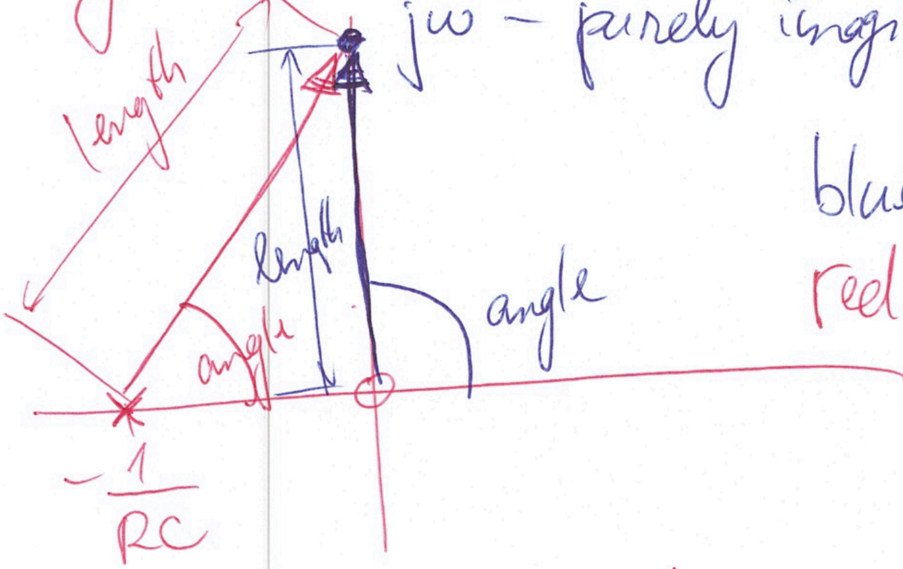
H(s) to H(jω) by simply re-(writing) all occurences of 's' by 'jω'

$$H(j\omega) = \frac{b_M}{a_N} \frac{\prod_{k=1}^{M} (j\omega - h_k)}{\prod_{k=1}^{N} (j\omega - p_k)} =$$

$$= \frac{j\omega - 0}{j\omega - \left(-\frac{1}{RC}\right)}$$

Numerator and denominator are complex numbers
They can be visualized as vectors! 10

jω – purely imaginary, imag. axis!



blue one: $j\omega - 0$

red one: $j\omega - \left(-\frac{1}{RC}\right)$

H(jω) is given by division of these complex numbers:

module is <u>division</u> of modules

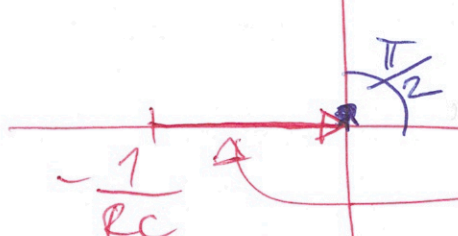
argument is <u>subtraction</u> of arguments

Module ≈ length of vector

Arg ≈ angle of vector

$\omega = 0{,}000001$:

$$|H(j0)| = \frac{\text{almost } 0}{\frac{1}{RC}} = 0$$

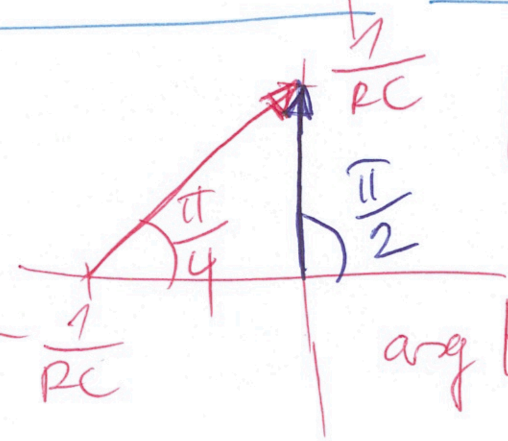


$$\arg H(j0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

angle 0

31.

$\omega = \frac{1}{RC}$

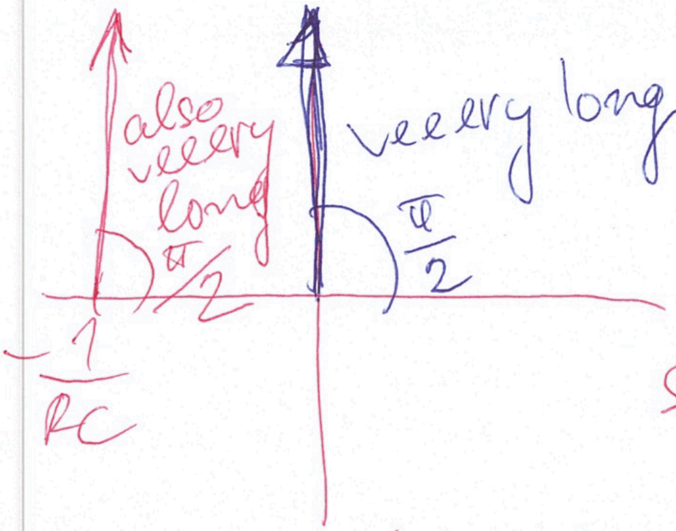


$$\left|H\left(j\frac{1}{RC}\right)\right| = \frac{\omega}{\sqrt{\omega^2+\omega^2}} = \frac{1}{\sqrt{2}}$$

$$\arg H\left(j\frac{1}{RC}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

32.

$\omega = \infty$

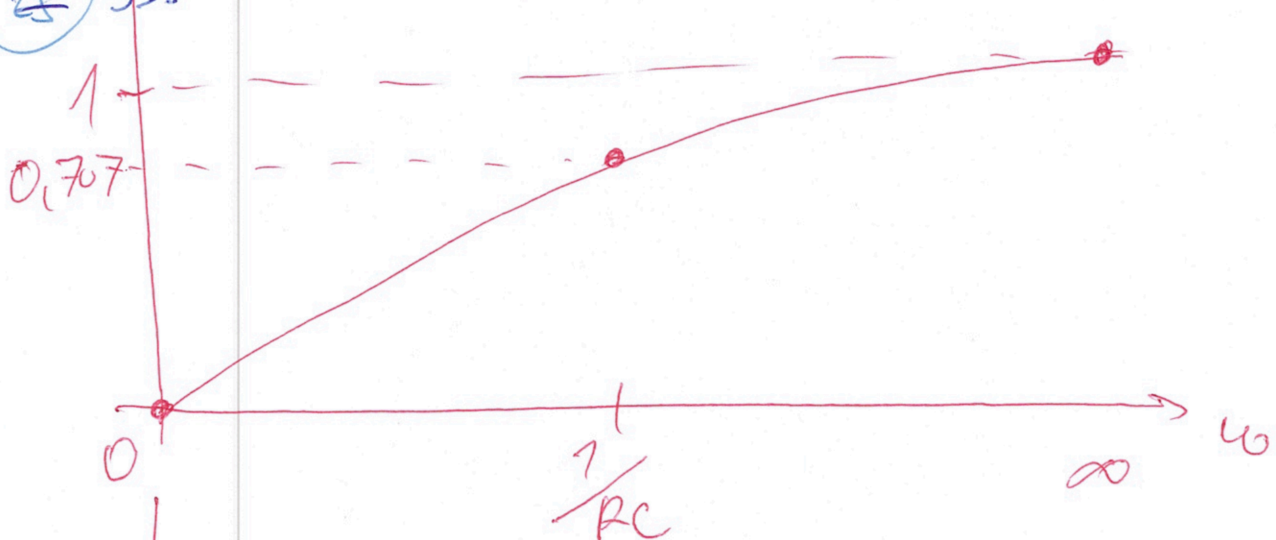


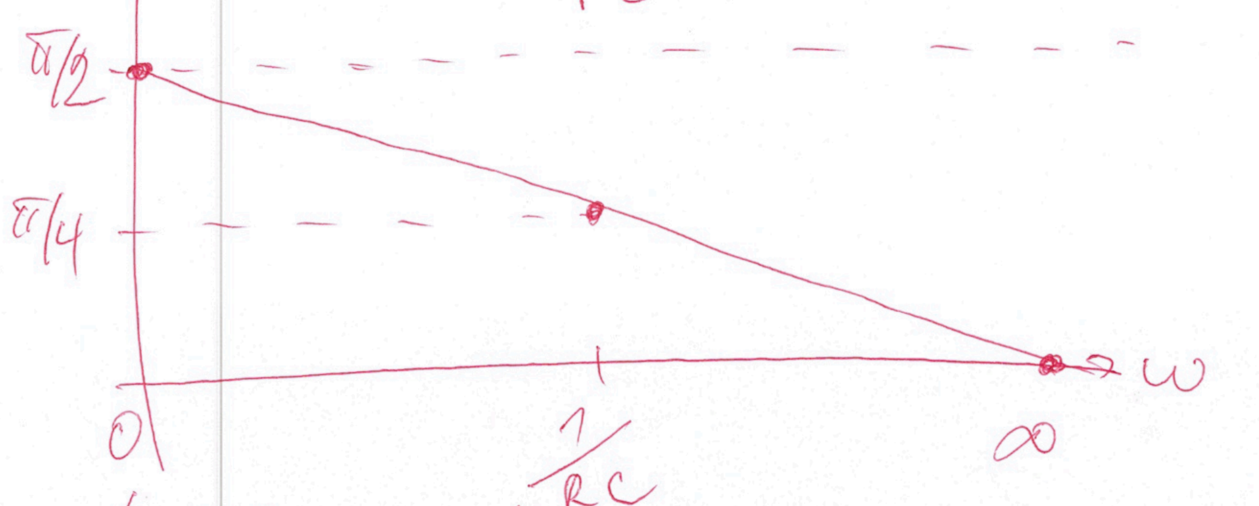
Their lengths are almost the same.

$$|H(j\infty)| = \frac{\text{veery long}}{\text{veeery long}} = 1$$

$$\arg H(j\infty) = \frac{\pi}{2} - \frac{\pi}{2} = 0$$

33.





$\Rightarrow$ high-pass!

$\Rightarrow$ look at the plot for $R = 1\,k\Omega$, $C = 1000\,pF$